Unit I.

(1) Prove that: $10n^2 + 4n + 2 = O(n^2)$.

let $10n^2 + 4n + 2 \leq 11n^2$

n	$10n^2 + 4n + 2$	$11n^2$
0	2	0
1	16	11
2	50	44
3	104	99
4	178	176
5	272	275
6	386	396

$\therefore 10n^2 + 4n + 2 \leq 11n^2$

$= O(n^2)$

where $n_0 = 5$ $c = 11$.

(2). Prove that: $6*2^n+n^2 = O(2^n)$ (2)

let $6*2^n+n^2 \leq 7*2^n$.

n.	$6*2^n+n^2$	$7*2^n$.
0	6	7.
1	13	14.
2.	28	28
3	57.	(56)
4.	112.	112
5	217.	224.

$\therefore 6*2^n+n^2 \leq 7*2^n$

$= O(2^n)$

where $n_0 = 4 \quad C = 7$.

$O(1)$ means computing time is Constant.

$O(n)$ " " " linear.

$O(n^2)$ " " " quadratic

$O(n^3)$ " " " Cubic.

$O(2^n)$ " " " Exponential.

Omega Notation [$\Omega(n)$]

The function $f(n) = \Omega(n)$ iff there exists positive constants c and n_0 such that $f(n) \geq c * g(n)$ for all n, $n \geq n_0$.

eg 1 Prove that $3n+2 = \Omega(n)$.

let $3n+2 \geq 3n$.

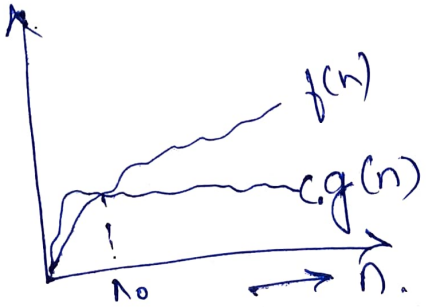
n	$3n+2$	$3n$
0	2	0
1	5	3
2	8	6
4	14	12
5	17	15

$\therefore 3n+2 > 3n$.

$= \Omega(n)$

where $n_0 = 1$ $c = 3$.

inequality holds for $n \geq 0$ but as per definition $n_0 > 0$.



Prove that $100n + 6 = \Omega(n)$.

let $100n + 6 \geq 100n$.

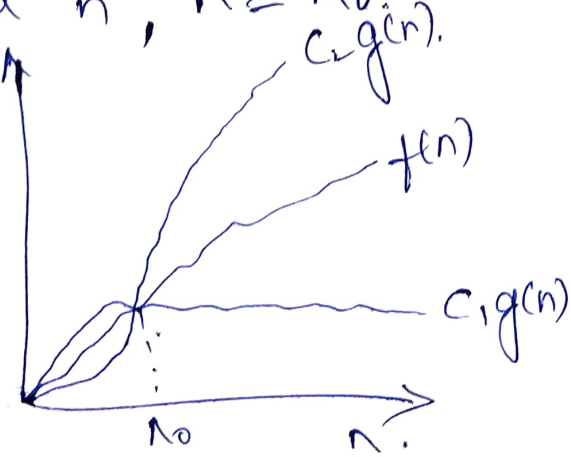
n	$100n + 6$	$100n$
1	106	100
2	206	200
3	306	300
4	406	400

$\therefore n_0 = 1 \quad C = 100$.

$\therefore 100n + 6 = \Omega(n)$.

Theta Notation. $\Theta(n)$

The function $f(n) = \Theta(g(n))$ iff there exist positive constants C_1, C_2 and n_0 such that $C_1 g(n) \leq f(n) \leq C_2 g(n)$ for all n, $n \geq n_0$.



eg: Prove that $3n+2 = \Theta(n)$.

let $3n+2 \geq 3n$ and $3n+2 \leq 4n$.

n	$3n$	$3n+2$	$4n$
1.	3	5	4.
2.	6.	8	8
3	9	11	12
4	12	14	16.

$\therefore \quad n_0 = 2 \quad C_1 = 3 \quad C_2 = 4.$

$3n \leq 3n+2 \leq 4n$

$\therefore \quad \boxed{3n+2 = \Theta(n)}$

Prove that: $10n^2+4n+2 = \Theta(n^2)$ (2).

let $10n^2 \le 10n^2+4n+2 \le 11n^2$

n	$10n^2$	$10n^2+4n+2$	$11n^2$
1	10	16	11
2	40	50	44
3	90	104	99
4	160	178	176
5	250	272	275
6	360	386	396.

$\therefore \quad 10n^2 \le 10n^2+4n+2 \le 11n^2$

$C_1 = 10 \qquad n_0 = 5 \qquad C_2 = 11.$

$\therefore \quad 10n^2+4n+2 = \Theta(n^2).$