Bellman Ford Algorithm.

Single Source All destinations with Negative weights.

Notation:

$dist^l[u]$: Represents distance from source V. to destination u with at most l edges and the distance is shortest.

$$dist^k[u] = \min\left\{ dist^{k-1}[u],\ \min_i \left[dist^{k-1}[i] + cost[i,u] \right] \right\}$$

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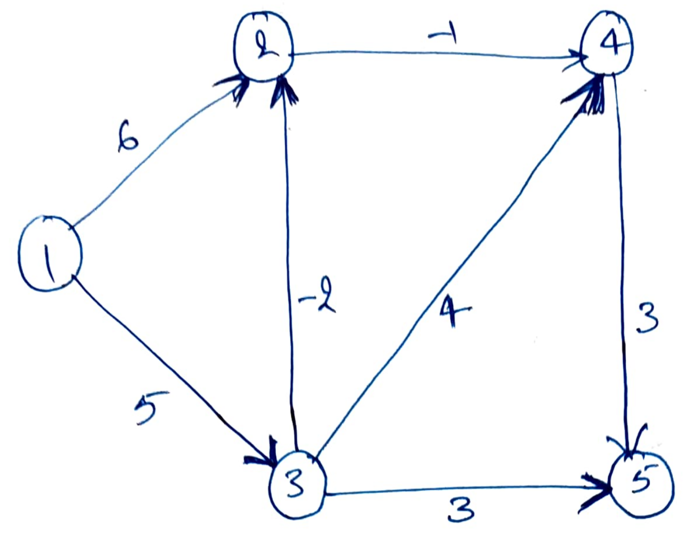
Algorithm Bellman ford (v, cost, dist, n)
{ for (i=1 to n do)
      dist[i] = cost [v,i]
   for k=2 to n-1 do
      for each u such that v≠u has
      atleast one incoming edge do
      for each (i,u) in the graph do
         if (dist[u] > dist[i] + cost [i,u])
            dist [u] = dist[i] + cost[i,u]
}
```



	1	2	3	4	5
1	0	6	5	∞	∞
2	0	3	5	5	8
3	0	3	5	2	8
4					

→ Destinations

No of edges included.

$dist^1[1] = 0$ $dist^1[2] = 6$ $dist^1[3] = 5$

$dist^1[4] = \infty$ $dist^1[5] = \infty$

Considering only one edge.

Consider 2 edges between v to u.

$dist^2[1] = 0.$

$dist^2[2] = dist^1[3] + cost[3,2]$

$= 5 + (-2)$

$= 3$

$dist^2[3] = 5$

$dist^2[4] = \min\{dist^1[2] + cost[2,4],$
$dist^1[3] + cost[3,4]\}$

$= \min\{6-1, 5+4\}$

$= \min\{5, 9\}$

$= 5.$

$dist^2[5] = dist^1[3] + cost[3,5]$

$= 5 + 3 = 8$

Consider 3 edges.

$dist^3[1] = 0$

$dist^3[2] = 3$

$dist^3[3] = 5$

$dist^3[4] = dist^2[2] + cost[2,4]$

$= 3 - 1$

$= 2.$

$dist^3[5] = dist^2[4] + cost[4,5]$

$= 5 + 3$

$= 8$

Consider 4 edges.

$dist^4[1] = 0$

$dist^4[2] = 3$

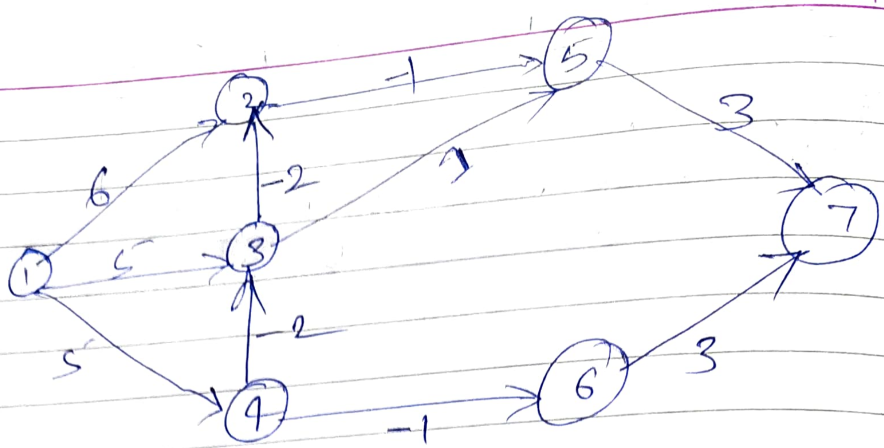
$dist^4[3] = 5$

$dist^4[4] = 2.$

$dist^4[5] = dist^3[4] + cost[4,5]$

$= 2 + 3$

$= 5.$



k	$dist^k[1 \ldots 7]$ 1	2	3	4	5	6	7
1	0	6	5	5	∞	∞	∞
2	0	3	3	5	5	4	∞
3	0	1	3	5	2	4	7
4	0	1	3	5	0	4	5
5	0	1	3	5	0	4	3
6	0	1	3	5	0	4	3

$dist^1[1] = 0$ $\quad dist^1[2] = 6$ $\quad dist^1[3] = 5$

$dist^1[4] = 5$ $\quad dist^1[5] = \infty$ $\quad dist^1[6] = \infty$ $\quad dist^1[7] = \infty$

$dist^2[1] = 0$

$dist^2[2] = \min\{dist^1[2], cost[3,2] + dist^1[3]\}$

$= \min\{6, -2+5\} = 3.$

$dist^2[3] = \min\{dist^1[3], cost[4,3] + dist^1[4]\}$

$= \min\{5, -2+5\} = 3.$

$dist^2[4] = 5$

$dist^2[5] = \min\{dist^1[5],\ cost(2,5) + dist^1[2],$
$cost(3,5) + dist^1[3]\}$

$= \min\{\infty,\ -1+6,\ 1+5\} = 5$

$dist^2[6] = \min\{dist^1[6],\ cost(4,6) + dist^1[4]\}$

$= \{\infty,\ -1+5\} = 4.$

$dist^2[7] = \infty.$

$dist^3[2] = \min\{dist^2[2],\ cost(3,2) + dist^2[3]\}$

$= \min\{3,\ -2+3\} = 1$

$dist^3[3] = \cancel{\min\{}\ 3$

$dist^3[4] = 5$

$dist^3[5] = \min\{dist^2[5],\ cost(3,5) + dist^2(3)\}$

$= \min\{5,\ 1+3\} = 4.$

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$dist^3[5] = \{dist^2[5], cost(3,5) + dist^2[3], cost(2,5) + dist^2[2]\}$

$= \{5, 1+3, -1+3\} = 2$//

$dist^3[6] = 4$.

$dist^3[7] = min\{dist^2[7], cost[5,7] + dist^2[5], cost(6,7) + dist^2[6]\}$

$= \{\infty, 3+5, 3+4\} = 7$//

$dist^4[2] = 1$ $dist^4[3] = 3$ $dist^4[4] = 5$

$dist^4[5] = min\{dist^3[5], cost(2,5) + dist^3[2]$

$= min\{2, -1+1\} = 0$.

$dist^4[6] = 4$.

$dist^4[7] = min\{dist^3[7], cost(5,7) + dist^3[5]\}$

$= min\{7, 3+2\} = 5$

$$dist^5[7] = \min\{dist^4[7],\ cost(5,7) + dist^4[5]\}$$

$$= \min\{5,\ 3+0\} = 3.$$