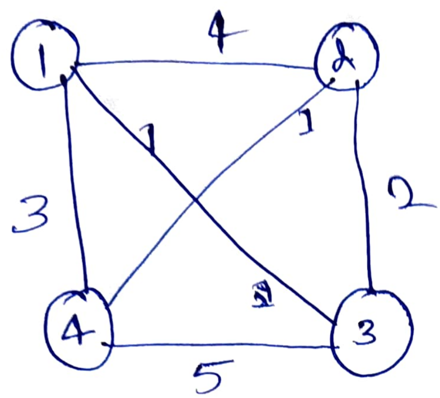
Travelling Salesman Problem.

(Dynamic Programming)



$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 4 & 1 & 3 \\ 2 & 4 & 0 & 2 & 1 \\ 3 & 1 & 2 & 0 & 5 \\ 4 & 3 & 1 & 5 & 0 \end{array}$$

$$\left.\begin{aligned} g(i, \{\}) &= C_{i1} \\ g(2, \{\}) &= C_{21} = 4 \\ g(3, \{\}) &= C_{31} = 1 \\ g(4, \{\}) &= C_{41} = 3. \end{aligned}\right\} \quad |S| = 0.$$

Consider groups with $|S| = 1$.

$$g(2, \{3\}) = C_{23} + g(3, \{\})$$
$$= 2 + 1 = 3$$

$$g(2, \{4\}) = C_{24} + g(4, \{\})$$
$$= 1 + 3 = 4$$

$$g(3, \{2\}) = C_{32} + g(2, \{\})$$
$$= 2 + 4 = 6.$$

$$g(3, \{4\}) = C_{34} + g(4, \{\})$$
$$= 5 + 3 = 8.$$

$$g(4, \{2\}) = C_{42} + g(2, \{\})$$
$$= 1 + 4 = 5$$

$$g(4, \{3\}) = C_{43} + g(3, \{\})$$
$$= 5 + 1 = 6.$$

Now consider groups with $|S| = 2$.

$$g(2, \{3, 4\}) = \min\{C_{23} + g(3, \{4\}), C_{24} + g(4, \{3\})\}$$

$$= \min\{2 + 8, 1 + 6\} = \min\{10, 7\}$$
$$= 7.$$

$$g(3, \{2, 4\}) = \min\{C_{32} + g(2, \{4\}), C_{34} + g(4, \{2\})\}$$
$$= \min\{2 + 4, 5 + 5\} = \min\{6, 10\}$$
$$= 6.$$

$g(4, \{2,3\}) = \min\{C_{42} + g(2,\{3\}), C_{43} + g(3,\{2\})\}$

$= \min\{1+3, 5+6\}$

$= \min\{4, 10\}$

$= 4.$

Now consider $|S| = 3$

$g(1, \{2,3,4\}) = \min\{C_{12} + g(2,\{3,4\}),$

$C_{13} + g(3, \{2,4\}),$

$C_{14} + g(4, \{2,3\})\}$

$= \min\{4+7, 1+6, 3+4\}$

$= \min\{11, 7, 7\}.$

$= 7.$

Minimum cost of the path $= 7.$

Finding the path.

$7 = C_{13} + g(3,\{2,4\})$

$\therefore 1 \rightarrow 3.$

$g(3, \{2,4\}) = C_{32} + g(2,\{4\})$

$\therefore 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$

final path $1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1.$

Travelling Salesman Problem

$$g(1, V-\{1\}) = \min_{2\le k\le n} \{C_{1k} + g(k, V-\{1,k\})\}$$

$g(1, V-\{1\})$ is the length of the shortest path starting at vertex 1 going through all vertices in $V-\{1\}$ and terminating at 1.

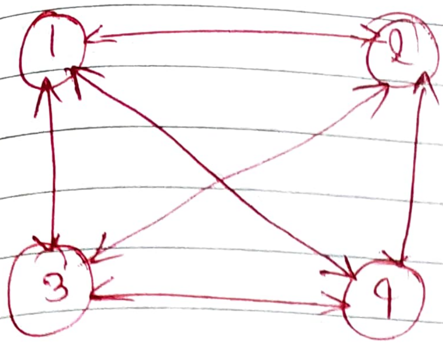
Generalizing the formula.

$$g(i, S) = \min_{j\in S} \{c_{ij} + g(j, S-\{j\})\}$$

eg: $g(2, \{3,4\})$ is the minimum dista. from 2-3-4-1 or
2-4-3-1

$g(1, \{2,3,4\})$ is

1-2-3-4-1 or
1-3-4-2-1 or
1-4-3-2-1.



$$\begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

$g(i, \phi) = C_{i,1}$

$\therefore g(2, \phi) = C_{21} = 5$ $\quad |S| = 0$

$g(3, \phi) = C_{31} = 6.$

$g(4, \phi) = C_{41} = 8.$

Now Consider groups with $|S| = 1$

$g(2,\{3\}) = C_{23} + g(3,\phi)$
$= 9 + 6 = 15$

$g(2,\{4\}) = C_{24} + g(4,\phi) = 10 + 8 = 18.$

$g(3,\{2\}) = C_{32} + g(2,\phi) = 13 + 5 = 18$

$g(3,\{4\}) = C_{34} + g(4,\phi) = 12 + 8 = 20.$

$g(4,\{2\}) = C_{42} + g(2,\phi) = 8 + 5 = 13$

$g(4,\{3\}) = C_{43} + g(3,\phi) = 9 + 6 = 15$

Now Consider groups with $|S| = 2$.

$g(2,\{3,4\}) = \min\{C_{23} + g(3,\{4\}),\ C_{24} + g(4,\{3\})\}$

$= \min\{9 + 20,\ 10 + 15\} = 25$

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$$g(3,\{2,4\}) = \min\{C_{32}+g(2,\{4\}),\ C_{34}+g(4,\{2\})\}$$
$$= \min\{18+18,\ 12+13\} = 25$$

$$g(4,\{2,3\}) = \min\{C_{42}+g(2,\{3\}),\ C_{43}+g(3,\{2\})\}$$
$$= \min\{8+15,\ 9+18\} = 23$$

Now $|S| = 3$

$$g(1,\{2,3,4\}) =$$
$$\min\{C_{12}+g(2,\{3,4\}),\ C_{13}+g(3,\{2,4\})+$$
$$C_{14}+g(4,\{2,3\})$$
$$= \min\{10+25,\ 15+25,\ 20+23\}$$
$$= \min\{35,\ 40,\ 43\}$$
$$= \underline{35}$$

Find the path.

$35 = C_{12} + g(2,\{3,4\})$

1 — 2

$g(2,\{3,4\}) = C_{24} + g(4,\{3\})$

2 — 4 — 3 — 1.

$\therefore$ final path $\boxed{1-2-4-3-1}$