Inhomogeneous recurrences.

Inhomogeneous recurrences are of the form

$a_0 t_n + a_1 t_{n-1} + \cdots \quad a_k t_{n-k} = b^n p(n)$

LHS is similar to homogeneous recurrence eq^n
RHS $b^n p(n)$ where b is a constant
$p(n)$ is polynomial in n of degree 'd'

3) Consider the recurrence
$t_n = 2t_{n-1} + 3^n$
$t_n - 2t_{n-1} = 3^n$ —— 1
$b = 3$ and $p(n) = 1$ of degree 0.

Multiply recurrence by 3.

$3t_n - 6t_{n-1} = 3^{n+1}$
Now replace n by $n-1$
$3t_{n-1} - 6t_{n-2} = 3^n$ —— 2
Substract eq^n 2 from 1

$t_n - 5t_{n-1} + 6t_{n-2} = 0.$
which is homogenious equation.
$x^2 - 5x + 6 = (x-2)(x-3) = 0$
$r_1 = 2$ and $r_2 = 3.$
$t_n = c_1 2^n + c_2 3^n$ —— 3
From eq^n 1
$t_1 = 2t_0 + 3$
From 3
$c_1 + c_2 = t_0.$ for $n=0$
$2c_1 + 3c_2 = t_1 = 2t_0 + 1$ for $n = 1$
By solving these equations

$c_1 = t_0 - 3$ & $C_2 = 3.$

$t_n = (t_0 - 3) 2^n + 3^{n+1}$

$t_n = \Theta(3^n)$ Provided $t_0 \geq 0.$

(4) $t_m = \begin{cases} 0 & \text{if } m = 0 \\ 2t(m-1) + 1 & \text{otherwise.} \end{cases}$

Now $t_m = 2t(m-1) + 1$

$t_{m-1} = 2t(m-2) + 1.$

$t_m - 2t(m-1) = 1$ ——— (1)

$t_{m-1} - 2t(m-2) = 1$ ——— (2).

subtract (2) from (1).

$t_m - 3t(m-1) + 2t(m-2) = 0.$

$x^2 - 3x + 2 = 0.$

$(x-2)(x-1) = 0.$

$\therefore r_1 = 2 \quad r_2 = 1.$

$t_m = \cancel{t_m} = c_1 2^m + c_2$

$t_0 = c_1 + c_2 = 0$ from $m = 0.$ ——— (3)

$t_m = 2t(m-1) + 1$ when $m = 1$

$= 2t(0) + 1$

$= 1.$

$\therefore 2c_1 + c_2 = 1$ ——— (4)

From (3) and (4)

$c_1 = 1 \quad c_2 = -1$

$t_m = 2^m - 1$

$\Theta = (2^m)!$