Assignment 5

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CS21BTECH11044

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Outline

Question

Papoulis Pillai Ch4 Ex 4-21:

Find the probability $P_n(k)$ for $p = 0.5, n = 10$ and $k = 5$:

(a) By Using Fundamental Theorem.

(b) By Using DeMoivre-Laplace Theorem.

Theory

Fundamental Theorem:
Suppose a trial is made repeatedly for $'n'$ number of times, where the probability of success is $'p'$ and of failure is $'q'$. Then the probability $\Pr(k)$ of the trial being successful for exactly $'k'$ times is given by:

$$\Pr(k) = {}^{n}C_k \times p^k \times q^{n-k}.$$

DeMoivre-Laplace Theorem:
When n is very large and k is in the $\sqrt{npq}$ neighbourhood of np, we can approximate,

$${}^{n}C_k \times p^k \times q^{n-k} \simeq \frac{1}{\sqrt{2\pi npq}} \cdot e^{\frac{-(k-np)^2}{2npq}}.$$

(This approximation is DeMoivre-Laplace Theorem).

Solution(a)

(a) Substituting given values,
$n = 10$, $k = 5$, $p = 0.5$ and $q = 1 - p = 0.5$ in Fundamental Theorem,

$$\Pr(5) = {}^{10}C_5 \cdot (0.5)^5 \cdot (0.5)^5$$
$$\Pr(5) = \frac{10!}{5!5!} \cdot (\frac{1}{2})^{10}$$
$$\Pr(5) = 0.246.$$

Calculating np and npq:

$$np = 10 \cdot \frac{1}{2} = 5$$
$$npq = 10 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{2}$$

Solution (b)

(b)Substituting $(np = 5)$ and $(npq = \frac{5}{2})$ in DeMoivre-Laplace Theorem,

$$\Pr(5) \simeq \frac{1}{\sqrt{2\pi(\frac{5}{2})}} \cdot e^{\frac{-(5-5)^2}{2(\frac{5}{2})}}$$

$$\Pr(5) \simeq \frac{1}{\sqrt{5\pi}}$$

$$\Pr(5) \simeq 0.252.$$

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pranav@XPS:~/Desktop/Acads/P&Rv/Assig5$ python3 -u "/home/pranav/Desktop/Acads/P&Rv/Assig5/code.py"
Value of Pr(5) =  0.24609375
The value of Approximate Pr(k) : 0.252313252202016
pranav@XPS:~/Desktop/Acads/P&Rv/Assig5$
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Figure 0: Verification Code