Assignment 7

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CS21BTECH11044

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Outline

Question

Papoulis Pillai Ch5 Ex 6-69:

Show that, if random variables x and y are $N(0, 0, {\sigma_1}^2, {\sigma_2}^2, r)$ then,

$$E\left\{\left|xy\right|\right\} = \frac{2}{\pi}\int_0^c arcsin\frac{\mu}{\sigma_1\sigma_2}d\mu + \frac{2\sigma_1\sigma_2}{\pi} = \frac{2\sigma_1\sigma_2}{\pi}(cos\alpha + \alpha sin\alpha)$$

where, $r = sin\alpha$ and $C = r\sigma_1\sigma_2$.

Theory

Given two jointly normal random variables x and y, we form the mean

$$I = E\{g(x,y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f(x,y)dxdy$$

of some function g(x, y) of (x, y). The above integral is a function $I(\mu)$ of the covariance μ of the random variables x and y and of four parameters specifying the joint density $f(x,y)$ of x and y. We shall show that if $g(x,y)f(x,y) \longrightarrow 0$ as $(x,y) \longrightarrow \infty$, then

$$\frac{\partial^n I(\mu)}{\partial \mu^n} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\partial^{2n} g(x,y)}{\partial x^n \partial y^n} f(x,y)dxdy = E\left(\frac{\partial^{2n} g(x,y)}{\partial x^n \partial y^n}\right)$$

Solution

Using Price's theorem,

$$\frac{\partial E\left\{\left|xy\right|\right\}}{\partial \mu} = E\left\{\frac{d\left|x\right|}{dx} + \frac{d\left|y\right|}{dy}\right\} = E\left\{sgnxsgny\right\} \tag{1}$$

$$= P\left\{xy > 0\right\} - P\left\{xy < 0\right\} \tag{2}$$

$$= \frac{2\alpha}{\pi} \tag{3}$$

$$= \frac{2}{\pi} arcsin \frac{\mu}{\sigma_1 \sigma_2} \tag{4}$$

if $\mu = 0$, then the RVs x and y are independent,

hence,

Solution

at $\mu = 0$

$$E\left\{\left|xy\right|\right\} = E\left\{\left|x\right|\right\} E\left\{\left|x\right|\right\}$$

$$= \frac{2}{\pi}\sigma_1\sigma_2$$

Integrating (4) using the above,we obtain

$$E\left\{\left|xy\right|\right\} = \frac{2}{\pi}\int_0^{\mu} arcsin\frac{\mu}{\sigma_1\sigma_2} + \frac{2}{\pi}\sigma_1\sigma_2$$

$$= \frac{2}{\pi}\sigma_1\sigma_2(cos\alpha + \alpha sin\alpha)$$