Assignment 9

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Outline

Question

Papoulis Pillai Ch8 Ex 8-33:

The number x of particles emitted from a radioactive substance in 1 second is a Poisson random variable with mean θ. In 50 seconds. 1058 particles are emitted. Test the hypothesis $\theta_0 = 20$ against $\theta \neq 20$ with $\alpha = 0.05$ using the asymptotic approximation.

Solution

$$f(x,\theta) = e^{-\theta}\frac{\theta^x}{x!}$$

$$f(X,\theta) = e^{-n\theta}\frac{\theta^{n\bar{x}}}{x_1!.....x_n!}$$

$f(X,\theta)$ is maximum for $\theta = \theta_m = \bar{x}$ and since $\theta_{m0} = \theta_0$ we can say that,

$$\lambda(X) = \frac{e^{-n\theta_0}\theta_0^{n\bar{x}}}{e^{-n\bar{x}}\bar{x}^{n\bar{x}}} \tag{1}$$

$$w = -2ln\lambda = 2n(\theta_0 - \bar{x}) + 2n\bar{x}ln(\bar{x}/\theta_0) \tag{2}$$

Solution

With $n = 50$,$\theta_0 = 20$ and $\bar{x} = 1058/50 = 21.16$

substuting them in eq(2):

$$w = 2(50)(20 - 21.16) + 2(50)(21.16) ln(21.16/20)$$
$$= -116 + 119.3007855508$$
$$= 3.3007$$

Thus $w = 3$,

Solution

and since $m_0 = 1$ and $m = 1$ and

$$\chi_{0.95} = 3.84$$
$$\chi_{0.95} > 3$$

$\therefore$ we accept H_0