

④ Table

Species	Green	Legs	Height	Smelly
M	N	3	S	Y
M	Y	2	T	N
M	Y	3	T	N
M	N	2	S	Y
M	Y	3	T	N
H	N	2	T	Y
H	N	2	S	N
H	N	2	T	N
H	Y	2	S	N
H	N	2	T	Y

Answer = 1

Entropy (species) = Formula:
$$-\frac{P}{P+N} \log_2 \left(\frac{P}{P+N} \right) - \frac{N}{P+N} \log_2 \left(\frac{N}{P+N} \right)$$

$P=5$ (M) $N=5$ (H)

$$= -\frac{5}{10} \log_2 \left(\frac{5}{10} \right) - \frac{5}{10} \log_2 \left(\frac{5}{10} \right)$$

$$= \boxed{1}$$

→ For Green Colⁿ:

	M	H	Entropy
Y	3	1	0.81126
N	2	4	0.91829

$$\rightarrow \text{Entropy}(Y) = \frac{-\frac{3}{4} \log_2 \left(\frac{3}{4} \right) - \frac{1}{4} \log_2 \left(\frac{1}{4} \right)}{\log_2} = \frac{0.09370 + 0.15051}{\log_2}$$

$$= \underline{\underline{0.81126}}$$

$$\rightarrow \text{Entropy}(N) = \frac{-\frac{2}{6} \log_2 \left(\frac{2}{6} \right) - \frac{4}{6} \log_2 \left(\frac{4}{6} \right)}{\log_2} = \frac{0.15904 + 0.11139}{\log_2}$$

$$= \underline{\underline{0.91829}}$$

$$\rightarrow \text{Avg. Info (Green)} = \text{Formula} = \sum \frac{P_i + N_i}{P + N} (\text{Entropy}(i))$$

$$= \frac{4}{10} (0.81126) + \frac{6}{10} (0.91829)$$

$$= 0.32450 + 0.55097$$

$$= \underline{\underline{0.87547}}$$

Answer = 2

$$\rightarrow \text{Information Gain (Green)} = \text{Entropy (Species)} - \text{Avg. Info (G)}$$

$$= 1 - 0.87547$$

$$= \underline{\underline{0.12452}}$$

→ For Legs Colⁿ

	M	H	Entropy
2	2	5	0.86309
3	3	0	0

$$\rightarrow \text{Entropy}(2)$$

$$= \frac{-\frac{2}{7} \log\left(\frac{2}{7}\right) - \frac{5}{7} \log\left(\frac{5}{7}\right)}{\log 2} = \frac{0.15544 + 0.10437}{\log 2}$$

$$= \underline{\underline{0.86309}}$$

$$\rightarrow \text{Avg. Gain (Legs)} = \frac{7}{10} (0.86309) + \frac{3}{10} (0)$$

$$= \underline{\underline{0.60416}}$$

Answer = 3

$$\rightarrow \text{Info Gain (Legs)} = 1 - 0.60416$$

$$= \underline{\underline{0.39583}}$$

→ For Height Col

	M	H	Entropy
S	2	2	1
T	3	3	1

$$\begin{aligned} \rightarrow \text{Avg Info (Height)} &= \frac{4}{10}(1) + \frac{6}{10}(1) \\ &= \boxed{1} \end{aligned}$$

$$\begin{aligned} \rightarrow IG_{(\text{Height})} &= 1 - 1 \\ &= \boxed{0} \end{aligned}$$

→ For Smelly Col

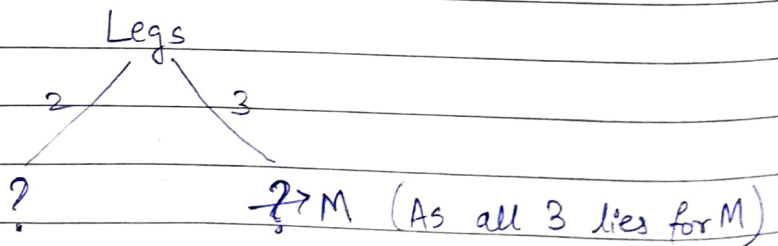
	M	H	Entropy
Y	2	2	1
N	3	3	1

$$\rightarrow \text{Avg Info} = \frac{4}{10}(1) + \frac{6}{10}(1) = 1$$

Answer = 4

$$\begin{aligned} \rightarrow IG_{(\text{Smelly})} &= 1 - 1 \\ &= \boxed{0} \end{aligned}$$

→ Maximum Information Gain is of "Legs" attribute. Thus Root node is "Legs"



→ Table for Legs = 2 branch (7 Rows)

Legs	Species	Green	Legs	Height	Smelly
	M	Y	2	T	N
	M	N	2	S	Y
	H	N	2	T	Y
	H	N	2	S	N
	H	N	2	T	N
	H	Y	2	S	N
	H	N	2	T	Y

$$\text{Entropy}_{\text{Species}} = \frac{-\frac{2}{7} \log\left(\frac{2}{7}\right) - \frac{5}{7} \log\left(\frac{5}{7}\right)}{\log 2} = \frac{0.15544 - 0.10437}{\log 2}$$

$P=2$ $N=5$
 (M) (H)

$$= \boxed{0.86309}$$

→ For Green Col :

	M	H	Entropy
Y	1	1	1
N	1	4	0.72191

$$\begin{aligned} \rightarrow \text{Entropy}_{(N)} &= \frac{-\frac{1}{5} \log\left(\frac{1}{5}\right) - \frac{4}{5} \log\left(\frac{4}{5}\right)}{\log 2} = 0.13979 + 0.07752 \\ &= \underline{\underline{0.72191}} \end{aligned}$$

$$\begin{aligned} \rightarrow \text{Avg Info}_{(\text{Green})} &= \sum \frac{P_i + N_i}{P + N} (\text{Entropy}_{(i)}) \\ &= \frac{2}{7} (1) + \frac{5}{7} (0.72191) \\ &= 0.28571 + 0.51565 \\ &= \underline{\underline{0.80136}} \end{aligned}$$

$$\rightarrow IG_{(Green)} = 0.86309 - 0.80136$$

$$= \boxed{0.06173}$$

→ For Height Colⁿ:

	M	H	Entropy
S	1	2	0.91829
T	1	3	0.81126

→ Entropy(S)

$$= \frac{-\frac{1}{3} \log\left(\frac{1}{3}\right) - \frac{2}{3} \log\left(\frac{2}{3}\right)}{\log 2} = \frac{0.15904 + 0.11739}{\log 2}$$

$$= \underline{0.91829}$$

$$\rightarrow Entropy(T) = \frac{-\frac{1}{4} \log\left(\frac{1}{4}\right) - \frac{3}{4} \log\left(\frac{3}{4}\right)}{\log 2} = \frac{0.15051 + 0.09370}{\log 2}$$

$$= \underline{0.81126}$$

$$Avg Info = \frac{3}{7} (0.91829) + \frac{4}{7} (0.81126)$$

$$= 0.39355 + 0.46357$$

$$= \underline{0.85712}$$

→ For Smelly Colⁿ

	M	H	Entropy
N	1	3	0.81126
Y	1	2	0.91829

} Calculations same as above.

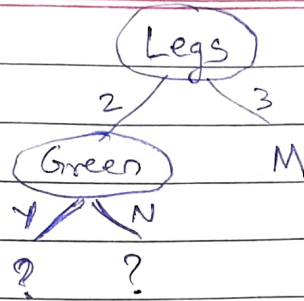
$$\rightarrow IG_{(Height)} = 0.86309 - 0.85712$$

$$= \boxed{0.00597}$$

$$Avg Info_{(Smelly)} = 0.85712$$

$$IG_{(smelly)} = \boxed{0.00597}$$

★ Max IG is of Green Colⁿ = 0.06173 thus next Node is Green



Reducing Table for Legs=2 & Green=Y

Species	Green	Legs	Height	Smelly
M	Y	2	T	N
H	Y	2	S	N

$$\text{Entropy (species)} = -\frac{1}{2} \log\left(\frac{1}{2}\right) - \frac{1}{2} \log\left(\frac{1}{2}\right)$$

$$= \boxed{1}$$

→ For Colⁿ Height

	M	H	Entropy
T	1	0	0
S	0	1	0

$$\text{Avg Info} = 0$$

$$\text{IG} = 1 - 0 = \boxed{1}$$

→ For Smelly Col IG=1

(★) Thus Next Node is Height

→ Likewise we can do all the calculations for rest of tree.
Finally we get a Decision tree like this :

★ Final Decision Tree

