10.5.2.14

EE23BTECH11003 - pranav

Question:Given that $\frac{dy}{dx} = 2x + y$ and $y = 1$,when $x = 0$ Using Runge-Kutta fourth order method,the value of y at $x = 0.2$ is (GATE 2023 AG 50)

Solution:

By using runge kutta 4 th order method

Variable	Description	Value
$x(n)$	value of x before runge kutta iteration	0
$y(n)$	value of y before runge kutta iteration	1
$y(n+1)$	value of y after runge kutta iteration	??
$x(n+1)$	value of x after runge kutta iteration	?
$f(x, y)$	derivative of y w.r.t to x	$2x + y$
h	step size	0.1

TABLE 1: Variables Used

$$\frac{y_n - y_{n-1}}{h} = 2x(n-1) + y(n-1)u(nh) \tag{1}$$

$$y(n) = y(n-1) + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{2}$$

$$x(n) = x(n-1) + hk_1 = 2x(n-1) + y(n-1)u(nh) \tag{3}$$

$$k_2 = 2(x(n-1) + \frac{h}{2}) + (y(n-1) + \frac{k_1}{2})u(n + \frac{h}{2}) \tag{4}$$

$$k_3 = 2(x(n-1) + \frac{h}{2}) + (y(n-1) + \frac{k_2}{2})u(n + \frac{h}{2}) \tag{5}$$

$$k_4 = 2(x(n-1) + h) + (y(n-1) + k_3)u(n + h)) \tag{6}$$

assume step size as 0.1 and initial conditions as $x = 0$ and $y = 1$

$$k_1 = 2(0) + 1 = 1 \tag{7}$$

$$k_2 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.1}{2}) \tag{8}$$

$$\implies k_2 = 1.15 \tag{9}$$

$$k_3 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.115}{2}) \tag{10}$$

$$\implies k_3 = 1.1575 \tag{11}$$

$$k_4 = 2(0 + 0.1) + (1 + 0.11575) \tag{12}$$

$$\implies k_4 = 1.3158 \tag{13}$$

$$y(n) = 1 + \frac{0.1}{6}(1 + 2.30 + 2.315 + 1.3158) \tag{14}$$

$$\implies y(n) = 1.1155 \tag{15}$$

$$x(n) = 0.1 \tag{16}$$

cosidering outputs of last iteration as inputs of next iteration

$$k_1 = 2(0.1) + 1.1155 = 1.3155 \tag{17}$$

$$k_2 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.12}{2}) \tag{18}$$

$$\implies k_2 = 1.4755 \tag{19}$$

$$k_3 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.1475}{2}) \tag{20}$$

$$\implies k_3 = 2.1532 \tag{21}$$

$$k_4 = 2(0.1 + 0.1) + (1.1155 + 2.1532) \tag{22}$$

$$\implies k_4 = 3.6687 \tag{23}$$

$$y(n) = 1.1155 + \frac{0.1}{6}(1.3155 + 7.014 + 3.6687) \tag{24}$$

$$\implies y(n) = 1.319 \tag{25}$$

$$x(n) = 0.2 \tag{26}$$

so at $x = 0.2$ value of y is 1.319

analysis

$$\frac{dy}{dx} = 2x + y \tag{27}$$

$$ye^{-x} = \int 2xe^{-x}dx \tag{28}$$

$$\implies ye^{-x} = -2(x+1)e^{-x} + c \tag{29}$$

by using intial conditions

$$c = 3 \tag{30}$$

$$\implies y = 3e^{x} - 2(x+1) \tag{31}$$

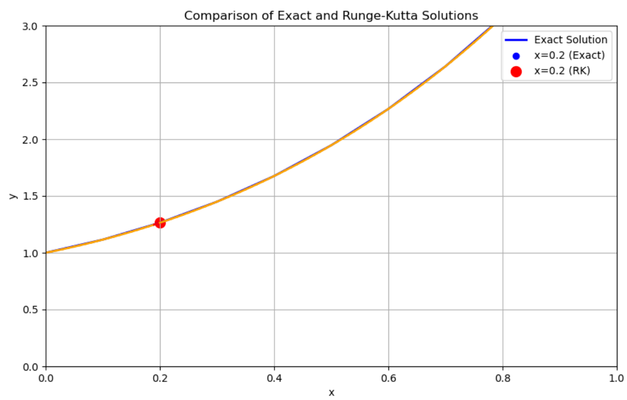

Fig. 1: analysis of runge kutta method