

# 10.5.2.14

EE23BTECH11003 - pranav

**Question:** Given that  $\frac{dy}{dx} = 2x + y$  and  $y = 1$ , when  $x = 0$  Using Runge-Kutta fourth order method, the value of  $y$  at  $x = 0.2$  is (GATE 2023 AG 50)

**Solution:**

By using runge kutta 4 th order method

Variable	Description	Value
$x_n$	value of $x$ before runge kutta iteration	0
$y_n$	value of $y$ before runge kutta iteration	1
$y_{n+1}$	value of $y$ after runge kutta iteration	??
$x_{n+1}$	value of $x$ after runge kutta iteration	?
$f(x, y)$	derivative of $y$ w.r.t to $x$	$2x + y$
$h$	step size	0.1

TABLE 1: Variables Used

$$k_1 = 2(0) + 1 = 1 \quad (7)$$

$$k_2 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.1}{2}) \quad (8)$$

$$\Rightarrow k_2 = 1.15 \quad (9)$$

$$k_3 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.115}{2}) \quad (10)$$

$$\Rightarrow k_3 = 1.1575 \quad (11)$$

$$k_4 = 2(0 + 0.1) + (1 + 0.11575) \quad (12)$$

$$\Rightarrow k_4 = 1.3158 \quad (13)$$

$$y_{n+1} = 1 + \frac{0.1}{6}(1 + 2.30 + 2.315 + 1.3158) \quad (14)$$

$$\Rightarrow y_{n+1} = 1.1155 \quad (15)$$

$$x_{n+1} = 0.1 \quad (16)$$

considering outputs of last iteration as inputs of next iteration

$$k_1 = 2(0.1) + 1 = 1.2 \quad (17)$$

$$k_2 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.12}{2}) \quad (18)$$

$$\Rightarrow k_2 = 1.4755 \quad (19)$$

$$k_3 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.1475}{2}) \quad (20)$$

$$\Rightarrow k_3 = 2.1532 \quad (21)$$

$$k_4 = 2(0.1 + 0.1) + (1 + 2.1532) \quad (22)$$

$$\Rightarrow k_4 = 3.5532 \quad (23)$$

$$y_{n+1} = 1.1155 + \frac{0.1}{6}(1.2 + 2 \cdot 1.4755 + 2 \cdot 2.1532 + 3.5532) \quad (24)$$

$$\Rightarrow y_{n+1} = 1.335 \quad (25)$$

$$x_{n+1} = 0.2 \quad (26)$$

so at  $x = 0.2$  value of  $y$  is 1.335

assume step size as 0.1 and initial conditions as  $x = 0$  and  $y = 1$