10.5.2.14

EE23BTECH11003 - pranav

Question:Given that $\frac{dy}{dx} = 2x + y$ and $y = 1$,when $x = 0$ Using Runge-Kutta fourth order method,the value of y at $x = 0.2$ is (GATE 2023 AG 50)

Solution:

By using runge kutta 4 th order method

Variable	Description	Value
x_n	value of x before runge kutta iteration	0
y_n	value of y before runge kutta iteration	1
y_{n+1}	value of y after runge kutta iteration	??
x_{n+1}	value of x after runge kutta iteration	?
$f(x,y)$	derivative of y w.r.t to x	$2x + y$
h	step size	0.1

TABLE 1: Variables Used

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{1}$$

$$x_{n+1} = x_n + h \tag{2}$$

$$k_1 = f(x_n, y_n) \tag{3}$$

$$k_2 = f(x_n + \frac{h}{2}, y_n + h\frac{k_1}{2}) \tag{4}$$

$$k_3 = f(x_n + \frac{h}{2}, y_n + h\frac{k_2}{2}) \tag{5}$$

$$k_4 = f(x_n + h, y_n + hk_3) \tag{6}$$

assume step size as 0.1 and initial conditions as $x = 0$ and $y = 1$

$$k_1 = 2(0) + 1 = 1 \tag{7}$$

$$k_2 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.1}{2}) \tag{8}$$

$$\implies k_2 = 1.15 \tag{9}$$

$$k_3 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.115}{2}) \tag{10}$$

$$\implies k_3 = 1.1575 \tag{11}$$

$$k_4 = 2(0 + 0.1) + (1 + 0.11575) \tag{12}$$

$$\implies k_4 = 1.3158 \tag{13}$$

$$y_{n+1} = 1 + \frac{0.1}{6}(1 + 2.30 + 2.315 + 1.3158) \tag{14}$$

$$\implies y_{n+1} = 1.1155 \tag{15}$$

$$x_{n+1} = 0.1 \tag{16}$$

cosidering outputs of last iteration as inputs of next iteration

$$k_1 = 2(0.1) + 1 = 1.2 \tag{17}$$

$$k_2 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.12}{2}) \tag{18}$$

$$\implies k_2 = 1.4755 \tag{19}$$

$$k_3 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.1475}{2}) \tag{20}$$

$$\implies k_3 = 2.1532 \tag{21}$$

$$k_4 = 2(0.1 + 0.1) + (1 + 2.1532) \tag{22}$$

$$\implies k_4 = 3.5532 \tag{23}$$

$$y_{n+1} = 1 + \frac{0.1}{6}(1.2 + 2 \cdot 1.4755 + 2 \cdot 2.1532 + 3.5532) \tag{24}$$

$$\implies y_{n+1} = 1.2201 \tag{25}$$

$$x_{n+1} = 0.2 \tag{26}$$

so at $x = 0.2$ value of y is 1.2201