10.5.2.14

EE23BTECH11003 - pranav

Question:Given that $\frac{dy}{dx} = 2x + y$ and $y = 1$,when $x = 0$ Using Runge-Kutta fourth order method,the value of y at $x = 0.2$ is (GATE 2023 AG 50)
Solution:
By using runge kutta 4 th order method

Variable	Description	Value
$x(n)$	value of x before runge kutta iteration	0
$y(n)$	value of y before runge kutta iteration	1
$y(n+1)$	value of y after runge kutta iteration	??
$x(n+1)$	value of x after runge kutta iteration	?
$f(x, y)$	derivative of y w.r.t to x	$2x + y$
h	step size	0.1

TABLE 1: Variables Used

$$y(n+1) = y(n) + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{1}$$
$$x(n+1) = x(n) + h \tag{2}$$
$$k_1 = f(x(n), y(n)) \tag{3}$$
$$k_2 = f(x(n) + \frac{h}{2}, y(n) + h\frac{k_1}{2}) \tag{4}$$
$$k_3 = f(x(n) + \frac{h}{2}, y(n) + h\frac{k_2}{2}) \tag{5}$$
$$k_4 = f(x(n) + h, y(n) + hk_3) \tag{6}$$

assume step size as 0.1 and initial conditions as $x = 0$ and $y = 1$

$$k_1 = 2(0) + 1 = 1 \tag{7}$$
$$k_2 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.1}{2}) \tag{8}$$
$$\implies k_2 = 1.15 \tag{9}$$
$$k_3 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.115}{2}) \tag{10}$$
$$\implies k_3 = 1.1575 \tag{11}$$
$$k_4 = 2(0 + 0.1) + (1 + 0.11575) \tag{12}$$
$$\implies k_4 = 1.3158 \tag{13}$$
$$y(n) = 1 + \frac{0.1}{6}(1 + 2.30 + 2.315 + 1.3158) \tag{14}$$
$$\implies y(n) = 1.1155 \tag{15}$$
$$x(n+1) = 0.1 \tag{16}$$

cosidering outputs of last iteration as inputs of next iteration

$$k_1 = 2(0.1) + 1.1155 = 1.3155 \tag{17}$$
$$k_2 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.12}{2}) \tag{18}$$
$$\implies k_2 = 1.4755 \tag{19}$$
$$k_3 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.1475}{2}) \tag{20}$$
$$\implies k_3 = 2.1532 \tag{21}$$
$$k_4 = 2(0.1 + 0.1) + (1.1155 + 2.1532) \tag{22}$$
$$\implies k_4 = 3.6687 \tag{23}$$
$$y(n) = 1.1155 + \frac{0.1}{6}(1.3155 + 7.014 + 3.6687) \tag{24}$$
$$\implies y(n) = 1.319 \tag{25}$$
$$x(n+1) = 0.2 \tag{26}$$

so at $x = 0.2$ value of y is 1.319

analysis

$$\frac{dy}{dx} = 2x + y \tag{27}$$

$$ye^{-x} = \int 2xe^{-x}dx \tag{28}$$

$$\implies ye^{-x} = -2(x+1)e^{-x} + c \tag{29}$$

by using intial conditions

$$c = 3 \tag{30}$$

$$\implies y = 3e^{x} - 2(x+1) \tag{31}$$

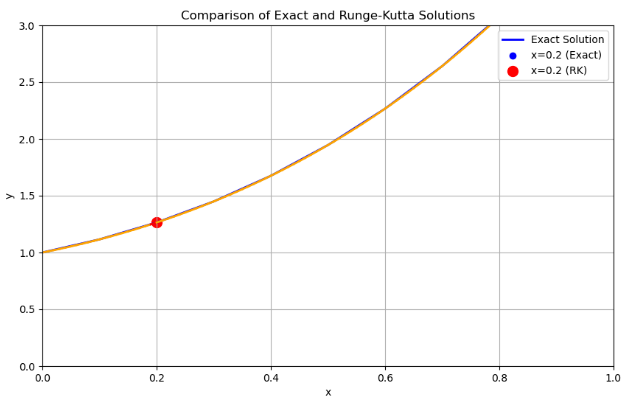

Fig. 1: analysis of runge kutta method