10.5.2.14

EE23BTECH11003 - pranav

Question:A spring having with a spring constant 1200 Nm^{-1} is mounted on a horizontal table as shown in Fig.A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released.
Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass

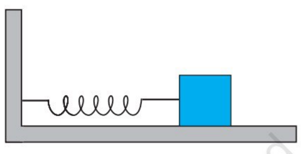

Fig. 1

Solution: at $t = 0$

Variable	Description	Value
k	spring constant	$1200N/m$
ω	angular frequency	$20rad/s$
A	amplitude	$0.02m$
$x(t)$	displasment function of the body	$0.02\cos 20t$

TABLE 1: Variables Used

$$A = A\sin(\omega(0) + \phi) \tag{1}$$
$$\implies \phi = \frac{\pi}{2} \tag{2}$$
$$\implies x(t) = A\cos\omega t \tag{3}$$

by appling laplace transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \tag{4}$$
$$\implies X(s) = -A\frac{s}{s^2+\omega^2} \tag{5}$$
$$\tag{6}$$

laplace transform of $x'(t) = sX(s) - x(0)$

$$\implies sX(s) - x(0) = A\frac{s^2}{s^2+\omega^2} - A \tag{7}$$
$$\implies sX(s) - x(0) = -A\frac{\omega^2}{s^2+\omega^2} \tag{8}$$
$$\tag{9}$$

laplace transform of $x''(t) = s^2X(s) - sx(0) - x'(0)$

$$s^2X(s) - sx(0) - x'(0) = A\frac{s^3}{s^2+\omega^2} - As - 0 \tag{10}$$
$$\implies s^2X(s) - sx(0) - x'(0) = -A\frac{s\omega^2}{s^2+\omega^2} \tag{11}$$
$$\implies sX(s) - x(0) = A\frac{\omega^2}{s^2+\omega^2} \tag{12}$$
$$\tag{13}$$

by using inverse laplace

$$\implies x'(t) = -A\omega\sin\omega t \tag{14}$$
$$\implies x''(t) = -A\omega^2\cos wt \tag{15}$$

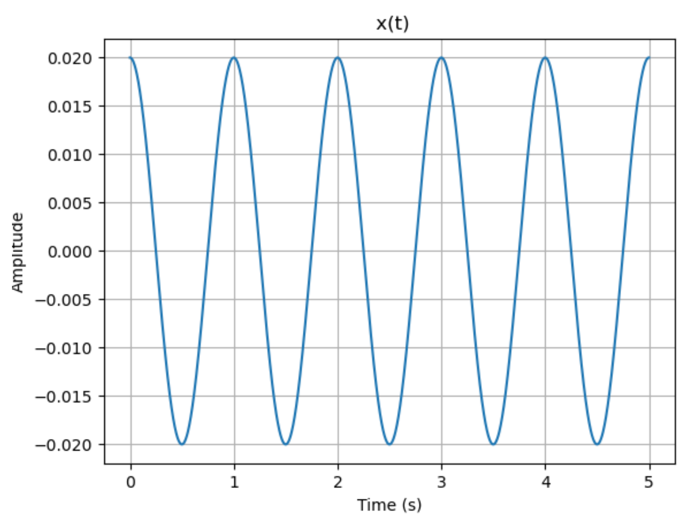

Fig. 2: plot of $x(t)$

(i) frequency of the oscillation

$$f = \frac{\omega}{2\pi} \tag{16}$$
$$\implies f = \frac{10}{\pi} \tag{17}$$

(iii)maximum speed of mass

$$x^{'}(t) = -A\omega \sin \omega t \tag{18}$$
$$x^{'}(t)_{\text{max}} = A\omega \tag{19}$$
$$\implies x^{'}(t)_{\text{max}} = 0.4m/s \tag{20}$$

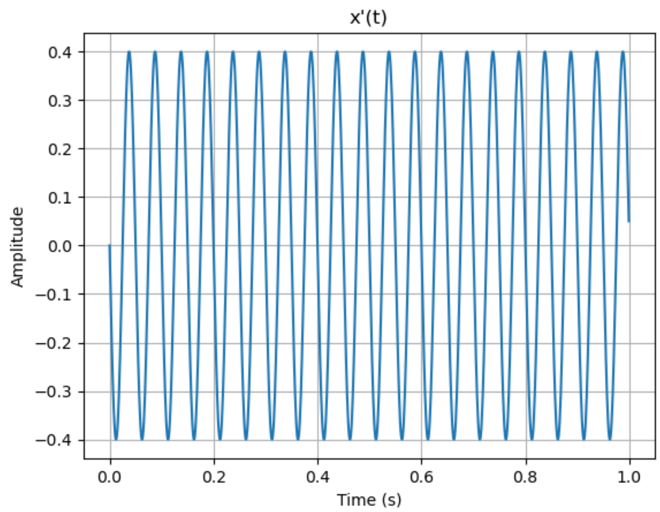

Fig. 3: plot of $x^{'}(t)$

(ii)maximum accelaration of mass

$$x^{''}(t) = -A\omega^2 \cos wt \tag{21}$$
$$x^{''}(t)_{max} = A\omega^2 \tag{22}$$
$$x^{''}(t)_{max} = 8m/s^2 \tag{23}$$

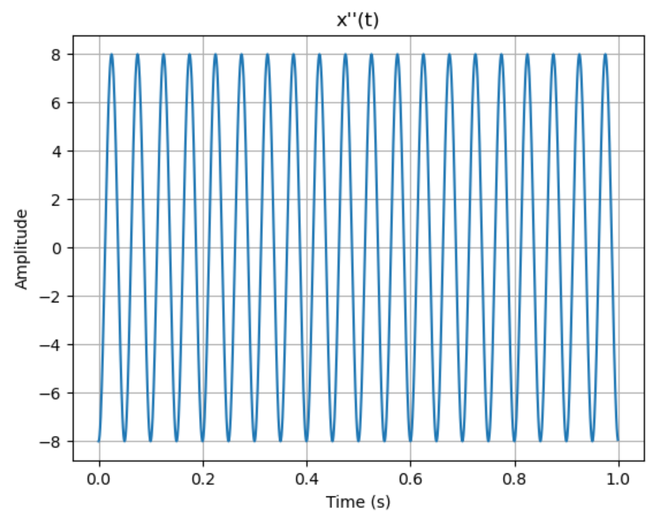

Fig. 4: plot of $x^{''}(t)$