10.5.2.14

EE23BTECH11003 - pranav

Question:Given that $\frac{dy}{dx} = 2x + y$ and $y = 1$,when $x = 0$ Using Runge-Kutta fourth order method,the value of y at $x = 0.2$ is (GATE 2023 AG 50)

Solution:

By using runge kutta 4 th order method

Variable	Description	Value
$x(n-1)$	value of x before runge kutta iteration	0
$y(n-1)$	value of y before runge kutta iteration	1
$y(n)$	value of y after runge kutta iteration	??
$x(n)$	value of x after runge kutta iteration	?
h	step size	0.1

TABLE 1: Variables Used

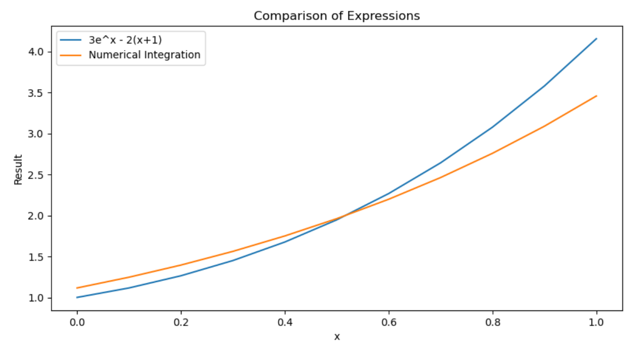

Fig. 1: simulation vs analysis

$$y(n) = y(n-1) + \frac{h}{6}[(2x(n-1) + y(n-1)(6 + 3h + h^2 + \frac{h^3}{4}) + (6h + 2h^2 + \frac{h^3}{2})] \tag{1}$$

assume step size as 0.1 and initial conditions as $x = 0$ and $y = 1$

$$y(n) = 1 + (6 + 3(0.1) + 0.1^2 + \frac{0.1^3}{4}) + (6(0.1) + 2(0.1)^2 + \frac{0.1^3}{2}) \tag{2}$$

$$\implies y_n = 1.115 \tag{3}$$

cosidering outputs of last iteration as inputs of next iteration

$$y(n) = 1.155 + \frac{0.1}{6}[(2(0.1) + 1.155(6 + 3(0.1) + (0.1)^2 + \frac{(0.1)^3}{4}) + (6(0.1) + 2(0.1)^2 + \frac{(0.1)^3}{2})] \implies y(n) = 1.29 \tag{4}$$

so at $x = 0.2$ value of y is 1.29

analysis

$$\frac{dy}{dx} = 2x + y \tag{5}$$

$$ye^{-x} = \int 2xe^{-x}dx \tag{6}$$

$$\implies ye^{-x} = -2(x+1)e^{-x} + c \tag{7}$$

by using intial conditions

$$c = 3 \tag{8}$$

$$\implies y = 3e^x - 2(x+1) \tag{9}$$