

10.5.2.14

EE23BTECH11003 - pranav

Question: Given that $\frac{dy}{dx} = 2x + y$ and $y = 1$, when $x = 0$ Using Runge-Kutta fourth order method, the value of y at $x = 0.2$ is (GATE 2023 AG 50)

Solution:

By using runge kutta 4 th order method

Variable	Description	Value
x_n	value of x before runge kutta iteration	0
y_n	value of y before runge kutta iteration	1
y_{n+1}	value of y after runge kutta iteration	??
x_{n+1}	value of x after runge kutta iteration	?
$f(x, y)$	derivative of y w.r.t to x	$2x + y$
h	step size	0.1

TABLE 1: Variables Used

$$k_1 = 2(0) + 1 = 1 \quad (7)$$

$$k_2 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.1}{2}) \quad (8)$$

$$\Rightarrow k_2 = 1.15 \quad (9)$$

$$k_3 = 2(0 + \frac{0.1}{2}) + (1 + \frac{0.115}{2}) \quad (10)$$

$$\Rightarrow k_3 = 1.1575 \quad (11)$$

$$k_4 = 2(0 + 0.1) + (1 + 0.11575) \quad (12)$$

$$\Rightarrow k_4 = 1.3158 \quad (13)$$

$$y_{n+1} = 1 + \frac{0.1}{6}(1 + 2.30 + 2.315 + 1.3158) \quad (14)$$

$$\Rightarrow y_{n+1} = 1.1155 \quad (15)$$

$$x_{n+1} = 0.1 \quad (16)$$

considering outputs of last iteration as inputs of next iteration

$$k_1 = 2(0.1) + 1.1155 = 1.3155 \quad (17)$$

$$k_2 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.12}{2}) \quad (18)$$

$$\Rightarrow k_2 = 1.4755 \quad (19)$$

$$k_3 = 2(0.1 + \frac{0.1}{2}) + (1.1155 + \frac{0.1475}{2}) \quad (20)$$

$$\Rightarrow k_3 = 2.1532 \quad (21)$$

$$k_4 = 2(0.1 + 0.1) + (1.1155 + 2.1532) \quad (22)$$

$$\Rightarrow k_4 = 3.6687 \quad (23)$$

$$y_{n+1} = 1.1155 + \frac{0.1}{6}(1.3155 + 2 \cdot 1.4755 + 2 \cdot 2.1532 + 3.6687) \quad (24)$$

$$\Rightarrow y_{n+1} = 1.319 \quad (25)$$

$$x_{n+1} = 0.2 \quad (26)$$

so at $x = 0.2$ value of y is 1.319

assume step size as 0.1 and initial conditions as $x = 0$ and $y = 1$