

# 10.5.2.14

EE23BTECH11003 - pranav

**Question:** A spring having with a spring constant  $1200 \text{ Nm}^{-1}$  is mounted on a horizontal table as shown in Fig. A mass of  $3 \text{ kg}$  is attached to the free end of the spring. The mass is then pulled sideways to a distance of  $2.0 \text{ cm}$  and released.

Determine (i) the frequency of oscillations, (ii) maximum acceleration of the mass, and (iii) the maximum speed of the mass

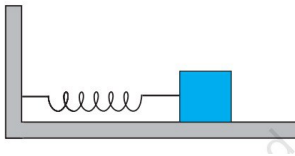


Fig. 1

**Solution:** at  $t = 0$

Variable	Description	Value
$k$	spring constant	$1200 \text{ N/m}$
$\omega$	angular frequency	$20 \text{ rad/s}$
$A$	amplitude	$0.02 \text{ m}$
$x(t)$	displacement function of the body	$0.02 \cos 20t$

TABLE 1: Variables Used

$$A = A \sin(\omega(0) + \phi) \quad (1)$$

$$\Rightarrow \phi = \frac{\pi}{2} \quad (2)$$

$$\Rightarrow x(t) = A \cos \omega t \quad (3)$$

by applying laplace transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt \quad (4)$$

$$\Rightarrow X(s) = A \frac{s}{s^2 + \omega^2} \quad (5)$$

laplace transform of  $x'(t) = sX(s) - x(0)$

$$\Rightarrow sX(s) - x(0) = A \frac{s^2}{s^2 + \omega^2} - A \quad (6)$$

$$\Rightarrow sX(s) - x(0) = -A \frac{\omega^2}{s^2 + \omega^2} \quad (7)$$

laplace transform of  $x'(t) = s^2X(s) - sx(0) - x'(0)$

$$s^2X(s) - sx(0) - x'(0) = A \frac{s^3}{s^2 + \omega^2} - As - 0 \quad (8)$$

$$\Rightarrow s^2X(s) - sx(0) - x'(0) = -A \frac{s\omega^2}{s^2 + \omega^2} \quad (9)$$

$$\Rightarrow sX(s) - x(0) = A \frac{\omega^2}{s^2 + \omega^2} \quad (10)$$

by using inverse laplace

$$\Rightarrow x'(t) = -A\omega \sin \omega t \quad (11)$$

$$\Rightarrow x''(t) = -A\omega^2 \cos \omega t \quad (12)$$

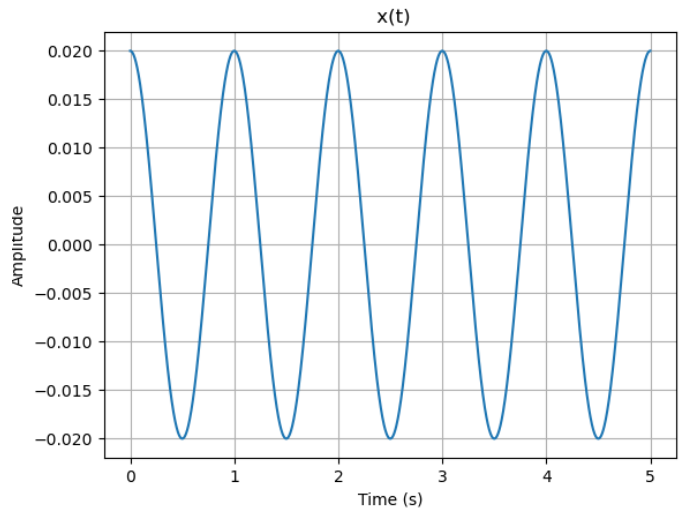


Fig. 2: plot of  $x(t)$

(i) frequency of the oscillation

$$f = \frac{\omega}{2\pi} \quad (13)$$

$$\Rightarrow f = \frac{10}{\pi} \quad (14)$$

(iii) maximum speed of mass

$$x'(t) = -A\omega \sin \omega t \quad (15)$$

$$x'(t) \Big|_{max} = A\omega \quad (16)$$

$$\Rightarrow x'(t) \Big|_{max} = 0.4m/s \quad (17)$$

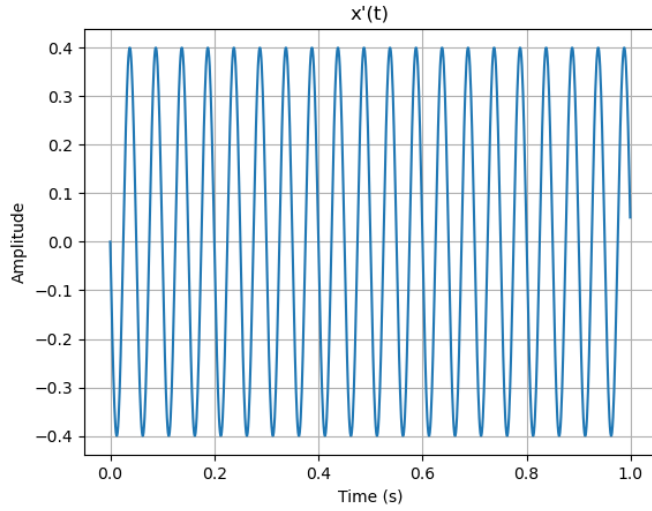


Fig. 3: plot of  $x'(t)$

(ii) maximum acceleration of mass

$$x''(t) = -A\omega^2 \cos \omega t \quad (18)$$

$$x''(t) \Big|_{max} = A\omega^2 \quad (19)$$

$$x''(t) \Big|_{max} = 8m/s^2 \quad (20)$$

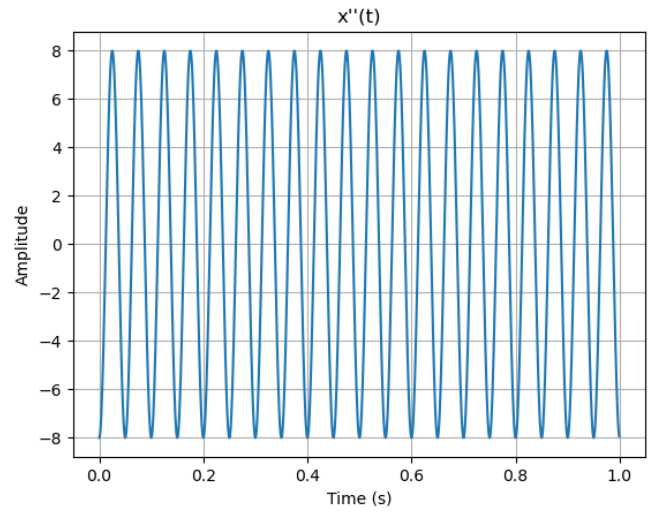


Fig. 4: plot of  $x''(t)$