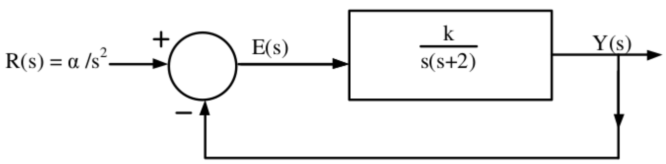
EE23BTECH11024 - G.Karthik Yadav*

GATE 2023 EC 41

1. A Closed loop systen is shown in the figure where $k > 0$ and $\alpha > 0$.
The Steady State error due to a ramp input $\left(R\left(s\right) = \alpha s^{-2}\right)$ is given by (GATE 2023 EC 41)



1) $\frac{2\alpha}{k}$
2) $\frac{\alpha}{k}$
3) $\frac{\alpha}{2k}$
4) $\frac{\alpha}{4k}$

Solution:
from table Open loop transfer function $G\left(s\right)$

Symbol	Parameters	Value
$R\left(s\right)$	Laplace transform Ramp input signal r(t)	αs^{-2}
$G\left(s\right)$	Open Loop transfer function	$\frac{Y(s)}{E(s)} = \frac{k}{s(s+2)}$
$Y\left(s\right)$	Laplace transform of the output signal y(t)	?
$E\left(s\right)$	Laplace transform of the error signal e(t)	R(s) - Y(s)
$E\left(s\right)$	Laplace transform of the error signal e(t)	R(s) - Y(s)
e_s	Steady State Error	?

TABLE I
PARAMETERS

$$G\left(s\right) = \frac{Y\left(s\right)}{E\left(s\right)} \tag{1}$$

$$= \frac{Y\left(s\right)}{R\left(s\right) - Y\left(s\right)} \tag{2}$$

$$Y\left(s\right) = \frac{R\left(s\right)G\left(s\right)}{1 + G\left(s\right)} \tag{3}$$

from eq 1 and eq (3)

$$G\left(s\right) = \frac{k}{s\left(s+2\right)} \tag{4}$$

$$Y\left(s\right) = \frac{\alpha k s^{-2}}{k + s\left(s+2\right)} \tag{5}$$

$$E\left(s\right) = R\left(s\right) - Y\left(s\right) \tag{6}$$

$$E\left(s\right) = \frac{\alpha\left(s+2\right)}{s\left(k + s\left(s+2\right)\right)} \tag{7}$$

By Taking Inverse Laplace Transform of eq (4) and eq(5)

$$g\left(t\right) = \frac{k\left(1 - e^{-2t}\right)}{2}u\left(t\right) \tag{8}$$

$$\begin{aligned} y\left(t\right) = {} & \alpha t u\left(t\right) - \frac{2\alpha}{k}u\left(t\right) \\ & + \frac{\alpha}{2k\sqrt{1-k}}\Big(2\sqrt{1-k}e^{\sqrt{1-k}t-1} \\ & + 2\sqrt{1-k}e^{-\sqrt{1-k}t-1} \\ & + \left(2-k\right)e^{\sqrt{1-k}t-1} - \left(2-k\right)e^{-\sqrt{1-k}t-1}\Big)u\left(t\right) \end{aligned} \tag{9}$$

$$e\left(t\right) = r\left(t\right) - y\left(t\right) \tag{10}$$

$$= \alpha t u\left(t\right) - y\left(t\right) \tag{11}$$

$$\begin{aligned} e\left(t\right) = {} & \frac{2\alpha}{k}u\left(t\right) \\ & - \frac{\alpha}{2k\sqrt{1-k}}\Big(2\sqrt{1-k}e^{\sqrt{1-k}t-1} \\ & + 2\sqrt{1-k}e^{-\sqrt{1-k}t-1} \\ & + \left(2-k\right)e^{\sqrt{1-k}t-1} - \left(2-k\right)e^{-\sqrt{1-k}t-1}\Big)u\left(t\right) \end{aligned} \tag{12}$$

$$e_s = \lim_{s \to 0} sE\left(s\right) \tag{13}$$

$$= \lim_{s \to 0} s\frac{R\left(s\right)}{1 + G\left(s\right)} \tag{14}$$

$$= \lim_{s \to 0} \frac{\alpha\left(s+2\right)}{s\left(s+2\right) + k} \tag{15}$$

$$e_s = \frac{2\alpha}{k} \tag{16}$$