GATE ECE 2023

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EE23BTECH11031

Consider a discrete-time signal with period $N = 5$. Let the discrete-time Fourier series (DTFS) representation be $x[n] = \sum_{k=0}^{4} a_k e^{\frac{jk2\pi n}{5}}$, where $a_0 = 1$, $a_1 = 3j$, $a_2 = 2j$, $a_3 = -2j$, $a_4 = -3j$. The value of the sum $\sum_{n=0}^{4} x[n] \sin\left(\frac{4\pi n}{5}\right)$ is

(A) -10
(B) 10
(C) -2
(D) 2

Gate 2023 EC 47

Solution:

1) Solving the question for N=5:

Parameter	Value	Description
N	5	Time period
$X(k)$	$\sum_{n=0}^{N-1} x(n)e^{\frac{-j2\pi kn}{N}}$	DFT formula
$X(0)$	5	DFT values
$X(1)$	$15j$	
$X(2)$	$10j$	
$X(3)$	$-10j$	
$X(4)$	$-15j$	

TABLE I
INPUT PARAMETERS

$$\sum_{n=0}^{4} x(n) \sin\left(\frac{4\pi n}{5}\right) = \sum_{n=0}^{4} x(n) \left[\frac{e^{\frac{j4\pi n}{5}} - e^{\frac{-j4\pi n}{5}}}{2j}\right] \tag{1}$$

$$= \frac{1}{2j}\left[\sum_{n=0}^{4} x(n)e^{\frac{j2\pi(2)n}{5}} - \sum_{n=0}^{4} x(n)e^{\frac{-j2\pi(2)n}{5}}\right] \tag{2}$$

Refering to the table I.

$$X(k) = \sum_{n=0}^{4} x(n)e^{\frac{-j2\pi kn}{5}} \tag{3}$$

Referencing from equation (3), equation (2) can be written as:

$$\sum_{n=0}^{4} x(n) \sin\left(\frac{4\pi n}{5}\right) = \frac{1}{2j}\left[X(-2) - X(2)\right] \tag{4}$$

From the property of discrete Fourier series.

$$X(k) = X(k + N) \tag{5}$$

So, equation (4) becomes,

$$\sum_{n=0}^{4} x(n) \sin\left(\frac{4\pi n}{5}\right) = \frac{1}{2j}\left[X(3) - X(2)\right] \tag{6}$$

$$\sum_{n=0}^{4} x(n) \sin\left(\frac{4\pi n}{5}\right) = -10 \tag{7}$$

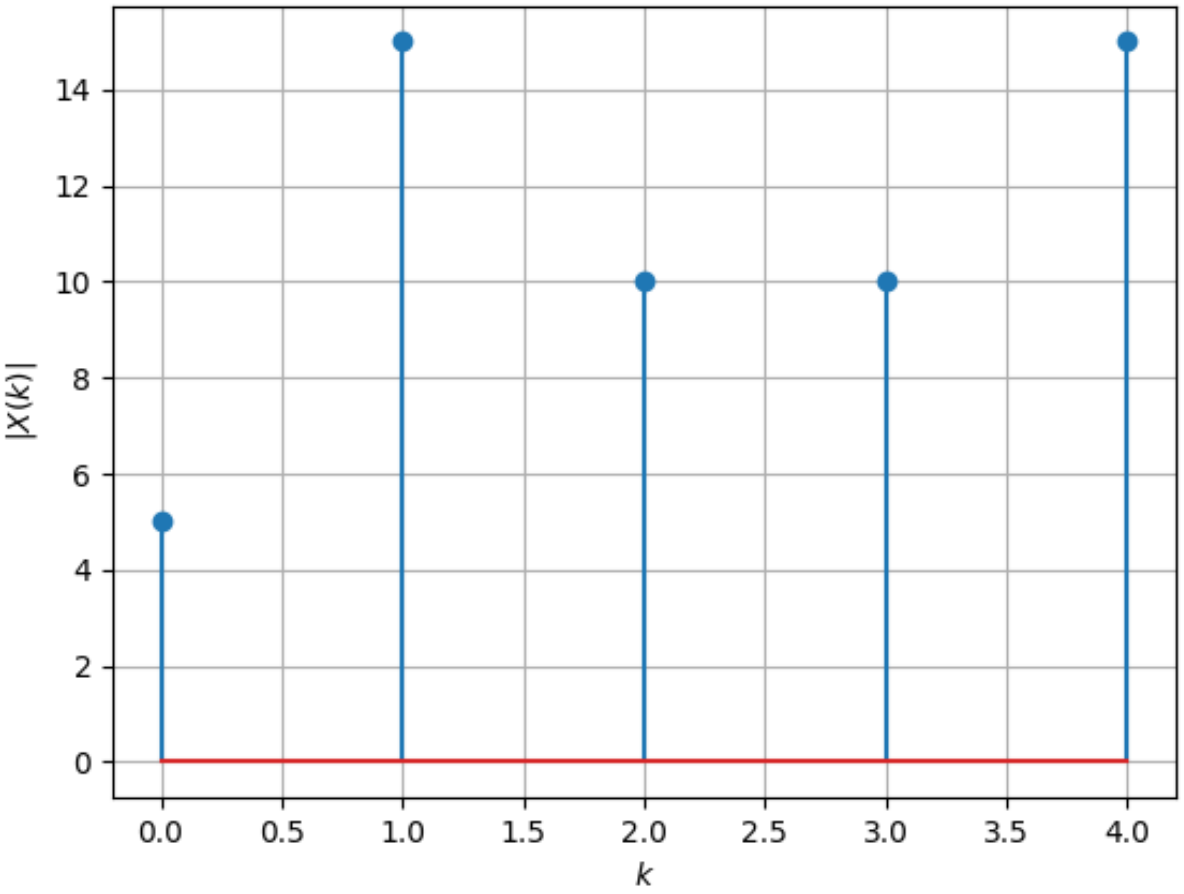

Fig. 1. Amplitude of equation (3)

2) Solving the question for N=8:

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = \sum_{n=0}^{7} x(n)\left[\frac{e^{\frac{j4\pi n}{8}} - e^{\frac{-j4\pi n}{8}}}{2j}\right] \tag{8}$$

$$= \frac{1}{2j}\left[\sum_{n=0}^{7} x(n) e^{\frac{j2\pi(2)n}{8}} - \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi(2)n}{8}}\right] \tag{9}$$

Refering to the table II.

$$X(k) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi kn}{8}} \tag{10}$$

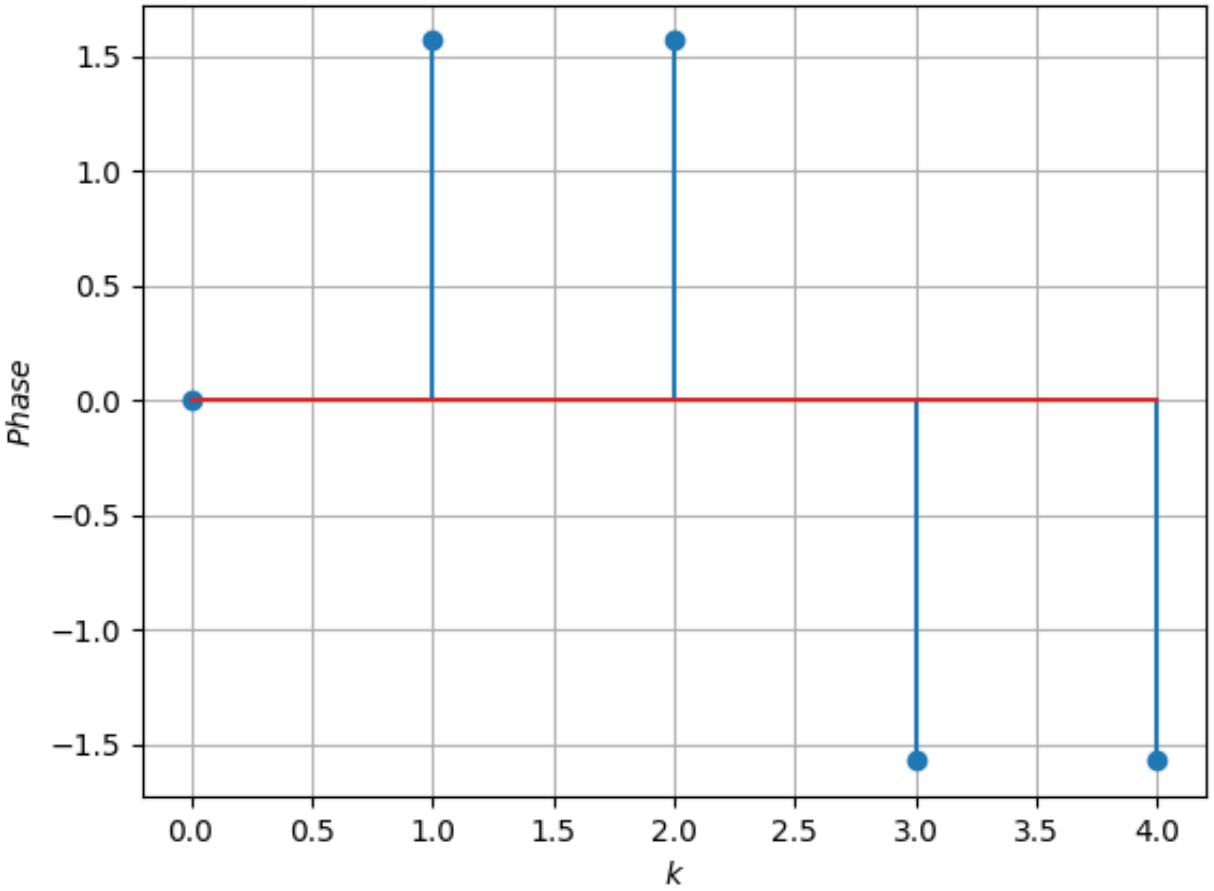

Fig. 2. Phase of equation (3)

Parameter	Value	Description
N	8	Time period
$X(k)$	$\sum_{n=0}^{N-1} x(n)e^{\frac{-j2\pi kn}{N}}$	DFT formula
$X(0)$	8	DFT values
$X(1)$	$24j$	
$X(2)$	$16j$	
$X(3)$	$-16j$	
$X(4)$	$-24j$	
$X(5)$	0	
$X(6)$	0	
$X(7)$	0	

TABLE II
INPUT PARAMETERS

Referencing from equation(10), equation(9) can be written as:

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = \frac{1}{2j}\left[X(-2) - X(2)\right] \tag{11}$$

From the property of discrete Fourier series.

$$X(k) = X(k+N) \tag{12}$$

So, equation(11) becomes,

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = \frac{1}{2j}\left[X(6) - X(2)\right] \tag{13}$$

$$\sum_{n=0}^{7} x(n) \sin\left(\frac{4\pi n}{8}\right) = -8 \tag{14}$$

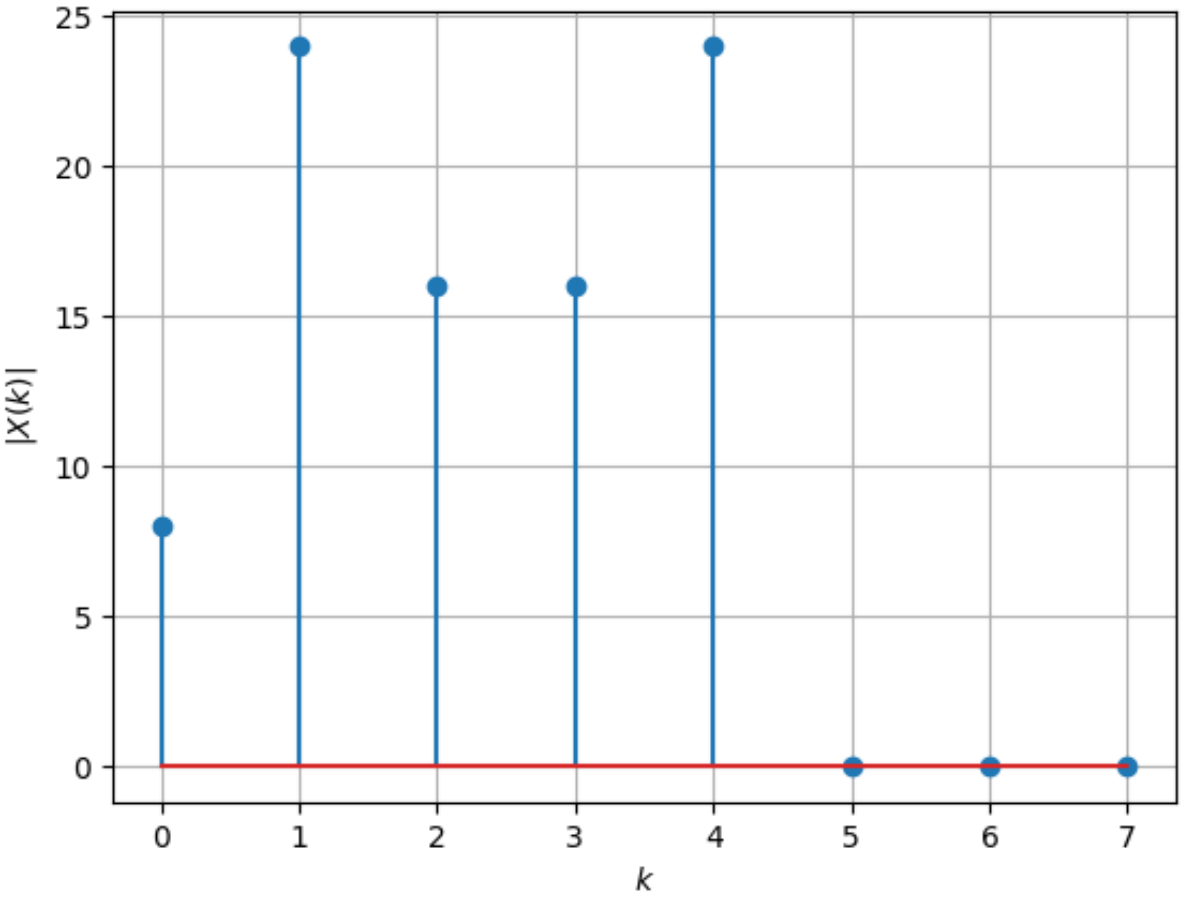

Fig. 3. Amplitude of equation (10)

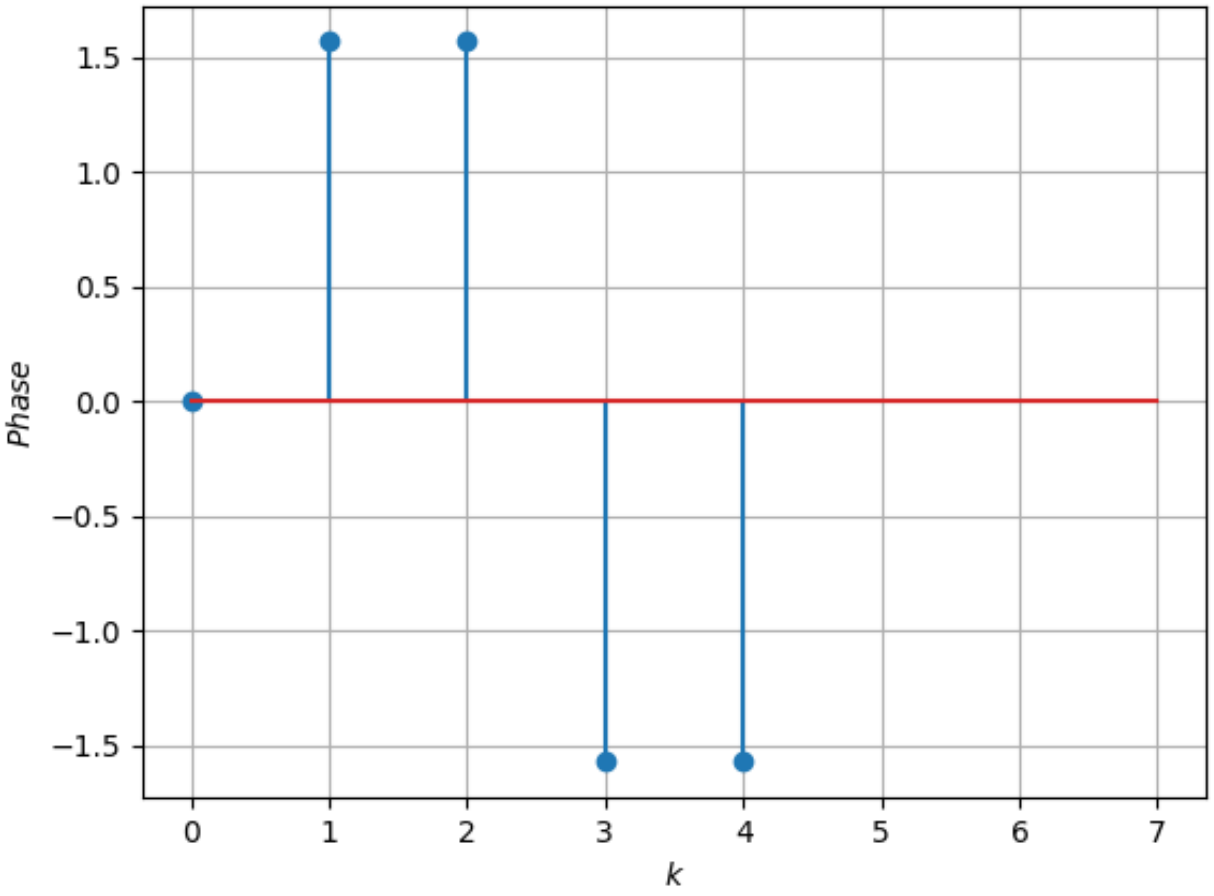

Fig. 4. Phase of equation (10)