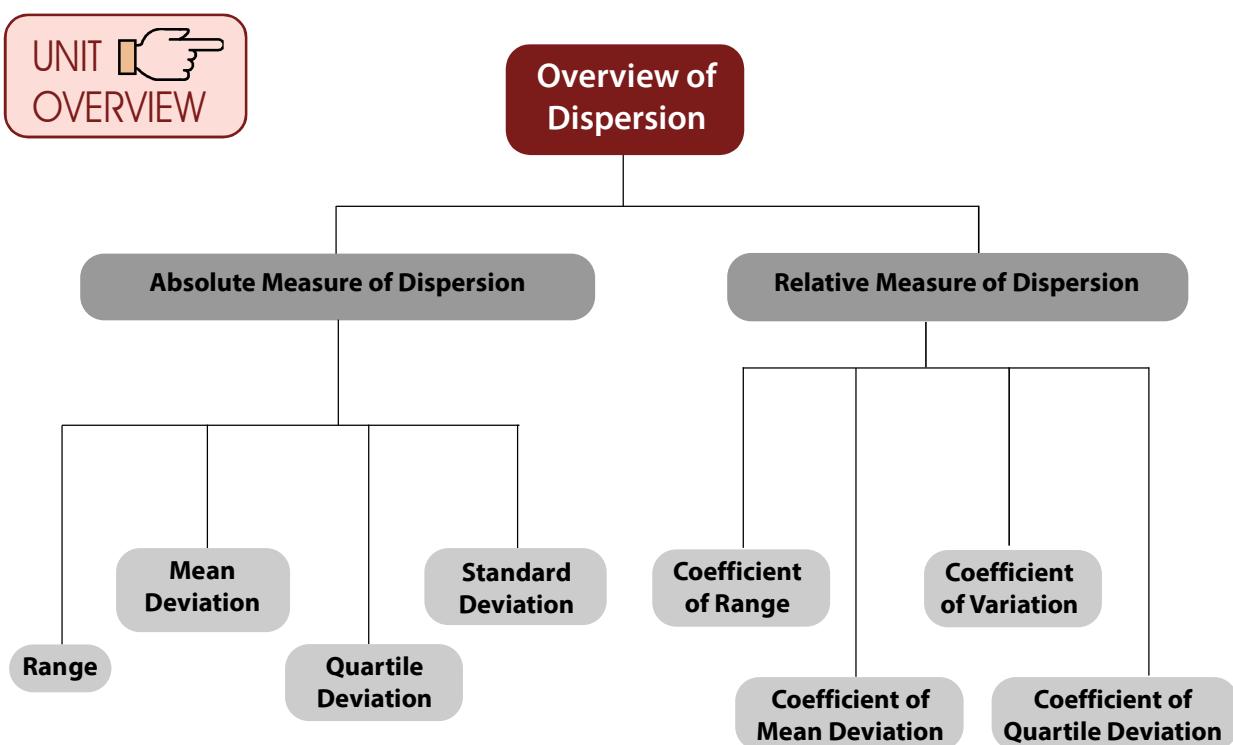


UNIT II: DISPERSION

LEARNING OBJECTIVES

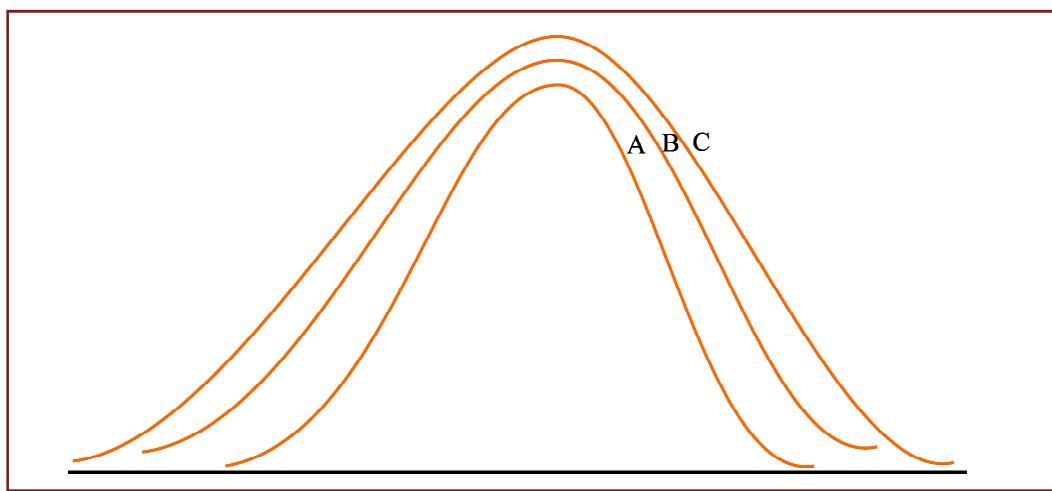
After reading this chapter, students will be able to understand:

- ◆ To understand different measures of Dispersion i.e Range, Quartile Deviation, Mean Deviation and Standard Deviation and computational techniques of these measures.
- ◆ To learn comparative advantages and disadvantages of these measures and therefore, which measures to use in which circumstance.
- ◆ To understand a set of observation, it is equally important to have knowledge of dispersion which indicates the volatility. In advanced stage of chartered accountancy course, volatility measures will be useful in understanding risk involved in financial decision making.



14.2.1 DEFINITION OF DISPERSION

The second important characteristic of a distribution is given by dispersion. Two distributions may be identical in respect of its first important characteristic i.e. central tendency and yet they may differ on account of scatterness. The following figure shows a number of distributions having identical measure of central tendency and yet varying measure of scatterness. Obviously, distribution is having the maximum amount of dispersion.

**Figure 14.2.1**

Showing distributions with identical measure of central tendency and varying amount of dispersion.

Dispersion for a given set of observations may be defined as the amount of deviation of the observations, usually, from an appropriate measure of central tendency. Measures of dispersion may be broadly classified into

1. Absolute measures of dispersion.

Absolute measures of dispersion are classified into

- | | |
|--------------------------|-------------------------|
| (i) Range | (ii) Mean Deviation |
| (iii) Standard Deviation | (iv) Quartile Deviation |

2. Relative measures of dispersion.

Likewise, we have the following relative measures of dispersion :

- | | |
|--------------------------------|---|
| (i) Coefficient of Range. | (ii) Coefficient of Mean Deviation |
| (iii) Coefficient of Variation | (iv) Coefficient of Quartile Deviation. |

We may note the following points of distinction between the absolute and relative measures of dispersion :

- I Absolute measures are dependent on the unit of the variable under consideration whereas the relative measures of dispersion are unit free.
- II For comparing two or more distributions, relative measures and not absolute measures of dispersion are considered.
- III Compared to absolute measures of dispersion, relative measures of dispersion are difficult to compute and comprehend.

Characteristics for an ideal measure of dispersion

As discussed in section 14.2.1 an ideal measure of dispersion should be properly defined, easy to comprehend, simple to compute, based on all the observations, unaffected by sampling fluctuations and amenable to some desirable mathematical treatment.



14.2.2 RANGE

For a given set of observations, range may be defined as the difference between the largest and smallest of observations. Thus if L and S denote the largest and smallest observations respectively then we have

$$\text{Range} = L - S$$

The corresponding relative measure of dispersion, known as coefficient of range, is given by

$$\text{Coefficient of range} = \frac{L-S}{L+S} \times 100$$

For a grouped frequency distribution, range is defined as the difference between the two extreme class boundaries. The corresponding relative measure of dispersion is given by the ratio of the difference between the two extreme class boundaries to the total of these class boundaries, expressed as a percentage.

We may note the following important result in connection with range:

Result:

Range remains unaffected due to a change of origin but affected in the same ratio due to a change in scale i.e., if for any two constants a and b, two variables x and y are related by $y = a + bx$,

Then the range of y is given by

$$R_y = |b| \times R_x \dots \dots \dots \quad (14.2.1)$$

Example 14.2.1: Following are the wages of 8 workers expressed in Rupees. 82, 96, 52, 75, 70, 65, 50, 70. Find the range and also its coefficient.

Solution: The largest and the smallest wages are $L = ₹ 96$ and $S = ₹ 50$
Thus range $= ₹ 96 - ₹ 50 = ₹ 46$

$$\begin{aligned}\text{Coefficient of range} &= \frac{96-50}{96+50} \times 100 \\ &= 31.51\end{aligned}$$

Example 14.2.2: What is the range and its coefficient for the following distribution of weights?

Weights in kgs. : 50 – 54 55 – 59 60 – 64 65 – 69 70 – 74

No. of Students : 12 18 23 10 3

Solution: The lowest class boundary is 49.50 kgs. and the highest class boundary is 74.50 kgs.
Thus we have

$$\text{Range} = 74.50 \text{ kgs.} - 49.50 \text{ kgs.}$$

$$= 25 \text{ kgs.}$$

$$\begin{aligned}\text{Also, coefficient of range} &= \frac{74.50 - 49.50}{74.50 + 49.50} \times 100 \\ &= \frac{25}{124} \times 100 \\ &= 20.16\end{aligned}$$

Example 14.2.3 : If the relationship between x and y is given by $2x+3y=10$ and the range of x is ₹ 15, what would be the range of y?

Solution: Since $2x+3y=10$

$$\text{Therefore, } y = \frac{10}{3} - \frac{2}{3}x$$

Applying (14.2.1) , the range of y is given by

$$R_y = |b| \times R_x = 2/3 \times ₹ 15 \\ = ₹ 10.$$

14.2.3 MEAN DEVIATION

Since range is based on only two observations, it is not regarded as an ideal measure of dispersion. A better measure of dispersion is provided by mean deviation which, unlike range, is based on all the observations. For a given set of observation, mean deviation is defined as the arithmetic mean of the absolute deviations of the observations from an appropriate measure of central tendency. Hence if a variable x assumes n values $x_1, x_2, x_3 \dots x_n$, then the mean deviation of x about an average A is given by

For a grouped frequency distribution, mean deviation about A is given by

Where x_i and f_i denote the mid value and frequency of the i -th class interval and

$$N = \sum f_i$$

In most cases we take A as mean or median and accordingly, we get mean deviation about mean or mean deviation about median.

A relative measure of dispersion applying mean deviation is given by

$$\text{Coefficient of mean deviation} = \frac{\text{Mean deviation about A}}{A} \times 100 \quad \dots \dots \dots \quad (14.2.3)$$

Mean deviation takes its minimum value when the deviations are taken from the median.
Also mean deviation remains unchanged due to a change of origin but changes in the same ratio due to a change in scale i.e. if $y = a + bx$, a and b being constants,

then $\text{MD of } y = |b| \times \text{MD of } x$ (14.2.4)

Example 14.2.4: What is the mean deviation about mean for the following numbers?

5, 8, 10, 10, 12, 9.

Solution:

The mean is given by

$$\bar{X} = \frac{5 + 8 + 10 + 10 + 12 + 9}{6} = 9$$

Table 14.2.1

Computation of MD about AM

| x_i | $ x_i - \bar{x} $ |
|-------|-------------------|
| 5 | 4 |
| 8 | 1 |
| 10 | 1 |
| 10 | 1 |
| 12 | 3 |
| 9 | 0 |
| Total | 10 |

Thus mean deviation about mean is given by

$$\frac{\sum|x_i - \bar{x}|}{n} = \frac{10}{6} = 1.67$$

Example. 14.2.5: Find mean deviations about median and also the corresponding coefficient for the following profits ('000 ₹) of a firm during a week.

82, 56, 75, 70, 52, 80, 68.

Solution:

The profits in thousand rupees is denoted by x . Arranging the values of x in an ascending order, we get

52, 56, 68, 70, 75, 80, 82.

Therefore, $Me = 70$. Thus, Median profit = ₹ 70,000.

Table 14.2.2

Computation of Mean deviation about median

| x_i | $ x_i - Me $ |
|-------|--------------|
| 52 | 18 |
| 56 | 14 |
| 68 | 2 |
| 70 | 0 |
| 75 | 5 |
| 80 | 10 |
| 82 | 12 |
| Total | 61 |

$$\text{Thus mean deviation about median} = \frac{\sum |x_i - \text{Median}|}{n}$$

$$= (\text{₹}) \frac{61}{7} \\ = \text{₹ } 8714.28$$

$$\begin{aligned}\text{Coefficient of mean deviation} &= \frac{\text{MD about median}}{\text{Median}} \times 100 \\ &= \frac{8714.28}{70000} \times 100 \\ &= 12.45\end{aligned}$$

Example 14.2.6 : Compute the mean deviation about the arithmetic mean for the following data:

| | | | | | |
|-----|---|---|---|---|---|
| x : | 1 | 3 | 5 | 7 | 9 |
| f : | 5 | 8 | 9 | 2 | 1 |

Also find the coefficient of the mean deviation about the AM.

Solution: We are to apply formula (14.1.2) as these data refer to a grouped frequency distribution the AM is given by

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$= \frac{5 \times 1 + 8 \times 3 + 9 \times 5 + 2 \times 7 + 1 \times 9}{5 + 8 + 9 + 2 + 1} = 3.88$$

Table 14.2.3
Computation of MD about the AM

| x (1) | f (2) | x - \bar{x} (3) | f x - \bar{x} (4) = (2) × (3) |
|----------|----------|------------------------|--------------------------------------|
| 1 | 5 | 2.88 | 14.40 |
| 3 | 8 | 0.88 | 7.04 |
| 5 | 9 | 1.12 | 10.08 |
| 7 | 2 | 3.12 | 6.24 |
| 9 | 1 | 5.12 | 5.12 |
| Total | 25 | - | 42.88 |

Thus, MD about AM is given by

$$\frac{\sum f |x - \bar{x}|}{N}$$

$$= \frac{42.88}{25}$$

$$= 1.72$$

| | |
|--------------------------------|---|
| Coefficient of MD about its AM | $= \frac{\text{MD about AM}}{\text{AM}} \times 100$ |
|--------------------------------|---|

$$= \frac{1.72}{3.88} \times 100$$

$$= 44.33$$

Example 14.2.7: Compute the coefficient of mean deviation about median for the following distribution:

| | | | | | |
|----------------|---|-------|-------|-------|-------|
| Weight in kgs. | : | 40-50 | 50-60 | 60-70 | 70-80 |
| No. of persons | : | 8 | 12 | 20 | 10 |

Solution: We need to compute the median weight in the first stage

Table 14.2.4
Computation of median weight

| Weight in kg (CB) | No. of Persons (Cumulative Frequency) |
|----------------------|--|
| 40 | 0 |
| 50 | 8 |
| 60 | 20 |
| 70 | 40 |
| 80 | 50 |

Hence,

$$M = l_1 + \left(\frac{\frac{N}{2} - N_l}{N_u - N_l} \right) \times C$$

$$= \left[60 + \frac{25 - 20}{40 - 20} \times 10 \right] \text{kg.} = 62.50 \text{kg.}$$

Table 14.2.5
Computation of mean deviation of weight about median

| weight (kgs.) (1) | mid-value (x _i) kgs. (2) | No. of persons (f _i) (3) | x _i - Me (kgs.) (4) | f _i x _i - Me (kgs.) (5) = (3) × (4) |
|-------------------------|--|--|---------------------------------------|---|
| 40-50 | 45 | 8 | 17.50 | 140 |
| 50-60 | 55 | 12 | 7.50 | 90 |
| 60-70 | 65 | 20 | 2.50 | 50 |
| 70-80 | 75 | 10 | 12.50 | 125 |
| Total | - | 50 | - | 405 |

$$\text{Mean deviation about median} = \frac{\sum f_i |x_i - \text{Median}|}{N}$$

$$= \frac{405}{50} \text{ kg.}$$

Coefficient of mean deviation about median = $\frac{\text{Mean deviation about median}}{\text{Median}} \times 100$

$$= \frac{8.10}{62.50} \times 100$$

$$= 12.96$$

Example 14.2.8: If x and y are related as $4x+3y+11 = 0$ and mean deviation of x is 5.40, what is the mean deviation of y?

Solution: Since $4x + 3y + 11 = 0$

$$\text{Therefore, } y = \left(\frac{-11}{3}\right) + \left(\frac{-4}{3}\right)x$$

Hence MD of $y = |b| \times \text{MD of } x$

$$= \frac{4}{3} \times 5.40 \\ = 7.20$$

14.2.4 STANDARD DEVIATION

Although mean deviation is an improvement over range so far as a measure of dispersion is concerned, mean deviation is difficult to compute and further more, it cannot be treated mathematically. The best measure of dispersion is, usually, standard deviation which does not possess the demerits of range and mean deviation.

Standard deviation for a given set of observations is defined as the root mean square deviation when the deviations are taken from the AM of the observations. If a variable x assumes n values $x_1, x_2, x_3, \dots, x_n$, then its standard deviation(s) is given by

For a grouped frequency distribution, the standard deviation is given by

$$s = \sqrt{\frac{\sum f_i(x_i - \bar{x})^2}{N}} \quad \dots \dots \dots \quad (14.2.6)$$

(14.2.5) and (14.2.6) can be simplified to the following forms

$$\begin{aligned}s &= \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} \text{ for unclassified data} \\ &= \sqrt{\frac{\sum f_i x_i^2}{N} - \bar{x}^2} \text{ for a grouped frequency distribution.} \quad \dots (14.2.7)\end{aligned}$$

Sometimes the square of standard deviation, known as variance, is regarded as a measure of dispersion. We have, then,

$$\begin{aligned}\text{Variance } s^2 &= \frac{\sum (x_i - \bar{x})^2}{n} \text{ for unclassified data} \\ &= \frac{\sum f_i (x_i - \bar{x})^2}{N} \text{ for a grouped frequency distribution.} \quad \dots (14.2.8)\end{aligned}$$

A relative measure of dispersion using standard deviation is given by coefficient of variation (cv) which is defined as the ratio of standard deviation to the corresponding arithmetic mean, expressed as a percentage.

$$\text{Coefficient of Variation (CV)} = \frac{SD}{AM} \times 100 \quad \dots (14.2.9)$$

ILLUSTRATIONS:

Example 14.2.9: Find the standard deviation and the coefficient of variation for the following numbers: 5, 8, 9, 2, 6

Solution: We present the computation in the following table.

Table 14.2.6
Computation of standard deviation

| x_i | x_i^2 |
|-------|--------------------|
| 5 | 25 |
| 8 | 64 |
| 9 | 81 |
| 2 | 4 |
| 6 | 36 |
| 30 | $\sum x_i^2 = 210$ |

Applying (14.2.7), we get the standard deviation as

$$s = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2}$$

$$\begin{aligned}
 &= \sqrt{\frac{210}{5} - \left(\frac{30}{5}\right)^2} \quad \left(\text{since } \bar{x} = \frac{\sum x_i}{n} \right) \\
 &= \sqrt{42 - 36} \\
 &= \sqrt{6} \\
 &= 2.45
 \end{aligned}$$

The coefficient of variation is

$$\begin{aligned}
 CV &= 100 \times \frac{SD}{AM} \\
 &= 100 \times \frac{2.45}{6} \\
 &= 40.83
 \end{aligned}$$

Example 14.2.10: Show that for any two numbers a and b, standard deviation is given by

$$\text{by } \frac{|a - b|}{2}.$$

Solution: For two numbers a and b, AM is given by $\bar{x} = \frac{a+b}{2}$

The variance is

$$\begin{aligned}
 s^2 &= \frac{\sum (x_i - \bar{x})^2}{2} \\
 &= \frac{\left(a - \frac{a+b}{2}\right)^2 + \left(b - \frac{a+b}{2}\right)^2}{2} \\
 &= \frac{\frac{(a-b)^2}{4} + \frac{(a-b)^2}{4}}{2} \\
 &= \frac{(a-b)^2}{4} \\
 \Rightarrow s &= \frac{|a - b|}{2}
 \end{aligned}$$

(The absolute sign is taken, as SD cannot be negative).

Example 14.2.11: Prove that for the first n natural numbers, SD is $\sqrt{\frac{n^2-1}{12}}$.

Solution: for the first n natural numbers AM is given by

$$\bar{x} = \frac{1+2+3+\dots+n}{n}$$

$$= \frac{n(n+1)}{2n}$$

$$= \frac{n+1}{2}$$

$$\begin{aligned}\therefore SD &= \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2} \\ &= \sqrt{\frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n} - \left(\frac{n+1}{2}\right)^2} \\ &= \sqrt{\frac{n(n+1)(2n+1)}{6n} - \frac{(n+1)^2}{4}} \\ &= \sqrt{\frac{(n+1)(4n+2-3n-3)}{12}} = \sqrt{\frac{n^2-1}{12}}\end{aligned}$$

Thus, SD of first n natural numbers is $SD = \sqrt{\frac{n^2-1}{12}}$

We consider the following formula for computing standard deviation from grouped frequency distribution with a view to saving time and computational labour:

$$S = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \quad \dots \dots \dots \quad (14.2.10)$$

Where $d_i = \frac{x_i - A}{C}$

Example 14.2.12: Find the SD of the following distribution:

| | | | | | | |
|-----------------|---|-------|-------|-------|-------|-------|
| Weight (kgs.) | : | 50-52 | 52-54 | 54-56 | 56-58 | 58-60 |
| No. of Students | : | 17 | 35 | 28 | 15 | 5 |

Solution:

Table 14.2.7
Computation of SD

| Weight (kgs.) (1) | No. of Students (f _i) (2) | Mid-value (x _i) (3) | d _i =x _i -55 2 (4) | f _i d _i (5)=(2)×(4) | f _i d _i ² (6)=(5)×(4) |
|-------------------------|---|---------------------------------------|--|--|---|
| 50-52 | 17 | 51 | -2 | -34 | 68 |
| 52-54 | 35 | 53 | -1 | -35 | 35 |
| 54-56 | 28 | 55 | 0 | 0 | 0 |
| 56-58 | 15 | 57 | 1 | 15 | 15 |
| 58-60 | 5 | 59 | 2 | 10 | 20 |
| Total | 100 | - | - | -44 | 138 |

Applying (14.2.7), we get the SD of weight as

$$= \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times C$$

$$= \sqrt{\frac{138}{100} - \frac{(-44)^2}{100}} \times 2 \text{ kgs.}$$

$$= \sqrt{1.38 - 0.1936} \times 2 \text{ kgs.}$$

$$= 2.18 \text{ kgs.}$$

Properties of standard deviation

- I. If all the observations assumed by a variable are constant i.e. equal, then the SD is zero. This means that if all the values taken by a variable x is k, say, then s = 0. This result applies to range as well as mean deviation.
- II. SD remains unaffected due to a change of origin but is affected in the same ratio due to a change of scale i.e., if there are two variables x and y related as y = a+bx for any two constants a and b, then SD of y is given by

$$s_y = |b| s_x \quad \dots \quad (14.2.11)$$

- III. If there are two groups containing n₁ and n₂ observations, \bar{x}_1 and \bar{x}_2 as respective AM's, s₁ and s₂ as respective SD's, then the combined SD is given by

$$s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}} \quad \dots \quad (14.2.12)$$

$$\text{where, } d_1 = \bar{x}_1 - \bar{x}$$

$$d_2 = \bar{x}_2 - \bar{x}$$

$$\text{and } \bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \text{combined AM}$$

This result can be extended to more than 2 groups. For $x \geq 2$ groups, we have

$$s = \sqrt{\frac{\sum n_i s_i^2 + \sum n_i d_i^2}{\sum n_i}} \quad \dots \dots \dots \quad (14.2.13)$$

$$\text{With } d_i = x_i - \bar{x}$$

$$\text{and } \bar{x} = \frac{\sum n_i \bar{x}_i}{\sum n_i}$$

Where $\bar{x}_1 = \bar{x}_2$ (14.2.13) is reduced to

$$s = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}}$$

Example 14.2.13: If AM and coefficient of variation of x are 10 and 40 respectively, what is the variance of $(15-2x)$?

Solution: let $y = 15 - 2x$

Then applying (14.2.4), we get,

$$s_y = 2 \times s_x \quad \dots \dots \dots \quad (1)$$

As given $cv_x = \text{coefficient of variation of } x = 40$ and $\bar{x} = 10$

$$\text{This } cv_x = \frac{s_x}{x} \times 100$$

$$\Rightarrow 40 = \frac{s_x}{10} \times 100$$

$$\Rightarrow s_x = 4$$

$$\text{From (1), } S_y = 2 \times 4 = 8$$

$$\text{Therefore, variance of } (15-2x) = S_y^2 = 64$$

Example 14.2.14: Compute the SD of 9, 5, 8, 6, 2.

Without any more computation, obtain the SD of

| | | | | | |
|------------|-----|-----|-----|-----|-----|
| Sample I | -1, | -5, | -2, | -4, | -8, |
| Sample II | 90, | 50, | 80, | 60, | 20, |
| Sample III | 23, | 15, | 21, | 17, | 9. |

Solution:

Table 14.2.7
Computation of SD

| x_i | x_i^2 |
|-------|---------|
| 9 | 81 |
| 5 | 25 |
| 8 | 64 |
| 6 | 36 |
| 2 | 4 |
| 30 | 210 |

The SD of the original set of observations is given by

$$s = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$= \sqrt{\frac{210}{5} - \left(\frac{30}{5}\right)^2}$$

$$= \sqrt{42 - 36}$$

$$= \sqrt{6}$$

$$= 2.45$$

If we denote the original observations by x and the observations of sample I by y , then we have

$$y = -10 + x$$

$$y = (-10) + (1)x$$

$$\therefore S_y = |1| \times S_x$$

$$= 1 \times 2.45$$

$$= 2.45$$

In case of sample II, x and y are related as

$$Y = 10x$$

$$= 0 + (15)x$$

$$\begin{aligned}\therefore s_y &= |10| \times s_x \\ &= 10 \times 2.45 \\ &= 24.50\end{aligned}$$

And lastly, $y = (5) + (2)x$

$$\begin{aligned}\Rightarrow s_y &= 2 \times 2.45 \\ &= 4.90\end{aligned}$$

Example 14.2.15: For a group of 60 boy students, the mean and SD of stats. marks are 45 and 2 respectively. The same figures for a group of 40 girl students are 55 and 3 respectively. What is the mean and SD of marks if the two groups are pooled together?

Solution: As given $n_1 = 60$, $\bar{x}_1 = 45$, $s_1 = 2$ $n_2 = 40$, $\bar{x}_2 = 55$, $s_2 = 3$

Thus the combined mean is given by

$$\begin{aligned}\bar{x} &= \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} \\ &= \frac{60 \times 45 + 40 \times 55}{60 + 40} \\ &= 49\end{aligned}$$

$$\text{Thus } d_1 = \bar{x}_1 - \bar{x} = 45 - 49 = -4$$

$$d_2 = \bar{x}_2 - \bar{x} = 55 - 49 = 6$$

Applying (14.2.13), we get the combined SD as

$$\begin{aligned}s &= \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}} \\ &= \sqrt{\frac{60 \times 2^2 + 40 \times 3^2 + 60 \times (-4)^2 + 40 \times 6^2}{60 + 40}} \\ &= \sqrt{30} \\ &= 5.48\end{aligned}$$

Example 14.2.16: The mean and standard deviation of the salaries of the two factories are provided below :

| Factory | No. of Employees | Mean Salary | SD of Salary |
|---------|------------------|-------------|--------------|
| A | 30 | ₹ 4800 | ₹ 10 |
| B | 20 | ₹ 5000 | ₹ 12 |

- i) Find the combined mean salary and standard deviation of salary.
- ii) Examine which factory has more consistent structure so far as satisfying its employees are concerned.

Solution: Here we are given

$$\begin{aligned} n_1 &= 30, \bar{x}_1 = ₹ 4800, s_1 = ₹ 10, \\ n_2 &= 20, \bar{x}_2 = ₹ 5000, s_2 = ₹ 12 \end{aligned}$$

i) $\frac{30 \times ₹ 4800 + 20 \times ₹ 5000}{30 + 20} = ₹ 4800$

$$d_1 = \bar{x}_1 - \bar{x} = ₹ 4,800 - ₹ 4880 = -₹ 80$$

$$d_2 = \bar{x}_2 - \bar{x} = ₹ 5,000 - ₹ 4880 = ₹ 120$$

hence, the combined SD in rupees is given by

$$s = \sqrt{\frac{30 \times 10^2 + 20 \times 12^2 + 30 \times (-80)^2 + 20 \times 120^2}{30 + 20}}$$

$$= \sqrt{9717.60}$$

$$= 98.58$$

thus the combined mean salary and the combined standard deviation of salary are ₹ 4880 and ₹ 98.58 respectively.

ii) In order to find the more consistent structure, we compare the coefficients of variation of the

two factories. Letting $CV_A = 100 \times \frac{S_A}{\bar{x}_A}$ and $CV_B = 100 \times \frac{S_B}{\bar{x}_B}$

We would say factory A is more consistent

if $CV_A < CV_B$. Otherwise factory B would be more consistent.

$$\text{Now } CV_A = 100 \times \frac{S_A}{\bar{x}_A} = 100 \times \frac{s_1}{\bar{x}_1} = \frac{100 \times 10}{4800} = 0.21$$

$$\text{and } CV_B = 100 \times \frac{S_B}{\bar{x}_B} = 100 \times \frac{S_2}{\bar{x}_2} = \frac{100 \times 12}{5000} = 0.24$$

Thus we conclude that factory A has more consistent structure.

Example 14.2.17: A student computes the AM and SD for a set of 100 observations as 50 and 5 respectively. Later on, she discovers that she has made a mistake in taking one observation as 60 instead of 50. What would be the correct mean and SD if

- i) The wrong observation is left out?
- ii) The wrong observation is replaced by the correct observation?

Solution: As given, $n = 100$, $\bar{x} = 50$, $S = 5$

Wrong observation = 60, correct observation = 50

$$\bar{x} = \frac{\sum x_i}{n}$$

$$\Rightarrow \sum x_i = n\bar{x} = 100 \times 50 = 5000$$

$$\text{and } s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$$

$$\Rightarrow \sum x_i^2 = n(\bar{x}^2 + s^2) = 100(50^2 + 5^2) = 252500$$

- i) Sum of the 99 observations = $5000 - 60 = 4940$

AM after leaving the wrong observation = $4940/99 = 49.90$

$$\begin{aligned} \text{Sum of squares of the observation after leaving the wrong observation} \\ = 252500 - 60^2 = 248900 \end{aligned}$$

Variance of the 99 observations = $248900/99 - (49.90)^2$

$$= 2514.14 - 2490.01$$

= 24,13

∴ SD of 99 observations = 4.91

- ii) Sum of the 100 observations after replacing the wrong observation by the correct observation
 $= 5000 - 60 + 50 = 4990$

$$AM = \frac{4990}{100} = 49.90$$

$$\text{Corrected sum of squares} = 252500 + 50^2 - 60^2 = 251400$$

$$\begin{aligned} \text{Corrected SD} &= \sqrt{\frac{251400}{100} - (49.90)^2} \\ &= \sqrt{23.94} = 4.90 \end{aligned}$$



14.2.5 QUARTILE DEVIATION

Another measure of dispersion is provided by **quartile deviation** or **semi-inter-quartile range** which is given by

$$Q_d = \frac{Q_3 - Q_1}{2} \quad \dots \dots \dots \quad (14.2.14)$$

A relative measure of dispersion using quartiles is given by coefficient of quartile deviation which is

$$\text{Coefficient of quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100 \quad \dots \dots \dots \quad (14.2.15)$$

Quartile deviation provides the best measure of dispersion for open-end classification. It is also less affected due to extreme observations or sampling fluctuations. Like other measures of dispersion, quartile deviation remains unaffected due to a change of origin but is affected in the same ratio due to change in scale.

Example 14.2.18 : Following are the marks of the 10 students : 56, 48, 65, 35, 42, 75, 82, 60, 55, 50. Find quartile deviation and also its coefficient.

Solution:

After arranging the marks in an ascending order of magnitude, we get 35, 42, 48, 50, 55, 56, 60, 65, 75, 82

$$\text{First quartile } (Q_1) = \frac{(n+1)}{4}^{\text{th}} \text{ observation}$$

$$= \frac{(10+1)}{4}^{\text{th}} \text{ observation}$$

$$= 2.75^{\text{th}} \text{ observation}$$

$$= 2^{\text{nd}} \text{ observation} + 0.75 \times \text{difference between the third and the } 2^{\text{nd}} \text{ observation.}$$

$$= 42 + 0.75 \times (48 - 42)$$

$$= 46.50$$

$$\text{Third quartile } (Q_3) = \frac{3(n+1)}{4}^{\text{th}} \text{ observation}$$

$$= 8.25^{\text{th}} \text{ observation}$$

$$= 65 + 0.25 \times 10$$

$$= 67.50$$

Thus applying (14.2.14), we get the quartile deviation as

$$\frac{Q_3 - Q_1}{2} = \frac{67.50 - 46.50}{2} = 10.50$$

Also, using (14.2.15), the coefficient of quartile deviation

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$$

$$= \frac{67.50 - 46.50}{67.50 + 46.50} \times 100$$

$$= 18.42$$

Example 14.2.19 : If the quartile deviation of x is 6 and $3x + 6y = 20$, what is the quartile deviation of y ?

Solution: $3x + 6y = 20$

$$\Rightarrow y = \left(\frac{20}{6} \right) + \left(\frac{-3}{6} \right)x$$

Therefore, quartile deviation of $y = \frac{|-3|}{6} \times \text{quartile deviation of } x$

$$\begin{aligned} &= \frac{1}{2} \times 6 \\ &= 3. \end{aligned}$$

Example 14.2.20: Find an appropriate measures of dispersion from the following data:

| | | | | | | |
|--------------------|---|---------|-------|-------|-------|--------|
| Daily wages (₹) | : | upto 20 | 20-40 | 40-60 | 60-80 | 80-100 |
| No. of workers (₹) | : | 5 | 11 | 14 | 7 | 3 |

Solution: Since this is an open-end classification, the appropriate measure of dispersion would be quartile deviation as quartile deviation does not take into account the first twenty five percent and the last twenty five per cent of the observations.

Table 14.2.8
Computation of Quartile

| Daily wages in (₹) (Class boundary) | No. of workers (less than cumulative frequency) |
|--|--|
| a | 0 |
| 20 | 5 |
| 40 | 16 |
| 60 | 30 |
| 80 | 37 |
| 100 | 40 |

Here a denotes the first Class Boundary

$$Q_1 = ₹ \left[20 + \frac{10 - 5}{16 - 5} \times 20 \right] = ₹ 29.09$$

$$Q_3 = ₹ \left[40 + \frac{30 - 16}{30 - 16} \times 20 \right] = ₹ 60$$

$$Q_3 = ₹ 60$$

Thus quartile deviation of wages is given by

$$\begin{aligned} &\frac{Q_3 - Q_1}{2} \\ &= \frac{₹ 60 - ₹ 29.09}{2} \\ &= ₹ 15.46 \end{aligned}$$

Example 14.2.21: The mean and variance of 5 observations are 4.80 and 6.16 respectively. If three of the observations are 2, 3 and 6, what are the remaining observations?

Solution: Let the remaining two observations be a and b , then as given

$$\begin{aligned} \frac{2+3+6+a+b}{5} &= 4.80 \\ \Rightarrow 11+a+b &= 24 \\ \Rightarrow a+b &= 13 \quad \dots \dots \dots (1) \end{aligned}$$

$$\text{and } \frac{2^2 + a^2 + b^2 + 3^2 + 6^2}{5} - (4.80)^2$$

$$\Rightarrow \frac{49 + a^2 + b^2}{5} - 23.04 = 6.16$$

$$\Rightarrow 49 + a^2 + b^2 = 146$$

$$\Rightarrow a^2 + b^2 = 97 \quad (2)$$

From (1), we get $a \equiv 13 - b$ (3)

Eliminating a from (2) and (3), we get

$$\begin{aligned} (13 - b)^2 + b^2 &= 97 \\ \Rightarrow 169 - 26b + 2b^2 &= 97 \\ \Rightarrow b^2 - 13b + 36 &= 0 \\ \Rightarrow (b-4)(b-9) &= 0 \\ \Rightarrow b &= 4 \text{ or } 9 \end{aligned}$$

From (3), $a \equiv 9$ or 4

Thus the remaining observations are 4 and 9.

Example 14.2.22: After shift of origin and change of scale, a frequency distribution of a continuous variable with equal class length takes the following form of the changed variable (d):

| | | | | | | |
|-----------|---|----|----|----|----|---|
| d | : | -2 | -1 | 0 | 1 | 2 |
| Frequency | : | 17 | 35 | 28 | 15 | 5 |

If the mean and standard deviation of the original frequency distribution are 54.12 and 2.1784 respectively, find the original frequency distribution.

Solution: We need find out the origin A and scale C from the given conditions.

Since $d_i = \frac{x_i - A}{C}$

$$\Rightarrow x_i = A + C d_i$$

Once A and C are known, the mid-values x_i 's would be known. Finally, we convert the mid-values to the corresponding class boundaries by using the formula:

$$LCB = x_i - C/2$$

$$\text{and } UCB = x_i + C/2$$

On the basis of the given data, we find that

$$\sum f_i d_i = -44, \sum f_i d_i^2 = 138 \text{ and } N = 100$$

$$\text{Hence } s = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2} \times C$$

$$\Rightarrow 2.1784 = \sqrt{\frac{138}{100} - \left(\frac{-44}{100}\right)^2} \times C$$

$$\Rightarrow 2.1784 = \sqrt{1.38 - 0.1936} \times C$$

$$\Rightarrow 2.1784 = 1.0892 \times C$$

$$\Rightarrow C = 2$$

$$\text{Further, } \bar{x} = A + \frac{\sum f_i d_i}{N} \times C$$

$$\Rightarrow 54.12 = A + \frac{-44}{100} \times 2$$

$$\Rightarrow 54.12 = A - 0.88$$

$$\Rightarrow A = 55$$

$$\text{Thus } x_i = A + Cd_i$$

$$\Rightarrow x_i = 55 + 2d_i$$

Table 14.2.9

Computation of the Original Frequency Distribution

| d_i | f_i | $x_i = 55 + 2d_i$ | Class interval $x_i \pm \frac{C}{2}$ |
|-------|-------|-------------------|---|
| -2 | 17 | 51 | 50-52 |
| -1 | 35 | 53 | 52-54 |
| 0 | 28 | 55 | 54-56 |
| 1 | 15 | 57 | 56-58 |
| 2 | 5 | 59 | 58-60 |

Example 14.2.23: Compute coefficient of variation from the following data:

| | | | | | | | |
|----------------|---|----------|----------|----------|----------|----------|----------|
| Age | : | under 10 | under 20 | under 30 | under 40 | under 50 | under 60 |
| No. of persons | | | | | | | |
| Dying | : | 10 | 18 | 30 | 45 | 60 | 80 |

Solution: What is given in this problem is less than cumulative frequency distribution. We need first convert it to a frequency distribution and then compute the coefficient of variation.

Table 14.2.10

Computation of coefficient of variation

| Age in years class Interval | No. of persons dying (f _i) | Mid-value (x _i) | $\frac{d_i = x_i - 25}{10}$ | f _i d _i | f _i d _i ² |
|--------------------------------|--|--------------------------------|-----------------------------|-------------------------------|--|
| 0-10 | 10 | 5 | -2 | -20 | 40 |
| 10-20 | 18-10=8 | 15 | -1 | -8 | 8 |
| 20-30 | 30-18=12 | 25 | 0 | 0 | 0 |
| 30-40 | 45-30=15 | 35 | 1 | 15 | 15 |
| 40-50 | 60-45=15 | 45 | 2 | 30 | 60 |
| 50-60 | 80-60=20 | 55 | 3 | 60 | 180 |
| Total | 80 | - | - | 77 | 303 |

The AM is given by:

$$\bar{x} = A + \frac{\sum f_i d_i}{N} \times C$$

$$= \left(25 + \frac{77}{80} \times 10 \right) \text{ years}$$

$$= 34.63 \text{ years}$$

The standard deviation is

$$s = \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N} \right)^2} \times C$$

$$= \sqrt{\frac{303}{80} - \left(\frac{77}{80} \right)^2} \times 10 \text{ years}$$

$$= \sqrt{3.79 - 0.93} \times 10 \text{ years}$$

$$= 16.91 \text{ years}$$

Thus the coefficient of variation is given by

$$CV = \frac{S}{X} \times 100$$

$$= \frac{16.91}{34.63} \times 100$$

$$= 48.83$$

Example 14.2.24: You are given the distribution of wages in two factories A and B

| | | | | | | | |
|---------------------|---|---------|---------|---------|---------|---------|---------|
| Wages in ₹ | : | 100-200 | 200-300 | 300-400 | 400-500 | 500-600 | 600-700 |
| No. of workers in A | : | 8 | 12 | 17 | 10 | 2 | 1 |
| No. of workers in B | : | 6 | 18 | 25 | 12 | 2 | 2 |

State in which factory, the wages are more variable.

Solution:

As explained in example 14.2.3, we need compare the coefficient of variation of A (i.e. v_A) and of B (i.e. v_B).

If $v_A > v_B$, then the wages of factory A would be more variable. Otherwise, the wages of factory B would be more variable where

$$V_A = 100 \times \frac{S_A}{\bar{X}_A} \quad \text{and} \quad V_B = 100 \times \frac{S_B}{\bar{X}_B}$$

Table 14.2.11

Computation of coefficient of variation of wages of Two Factories A and B

| Wages in rupees (1) | Mid-value (2) | $d =$ (3) | No. of workers of A f_A (4) | No. of workers of B f_B (5) | $f_A d$ (6)=(3)×(4) | $f_A d^2$ (7)=(3)×(6) | $f_B d$ (8)=(3)×(5) | $f_B d^2$ (9)=(3)×(8) |
|------------------------|------------------|--------------|--|--|------------------------|--------------------------|------------------------|--------------------------|
| 100-200 | 150 | -2 | 8 | 6 | -16 | 32 | -12 | 24 |
| 200-300 | 250 | -1 | 12 | 18 | -12 | 12 | -18 | 18 |
| 300-400 | 350 | 0 | 17 | 25 | 0 | 0 | 0 | 0 |
| 400-500 | 450 | 1 | 10 | 12 | 10 | 10 | 12 | 12 |
| 500-600 | 550 | 2 | 2 | 2 | 4 | 8 | 4 | 8 |
| 600-700 | 650 | 3 | 1 | 2 | 3 | 9 | 6 | 18 |
| Total | - | - | 50 | 65 | -11 | 71 | -8 | 80 |

For Factory A

$$\bar{x}_A = ₹ \left(350 + \frac{-11}{50} \times 100 \right) = ₹ 328$$

$$S_A = ₹ \sqrt{\frac{71}{50} - \left(\frac{-11}{50} \right)^2} \times 100 = ₹ 117.12$$

$$\therefore V_A = \frac{S_A}{\bar{x}_A} \times 100 = 35.71$$

For Factory B

$$\bar{x}_B = ₹ \left(350 + \frac{-8}{65} \times 100 \right) = ₹ 337.69$$

$$S_B = ₹ \sqrt{\frac{80}{65} - \left(\frac{-8}{65} \right)^2} \times 100$$

$$= ₹ 110.25$$

$$\therefore V_B = \frac{110.25}{337.69} \times 100 = 32.65$$

As $V_A > V_B$, the wages for factory A is more variable.



SUMMARY

- ◆ Standard deviation is the most widely and commonly used measure of dispersion
- ◆ Range is the quickest to compute and as such, has its application in statistical quality control. However, range is based on only two observations and affected too much by the presence of extreme observation(s).
- ◆ Mean deviation is rigidly defined, based on all the observations and not much affected by sampling fluctuations. However, mean deviation is difficult to comprehend and its computation is also time consuming and laborious. Furthermore, unlike SD, mean deviation does not possess mathematical properties.
- ◆ Quartile deviation is also rigidly defined, easy to compute and not much affected by sampling fluctuations. The presence of extreme observations has no impact on quartile deviation since quartile deviation is based on the central fifty-percent of the observations. However, quartile deviation is not based on all the observations and it has no desirable mathematical properties. Nevertheless, quartile deviation is the best measure of dispersion for open-end classifications.



EXERCISE — UNIT-II

Set A

Write down the correct answers. Each question carries one mark.

1. Which of the following statements is correct?
 - (a) Two distributions may have identical measures of central tendency and dispersion.
 - (b) Two distributions may have the identical measures of central tendency but different measures of dispersion.
 - (c) Two distributions may have the different measures of central tendency but identical measures of dispersion.
 - (d) All the statements (a), (b) and (c).
2. Dispersion measures
 - (a) The scatterness of a set of observations
 - (b) The concentration of a set of observations
 - (c) Both (a) and (b)
 - (d) Neither (a) and (b).
3. When it comes to comparing two or more distributions we consider
 - (a) Absolute measures of dispersion
 - (b) Relative measures of dispersion
 - (c) Both (a) and (b)
 - (d) Either (a) or (b).
4. Which one is easiest to compute?
 - (a) Relative measures of dispersion
 - (b) Absolute measures of dispersion
 - (c) Both (a) and (b)
 - (d) Range
5. Which one is an absolute measure of dispersion?
 - (a) Range
 - (b) Mean Deviation
 - (c) Standard Deviation
 - (d) All these measures
6. Which measure of dispersion is most useful?
 - (a) Standard deviation
 - (b) Quartile deviation
 - (c) Mean deviation
 - (d) Range
7. Which measures of dispersions is not affected by the presence of extreme observations?
 - (a) Range
 - (b) Mean deviation
 - (c) Standard deviation
 - (d) Quartile deviation
8. Which measure of dispersion is based on the absolute deviations only?
 - (a) Standard deviation
 - (b) Mean deviation
 - (c) Quartile deviation
 - (d) Range

9. Which measure is based on only the central fifty percent of the observations?
- (a) Standard deviation
 - (b) Quartile deviation
 - (c) Mean deviation
 - (d) All these measures
10. Which measure of dispersion is based on all the observations?
- (a) Mean deviation
 - (b) Standard deviation
 - (c) Quartile deviation
 - (d) (a) and (b) but not (c)
11. The appropriate measure of dispersion for open-end classification is
- (a) Standard deviation
 - (b) Mean deviation
 - (c) Quartile deviation
 - (d) All these measures.
12. The most commonly used measure of dispersion is
- (a) Range
 - (b) Standard deviation
 - (c) Coefficient of variation
 - (d) Quartile deviation.
13. Which measure of dispersion has some desirable mathematical properties?
- (a) Standard deviation
 - (b) Mean deviation
 - (c) Quartile deviation
 - (d) All these measures
14. If the profits of a company remains the same for the last ten months, then the standard deviation of profits for these ten months would be ?
- (a) Positive
 - (b) Negative
 - (c) Zero
 - (d) (a) or (c)
15. Which measure of dispersion is considered for finding a pooled measure of dispersion after combining several groups?
- (a) Mean deviation
 - (b) Standard deviation
 - (c) Quartile deviation
 - (d) Any of these
16. A shift of origin has no impact on
- (a) Range
 - (b) Mean deviation
 - (c) Standard deviation
 - (d) All these and quartile deviation.
17. The range of 15, 12, 10, 9, 17, 20 is
- (a) 5
 - (b) 12
 - (c) 13
 - (d) 11.
18. The standard deviation of 10, 16, 10, 16, 10, 10, 16, 16 is
- (a) 4
 - (b) 6
 - (c) 3
 - (d) 0.
19. For any two numbers SD is always
- (a) Twice the range
 - (b) Half of the range
 - (c) Square of the range
 - (d) None of these.

20. If all the observations are increased by 10, then

 - (a) SD would be increased by 10
 - (b) Mean deviation would be increased by 10
 - (c) Quartile deviation would be increased by 10
 - (d) All these three remain unchanged.

21. If all the observations are multiplied by 2, then

 - (a) New SD would be also multiplied by 2
 - (b) New SD would be half of the previous SD
 - (c) New SD would be increased by 2
 - (d) New SD would be decreased by 2.

Set B

Write down the correct answers. Each question carries two marks.

8. If the relation between x and y is $5y - 3x = 10$ and the mean deviation about mean for x is 12, then the mean deviation of y about mean is
(a) 7.20 (b) 6.80 (c) 20 (d) 18.80.

9. If two variables x and y are related by $2x + 3y - 7 = 0$ and the mean and mean deviation about mean of x are 1 and 0.3 respectively, then the coefficient of mean deviation of y about its mean is
(a) -5 (b) 12 (c) 50 (d) 4.

10. The mean deviation about mode for the numbers $4/11, 6/11, 8/11, 9/11, 12/11, 8/11$ is
(a) $1/6$ (b) $1/11$ (c) $6/11$ (d) $5/11$.

11. What is the standard deviation of $5, 5, 9, 9, 9, 10, 5, 10, 10$?
(a) $\sqrt{14}$ (b) $\frac{\sqrt{42}}{3}$ (c) 4.50 (d) 8

12. If the mean and SD of x are a and b respectively, then the SD of $\frac{x-a}{b}$ is
(a) -1 (b) 1 (c) ab (d) a/b .

13. What is the coefficient of variation of the following numbers?
 $53, 52, 61, 60, 64$.
(a) 8.09 (b) 18.08 (c) 20.23 (d) 20.45

14. If the SD of x is 3, what is the variance of $(5-2x)$?
(a) 36 (b) 6 (c) 1 (d) 9

15. If x and y are related by $2x+3y+4=0$ and SD of x is 6, then SD of y is
(a) 22 (b) 4 (c) $\sqrt{5}$ (d) 9.

16. The quartiles of a variable are 45, 52 and 65 respectively. Its quartile deviation is
(a) 10 (b) 20 (c) 25 (d) 8.30.

17. If x and y are related as $3x+4y = 20$ and the quartile deviation of x is 12, then the quartile deviation of y is
(a) 16 (b) 14 (c) 10 (d) 9.

18. If the SD of the 1st n natural numbers is 2, then the value of n must be
(a) 2 (b) 7 (c) 6 (d) 5.

19. If x and y are related by $y = 2x + 5$ and the SD and AM of x are known to be 5 and 10 respectively, then the coefficient of variation of y is
(a) 25 (b) 30 (c) 40 (d) 20.

Set C

Write down the correct answer. Each question carries 5 marks.

1. What is the mean deviation about mean for the following distribution?

| | | | | | | |
|------------|---|----------|----|----------|----|----------|
| Variable: | 5 | 10 | 15 | 20 | 25 | 30 |
| Frequency: | 3 | 4 | 6 | 5 | 3 | 2 |
| (a) 6.00 | | (b) 5.93 | | (c) 6.07 | | (d) 7.20 |

2. What is the mean deviation about median for the following data?

| | | | | | | |
|----------|----------|----------|----------|----|----|----|
| X: 3 | 5 | 7 | 9 | 11 | 13 | 15 |
| F: 2 | 8 | 9 | 16 | 14 | 7 | 4 |
| (a) 2.50 | (b) 2.46 | (c) 2.43 | (d) 2.37 | | | |

3. What is the coefficient of mean deviation for the following distribution of heights? Take deviation from AM.

| | | | | | |
|-------------------|-----------------|-----------------|-----------------|-------|-------|
| Height in inches: | 60-62 | 63-65 | 66-68 | 69-71 | 72-74 |
| No. of students: | 5 | 22 | 28 | 17 | 3 |
| (a) 2.31 inches | (b) 3.45 inches | (c) 3.82 inches | (d) 2.48 inches | | |

4. The mean deviation of weights about median for the following data:

| | | | | | | |
|------------------|----------|----------|------------|---------|---------|---------|
| Weight (lb) : | 131-140 | 141-150 | 151-160 | 161-170 | 171-180 | 181-190 |
| No. of persons : | 3 | 8 | 13 | 15 | 6 | 5 |
| Is given by | | | | | | |
| (a) 10.97 | (b) 8.23 | (c) 9.63 | (d) 11.45. | | | |

5. What is the standard deviation from the following data relating to the age distribution of 200 persons?

| | | | | | | |
|-----------------|-----------|-----------|-----------|----|----|----|
| Age (year) : 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| No. of people: | 13 | 28 | 31 | 46 | 39 | 23 |
| (a) 15.29 | (b) 16.87 | (c) 18.00 | (d) 17.52 | | | |

6. What is the coefficient of variation for the following distribution of wages?

| | | | | | | |
|------------------|-----------|-----------|-----------|---------|---------|---------|
| Daily Wages (₹): | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 | 70 – 80 | 80 – 90 |
| No. of workers | 17 | 28 | 21 | 15 | 13 | 6 |
| (a) ₹ 14.73 | (b) 14.73 | (c) 26.93 | (d) 20.82 | | | |

7. Which of the following companies A and B is more consistent so far as the payment of dividend is concerned?

UNIT-II: ANSWERS

Set A

- | | | | | | |
|---------|---------|---------|---------|---------|---------|
| 1. (d) | 2. (a) | 3. (b) | 4. (d) | 5. (d) | 6. (a) |
| 7. (d) | 8. (b) | 9. (b) | 10. (d) | 11. (c) | 12. (b) |
| 13. (a) | 14. (c) | 15. (b) | 16. (d) | 17. (d) | 18. (c) |
| 19. (b) | 20. (d) | 21. (a) | | | |

Set B

- | | | | | | | | | | | | |
|------------|-----|------------|-----|------------|-----|------------|-----|------------|-----|------------|-----|
| 1. | (d) | 2. | (c) | 3. | (c) | 4. | (b) | 5. | (c) | 6. | (a) |
| 7. | (c) | 8. | (a) | 9. | (b) | 10. | (a) | 11. | (b) | 12. | (b) |
| 13. | (a) | 14. | (a) | 15. | (b) | 16. | (a) | 17. | (d) | 18. | (b) |
| 19. | (c) | 20. | (c) | | | | | | | | |

Set C

- 1.** (c) **2.** (d) **3.** (a) **4.** (a) **5.** (b) **6.** (c)
7. (a) **8.** (c) **9.** (b) **10.** (b) **11.** (a)

ADDITIONAL QUESTION BANK

1. The number of measures of central tendency is

| | | | |
|---------|-----------|----------|----------|
| (a) two | (b) three | (c) four | (d) five |
|---------|-----------|----------|----------|
2. The words "mean" or "average" only refer to

| | | | |
|---------|---------|---------|----------|
| (a) A.M | (b) G.M | (c) H.M | (d) none |
|---------|---------|---------|----------|
3. _____ is the most stable of all the measures of central tendency.

| | | | |
|---------|---------|---------|-----------|
| (a) G.M | (b) H.M | (c) A.M | (d) none. |
|---------|---------|---------|-----------|
4. Mean is of _____ types.

| | | | |
|-------|-------|-------|-------|
| (a) 3 | (b) 4 | (c) 8 | (d) 5 |
|-------|-------|-------|-------|
5. Weighted A.M is related to

| | | | |
|---------|---------------|---------|-----------|
| (a) G.M | (b) frequency | (c) H.M | (d) none. |
|---------|---------------|---------|-----------|
6. Frequencies are also called as weights.

| | | | |
|----------|-----------|----------|----------|
| (a) True | (b) false | (c) both | (d) none |
|----------|-----------|----------|----------|
7. The algebraic sum of deviations of observations from their A.M is

| | | | |
|-------|--------|-------|-------|
| (a) 2 | (b) -1 | (c) 1 | (d) 0 |
|-------|--------|-------|-------|
8. G.M of a set of n observations is the _____ root of their product.

| | | | |
|--------------|----------------|---------|------------------|
| (a) $n/2$ th | (b) $(n+1)$ th | (c) nth | (d) $(n - 1)$ th |
|--------------|----------------|---------|------------------|
9. The algebraic sum of deviations of 8, 1, 6 from the A.M viz.5 is

| | | | |
|--------|-------|-------|----------|
| (a) -1 | (b) 0 | (c) 1 | (d) none |
|--------|-------|-------|----------|
10. G.M of 8, 4, 2 is

| | | | |
|-------|-------|-------|----------|
| (a) 4 | (b) 2 | (c) 8 | (d) none |
|-------|-------|-------|----------|
11. _____ is the reciprocal of the A.M of reciprocal of observations.

| | | | |
|---------|---------|----------|----------|
| (a) H.M | (b) G.M | (c) both | (d) none |
|---------|---------|----------|----------|
12. A.M is never less than G.M

| | | | |
|----------|-----------|----------|----------|
| (a) True | (b) false | (c) both | (d) none |
|----------|-----------|----------|----------|
13. G.M is less than H.M

| | | | |
|----------|-----------|----------|----------|
| (a) true | (b) false | (c) both | (d) none |
|----------|-----------|----------|----------|
14. The value of the middlemost item when they are arranged in order of magnitude is called

| | | | |
|------------------------|----------|----------|------------|
| (a) standard deviation | (b) mean | (c) mode | (d) median |
|------------------------|----------|----------|------------|
15. Median is unaffected by extreme values.

| | | | |
|----------|-----------|----------|----------|
| (a) true | (b) false | (c) both | (d) none |
|----------|-----------|----------|----------|

43. _____ can be calculated from a frequency distribution with open end intervals
 (a) Median (b) Mean (c) Mode (d) none
44. The values of all items are taken into consideration in the calculation of
 (a) median (b) mean (c) mode (d) none
45. The values of extreme items do not influence the average in case of
 (a) median (b) mean (c) mode (d) none
46. In a distribution with a single peak and moderate skewness to the right, it is closer to the concentration of the distribution in case of
 (a) mean (b) median (c) both (d) none
47. If the variables x & z are so related that $z = ax + b$ for each $x = x_i$ where a & b are constants, then $\bar{z} = a\bar{x} + b$
 (a) true (b) false (c) both (d) none
48. G.M is defined only when
 (a) all observations have the same sign and none is zero
 (b) all observations have the different sign and none is zero
 (c) all observations have the same sign and one is zero
 (d) all observations have the different sign and one is zero
49. _____ is useful in averaging ratios, rates and percentages.
 (a) A.M (b) G.M (c) H.M (d) Both (b) and (c)
50. G.M is useful in construction of index number.
 (a) true (b) false (c) both (d) none
51. More laborious numerical calculations involves in G.M than A.M
 (a) True (b) false (c) both (d) none
52. H.M is defined when no observation is
 (a) 3 (b) 2 (c) 1 (d) 0
53. When all values occur with equal frequency, there is no
 (a) mode (b) mean (c) median (d) none
54. _____ cannot be treated algebraically
 (a) mode (b) mean (c) median (d) Both (a) and (c)
55. For the calculation of _____, the data must be arranged in the form of a frequency distribution.
 (a) median (b) mode (c) mean (d) none

66. _____ is equal to the value corresponding to cumulative frequency
(a) mode (b) mean (c) median (d) none

67. _____ is the value of the variable corresponding to the highest frequency
(a) mode (b) mean (c) median (d) none

68. The class in which mode belongs is known as
(a) median class (b) mean class (c) modal class (d) none

69. The formula of mode is applicable if classes are of _____ width.
(a) equal (b) unequal (c) both (d) none

70. For calculation of _____ we have to construct cumulative frequency distribution
(a) mode (b) median (c) mean (d) none

71. For calculation of _____ we have to construct a grouped frequency distribution
(a) median (b) mode (c) mean (d) none

72. Relation between mean, median & mode is
(a) mean - mode = 2 (mean - median) (b) mean - median = 3 (mean - mode)
(c) mean - median = 2 (mean - mode) (d) mean - mode = 3 (mean - median)

73. When the distribution is symmetrical, mean, median and mode
(a) coincide (b) do not coincide (c) both (d) none

74. Mean, median & mode are equal for the
(a) Binomial distribution (b) Normal distribution
(c) both (d) none

75. In most frequency distributions, it has been observed that the three measures of central tendency viz. mean, median & mode, obey the approximate relation, provided the distribution is
(a) very skew (b) not very skew (c) both (d) none

76. _____ divides the total number of observations into two equal parts.
(a) mode (b) mean (c) median (d) none

77. Measures which are used to divide or partition the observations into a fixed number of parts are collectively known as
(a) partition values (b) quartiles (c) both (d) none

78. The middle most value of a set of observations is
(a) median (b) mode (c) mean (d) none

79. The number of observations smaller than _____ is the same as the number larger than it.
(a) median (b) mode (c) mean (d) none

* Question no. 64 is based on theoretical distribution.

* Question no. 78 is based on skewness, which is not in syllabus.

114. _____ is equal to the value corresponding to cumulative frequency $k(N + 1)/100$ from simple frequency distribution
 (a) k^{th} decile (b) k^{th} percentile (c) both (d) none
115. For grouped frequency distribution _____ is equal to the value corresponding to cumulative frequency $N/2$
 (a) median (b) 1st quartile (c) 3rd quartile (d) none
116. For grouped frequency distribution _____ is equal to the value corresponding to cumulative frequency $N/4$
 (a) median (b) 1st quartile (c) 3rd quartile (d) none
117. For grouped frequency distribution _____ is equal to the value corresponding to cumulative frequency $3N/4$
 (a) median (b) 1st quartile (c) 3rd quartile (d) none
118. For grouped frequency distribution _____ is equal to the value corresponding to cumulative frequency $kN/10$
 (a) median (b) k^{th} percentile (c) k^{th} decile (d) none
119. For grouped frequency distribution _____ is equal to the value corresponding to cumulative frequency $kN/100$
 (a) k^{th} quartile (b) k^{th} percentile (c) k^{th} decile (d) none
120. In Ogive, abscissa corresponding to ordinate $N/2$ is
 (a) median (b) 1st quartile (c) 3rd quartile (d) none
121. In Ogive, abscissa corresponding to ordinate $N/4$ is
 (a) median (b) 1st quartile (c) 3rd quartile (d) none
122. In Ogive, abscissa corresponding to ordinate $3N/4$ is
 (a) median (b) 3rd quartile (c) 1st quartile (d) none
123. In Ogive, abscissa corresponding to ordinate _____ is k^{th} decile.
 (a) $kN/10$ (b) $kN/100$ (c) $kN/50$ (d) none
124. In Ogive, abscissa corresponding to ordinate _____ is k^{th} percentile.
 (a) $kN/10$ (b) $kN/100$ (c) $kN/50$ (d) none
125. For 899, 999, 391, 384, 590, 480, 485, 760, 111, 240
 Rank of median is
 (a) 2.75 (b) 5.5 (c) 8.25 (d) none
126. For 333, 999, 888, 777, 666, 555, 444
 Rank of 1st quartile is
 (a) 3 (b) 1 (c) 2 (d) 7

153. In the problem

| | | | | | |
|-----------------|-------|-------|-------|-------|-------|
| No. of shirts: | 30–32 | 33–35 | 36–38 | 39–41 | 42–44 |
| No. of persons: | 15 | 14 | 42 | 27 | 18 |

The assumed mean is

154. In the problem

| | | | | |
|----------------|-----|-----|------|-------|
| Size of items: | 1-3 | 3-8 | 8-15 | 15-26 |
| Frequency: | 5 | 10 | 16 | 15 |

The assumed mean is

155. The average of a series of overlapping averages, each of which is based on a certain number of item within a series is known as

156. _____ averages is used for smoothening a time series.

157. Pooled Mean is also called

- (a) Mean (b) Geometric Mean (c) Grouped Mean (d) none

158. Half of the numbers in an ordered set have values less than the _____ and half will have values greater than the _____.

- (a) mean, median (b) median, median (c) mode, mean (d) none.

159. The median of 27, 30, 26, 44, 42, 51, 37 is

160. For an even number of values the median is the

- (a) average of two middle values (b) middle value
(c) both (d) none

161. In the case of a continuous frequency distribution, the size of the _____ item indicates class interval in which the median lies.

- (a) $(n-1)/2^{\text{th}}$ (b) $(n+1)/2^{\text{th}}$ (c) $n/2^{\text{th}}$ (d) none

162. The deviations from median are _____ if negative signs are ignored as compared to other measures of central tendency.

- (a) minimum (b) maximum (c) same (d) none

* Question no. 155 and 156 is based on moving averages, which is not in foundation syllabus.

163. Ninth Decile lies in the class interval of the item
(a) $n/9$ (b) $9n/10$ (c) $9n/20$ (d) none item.

164. Ninety Ninth Percentile lies in the class interval of the item
(a) $99n/100$ (b) $99n/10$ (c) $99n/200$ (d) none item.

165. _____ is the value of the variable at which the concentration of observation is the densest.
(a) mean (b) median (c) mode (d) none

166. Height in cms: 60–62 63–65 66–68 69–71 72–74
No. of students: 15 118 142 127 18
Modal group is
(a) 66–68 (b) 69–71 (c) 63–65 (d) none

167. A distribution is said to be symmetrical when the frequency rises & falls from the highest value in the _____ proportion.
(a) unequal (b) equal (c) both (d) none

168. _____ always lies in between the arithmetic mean & mode.
(a) G.M (b) H.M (c) Median (d) none

169. Logarithm of G.M is the _____ of logarithms of the different values.
(a) weighted mean (b) simple mean (c) both (d) none

170. _____ is not much affected by fluctuations of sampling.
(a) A.M (b) G.M (c) H.M (d) none

171. The data 1, 2, 4, 8, 16 are in
(a) Arithmetic progression (b) Geometric progression
(c) Harmonic progression (d) none

172. _____ & _____ can not be calculated if any observation is zero.
(a) G.M & A.M (b) H.M & A.M (c) H.M & G. M (d) None.

173. _____ & _____ are called ratio averages.
(a) H.M & G.M (b) H. M & A.M (c) A.M & G.M (d) none

174. _____ is a good substitute to a weighted average.
(a) A.M (b) G.M (c) H.M (d) none

175. For ordering shoes of various sizes for resale, a _____ size will be more appropriate.
(a) median (b) modal (c) mean (d) none

176. _____ is called a positional measure.
(a) mean (b) mode (c) median (d) none

* Question no. 174 is not in foundation syllabus.

192. The average discovers

- (a) uniformity in variability
- (c) both
- (b) variability in uniformity of distribution
- (d) none

193. The average has relevance for

- (a) homogeneous population
- (c) both
- (b) heterogeneous population
- (d) none

194. The correction factor is applied in

- (a) inclusive type of distribution
- (c) both
- (b) exclusive type of distribution
- (d) none

195. "Mean has the least sampling variability" prove the mathematical property of mean

- (a) True
- (b) false
- (c) both
- (d) none

196. "The sum of deviations from the mean is zero" — is the mathematical property of mean

- (a) True
- (b) false
- (c) both
- (d) none

197. "The mean of the two samples can be combined" — is the mathematical property of mean

- (a) True
- (b) false
- (c) both
- (d) none

198. "Choices of assumed mean does not affect the actual mean"— prove the mathematical property of mean

- (a) True
- (b) false
- (c) both
- (d) none

199. "In a moderately asymmetric distribution mean can be found out from the given values of median & mode"— is the mathematical property of mean

- (a) True
- (b) false
- (c) both
- (d) none

200. The mean wages of two companies are equal. It signifies that the workers of both the companies are equally well-off.

- (a) True
- (b) false
- (c) both
- (d) none

201. The mean wage in factory A is ₹ 6,000 whereas in factory B it is ₹ 5,500. It signifies that factory A pays more to all its workers than factory B.

- (a) True
- (b) false
- (c) both
- (d) none

202. Mean of 0, 3, 5, 6, 7, 9, 12, 0, 2 is

- (a) 4.9
- (b) 5.7
- (c) 5.6
- (d) none

203. Median of 15, 12, 6, 13, 12, 15, 8, 9 is

- (a) 13
- (b) 8
- (c) 12
- (d) 9

204. Median of 0, 3, 5, 6, 7, 9, 12, 0, 2 is

- (a) 7
- (b) 6
- (c) 3
- (d) 5

247. In _____ range has the greatest use.

- (a) Time series (b) quality control (c) both (d) none

248. Mean is an absolute measure & standard deviation is based upon it. Therefore standard deviation is a relative measure.

- (a) true (b) false (c) both (d) none

249. Semi-quartile range is one-fourth of the range in a normal symmetrical distribution.

- (a) Yes (b) No (c) both (d) none

250. Whole frequency table is needed for the calculation of

- (a) range (b) variance (c) both (d) none

251. Relative measures of dispersion make deviations in similar units comparable.

- (a) true (b) false (c) both (d) none

252. Quartile deviation is based on the

- | | |
|-----------------|-----------------------------|
| (a) highest 50% | (b) lowest 25% |
| (c) highest 25% | (d) middle 50% of the item. |

253. S.D is less than Mean deviation

- (a) true (b) false (c) both (d) none

254. Coefficient of variation is independent of the unit of measurement.

- (a) true (b) false (c) both (d) none

255. Coefficient of variation is a relative measure of

- (a) mean (b) deviation (c) range (d) dispersion.

256. Coefficient of variation is equal to

- | | |
|---------------------------------------|-------------------------------------|
| (a) Standard deviation x 100 / median | (b) Standard deviation x 100 / mode |
| (c) Standard deviation x 100 / mean | (d) none |

257. Coefficient of Quartile Deviation is equal to

- | | |
|---------------------------------------|-------------------------------------|
| (a) Quartile deviation x 100 / median | (b) Quartile deviation x 100 / mean |
| (c) Quartile deviation x 100 / mode | (d) none |

258. If each item is reduced by 15 A.M is

- (a) reduced by 15 (b) increased by 15 (c) reduced by 10 (d) none

259. If each item is reduced by 10, the range is

- (a) increased by 10 (b) decreased by 10 (c) unchanged (d) none

260. If each item is reduced by 20, the standard deviation

- (a) increased (b) decreased (c) unchanged (d) none

275. The class having maximum frequency is called
 (a) modal class (b) median class (c) mean class (d) none
276. For determination of mode, the class intervals should be
 (a) overlapping (b) maximum (c) minimum (d) none
277. First Quartile lies in the class interval of the
 (a) $n/2^{\text{th}}$ item (b) $n/4^{\text{th}}$ item (c) $3n/4^{\text{th}}$ item (d) $n/10^{\text{th}}$ item
278. The value of a variate that occur most often is called
 (a) median (b) mean (c) mode (d) none
279. For the values of a variable 3, 1, 5, 2, 6, 8, 4 the median is
 (a) 3 (b) 5 (c) 4 (d) none
280. If $y = 5x - 20$ & $\bar{x} = 30$ then the value of \bar{y} is
 (a) 130 (b) 140 (c) 30 (d) none
281. If $y = 3x - 100$ and $\bar{x} = 50$ then the value of \bar{y} is
 (a) 60 (b) 30 (c) 100 (d) 50
282. The median of the numbers 11, 10, 12, 13, 9 is
 (a) 12.5 (b) 12 (c) 10.5 (d) 11
283. The mode of the numbers 7, 7, 7, 9, 10, 11, 11, 11, 12 is
 (a) 11 (b) 12 (c) 7 (d) 7 & 11
284. In a symmetrical distribution when the 3^{rd} quartile plus 1^{st} quartile is halved, the value would give
 (a) mean (b) mode (c) median (d) none
285. In Zoology _____ is used.
 (a) median (b) mean (c) mode (d) none
286. For calculation of Speed & Velocity
 (a) G.M (b) A.M (c) H.M (d) none is used.
287. The S.D is always taken from
 (a) median (b) mode (c) mean (d) none
288. Coefficient of Standard deviation is equal to
 (a) S.D/A.M (b) A.M/S.D (c) S.D/GM (d) none
289. The distribution, for which the coefficient of variation is less, is _____ consistent.
 (a) less (b) more (c) moderate (d) none

ANSWERS

- | | | | | |
|----------|----------|----------|----------|----------|
| 1. (b) | 2. (a) | 3. (c) | 4. (a) | 5. (b) |
| 6. (a) | 7. (d) | 8. (c) | 9. (b) | 10. (a) |
| 11. (a) | 12. (a) | 13. (b) | 14. (d) | 15. (a) |
| 16. (d) | 17. (b) | 18. (a) | 19. (b) | 20. (a) |
| 21. (b) | 22. (d) | 23. (a) | 24. (c) | 25. (b) |
| 26. (a) | 27. (a) | 28. (b) | 29. (b) | 30. (a) |
| 31. (c) | 32. (a) | 33. (b) | 34. (a) | 35. (a) |
| 36. (a) | 37. (a) | 38. (a) | 39. (a) | 40. (a) |
| 41. (d) | 42. (a) | 43. (a) | 44. (b) | 45. (a) |
| 46. (b) | 47. (a) | 48. (a) | 49. (d) | 50. (a) |
| 51. (a) | 52. (d) | 53. (a) | 54. (d) | 55. (b) |
| 56. (c) | 57. (a) | 58. (c) | 59. (a) | 60. (b) |
| 61. (b) | 62. (d) | 63. (a) | 64. (b) | 65. (b) |
| 66. (c) | 67. (a) | 68. (a) | 69. (a) | 70. (c) |
| 71. (c) | 72. (c) | 73. (a) | 74. (a) | 75. (d) |
| 76. (a) | 77. (b) | 78. (b) | 79. (a) | 80. (c) |
| 81. (b) | 82. (a) | 83. (b) | 84. (b) | 85. (a) |
| 86. (c) | 87. (c) | 88. (a) | 89. (b) | 90. (a) |
| 91. (b) | 92. (d) | 93. (a) | 94. (d) | 95. (c) |
| 96. (c) | 97. (a) | 98. (a) | 99. (c) | 100. (b) |
| 101. (a) | 102. (b) | 103. (c) | 104. (a) | 105. (a) |
| 106. (b) | 107. (a) | 108. (d) | 109. (c) | 110. (a) |
| 111. (b) | 112. (c) | 113. (b) | 114. (b) | 115. (a) |
| 116. (b) | 117. (c) | 118. (c) | 119. (b) | 120. (a) |
| 121. (b) | 122. (b) | 123. (a) | 124. (b) | 125. (b) |
| 126. (c) | 127. (d) | 128. (d) | 129. (a) | 130. (c) |
| 131. (b) | 132. (a) | 133. (a) | 134. (a) | 135. (b) |
| 136. (c) | 137. (a) | 138. (d) | 139. (a) | 140. (b) |
| 141. (a) | 142. (d) | 143. (c) | 144. (c) | 145. (a) |
| 146. (b) | 147. (a) | 148. (b) | 149. (a) | 150. (d) |
| 151. (c) | 152. (b) | 153. (b) | 154. (c) | 155. (a) |

- | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 156. (a) | 157. (c) | 158. (b) | 159. (d) | 160. (a) |
| 161. (c) | 162. (a) | 163. (b) | 164. (a) | 165. (c) |
| 166. (a) | 167. (b) | 168. (c) | 169. (b) | 170. (b) |
| 171. (b) | 172. (c) | 173. (a) | 174. (c) | 175. (b) |
| 176. (c) | 177. (b) | 178. (c) | 179. (c) | 180. (a) |
| 181. (a) | 182. (c) | 183. (a) | 184. (c) | 185. (a) |
| 186. (a) | 187. (b) | 188. (a) | 189. (c) | 190. (c) |
| 191. (b) | 192. (a) | 193. (b) | 194. (a) | 195. (b) |
| 196. (a) | 197. (a) | 198. (a) | 199. (b) | 200. (b) |
| 201. (b) | 202. (a) | 203. (c) | 204. (d) | 205. (b) |
| 206. (a) | 207. (b) | 208. (b) | 209. (a) | 210. (a) |
| 211. (b) | 212. (a) | 213. (b) | 214. (a) | 215. (b) |
| 216. (a) | 217. (b) | 218. (c) | 219. (a) | 220. (a) |
| 221. (a) | 222. (a) | 223. (b) | 224. (c) | 225. (a) |
| 226. (d) | 227. (a) | 228. (b) | 229. (a) | 230. (d) |
| 231. (c) | 232. (a) | 233. (b) | 234. (a) | 235. (d) |
| 236. (b) | 237. (b) | 238. (c) | 239. (a) | 240. (a) |
| 241. (b) | 242. (b) | 243. (a) | 244. (d) | 245. (a) |
| 246. (c) | 247. (b) | 248. (b) | 249. (a) | 250. (c) |
| 251. (a) | 252. (d) | 253. (b) | 254. (a) | 255. (d) |
| 256. (c) | 257. (a) | 258. (a) | 259. (c) | 260. (c) |
| 261. (c) | 262. (a) | 263. (c) | 264. (b) | 265. (a) |
| 266. (b) | 267. (a) | 268. (c) | 269. (b) | 270. (b) |
| 271. (a) | 272. (b) | 273. (c) | 274. (c) | 275. (a) |
| 276. (a) | 277. (b) | 278. (c) | 279. (c) | 280. (a) |
| 281. (d) | 282. (d) | 283. (d) | 284. (c) | 285. (c) |
| 286. (c) | 287. (c) | 288. (a) | 289. (b) | |

