

ADVANCED CAPITAL BUDGETING DECISIONS



LEARNING OUTCOMES

After going through the chapter student shall be able to understand

- Current trends in Capital Budgeting
- Dealing with Risk in Investment Decisions
- Internal and External Factors affecting capital budgeting decision
- Methods of incorporating risk in Capital Budgeting
- Adjusted Present Value
- Optimum Replacement Cycle



1. CURRENT TRENDS IN CAPITAL BUDGETING

While discussing the capital budgeting or investment evaluation techniques at Intermediate Level, we have assumed that the investment proposals do not involve any risk and cash flows of the project are known with certainty. This assumption was taken to simplify the understanding of the capital budgeting techniques. However, in practice, this assumption is not correct. In-fact, investment projects are exposed to various types of factors some of which are as follows:

- (i) Inflation
- (ii) Change in technology
- (iii) Change in Government Policies

Now let us discuss the impact of each factor in a detailed manner.

1.1 Impact of Inflation on Capital Budgeting Decisions

Adjustment for inflation is a necessity for capital investment appraisal. This is because inflation will raise the revenues & costs of the project. The net revenues after adjustment for inflation shall be equal to net revenues in current terms. The considerations, which cause distortion, are:

- (1) Depreciation charges are based on historical costs. Tax benefits accruing from depreciation charges do not keep parity with inflation.

As annual after-tax cash inflow of a project is equal to

$$(R - C - D)(1 - T) + D = (R - C)(1 - T) + DT$$

Where,

R → Revenue from project

C → Costs (apart from depreciation) relating to the project

D → Depreciation charges

T → Tax Rate

Here $(R - C)(1 - T)$ tends to move in line with inflation as inflation influences revenues & costs similarly. DT does not depend on inflation as depreciation charges are based on historical costs. The effect of inflation is to reduce the actual rate of return.

Example:

Initial outlay of a project	– ₹ 80,000
Expected life	– 4 years
Salvage value	– Nil
Annual revenues	– ₹ 60,000
Annual costs other than depreciation	– ₹ 20,000
Tax Rate	– 50%

Depreciation on straight-line basis presuming as if there is no inflation.

Year	1	2	3	4
Revenues	₹ 60,000	₹ 60,000	₹ 60,000	₹ 60,000
Costs other than depreciation	₹ 20,000	₹ 20,000	₹ 20,000	₹ 20,000
Depreciation	₹ 20,000	₹ 20,000	₹ 20,000	₹ 20,000
Taxable profit	₹ 20,000	₹ 20,000	₹ 20,000	₹ 20,000

Tax	₹ 10,000	₹ 10,000	₹ 10,000	₹ 10,000
Profit after tax	₹ 10,000	₹ 10,000	₹ 10,000	₹ 10,000
Net cash inflow	₹ 30,000	₹ 30,000	₹ 30,000	₹ 30,000

If there is inflation @ 10% applicable to revenues & cost of project.

Year	1	2	3	4
Revenues	₹ 66,000	₹ 72,600	₹ 79,860	₹ 87,846
Costs other than depreciation	₹ 22,000	₹ 24,200	₹ 26,620	₹ 29,282
Depreciation	₹ 20,000	₹ 20,000	₹ 20,000	₹ 20,000
Taxable profit	₹ 24,000	₹ 28,400	₹ 33,240	₹ 38,564
Tax	₹ 12,000	₹ 14,200	₹ 16,620	₹ 19,282
Profit after tax	₹ 12,000	₹ 14,200	₹ 16,620	₹ 19,282
Net cash inflow	₹ 32,000	₹ 34,200	₹ 36,620	₹ 39,282

The actual net cash flow stream after deflating for inflation rate of 10% .

Real Net Cash Flow	₹ 29,091	₹ 28,264	₹ 27,513	₹ 26,830
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So actual net cash flows are less than net cash flow if there is no inflation.

(2) Costs of capital considered for investment appraisals contain a premium for anticipated inflation.

Due to inflation investors require the nominal rate of return to be equal to:

Required Rate of Return in real terms plus Rate of Inflation.

Formula

$$R_N = R_R + P$$

$R_N \rightarrow$ Required rate of return in nominal terms.

$R_R \rightarrow$ Required rate of return in real terms.

$P \rightarrow$ Anticipated inflation rate.

If cost of capital (required rate of return) contains a premium for anticipated inflation, the inflation factor has to be reflected in the projected cash flows.

If there is no inflation, then it has to be discounted at required rate of return in real terms.

Illustration 1

Determine NPV of the project with the following information:

Initial Outlay of project	₹ 40,000
Annual Cash Flow from the Project (Without inflation)	₹ 15,000
Useful life	4 years
Cost of Capital (Including inflation premium of 10%)	12%

Solution

Annual Cash Flow of project is ₹ 15,000.

It would be inconsistent to discount these real cash flows at 12% (nominal rate of return).

There are two alternatives:

- (i) Either to restate the cash flow in nominal term and discount at 12% or
- (ii) Restate the discount rate in real terms and use this to discount the real cash flows.

NPV using (i) approach

Since inflation rate is 10% a year, real cash flows may be stated in nominal cash flows as follows:

$$\text{Nominal Cash Flow} = (1 + \text{Inflation Rate}) \text{ Real Cash Flows}$$

Year	Real Cash Flows	Nominal Cash flows
1	15000	$15,000 \times 1.10 = 16,500$
2	15,000	$15,000 \times (1.10)^2 = 18,150$
3	15,000	$15,000 \times (1.10)^3 = 19,965$
4	15,000	$15,000 \times (1.10)^4 = 21,962$

NPV using nominal discounting rate 12%

$$\frac{16,500}{(1.12)} + \frac{18,150}{(1.12)^2} + \frac{19,965}{(1.12)^3} + \frac{21,962}{(1.12)^4} - 40,000$$

$$= ₹ 14,732 + ₹ 14,469 + ₹ 14,211 + ₹ 13,957 - ₹ 40,000$$

$$= ₹ 17,369 \text{ (Approx)}$$

NPV using (ii) approach

To compute NPV using (ii) approach, we shall need real discount rate, which shall be computed as follows:

$$\text{Real Discount Rate} = \frac{1 + \text{Nominal Discount Rate}}{1 + \text{Inflation Rate}} - 1$$

$$\text{Real Discount Rate} = \frac{1 + 0.12}{1 + 0.10} - 1 = 0.0182 \text{ i.e. } 1.8\%$$

$$NPV = \sum_{t=1}^n cf_t - I_0$$

Where t = Time Period

cf_t = Annual Cash Flow

I_0 = Initial Outlay

Accordingly NPV of the project

$$\begin{aligned} & \frac{15,000}{(1.0182)} + \frac{15,000}{(1.0182)^2} + \frac{15,000}{(1.0182)^3} + \frac{15,000}{(1.0182)^4} - 40,000 \\ & = ₹ 14,732 + ₹ 14,469 + ₹ 14,210 + ₹ 13,956 - ₹ 40,000 \\ & = ₹ 57,367 - ₹ 40,000 = ₹ 17,367(\text{Approx}) \end{aligned}$$

NPV based on consideration that inflation rate for revenue and cost are different shall be computed as follows:

$$N.P.V. = \sum_{t=1}^n [(R_t(1+i_r) - C_t(1+i_c)) (1-T) + D_t T] / (1+k)^t - I_0$$

R_t → revenues for the year 't' with no inflation.

i_r → annual inflation rate in revenues for 'r th' year.

C_t → costs for year 't' with no inflation.

i_c → annual inflation rate of costs for year 'r'.

T → tax rate.

D_t → depreciation charge for year 't'.

I_0 → initial outlay.

k → cost of capital (with inflation premium).

Illustration 2

XYZ Ltd. requires ₹ 8,00,000 for a project. Useful life of project - 4 years. Salvage value - Nil. Depreciation Charge ₹ 2,00,000 p.a. Expected revenues & costs (excluding depreciation) ignoring inflation.

Year	1	2	3	4
Revenues	₹ 6,00,000	₹ 7,00,000	₹ 8,00,000	₹ 8,00,000
Costs	₹ 3,00,000	₹ 4,00,000	₹ 4,00,000	₹ 4,00,000

Applicable Tax Rate is 60% and cost of capital is 10% (including inflation premium).

Calculate NPV of the project if inflation rates for revenues & costs are as follows:

Year	Revenues	Costs
1	10%	12%
2	9%	10%
3	8%	9%
4	7%	8%

Solution

Computation of Annual Cash Flow

(i) Inflation adjusted Revenues

Year	Revenues (₹)	Revenues (Inflation Adjusted) (₹)
1	6,00,000	6,00,000(1.10) = 6,60,000
2	7,00,000	7,00,000(1.10)(1.09) = 8,39,300
3	8,00,000	8,00,000(1.10)(1.09)(1.08) = 10,35,936
4	8,00,000	8,00,000(1.10)(1.09)(1.08)(1.07) = 11,08,452

(ii) Inflation adjusted Costs

Year	Revenues (₹)	Revenues (Inflation Adjusted) (₹)
1	3,00,000	3,00,000(1.12) = 3,36,000
2	4,00,000	4,00,000(1.12)(1.10) = 4,92,800
3	4,00,000	4,00,000(1.12)(1.10)(1.09) = 5,37,152
4	4,00,000	4,00,000(1.12)(1.10)(1.09)(1.08) = 5,80,124

(iii) Tax Benefit on Depreciation = ₹ 2,00,000 x 0.60 = ₹ 1,20,000

(iv) Net Profit after Tax

Year	Revenues (Inflation Adjusted) (₹)(1)	Costs (Inflation Adjusted) (₹)(2)	Net Profit (₹) (3) = (1) - (2)	Tax (₹) (4) = 60% of (3)	Net after Profit (₹) (3) - (4)
1	6,60,000	3,36,000	3,24,000	1,94,400	1,29,600
2	8,39,300	4,92,800	3,46,500	2,07,900	1,38,600

3	10,35,936	5,37,152	4,98,784	2,99,270	1,99,514
4	11,08,452	5,80,124	5,28,328	3,16,997	2,11,331

(iv) Present Value of Cash Inflows

Year	Net after Profit ₹)	Tax Benefit on Depreciation ₹)	Cash Inflow ₹)	PVF@ 10%	PV ₹)
1	1,29,600	1,20,000	2,49,600	0.909	2,26,886
2	1,38,600	1,20,000	2,58,600	0.826	2,13,604
3	1,99,514	1,20,000	3,19,514	0.751	2,39,955
4	2,11,331	1,20,000	3,31,331	0.683	2,26,299
					9,06,744

$$\text{NPV} = ₹ 9,06,744 - ₹ 8,00,000 = ₹ 1,06,744$$

1.2 Impact of change in technology on Capital Budgeting Decisions

Generally it has been observed that those making capital Budgeting decision evaluates the proposals in monetary terms i.e. quantitative values and normally fails to consider critical factors i.e. qualitative factor that can affect the future cash flows one of such factor is technology. It is important to note that here we are not simply talking about decision to replace existing machinery with new machine having improved technology rather we are talking about the impact of technology change on capital budgeting. Now the question arises why it is important to analyze the impacts of change in technology it is because of following reason:

- ❖ Change in technology can significantly alter production process.
- ❖ Changes can also yield benefits such as improved quality, delivery time greater flexibility, etc.
- ❖ Changed technology can also result in reduction in cost of capital
- ❖ Improved cash inflows can be achieved through technological changes.
- ❖ There may be need to incur additional cost in the form of additional capital expenditure.
- ❖ The sale volume can be impacted as the anticipated life cycle of the product can be shortened because of change in consumer preference.

Now next question arises how to incorporate impact in capital budgeting decision. For this purpose it is very necessary that once the project has been launched it should be reviewed on continuous basis and if required it need to be revised in light of changes in the technology.

The various ways in which the impact of change in technology can be incorporated in capital Budgeting decisions are as follows.

1. At the time of making Capital Budgeting decisions the risk of change in technology should be considered using various techniques such as Sensitivity Analysis, Scenario Analysis, Simulation Analysis etc. (discussed later in this chapter)
2. Once project has been launched analyse the impact of change in technology both positive or negative and revise estimates in monetary terms.
3. If continuation of project is proving to be unviable then look for abandonment option and evaluate the same (discussed later).
4. Suitably adjusting the discounting rate.

1.3 Impact of change in Government Policies on Capital Budgeting Decisions

Government Policies are important external factors that impacts the capital budgeting decision because directly or indirectly they affect the future cash flows of the firm that forms the basis of capital budgeting decisions. It might be possible that Government Policy may not affect us, but it may affect our supplier, buyers, customers, service providers etc.

The impact of changes in these policies can be positive as well as negative. What is more important is that the impact of such should be analysed and if required the estimation should be revised adequately. If required, the firm should consider the option to abandon the project (discussed in later chapter of study material).

The change in Government Policy can be analysed under two headings:

- i. Impact of change of Policies on Domestic Capital Budgeting Decision.
- ii. Impact of change of Policies on International Capital Budgeting Decision.

While some Government policies are changed after a longer period, say five to ten years, some change from quarter to a year. The impact of each policy may vary from each other. For example, the policies such as New Industrial Policy 1991, might had drastically impacted the Capital Budgeting decisions of various firms in the beginning period of 1990s due to opening of the doors of the Indian Economy for the Global world. However, such types of policies normally come out after a longer period. On the other hand, there are some policies of the Government that are announced/ reviewed within a period of one year. Some of these are as follows:

- ❖ **Fiscal policy:** The use of government spending, taxation and borrowing to influence both the pattern of economic activity and level of growth of aggregate demand, output and employment.

- ❖ **Monetary Policy:** Monetary policy refers to the use of monetary policy instruments which are at the disposal of the central bank to regulate the availability, cost and use of money and credit to promote economic growth, price stability, optimum levels of output and employment, balance of payments equilibrium, stable currency or any other goal of government's economic policy.

Generally, the change in monetary policy depends on the economic status of the nation. In India, the monetary policy includes decisions on open market operations, variation in reserve requirements, selective credit controls, supply of currency, bank rates (Repo Rates) and other rates.

Since in India members of Monetary Policy Committee (MPC) are required to meet at least four times in a year generally changes in the policies related to above mentioned matters takes at least two to three times in a year.

Now let us discuss how changes in Government Policies affect the Capital Budgeting decision under two broad heads:

1.3.1 Impact of changes in Government Policies on Domestic Capital Budgeting Decision.

- (a) Since the change in interest rate are decided by Government through its Monetary Policy. This can affect the Cost of Capital because the Cost of Debt is normally dependent on the bank rate of interest as they are considered as one of the important factors to compute YTM. Though this rate change may not much affect Capital Budgeting decision because they are financed from long term source of finance but they may impact working capital decisions to a great extent. The main reason behind is that the Bank Overdraft as one of the important constituents of Working Capital and it may lead to change in cash flow estimation. Hence, it is important that though small change in Bank Interest can be ignored but a major change say about 100 basis points or so can impact cash flows of the firm and may call for revision of estimations.
- (b) Another important change (Government Policy) is related to Fiscal Policy, Since Fiscal Policy forms the basis of Tax Rate and Annual Cash Flows are dependent on Rate of Depreciation of Tax Rate, any drastic change in any of these two items may call for revision of estimated cash flows.

1.3.2 Impact of changes in Government Policies on International Capital Budgeting Decision.

- (a) In International Capital Budgeting Decisions, the foreign exchange rate play a very important role. As mentioned above the change in bank rate and money supply is decided as per Monetary Policy, the change in any of these two impacts the rate of Foreign Exchange and it may call for revision of estimates.

- (b) Change in Tax Rates relating to Foreign Income or changes in provisions of Double Tax Avoiding Agreement (DTAA) as decided in Fiscal Policy may call revision of estimates.

Thus, from above discussion it can be concluded that while estimating future cash inflows change in the policies be forecasted and a proper provision should be incorporated in the expected cash flows.



2. DEALING WITH RISK IN INVESTMENT DECISIONS

While discussing the capital budgeting or investment evaluation techniques at Intermediate Level in the paper of Financial Management, we have assumed that the investment proposals do not involve any risk and cash flows of the project are known with certainty. This assumption was taken to simplify the understanding of the capital budgeting techniques. However, in practice, this assumption is not correct. In-fact, investment projects are exposed to various degrees of risk.

There can be three types of decision making:

- (i) Decision making under certainty: When cash flows are certain.
- (ii) Decision making involving risk: When cash flows involves risk and probability can be assigned.
- (iii) Decision making under uncertainty: When the cash flows are uncertain and probability cannot be assigned.

2.1 Risk and Uncertainty

Risk is the variability in terms of actual returns comparing with the estimated returns. Most common techniques of risk measurement are Standard Deviation and Coefficient of Variation. There is a thin difference between risk and uncertainty. In case of risk, probability distribution of cash flow is known. When no information is known to formulate probability distribution of cash flows, the situation is referred as uncertainty. However, these two terms are used interchangeably.

2.2 Reasons for adjustment of Risk in Capital Budgeting decisions

Main reasons for considering risk in capital budgeting decisions are as follows:

1. There is an opportunity cost involved while investing in a project for the level of risk. Adjustment of risk is necessary to help make the decision as to whether the returns out of the project are proportionate with the risks borne and whether it is worth investing in the project over the other investment options available.

2. Risk adjustment is required to know the real value of the Cash Inflows. Higher risk will lead to higher risk premium and also expectation of higher return.



3. INTERNAL AND EXTERNAL FACTORS AFFECTING CAPITAL BUDGETING DECISION

Risk arises from different factors, depending on the type of investment being considered, as well as the circumstances and the industry in which the organisation is operating. Accordingly it these factors can be divided following two broad categories:

3.1 Internal Factors

These factors are internal to the company, and they can further be divided into following categories:

3.1.1 Project-specific risk

Risks which are related to a particular project and affects the project's cash flows. It includes completion of the project in scheduled time, error of estimation in resources and allocation, estimation of cash flows etc. For example, a nuclear power project of a power generation company has different risks than hydel projects.

3.1.2 Company-specific risk

Risk which arise due to company specific factors like downgrading of credit rating, changes in key managerial persons, cases for violation of intellectual property rights (IPR) and other laws and regulations, dispute with workers etc. All these factors affect the cash flows of an entity and access to funds for capital investments. For example, two banks have different exposure to default risk.

3.2 External Factors

These factors are external to the company, and they can further be divided into following categories:

3.2.1 Industry-specific risk

These are the risks which effect the whole industry in which the company operates. These risks include regulatory restrictions on industry, changes in technologies etc. For example, regulatory restriction imposed on leather and breweries industries.

3.2.2 Market risk

The risk which arise due to market related conditions like entry of substitute, changes in demand conditions, availability and access to resources etc. For example, a thermal power project gets affected if the coal mines are unable to supply coal requirements of a thermal power company etc.

3.2.3 Competition risk

These are risks related with competition in the market in which a company operates. These risks are risk of entry of rival, product dynamism and change in taste and preference of consumers etc.

3.2.4 Risk due to Economic conditions

These are the risks which are related with macro-economic conditions like changes in monetary policies by central banks, changes in fiscal policies like introduction of new taxes and cess, inflation, changes in GDP, changes in savings and net disposable income etc.

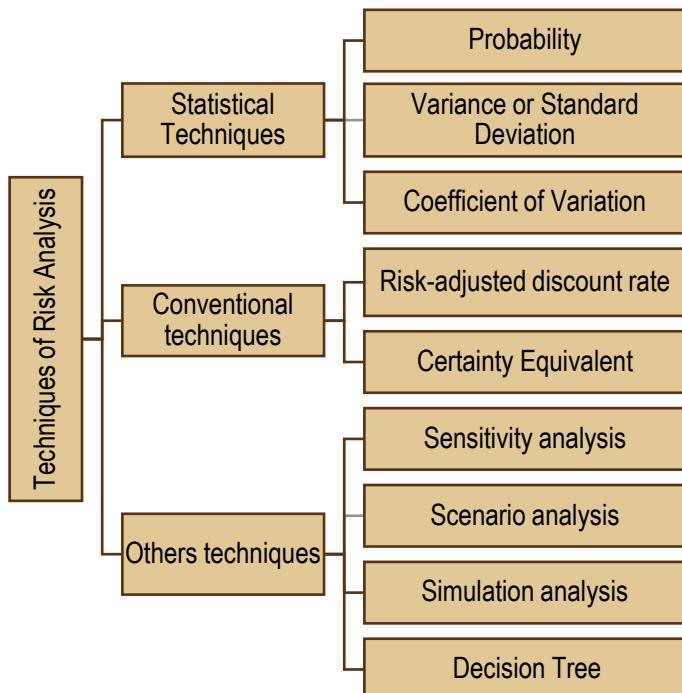
3.2.5 International risk

These are risk which are related with conditions which are caused by global economic conditions like restriction on free trade, restrictions on market access, recessions, bilateral agreements, political and geographical conditions etc. For example, restriction on outsourcing of jobs to overseas markets.



4. METHODS OF INCORPORATING RISK IN CAPITAL BUDGETING

Techniques of risk analysis in capital budgeting can be classified as below:



4.1 Statistical Techniques

4.1.1 Probability

Probability is a measure about the chances that an event will occur. When an event is certain to occur, probability will be 1 and when there is no chance of happening an event, probability will be 0.

Example:

Assumption	Cash Flows (₹)	Probability
Best guess	3,00,000	0.3
High guess	2,00,000	0.6
Low guess	1,20,000	0.1

In the above example chances that cash flow will be ₹ 3,00,000, ₹ 2,00,000 and ₹ 1,00,000 are 30%, 60% and 10% respectively.

(i) Expected Net Cash Flows

Expected Net Cash flows are calculated as the sum of the likely Cash flows of the Project multiplied by the probability of cash flows. Expected Cash flows are calculated as below:

$$E(R)/ENCF = \sum_{i=1}^n NCF_i \times P_i$$

Where, $E(R)/ENCF$ = Expected Net Cash flows

P_i = Probability of Cash flows

NCF_i = Net Cash flows

Example:

Assumption (1)	Cash Flows (₹) (2)	Probability (3)	Expected cash flow (₹) (2×3)
Best guess	3,00,000	0.3	$3,00,000 \times 0.3 = 90,000$
High guess	2,00,000	0.6	$2,00,000 \times 0.6 = 1,20,000$
Low guess	1,20,000	0.1	$1,20,000 \times 0.1 = 12,000$
Expected Net cash flow (ENCF)			2,22,000

(ii) Expected Net Present Value

Expected net present value =

$$ENPV = \sum_{t=1}^n \frac{ENCF}{(1+k)^t}$$

Where, ENPV = Expected Net Present Value

ENCF = Expected Net Cash Flows(including both inflows and outflows)

t = Period

k = Discount rate.

(a) Expected Net Present Value - Single period

Let us understand the calculation of Expected Net Present Value (ENPV) for a single period through an illustration as follows:

Illustration 3

Possible net cash flows of Projects A and B at the end of first year and their probabilities are given below. Discount rate is 10 per cent. For both the projects, initial investment is ₹ 10,000. Calculate the expected net present value for each project. State which project is preferable?

Possible Event	Project A		Project B	
	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
A	8,000	0.10	24,000	0.10
B	10,000	0.20	20,000	0.15
C	12,000	0.40	16,000	0.50
D	14,000	0.20	12,000	0.15
E	16,000	0.10	8 ,000	0.10

Solution

Calculation of Expected Value for Project A and Project B

Project A				Project B		
Possible Event	Cash Flow (₹)	Probability	Expected Value (₹)	Cash Flow (₹)	Probability	Expected Value (₹)
A	8,000	0.10	800	24,000	0.10	2,400

B	10,000	0.20	2,000	20,000	0.15	3,000
C	12,000	0.40	4,800	16,000	0.50	8,000
D	14,000	0.20	2,800	12,000	0.15	1,800
E	16,000	0.10	1,600	8,000	0.10	800
ENCF			12,000			16,000

The Net Present Value for Project A is $(0.909 \times ₹ 12,000 - ₹ 10,000) = ₹ 908$

The Net Present Value for Project B is $(0.909 \times ₹ 16,000 - ₹ 10,000) = ₹ 4,544.$

(b) Expected Net Present Value- Multiple period

Let us understand the calculation of Expected Net Present Value (ENPV) for multiple periods through an illustration as follows:

Illustration 4

Probabilities for net cash flows for 3 years of a project are as follows:

Year 1		Year 2		Year 3	
Cash Flow ₹)	Probability	Cash Flow ₹)	Probability	Cash Flow ₹)	Probability
2,000	0.1	2,000	0.2	2,000	0.3
4,000	0.2	4,000	0.3	4,000	0.4
6,000	0.3	6,000	0.4	6,000	0.2
8,000	0.4	8,000	0.1	8,000	0.1

Calculate the expected net present value of the project using 10 per cent discount rate if the Initial Investment of the project is ₹ 10,000.

Solution

Calculation of Expected Value

Year 1			Year 2			Year 3		
Cash Flow ₹)	Prob.	Expected Value ₹)	Cash Flow ₹)	Prob.	Expected Value ₹)	Cash Flow ₹)	Prob.	Expected Value ₹)
2,000	0.1	200	2,000	0.2	400	2,000	0.3	600

4,000	0.2	800	4,000	0.3	1200	4,000	0.4	1,600
6,000	0.3	1,800	6,000	0.4	2400	6,000	0.2	1,200
8,000	0.4	3,200	8,000	0.1	800	8,000	0.1	800
ENCF		6,000			4,800			4,200
ENCF								

The present value of the expected value of cash flow at 10 per cent discount rate has been determined as follows:

$$\begin{aligned}
 \text{Present Value of cash flow} &= \frac{\text{ENCF}_1}{(1+k)^1} + \frac{\text{ENCF}_2}{(1+k)^2} + \frac{\text{ENCF}_3}{(1+k)^3} \\
 &= \frac{6,000}{(1.1)} + \frac{4,800}{(1.1)^2} + \frac{4,200}{(1.1)^3} \\
 &= (6,000 \times 0.909) + (4,800 \times 0.826) + (4,200 \times 0.751) \\
 &= ₹ 12,573
 \end{aligned}$$

Expected Net Present value = Present Value of cash flow - Initial Investment

$$= ₹ 12,573 - ₹ 10,000 = ₹ 2,573.$$

4.1.2 Variance

Variance is a measurement of the degree of dispersion between numbers in a data set from its average. In very simple words, variance is the measurement of difference between the average of the data set from every number of the data set. Variance is calculated as below:

$$\text{Variance}(\sigma^2) = \sum_{j=1}^n (\text{NCF}_j - \text{ENCF})^2 P_j$$

Where, NCF_j = Net Cash Flow

ENCF = Expected Net Cash Flow

P_j = Probability

Variance measures the uncertainty of a value from its average. Thus, variance helps an organization to understand the level of risk it might face on investing in a project. A variance value of zero would indicate that the cash flows that would be generated over the life of the project would be same. This might happen in a case where the company has entered into a contract of providing services in return of a specific sum. A large variance indicates that there will be a large variability between the cash flows of the different years. This can happen in a case where the project being undertaken is very innovative and would require a certain time frame to market the product and enable to develop

a customer base and generate revenues. A small variance would indicate that the cash flows would be somewhat stable throughout the life of the project. This is possible in case of products which already have an established market.

4.1.3 Standard Deviation

Standard Deviation (SD) is a degree of variation of individual items of a set of data from its average. The square root of variance is called Standard Deviation. For Capital Budgeting decisions, Standard Deviation is used to calculate the risk associated with the estimated cash flows from the project.

Importance of Variance and Standard Deviation in Capital Budgeting: For making capital budgeting decisions, these two concepts are important to measure the volatility in estimated cash flows and profitability in an investment proposal. Both the concepts measures the difference between the expected cash flows and estimated cash flows (mean or average). Variance measures the range of variability (difference) in cash flows data while Standard deviation determines risk in an investment proposal. An investment proposal in which expected cash flows are close to the estimated net cash flow are seen as less risky and has the potential to make profit.

Standard deviation and Variance are two different statistical concepts but are closely interrelated. Standard deviation is calculated as square root of variance, hence, variance is prerequisite for calculation of SD.

Illustration 5

Calculate Variance and Standard Deviation of Project A and Project B on the basis of following information:

Possible Event	Project A		Project B	
	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
A	8,000	0.10	24,000	0.10
B	10,000	0.20	20,000	0.15
C	12,000	0.40	16,000	0.50
D	14,000	0.20	12,000	0.15
E	16,000	0.10	8,000	0.10

Solution

Calculation of Expected Value for Project A and Project B

Project A				Project B		
Possible Event	Cash Flow (₹)	Probability	Expected Value (₹)	Cash Flow (₹)	Probability	Expected Value (₹)
A	8,000	0.10	800	24,000	0.10	2,400
B	10,000	0.20	2,000	20,000	0.15	3,000
C	12,000	0.40	4,800	16,000	0.50	8,000
D	14,000	0.20	2,800	12,000	0.15	1,800
E	16,000	0.10	1,600	8,000	0.10	800
ENCF			12,000			16,000

Project A:

$$\begin{aligned}
 \text{Variance } (\sigma^2) &= (8,000 - 12,000)^2 \times (0.1) + (10,000 - 12,000)^2 \times (0.2) + (12,000 - 12,000)^2 \times (0.4) \\
 &\quad + (14,000 - 12,000)^2 \times (0.2) + (16,000 - 12,000)^2 \times (0.1) \\
 &= 16,00,000 + 8,00,000 + 0 + 8,00,000 + 16,00,000 = 48,00,000
 \end{aligned}$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}(\sigma^2)} = \sqrt{48,00,000} = 2,190.90$$

Project B:

$$\begin{aligned}
 \text{Variance}(\sigma^2) &= (24,000 - 16,000)^2 \times (0.1) + (20,000 - 16,000)^2 \times (0.15) + (16,000 - 16,000)^2 \times \\
 &\quad (0.5) + (12,000 - 16,000)^2 \times (0.15) + (8,000 - 16,000)^2 \times (0.1) \\
 &= 64,00,000 + 24,00,000 + 0 + 24,00,000 + 64,00,000 = 1,76,00,000
 \end{aligned}$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}(\sigma^2)} = \sqrt{1,76,00,000} = 4195.23$$

4.1.4 The Coefficient of Variation

The standard deviation is a useful measure of calculating the risk associated with the estimated cash inflows from an Investment. However, in Capital Budgeting decisions, the management is several times faced with choosing between many investments' avenues. Under such situations, it becomes difficult for the management to compare the risk associated with different projects using Standard Deviation as each project has different estimated cash flow values. In such cases, the Coefficient of Variation becomes useful.

The Coefficient of Variation calculates the risk borne for every percent of expected return. It is calculated as:

$$\text{Coefficient of variation} = \frac{\text{Standard Deviation}}{\text{Expected Return/ Expected Cash Flow}}$$

The Coefficient of Variation enables the management to calculate the risk borne by the concern for every unit of estimated return from a particular investment. Simply put, the investment avenue which has a lower ratio of standard deviation to expected return will provide a better risk – return trade off. Thus, when a selection has to be made between two projects, the management would select a project which has a lower Coefficient of Variation.

Illustration 6

Calculate Coefficient of Variation of Project A and Project B based on the following information:

Possible Event	Project A		Project B	
	Cash Flow (₹)	Probability	Cash Flow (₹)	Probability
A	10000	0.10	26,000	0.10
B	12,000	0.20	22,000	0.15
C	14,000	0.40	18,000	0.50
D	16,000	0.20	14,000	0.15
E	18,000	0.10	10,000	0.10

Solution

Calculation of Expected Value for Project A and Project B

Project A				Project B		
Possible Event	Cash Flow (₹)	Probability	Expected Value (₹)	Cash Flow (₹)	Probability	Expected Value (₹)
A	10,000	0.10	1,000	26,000	0.10	2,600
B	12,000	0.20	2,400	22,000	0.15	3,300
C	14,000	0.40	5,600	18,000	0.50	9,000
D	16,000	0.20	3,200	14,000	0.15	2,100
E	18,000	0.10	1,800	10,000	0.10	1,000
ENCF			14,000			18,000

Project A

$$\begin{aligned}\text{Variance } (\sigma^2) &= (10,000 - 14,000)^2 \times (0.1) + (12,000 - 14,000)^2 \times (0.2) + (14,000 - 14,000)^2 \times (0.4) \\ &\quad + (16,000 - 14,000)^2 \times (0.2) + (18,000 - 14,000)^2 \times (0.1) \\ &= 16,00,000 + 8,00,000 + 0 + 8,00,000 + 16,00,000 = 48,00,000\end{aligned}$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}(\sigma^2)} = \sqrt{48,00,000} = 2,190.90$$

Project B:

$$\begin{aligned}\text{Variance}(\sigma^2) &= (26,000 - 18,000)^2 \times (0.1) + (22,000 - 18,000)^2 \times (0.15) + (18,000 - 18,000)^2 \times (0.5) \\ &\quad + (14,000 - 18,000)^2 \times (0.15) + (10,000 - 18,000)^2 \times (0.1) \\ &= 64,00,000 + 24,00,000 + 0 + 24,00,000 + 64,00,000 = 1,76,00,000\end{aligned}$$

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}(\sigma^2)} = \sqrt{1,76,00,000} = 4195.23$$

Projects	Coefficient of variation	Risk	Expected Value
A	$\frac{2190.90}{14000} = 0.1565$	Less	Less
B	$\frac{4195.23}{18000} = 0.2331$	More	More

In project A, risk per rupee of cash flow is ₹ 0.16 while in project B, it is ₹ 0.23. Therefore, Project A is better than Project B.

4.2 Conventional Techniques

4.2.1 Risk Adjusted Discount Rate

The use of risk adjusted discount rate (RADR) is based on the concept that investors demand higher returns from the risky projects. The required rate of return on any investment should include compensation for delaying consumption plus compensation for inflation equal to risk free rate of return, plus compensation for any kind of risk taken. If the risk associated with any investment project is higher than risk involved in a similar kind of project, discount rate is adjusted upward in order to compensate this additional risk borne. Under this method, NPV is calculated as follows:

$$\text{NPV} = \sum_{t=1}^n \frac{\text{NCF}_t}{(1+k)^t} - I$$

Where, NCF_t = Net cash flow

k = Risk adjusted discount rate (RADR)

I = Initial Investment

t = Period

A risk adjusted discount rate is a sum of risk-free rate and risk premium. The Risk Premium depends on the perception of risk by the investor of a particular investment and risk aversion of the Investor.

So, Risk adjusted discount rate (RADR) = Risk free rate + Risk premium

Risk Free Rate: It is the rate of return on Investments that bear no risk. For e.g., Government securities yield a return of 6% and bear no risk. In such case, 6% is the risk-free rate.

Risk Premium: It is the rate of return over and above the risk free rate, expected by the Investors as a reward for bearing extra risk. For high risk projects, the risk premium will be high and for low risk projects, the risk premium would be lower.

Illustration 7

An enterprise is investing ₹ 100 lakhs in a project. The risk-free rate of return is 7%. Risk premium expected by the Management is 7%. The life of the project is 5 years. Following are the cash flows that are estimated over the life of the project:

Year	Cash flows (₹ in lakhs)
1	25
2	60
3	75
4	80
5	65

Calculate Net Present Value of the project based on Risk free rate and also on the basis of Risks adjusted discount rate.

Solution

The Present Value of the Cash Flows for all the years by discounting the cash flow at 7% is calculated as below:

Year	Cash flows (₹ in lakhs)	Discounting Factor @ 7%	Present value of Cash Flows (₹ In Lakhs)
1	25	0.935	23.38
2	60	0.873	52.38
3	75	0.816	61.20
4	80	0.763	61.04
5	65	0.713	46.35
Total of Present value of Cash flows			244.34
Less: Initial investment			100.00
Net Present Value (NPV)			144.34

Now, when the risk-free rate is 7% and the risk premium expected by the Management is 7%, then risk adjusted discount rate is $7\% + 7\% = 14\%$.

Discounting the above cash flows using the Risk Adjusted Discount Rate would be as below:

Year	Cash flows ₹ in Lakhs)	Discounting Factor @ 14%	Present Value of Cash Flows ₹ in lakhs)
1	25	0.877	21.93
2	60	0.769	46.14
3	75	0.675	50.63
4	80	0.592	47.36
5	65	0.519	33.74
Total of Present value of Cash flows			199.79
Less: Initial investment			100.00
Net present value (NPV)			99.79

Advantages of Risk-adjusted discount rate

- (1) It is easy to understand.
- (2) It incorporates risk premium in the discounting factor.

Limitations of Risk-adjusted discount rate

- (1) Difficulty in finding risk premium and risk-adjusted discount rate.
- (2) Though NPV can be calculated but it is not possible to calculate Standard Deviation of a given project.

4.2.2 Certainty Equivalent (CE)

As per CIMA terminology, “Certainty Equivalent is an approach dealing with risk in a capital budgeting context. It involves expressing risky future cash flows in terms of the certain cashflow which would be considered, by the decision maker, as their equivalent, that is the decision maker would be indifferent between the risky amount and the (lower) riskless amount considered to be its equivalent.”

The certainty equivalent is a guaranteed return that the management would accept rather than accepting a higher but uncertain return. This approach allows the decision maker to incorporate his or her utility function into the analysis. In this approach a set of risk less cash flow is generated in place of the original cash flows.

Steps in the Certainty Equivalent (CE) approach

Step 1: Remove risks by substituting equivalent certain cash flows from risky cash flows. This can be done by multiplying each risky cash flow by the appropriate α_t value (CE coefficient)

$$\alpha_t = \frac{\text{Certain cash flow}}{\text{Risky or expected cash flow}_t}$$

Suppose on tossing out a coin, if it comes head, you will win ₹ 10,000 and if it comes out to be tail, you will win nothing. Thus, you have 50% chance of winning and expected value is ₹ 5,000 ($\text{₹ } 10,000 \times 0.50$). In such case, if you are indifferent at receiving ₹ 3,000 for a certain amount and not playing then ₹ 3,000 will be certainty equivalent and 0.3 (i.e. ₹ 3,000/₹ 10,000) will be certainty equivalent coefficient.

Step 2: Discounted value of cash flow is obtained by applying risk less rate of interest. Since you have already accounted for risk in the numerator using CE coefficient, using the cost of capital to discount cash flows will tantamount to double counting of risk.

Step 3: After that, normal capital budgeting method is applied except in case of IRR method, where IRR is compared with risk free rate of interest rather than the firm's required rate of return.

Certainty Equivalent Coefficient transforms expected values of uncertain flows into their Certainty Equivalents. It is important to note that the value of Certainty Equivalent Coefficient lies between 0 & 1. Certainty Equivalent Coefficient 1 indicates that the cash flow is certain or management is risk neutral. In industrial situation, cash flows are generally uncertain and managements are usually risk averse. Under this method, NPV is calculated as follows:

$$NPV = \sum_{t=1}^n \frac{\alpha_t \times NCF_t}{(1+k)^t} - I$$

Where,

α_t = Risk-adjustment factor or the certainty equivalent coefficient

NCF_t = Forecasts of net cash flow for year 't' without risk-adjustment

k = Risk free rate assumed to be constant for all periods

I = Initial Investment

Illustration 8

If Investment proposal costs ₹ 45,00,000 and risk free rate is 5%, calculate net present value under certainty equivalent technique.

Year	Expected cash flow (₹)	Certainty Equivalent coefficient
1	10,00,000	0.90
2	15,00,000	0.85
3	20,00,000	0.82
4	25,00,000	0.78

Solution

$$\begin{aligned}
 NPV &= \frac{10,00,000 \times (0.90)}{(1.05)} + \frac{15,00,000 \times (0.85)}{(1.05)^2} + \frac{20,00,000 \times (0.82)}{(1.05)^3} + \frac{25,00,000 \times (0.78)}{(1.05)^4} - 45,00,000 \\
 &= ₹ 5,34,570
 \end{aligned}$$

Advantages of Certainty Equivalent Method

1. The certainty equivalent method is **simple and easy** to understand and apply.
2. It can **easily be calculated for different risk levels** applicable to different cash flows. For example, if in a particular year, a higher risk is associated with the cash flow, it can be easily adjusted and the NPV can be recalculated accordingly.

Disadvantages of Certainty Equivalent Method

1. There is **no objective** or mathematical method to estimate certainty equivalents. Certainty Equivalents are subjective and vary as per each individual's estimate.
2. Certainty equivalents are decided by the management based on their perception of risk. However, the **risk perception of the shareholders** who are the money lenders for the project is **ignored**. Hence, it is not used often in corporate decision making.

Risk-adjusted Discount Rate Vs. Certainty-Equivalent

Certainty Equivalent Method is superior to Risk Adjusted Discount Rate Method as it does not assume that risk increases with time at constant rate. Each year's Certainty Equivalent Coefficient is based on level of risk impacting its cash flow. Despite its soundness, it is not preferable like Risk Adjusted Discount Rate Method. It is difficult to specify a series of Certainty Equivalent Coefficients but simple to adjust discount rates.

4.3 Other Techniques

4.3.1 Sensitivity Analysis

As per CIMA terminology, "Sensitivity Analysis a modelling and risk assessment procedure in which changes are made to significant variables in order to determine the effect of these changes on the

planned outcome. Particular attention is thereafter paid to variables identified as being of special significance".

Sensitivity analysis put in simple terms is a modelling technique which is used in Capital Budgeting decisions, to study the impact of changes in the variables on the outcome of the project. In a project, several variables like weighted average cost of capital, consumer demand, price of the product, cost price per unit etc. operate simultaneously. The changes in these variables impact the outcome of the project. Therefore, it becomes very difficult to assess, change in which variable impacts the project outcome in a significant way. In Sensitivity Analysis, the project outcome is studied after taking into account change in only one variable. The more sensitive is the NPV (or IRR), the more critical is that variable. So, Sensitivity analysis is a way of finding impact on the project's NPV (or IRR) for a given change in one of the variables.

Steps involved in Sensitivity Analysis

Sensitivity Analysis is conducted by following the steps as below:

1. Finding variables, which have an influence on the NPV (or IRR) of the project.
2. Establishing mathematical relationship between the variables.
3. Analysing the effect of the change in each of the variables on the NPV (or IRR) of the project.

Illustration 9

X Ltd. is considering its new project with the following details:

Sr. No.	Particulars	Figures
1	Initial capital cost	₹ 400 Cr.
2	Annual unit sales	5 Cr.
3	Selling price per unit	₹ 100
4	Variable cost per unit	₹ 50
5	Fixed costs per year	₹ 50 Cr.
6	Discount Rate	6%

Required:

1. Calculate the NPV of the project.
2. Compute the impact on the project's NPV considering a 2.5 per cent adverse variance in each variable. Which variable is having maximum effect?

Consider Life of the project as 3 years.

Solution**1. Calculation of Net Cash Inflow per year**

	Particulars	Amount (₹)
A	Selling price per unit	100
B	Variable cost per unit	50
C	Contribution per unit (A - B)	50
D	Number of units sold per year	5 Cr.
E	Total Contribution (C × D)	₹ 250 Cr.
F	Fixed cost per year	₹ 50 Cr.
G	Net cash inflow per year (E - F)	₹ 200 Cr.

Calculation of Net Present Value (NPV) of the Project

Year	Year Cash Flow (₹ in Cr.)	PV factor @ 6%	Present Value (PV) (₹ in Cr.)
0	(400.00)	1.000	(400.00)
1	200.00	0.943	188.60
2	200.00	0.890	178.00
3	200.00	0.840	168.00
Net Present Value			134.60

Here, NPV represent the most likely outcomes and not the actual outcomes. The actual outcome can be lower or higher than the expected outcome.

2. Sensitivity Analysis considering 2.5 % Adverse Variance in each variable

Particulars	Base	Initial capital cost increased to ₹ 410 crore	Selling Price per Unit Reduced to ₹ 97.5	Variable Cost Per Unit increased to ₹ 51.25	Fixed Cost Per Unit increased to ₹ 51.25	Units sold per year reduced to 4.875 crore
	(₹)	(₹)	(₹)	(₹)	(₹)	(₹)
A Selling price per unit	100	100	97.5	100	100	100

B Variable cost per unit	50	50	50	51.25	50	50
C Contribution per unit (A - B)	50	50	47.5	48.75	50	50
	(₹ in Cr.)					
D Number of units sold per year (units in Crores)	5	5	5	5	5	4.875
E Total Contribution (C × D)	250	250	237.5	243.75	250	243.75
F Fixed cost per year	50	50	50	50	51.25	50
G Net Cash Inflow per year (E - F)	200	200	187.5	193.75	198.75	193.75
H PV of Net cash Inflow per year (G × 2.673)	534.60	534.60	501.19	517.89	531.26	517.89
I Initial capital cost	400	410	400	400	400	400
J NPV (H - I)	134.60	124.60	101.19	117.89	131.26	117.89
K Percentage Change in NPV	-	-7.43%	-24.82%	-12.41%	-2.48%	-12.41%

The above table shows that by changing one variable at a time by 2.5% (adverse) while keeping the others constant, the impact in percentage terms on the NPV of the project can be calculated. Thus, the change in selling price has the maximum effect on the NPV by 24.82%.

Advantages of Sensitivity Analysis:

Following are the main advantages of Sensitivity Analysis:

- (1) **Critical Issues:** This analysis identifies critical factors that impinge on a project's success or failure.
- (2) **Simplicity:** It is a simple technique.

Disadvantage of Sensitivity Analysis

Following are the main disadvantages of Sensitivity Analysis:

- (1) **Assumption of Independence:** This analysis assumes that all variables are independent i.e. they are not related to each other, which is unlikely in real life.
- (2) **Ignore probability:** This analysis does not look to the probability of changes in the variables.

4.3.2 Scenario Analysis

Although sensitivity analysis is probably the most widely used risk analysis technique, it does have limitations. Therefore, we need to extend sensitivity analysis to deal with the probability distributions of the inputs. In addition, it would be useful to vary more than one variable at a time so we could see the combined effects of changes in the variables.

Scenario analysis provides answer to these situations of extensions. This analysis brings in the probabilities of changes in key variables and also allows us to change more than one variable at a time.

This analysis begins with base case or most likely set of values for the input variables. Then, go for worst case scenario (low unit sales, low sale price, high variable cost, etc.) and best case scenario (high unit sales, high sale price, low variable cost, etc.). Alternatively, Scenarios analysis is possible where some factors are changed positively and some factors are changed negatively.

So, in a nutshell Scenario analysis examine the risk of investment, to analyse the impact of alternative combinations of variables, on the project's NPV (or IRR).

Illustration 10

XYZ Ltd. is considering a project "A" with an initial outlay of ₹ 14,00,000 and the possible three cash inflow attached with the project as follows:

Particulars	Year 1	Year 2	Year 3
Worst case	450	400	700
Most likely	550	450	800
Best case	650	500	900

Assuming the cost of capital as 9%, determine NPV in each scenario. If XYZ Ltd is certain about the most likely result in first two years but uncertain about the third year's cash flow, analyze what will be the NPV expecting worst scenario in the third year.

Solution

The possible outcomes will be as follows:

Year	PVF @ 9%	Worst Case		Most likely		Best case	
		Cash Flow	PV	Cash Flow	PV	Cash Flow	PV
		(₹ '000)	(₹ '000)	(₹ '000)	(₹ '000)	(₹ '000)	(₹ '000)
0	1	(1,400)	(1,400)	(1,400)	(1,400)	(1,400)	(1,400)
1	0.917	450	412.65	550	504.35	650	596.05
2	0.842	400	336.80	450	378.90	500	421.00
3	0.772	700	540.40	800	617.60	900	694.80
NPV		-110.15		100.85		311.85	

If XYZ Ltd. is certain about the most likely result in first two years but uncertain about the third year's cash flow, then, NPV expecting worst case scenario is expected in the third year will be as follows:

$$= - ₹ 14,00,000 + \frac{₹ 5,50,000}{(1+0.09)} + \frac{₹ 4,50,000}{(1+0.09)^2} + \frac{₹ 7,00,000}{(1+0.09)^3}$$

$$= - ₹ 14,00,000 + ₹ 5,04,587 + ₹ 3,78,756 + ₹ 5,40,528 = ₹ 23,871$$

Scenario Analysis Vs Sensitivity Analysis

- ❖ Sensitivity analysis and Scenario analysis both help to understand the impact of the change in input variable on the outcome of the project. However, there are certain basic differences between the two.
- ❖ Sensitivity analysis calculates the impact of the change of a single input variable on the outcome of the project viz., NPV or IRR. The sensitivity analysis thus enables to identify that single critical variable which can impact the outcome in a huge way and the range of outcomes of the project given the change in the input variable.
- ❖ Scenario analysis, on the other hand, is based on a scenario. The scenario may be recession or a boom wherein depending on the scenario, all input variables change. Scenario Analysis calculates the outcome of the project considering this scenario where the variables have changed simultaneously. Similarly, the outcome of the project would also be considered for the normal and recessionary situation. The variability in the outcome under the three different scenarios would help the management to assess the risk a project carries. Higher deviation in the outcome can be assessed as higher risk and lower to medium deviation can be assessed accordingly.

- ❖ Scenario analysis is far more complex than sensitivity analysis because in scenario analysis all inputs are changed simultaneously, considering the situation in hand while in sensitivity analysis, only one input is changed and others are kept constant.

4.3.3 Simulation Analysis (Monte Carlo)

Simulation is the exact replica of the actual situation. To simulate an actual situation, a model shall be prepared. The simulation Analysis is a technique, in which infinite calculations are made to obtain the possible outcomes and probabilities for any given action.

Monte Carlo simulation ties together sensitivities and probability distributions. The method came out of the work of first nuclear bomb and was so named because it was based on mathematics of Casino gambling. Fundamental appeal of this analysis is that it provides decision makers with a probability distribution of NPVs rather than a single point estimates of the expected NPV.

This analysis starts with carrying out a simulation exercise to model the investment project. It involves identifying the key factors affecting the project and their inter relationships. It involves modelling of cash flows to reveal the key factors influencing both cash receipt and payments and their inter relationship.

This analysis specifies a range for a probability distribution of potential outcomes for each of model's assumptions.

4.3.3.1 Steps for Simulation Analysis:

1. Modelling the project: The model shows the relationship of NPV with parameters and exogenous variables. (Parameters are input variables specified by decision maker and held constant over all simulation runs. Exogenous variables are input variables, which are stochastic in nature and outside the control of the decision maker).
2. Specify values of parameters and probability distributions of exogenous variables.
3. Select a value at random from probability distribution of each of the exogenous variables.
4. Determine NPV corresponding to the randomly generated value of exogenous variables and pre-specified parameter variables.
5. Repeat steps (3) & (4) a large number of times to get a large number of simulated NPVs.
6. Plot probability distribution of NPVs and compute a mean and Standard Deviation of returns to gauge the project's level of risk.

Example: Uncertainty associated with two aspects of the project: Annual Net Cash Flow & Life of the project. NPV model for the project is

$$\sum_{t=1}^n [CF_t / (1+i)^t] - I$$

Where $i \rightarrow$ Risk free interest rate, $I \rightarrow$ initial investment are parameters, $CF =$ Annual Cash Flow

With $i = 10\%$, $I = ₹ 1,30,000$, CF_t & n stochastic exogenous variables with the following distribution will be as under:

Annual Cash Flow		Project Life	
Value (₹)	Probability	Value (Year)	Probability
10,000	0.02	3	0.05
15,000	0.03	4	0.10
20,000	0.15	5	0.30
25,000	0.15	6	0.25
30,000	0.30	7	0.15
35,000	0.20	8	0.10
40,000	0.15	9	0.03
		10	0.02

Ten manual simulation runs are performed for the project. To perform this operation, values are generated at random for the two exogenous variables viz., Annual Cash Flow and Project Life. For this purpose, we take following steps

- (1) set up correspondence between values of exogenous variables and random numbers
- (2) choose some random number generating device.

Correspondence between Values of Exogenous Variables and two Digit Random Numbers:

Annual Cash Flow				Project Life			
Value (₹)	Probability	Cumulative Probability	Two Digit Random No.	Value (Year)	Probability	Cumulative Probability	Two Digit Random No.
10,000	0.02	0.02	00 – 01	3	0.05	0.05	00 – 04
15,000	0.03	0.05	02 – 04	4	0.10	0.15	05 – 14
20,000	0.15	0.20	05 – 19	5	0.30	0.45	15 – 44
25,000	0.15	0.35	20 – 34	6	0.25	0.70	45 – 69
30,000	0.30	0.65	35 – 64	7	0.15	0.85	70 – 84

35,000	0.20	0.85	65 – 84	8	0.10	0.95	85 – 94
40,000	0.15	1.00	85 - 99	9	0.03	0.98	95 – 97

Random Number

53479	81115	98036	12217	59526
97344	70328	58116	91964	26240
66023	38277	74523	71118	84892
99776	75723	03172	43112	83086
30176	48979	92153	38416	42436
81874	83339	14988	99937	13213
19839	90630	71863	95053	55532
09337	33435	53869	52769	18801
31151	58295	40823	41330	21093
67619	52515	03037	81699	17106

For random numbers, we can begin from any-where taking at random from the table and read any pair of adjacent columns, column/row wise. For the first simulation run we need two digit random numbers (1) For Annual Cash Flow (2) For Project Life. The numbers are 53 & 97 and corresponding value of Annual Cash Flow and Project Life are ₹ 3,000 and 9 years respectively.

Simulation Results

Annual Cash Flow			Project Life			
Run	Random No.	Corres. Value of Annual Cash Flow (1)	Random No.	Corres. Value of Project Life	PVAF @ 10% (2)	NPV (1)x(2) – 1,30,000
1	53	30,000	97	9	5.759	42,770
2	66	35,000	99	10	6.145	85,075
3	30	25,000	81	7	4.868	(8,300)
4	19	20,000	09	4	3.170	(66,600)
5	31	25,000	67	6	4.355	(21,125)
6	81	35,000	70	7	4.868	40,380
7	38	30,000	75	7	4.868	16,040
8	48	30,000	83	7	4.868	16,040
9	90	40,000	33	5	3.791	21,640
10	58	30,000	52	6	4.355	650

4.3.3.2. Advantages of Simulation Analysis: Strength lies in Variability.

- (1) We can predict all type of bad market situation beforehand.
- (2) Handle problems characterised by:
 - (a) numerous exogenous variables following any kind of distribution.
 - (b) complex inter-relationships among parameters, exogenous variables and endogenous variables. Such problems defy capabilities of analytical methods.
 - (c) compels decision maker to explicitly consider the inter-dependencies and uncertainties featuring the project.

4.3.3.3 Shortcomings

- (1) Difficult to model the project and specify probability distribution of exogenous variables.
- (2) Simulation is inherently imprecise. Provides rough approximation of probability distribution of NPV Due to its imprecision, simulation probability distribution may be misleading when a tail of distribution is critical.
- (3) Realistic simulation model being likely to be complex would probably be constructed by management expert and not by the decision maker. Decision maker lacking understanding of the model may not use it.
- (4) Determine NPV in simulation run, risk free discount rate is used. It is done to avoid pre-judging risk, which is reflected in the dispersion of the distribution of N.P.V. This derived measure of NPV takes a different meaning from its original value, and, therefore, is difficult to interpret.

4.3.4. Decision Tree Analysis

Till now we have discussed simple accept-or-reject decisions which view current investments in isolation of subsequent decisions. However, practically investment decisions may have implications for future or further investment decisions and may also impact future decision and events. Such situation can be handled by taking a sequence of decisions over a period. The technique to handle this type of sequential decisions is done through “Decision Tree” technique.

Basically, decision tree is a graphic display of the relationship between a present decision and future events, future decision, and their consequences.

This approach assumes that there are only two types of situations that a finance manager has to face. The first situation is where the manager has control or power to determine what happens next. This is known as “Decision”, as he can do what he desires to do.

The second situation is where finance manager has no control over what happens next. This is known as "Event". Since the outcome of the events is not known, a probability distribution needs to be assigned to the various outcomes or consequences. It should, however, be noted when a finance manager faced with a decision situation, he is assumed to act rationally. For example, in a commercial business, he will choose the most profitable course of action and in non-profit organization, the lowest cost may be rational choice.

Steps involved in Decision Tree analysis:

Step 1- Define Investment: Decision tree analysis can be applied to a variety of business decision-making scenarios. Normally it includes following types of decisions.

- Whether or not to launch a new product, if so, whether this launch should be local, national, or international.
- Whether extra production requirement should be met by extending the factory or by outsourcing it to an external supplier.
- Whether to dig for oil or not if so, upto what height and continue to dig even after finding no oil upto a certain depth.

Step 2- Identification of Decision Alternatives: It is very essential to clearly identify decision alternatives. For example if a company is planning to introduce a new product, it may be local launch, national launch or international launch.

Step 3- Drawing a Decision Tree: After identifying decision alternatives, all the relevant data such as the projected cash flows, probability distribution expected present value etc. should be put in diagrammatic form called decision tree.

While drawing a decision tree, it should be noted that NPVs etc. should be placed on the branches of decision tree, coming out of the decisions identified.

While drawing a decision tree, it should be noted that the:-

- The decision point (traditionally represented by square) is the option available for manager to take or not to take - in other words action at these points.
- The event or chance or outcome (traditionally represented by circle) which are dependent on chance process, along with the probabilities thereof, and monetary value associated with them.
- This diagram is drawn from left to right.

Step 4- Evaluating the Alternatives: After drawing out the decision the next step is the evaluation of alternatives. The various alternatives can be evaluated as follows:

- (i) This procedure is carried out from the last decision in the sequence (extreme right) and goes on working back to the first (left) for each of the possible decision.
- (ii) At each final stage decision point, select the alternative which has the highest NPV and truncate the other alternatives. Each decision point is assigned a value equal to the NPV of the alternative selected at the decision point.
- (iii) Proceed backward in the same manner calculating the NPV at chance or event or outcome points (○) selecting the decisions alternative which has highest NPV at various decision points [□] rejecting the inferior decision option, assigning NPV to the decision point, till the first decision point is reached.

In Capital Budgeting, the decision taker has to identify and find out the various alternatives available to an investment decision. By drawing a decision tree, the alternatives are highlighted through a diagram, giving the range of possible outcomes. The stages set for drawing a decision tree is based on the following rules.

1. It begins with a decision point, also known as decision node, represented by a rectangle while the outcome point, also known as chance node, denoted by a circle.
2. Decision alternatives are shown by a straight line starting from the decision node.
3. The Decision Tree Diagram is drawn from left to right. Rectangles and circles have to be sequentially numbered.
4. Values and Probabilities for each branch are to be incorporated next.

The Value of each circle and each rectangle is computed by evaluating from right to left. This procedure is carried out from the last decision in the sequence and goes on working back to the first for each of the possible decisions. The following rules have been set for such evaluation.

- (a) The expected monetary value (EMV) at the chance node with branches emanating from a circle is the aggregate of the expected values of the various branches that emanate from the chance node.
- (b) The expected value at a decision node with branches emanating from a rectangle is the highest amongst the expected values of the various branches that emanate from the decision node.

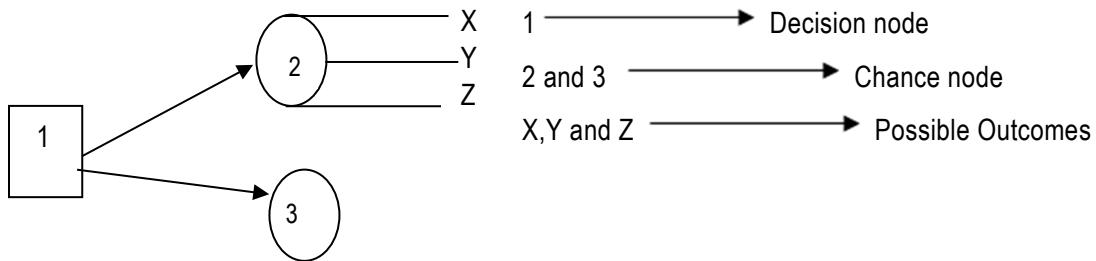
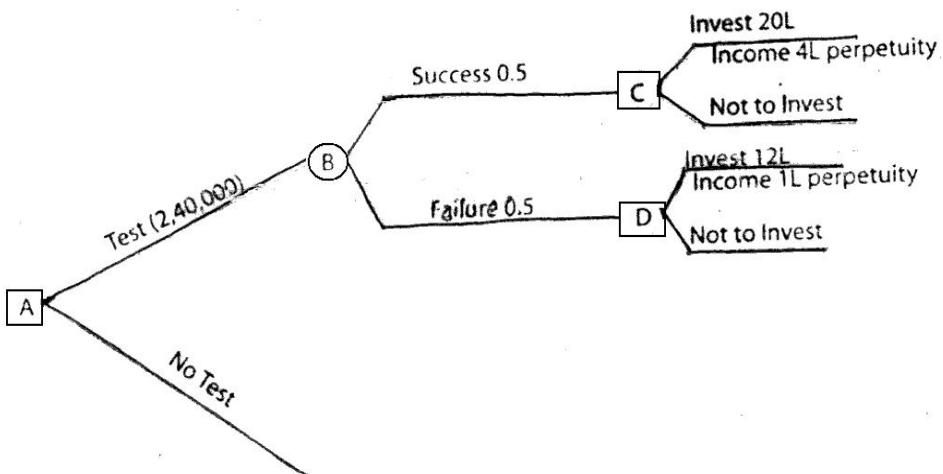


Illustration 11

L & R Limited wishes to develop new virus-cleaner software. The cost of the pilot project would be ₹ 2,40,000. Presently, the chances of the product being successfully launched on a commercial scale are rated at 50%. In case it does succeed, L&R can invest a sum of ₹ 20 lacs to market the product. Such an effort can generate perpetually, an annual net after tax cash income of ₹ 4 lacs. Even if the commercial launch fails, they can make an investment of a smaller amount of ₹ 12 lacs with the hope of gaining perpetually a sum of ₹ 1 lac. Evaluate the proposal, adopting decision tree approach. The discount rate is 10%.

Solution

Decision tree diagram is given below:



Evaluation

At Decision Point C: The choice is between investing ₹ 20 lacs for a perpetual benefit of ₹ 4 lacs and not to invest. The preferred choice is to invest, since the capitalized value of benefit of ₹ 4 lacs (at 10%) adjusted for the investment of ₹ 20 lacs, yields a net benefit of ₹ 20 lacs.

At Decision Point D: The choice is between investing ₹ 12 lacs, for a similar perpetual benefit of ₹ 1 lac. and not to invest. Here the invested amount is greater than capitalized value of benefit at ₹ 10 lacs. There is a negative benefit of ₹ 2 lacs. Therefore, it would not be prudent to invest.

At Outcome Point B: Evaluation of EMV is as under (₹ in lacs).

Outcome	Amount (₹)	Probability	Result (₹)
Success	20.00	0.50	10.00
Failure	0.00	0.50	<u>00.00</u>
Net result			10.00

EMV at B is, therefore, ₹10 lacs.

At A: Decision is to be taken based on preferences between two alternatives. The first is to test, by investing ₹2,40,000 and reap a benefit of ₹ 10 lacs. The second is not to test, and thereby losing the opportunity of a possible gain.

The preferred choice is, therefore, investing a sum of ₹ 2,40,000 and undertaking the test.

5. REPLACEMENT DECISION

Capital budgeting refers to the process we use to make decisions concerning investments in the long-term assets of the firm. The general idea is that the capital, or long-term funds, raised by the firms are used to invest in assets that will enable the firm to generate revenues several years into the future. Often the funds raised to invest in such assets are not unrestricted, or infinitely available; thus the firm must budget how these funds are invested. Among various capital budgeting decision, Replacement decision is one of the most important classifications of capital budgeting. The replacement decision can be divided into following two types of decisions:

5.1 Replacement of Existing Machine

This is a decision concerning whether an existing asset should be replaced by a newer version of the same machine or even a different type of machine that has the same functionality as the existing machine. Such replacements are generally made to maintain existing levels of operations, although profitability might change due to changes in expenses (that is, the new machine might be either more expensive or cheaper to operate than the existing machine).

Evaluation of replacement projects is slightly more complicated comparing expansion projects because an existing asset is being replaced. When identifying the cash flows for replacement projects, keep in mind that the cash flows associated with the existing (replaced) asset will no longer exist if the new asset is purchased. Therefore, we must not only determine the cash flows that the

new asset will generate, but we must also determine the effect of eliminating the cash flows generated by the replaced asset. For example, if a new asset that will produce cash sales equal to ₹ 100,000 per year is purchased to replace an existing asset that is generating cash sales equal to ₹ 75,000, then the incremental, or marginal, cash flow related to sales is ₹ 25,000. Likewise, if the asset that is replaced can be sold for ₹ 350,000, then the purchase price of the new asset effectively is ₹ 350,000 less than its invoice price. In other words, for replacement decisions, we must determine the overall net effect of purchasing a new asset to replace an existing asset—the cash flows associated with the old asset will be replaced with the cash flows associated with the new asset. Two items that you must remember to include when determining the incremental cash flows are depreciation — not because it is a cash flow, but because it affects cash flows through taxes and taxes — both of which generally change when an older asset is replaced with a newer asset.

Therefore analysis of replacement decision follows certain steps:

Step I. Net cash outflow (assumed at current time /[Present value of cost]):

- (Book value of old equipment - market value of old equipment) × Tax Rate = Tax payable/savings from sale
- Cost of new equipment – [Tax payable/savings from sale + market value of old equipment] = Net cash outflow

Step II. Estimate change in cash flow per year, if replacement decision is implemented.

Change in cash flow = [(Change in sales ± Change in operating costs) – Change in depreciation]
(1 – tax rate) + Change in depreciation

Step III. Present value of benefits = Present value of yearly cash flows + Present value of estimated salvage of new system

Step IV. Net present value = Present value of benefits – Present value of costs

Step V. Decision rule. Accept when present value of benefits > present value of costs.

Reject when the opposite is true.

Illustration 12

A Company named Roby's cube decided to replace the existing Computer system of their organisation. Original cost of old system was ₹ 25,000 and it was installed 5 years ago. Current market value of old system is ₹ 5,000. Depreciation of the old system was charged with life of 10 years with Estimated Salvage value as Nil. Depreciation of the new system will be charged with life over 5 years. Present cost of the new system is ₹ 50,000. Estimated Salvage value of the new system is ₹ 1,000. Estimated cost savings with new system is ₹ 5,000 per year. Increase in sales

with new system is assumed at 10% per year based on original total sales of ₹ 10,00,000. Company follows straight line method of depreciation. Cost of capital of the company is 10% whereas tax rate is 30%.

Solution

Step I. Net cash outflow (assumed at current time) [Present values of cost]:

- a. $(\text{Book value of old system} - \text{market value of old system}) \times \text{Tax Rate}$
 $= \text{Tax payable/savings from sale}$
 $= [(\text{₹ } 25,000 - 5 \times \text{₹ } 2,500) - \text{₹ } 5,000] \times 0.30 = \text{₹ } 7,500 \times 0.30$
 $= \text{₹ } 2,250$
- b. $\text{Cost of new system} - [\text{Tax payable/savings from sale} + \text{Market value of old system}]$
 $= \text{Net cash outflow}$
Or, $\text{₹ } 50,000 - [\text{₹ } 2,250 + \text{₹ } 5,000] = \text{₹ } 42,750$

Step II. Estimated change in cash flows per year if replacement decision is implemented.

$$\begin{aligned}\text{Change in cash flow} &= [(\text{Change in sales} \pm \text{Change in operating costs}) - \text{Change in depreciation}] (1 - \text{tax rate}) + \text{Change in depreciation} \\ &= [\text{₹ } 1,00,000 \times 0.1 + \text{₹ } 5,000 - (\text{₹ } 49,000/5 - \text{₹ } 25,000/10)] (1-0.30) + (\text{₹ } 49,000/5 - \text{₹ } 25,000/10) \\ &= \text{₹ } 12,690\end{aligned}$$

Step III. Present value of benefits = Present value of yearly cash flows + Present value of estimated salvage of new system

$$\begin{aligned}&= \text{₹ } 12,690 \times \text{PVIFA}(10\%, 5) + \text{₹ } 1,000 \times \text{PVIF}(10\%, 5) \\ &= \text{₹ } 48,723\end{aligned}$$

Step IV. Net present value = Present value of benefits - Present value of costs

$$\begin{aligned}&= \text{₹ } 48,723 - \text{₹ } 42,750 \\ &= \text{₹ } 5,973\end{aligned}$$

Step V. Decision rule: Since NPV is positive we should accept the proposal to replace the machine.

5.2 Optimum Replacement Cycle

Case discussed above is a simple example replacement decision based on NPV. This decision was based on assumption that the projects do not form part of continuous replacement cycle.

However, sometimes, project may involve continuous replacement cycle. In such cases NPV decision rules needs modification. To determine optimal replacement cycle, concept of Equivalent Annual Cost (EAC), discussed at Intermediate (IPC) Level is used.

The formula to compute EAC is as follows:

$$\frac{\text{PV of Cash Outflow}}{\text{PVAF}}$$

This decision is based on assumption that as the machine (asset) becomes older its efficiency decreases and leading to increase in operating cost and reduction in resale value.

Illustration 13

X Ltd. is a taxi operator. Each taxi cost to company ₹ 4,00,000 and has a useful life of 3 years. The taxi's operating cost for each of 3 years and salvage value at the end of year is as follows:

	Year 1	Year 2	Year 3
Operating Cost	₹ 1,80,000	₹ 2,10,000	₹ 2,38,000
Resale Value	₹ 2,80,000	₹ 2,30,000	₹ 1,68,000

You are required to determine the optimal replacement period of taxi if cost of capital of X Ltd. is 10%.

Solution

NPV if taxi is kept for 1 Year

$$\begin{aligned}
 &= - ₹ 4,00,000 - ₹ 1,80,000 (0.909) + ₹ 2,80,000 (0.909) \\
 &= - ₹ 3,09,100
 \end{aligned}$$

NPV if taxi is kept for 2 Year

$$\begin{aligned}
 &= - ₹ 4,00,000 - ₹ 1,80,000 \times 0.909 + ₹ 20,000 \times 0.826 \\
 &= - ₹ 5,47,100
 \end{aligned}$$

NPV if taxi is kept for 3 Year

$$\begin{aligned}
 &= - ₹ 4,00,000 - ₹ 1,80,000 \times 0.909 - ₹ 2,10,000 \times 0.826 - ₹ 70,000 \times 0.751 \\
 &= - ₹ 7,89,650
 \end{aligned}$$

Since above NPV figures relate to different periods, there are not comparable. to make them comparable we shall use concept of EAC as follows:

EAC of 1 year

$$\frac{3,09,100}{0.909} = ₹ 3,40,044$$

EAC of 2 year

$$\frac{5,47,100}{1.735} = ₹ 3,15,331$$

EAC of 3 year

$$\frac{7,89,650}{2.486} = ₹ 3,17,639$$

Since lowest EAC incur if taxi for 2 year; Hence the optimum replacement cycle to replace taxi in 2 years.



6. ADJUSTED PRESENT VALUE

As we are well aware that to evaluate a capital project we discount the expected cash flows by overall Cost of Capital i.e. WACC. Further, as discussed earlier to incorporate risk in the evaluation of any project we can adjust the same discount rate.

However instead of adjusting the cost of capital we can use an alternative approach called Adjusted Present Value (APV) Method. This approach separates the investment decision and financing decision.

Following formula is used to evaluate a project as per this approach:

Base Case NPV + PV of Tax Benefit on Interest

Base Case NPV is calculated using cost of equity assuming the company is unlevered i.e., all equity financed. Now question arises how to calculate the Unlevered Cost of Equity. It has been discussed in the chapter of Business Valuation of this Study Material.

Since viability of the project is partly dependent on how project is financed the PV of Tax Benefits on Interest payment allows for such adjustment. Thus, this method provides a broader view to evaluate a project considering the benefit of increased use of debt in financing of any project.

TEST YOUR KNOWLEDGE

Theoretical Questions

1. Write short note on Certainty Equivalent Approach.
2. What is the sensitivity analysis in Capital Budgeting?
3. Write a note on project appraisal under inflationary conditions.
4. Explain the steps involved in Simulation Analysis.

Practical Questions

1. Skylark Airways is planning to acquire a light commercial aircraft for flying class clients at an investment of ₹ 50,00,000. The expected cash flow after tax for the next three years is as follows:

Year 1		Year 2		Year 3	
CFAT	Probability	CFAT	Probability	CFAT	Probability
14,00,000	0.1	15,00,000	0.1	18,00,000	0.2
18,00,000	0.2	20,00,000	0.3	25,00,000	0.5
25,00,000	0.4	32,00,000	0.4	35,00,000	0.2
40,00,000	0.3	45,00,000	0.2	48,00,000	0.1

The Company wishes to take into consideration all possible risk factors relating to airline operations. The company wants to know:

- (i) The expected NPV of this venture assuming independent probability distribution with 6 per cent risk free rate of interest.
- (ii) The possible deviation in the expected value.
- (iii) How would standard deviation of the present value distribution help in Capital Budgeting decisions?
2. Cyber Company is considering two mutually exclusive projects. Investment outlay of both the projects is ₹ 5,00,000 and each is expected to have a life of 5 years. Under three possible situations their annual cash flows and probabilities are as under:

Situation	Probabilities	Cash Flow (₹)	
		Project A	ProjectB
Good	0.3	6,00,000	5,00,000

Normal	0.4	4,00,000	4,00,000
Worse	0.3	2,00,000	3,00,000

The cost of capital is 7 per cent, which project should be accepted? Explain with workings.

3. A company is considering Projects X and Y with following information:

Project	Expected NPV (₹)	Standard deviation
X	1,22,000	90,000
Y	2,25,000	1,20,000

- (i) Which project will you recommend based on the above data?
 - (ii) Explain whether your opinion will change, if you use coefficient of variation as a measure of risk.
 - (iii) Which measure is more appropriate in this situation and why?
4. KLM Ltd., is considering taking up one of the two projects-Project-K and Project-S. Both the projects having same life and require equal investment of ₹ 80 lakhs each. Both are estimated to have almost the same yield. As the company is new to this type of business, the cash flow arising from the projects cannot be estimated with certainty. An attempt was therefore, made to use probability to analyse the pattern of cash flow from other projects during the first year of operations. This pattern is likely to continue during the life of these projects. The results of the analysis are as follows:

Project K		Project S	
Cash Flow (in ₹)	Probability	Cash Flow (in ₹)	Probability
11	0.10	09	0.10
13	0.20	13	0.25
15	0.40	17	0.30
17	0.20	21	0.25
19	0.10	25	0.10

Required:

- (i) Calculate variance, standard deviation and co-efficient of variation for both the projects.
 - (ii) Which of the two projects is riskier?
5. Project X and Project Y are under the evaluation of XY Co. The estimated cash flows and their probabilities are as below:

Project X : Investment (year 0) ₹ 70 lakhs

Probability weights	0.30	0.40	0.30
Years	₹ lakhs	₹ lakhs	₹ lakhs
1	30	50	65
2	30	40	55
3	30	40	45

Project Y: Investment (year 0) ₹ 80 lakhs.

Probability weighted	Annual cash flows through life
	₹ lakhs
0.20	40
0.50	45
0.30	50

- (a) Which project is better based on NPV, criterion with a discount rate of 10%?
 - (b) Compute the standard deviation of the present value distribution and analyse the inherent risk of the projects.
6. Shivam Ltd. is considering two mutually exclusive projects A and B. Project A costs ₹ 36,000 and project B ₹ 30,000. You have been given below the net present value probability distribution for each project.
- | Project A | | Project B | |
|-------------------|-------------|-------------------|-------------|
| NPV estimates (₹) | Probability | NPV estimates (₹) | Probability |
| 15,000 | 0.2 | 15,000 | 0.1 |
| 12,000 | 0.3 | 12,000 | 0.4 |
| 6,000 | 0.3 | 6,000 | 0.4 |
| 3,000 | 0.2 | 3,000 | 0.1 |
- (i) Compute the expected net present values of projects A and B.
 - (ii) Compute the risk attached to each project i.e. standard deviation of each probability distribution.
 - (iii) Compute the profitability index of each project.
 - (iv) Which project do you recommend? State with reasons.
7. Following are the estimates of the net cash flows and probability of a new project of M/s X Ltd.:

	Year	P = 0.3	P = 0.5	P = 0.2
Initial investment	0	4,00,000	4,00,000	4,00,000
Estimated net after tax cash inflows per year	1 to 5	1,00,000	1,10,000	1,20,000
Estimated salvage value (after tax)	5	20,000	50,000	60,000

Required rate of return from the project is 10%. Find:

- (i) The expected NPV of the project.
- (ii) The best case and the worst case NPVs.
- (iii) The probability of occurrence of the worst case if the cash flows are perfectly dependent overtime and independent overtime.
- (iv) Standard deviation and coefficient of variation assuming that there are only three streams of cash flow, which are represented by each column of the table with the given probabilities.
- (v) Coefficient of variation of X Ltd. on its average project which is in the range of 0.95 to 1.0. If the coefficient of variation of the project is found to be less risky than average, 100 basis points are deducted from the Company's cost of Capital

Should the project be accepted by X Ltd?

8. XY Ltd. has under its consideration a project with an initial investment of ₹ 1,00,000. Three probable cash inflow scenarios with their probabilities of occurrence have been estimated as below:

Annual cash inflow (₹)	20,000	30,000	40,000
Probability	0.1	0.7	0.2

The project life is 5 years and the desired rate of return is 20%. The estimated terminal values for the project assets under the three probability alternatives, respectively, are ₹ 0, 20,000 and 30,000.

You are required to:

- (i) Find the probable NPV;
- (ii) Find the worst-case NPV and the best-case NPV; and
- (iii) State the probability occurrence of the worst case, if the cash flows are perfectly positively correlated over time.

9. XYZ Ltd. is considering a project for which the following estimates are available:

	₹
Initial Cost of the project	10,00,000
Sales price/unit	60
Cost/unit	40
Sales volumes	
Year 1	20000 units
Year 2	30000 units
Year 3	30000 units

Discount rate is 10% p.a.

You are required to measure the sensitivity of the project in relation to each of the following parameters:

- (a) Sales Price/unit
- (b) Unit cost
- (c) Sales volume
- (d) Initial outlay and
- (e) Project lifetime

Taxation may be ignored.

10. From the following details relating to a project, analyse the sensitivity of the project to changes in initial project cost, annual cash inflow and cost of capital:

Initial Project Cost (₹)	1,20,000
Annual Cash Inflow (₹)	45,000
Project Life (Years)	4
Cost of Capital	10%

To which of the three factors, the project is most sensitive? (Use annuity factors: for 10% 3.169 and 11% 3.103).

11. Red Ltd. is considering a project with the following Cash flows:

Years	Cost of Plant	Recurring Cost	Savings	₹
0	10,000			
1		4,000	12,000	
2		5,000	14,000	

The cost of capital is 9%. Measure the sensitivity of the project to changes in the levels of plant value, running cost and savings (considering each factor at a time) such that the NPV becomes zero. The P.V. factor at 9% are as under:

<u>Year</u>	<u>Factor</u>
0	1
1	0.917
2	0.842

Which factor is the most sensitive to affect the acceptability of the project?

12. The Easygoing Company Limited is considering a new project with initial investment, for a product "Survival". It is estimated that IRR of the project is 16% having an estimated life of 5 years.

Financial Manager has studied that project with sensitivity analysis and informed that annual fixed cost sensitivity is 7.8416%, whereas cost of capital (discount rate) sensitivity is 60%.

Other information available are:

Profit Volume Ratio (P/V) is 70%,

Variable cost ₹ 60/- per unit

Annual Cash Flow ₹ 57,500/-

Ignore Depreciation on initial investment and impact of taxation.

Calculate

- (i) Initial Investment of the Project
- (ii) Net Present Value of the Project
- (iii) Annual Fixed Cost
- (iv) Estimated annual unit of sales
- (v) Break Even Units

Cumulative Discounting Factor for 5 years

8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%
3.993	3.890	3.791	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127

13. Unnat Ltd. is considering investing ₹ 50,00,000 in a new machine. The expected life of machine is five years and has no scrap value. It is expected that 2,00,000 units will be produced and sold each year at a selling price of ₹ 30.00 per unit. It is expected that the variable costs to be ₹ 16.50 per unit and fixed costs to be ₹ 10,00,000 per year. The cost of capital of Unnat Ltd. is 12% and acceptable level of risk is 20%.

You are required to measure the sensitivity of the project's net present value to a change in the following project variables:

- (a) sale price;
- (b) sales volume;
- (c) variable cost;
- (d) On further investigation it is found that there is a significant chance that the expected sales volume of 2,00,000 units per year will not be achieved. The sales manager of Unnat Ltd. suggests that sales volumes could depend on expected economic states which could be assigned the following probabilities:

State of Economy	Annual Sales (in Units)	Prob.
Poor	1,75000	0·30
Normal	2,00,000	0·60
Good	2,25,000	0·10

Calculate expected net present value of the project and give your decision whether company should accept the project or not.

14. The Textile Manufacturing Company Ltd., is considering one of two mutually exclusive proposals, Projects M and N, which require cash outlays of ₹ 8,50,000 and ₹ 8,25,000 respectively. The certainty-equivalent (C.E) approach is used in incorporating risk in capital budgeting decisions. The current yield on government bonds is 6% and this is used as the risk free rate. The expected net cash flows and their certainty equivalents are as follows:

Project M			Project N	
Year-end	Cash Flow ₹	C.E.	Cash Flow ₹	C.E.
1	4,50,000	0.8	4,50,000	0.9

2	5,00,000	0.7	4,50,000	0.8
3	5,00,000	0.5	5,00,000	0.7

Present value factors of ₹ 1 discounted at 6% at the end of year 1, 2 and 3 are 0.943, 0.890 and 0.840 respectively.

Required:

- (i) Which project should be accepted?
- (ii) If risk adjusted discount rate method is used, which project would be appraised with a higher rate and why?

15. Determine the risk adjusted net present value of the following projects:

	X	Y	Z
Net cash outlays (₹)	2,10,000	1,20,000	1,00,000
Project life	5 years	5 years	5 years
Annual Cash inflow (₹)	70,000	42,000	30,000
Coefficient of variation	1.2	0.8	0.4

The Company selects the risk-adjusted rate of discount on the basis of the coefficient of variation:

Coefficient of Variation	Risk-Adjusted Rate of Return	P.V. Factor 1 to 5 years At risk adjusted rate of discount
0.0	10%	3.791
0.4	12%	3.605
0.8	14%	3.433
1.2	16%	3.274
1.6	18%	3.127
2.0	22%	2.864
More than 2.0	25%	2.689

16. New Projects Ltd. is evaluating 3 projects, P-I, P-II, P-III. Following information is available in respect of these projects:

	P-I	P-II	P-III
Cost	₹ 15,00,000	₹ 11,00,000	₹ 19,00,000
Inflows-Year 1	6,00,000	6,00,000	4,00,000
Year 2	6,00,000	4,00,000	6,00,000
Year 3	6,00,000	5,00,000	8,00,000
Year 4	6,00,000	2,00,000	12,00,000
Risk Index	1.80	1.00	0.60

Minimum required rate of return of the firm is 15% and applicable tax rate is 40%. The risk free interest rate is 10%.

Required:

- (i) Find out the risk-adjusted discount rate (RADR) for these projects.
- (ii) Which project is the best?

17. A firm has projected the following cash flows from a project under evaluation:

Year	₹ lakhs
0	(70)
1	30
2	40
3	30

The above cash flows have been made at expected prices after recognizing inflation. The firm's cost of capital is 10%. The expected annual rate of inflation is 5%.

Show how the viability of the project is to be evaluated.

18. Shashi Co. Ltd has projected the following cash flows from a project under evaluation:

Year	0	1	2	3
₹ (in lakhs)	(72)	30	40	30

The above cash flows have been made at expected prices after recognizing inflation. The firm's cost of capital is 10%. The expected annual rate of inflation is 5%. Show how the viability of the project is to be evaluated. PVF at 10% for 1-3 years are 0.909, 0.826 and 0.751.

19. KLM Ltd. requires ₹ 15,00,000 for a new project.

Useful life of project is 3 years.

Salvage value - NIL.

Depreciation is ₹ 5,00,000 p.a.

Given below are projected revenues and costs (excluding depreciation) ignoring inflation:

Year →	1	2	3
Revenues in ₹	10,00,000	13,00,000	14,00,000
Costs in ₹	5,00,000	6,00,000	6,50,000

Applicable tax rate is 35%. Assume nominal cost of capital to be 14% (after tax). The inflation rates for revenues and costs are as under:

Year	Revenues %	Costs %
1	9	10
2	8	9
3	6	7

PVF at 14%, for 3 years = 0.877, 0.769 and 0.675

Show amount to the nearest rupee in calculations.

You are required to calculate net present value of the project.

20. A firm has an investment proposal, requiring an outlay of ₹ 80,000. The investment proposal is expected to have two years economic life with no salvage value. In year 1, there is a 0.4 probability that cash inflow after tax will be ₹ 50,000 and 0.6 probability that cash inflow after tax will be ₹ 60,000. The probability assigned to cash inflow after tax for the year 2 is as follows:

The cash inflow year 1	₹ 50,000	₹ 60,000		
The cash inflow year 2	Probability	Probability		
	₹ 24,000	0.2	₹ 40,000	0.4
	₹ 32,000	0.3	₹ 50,000	0.5
	₹ 44,000	0.5	₹ 60,000	0.1

The firm uses a 10% discount rate for this type of investment.

Required:

- (i) Construct a decision tree for the proposed investment project and calculate the expected net present value (NPV).
- (ii) What net present value will the project yield, if worst outcome is realized? What is the probability of occurrence of this NPV?

- (iii) What will be the best outcome and the probability of that occurrence?
- (iv) Will the project be accepted?

(Note: 10% discount factor 1 year 0.909; 2 year 0.826)

21. Jumble Consultancy Group has determined relative utilities of cash flows of two forthcoming projects of its client company as follows:

Cash Flow in ₹	-15000	-10000	-4000	0	15000	10000	5000	1000
Utilities	-100	-60	-3	0	40	30	20	10

The distribution of cash flows of project A and Project B are as follows:

Project A

Cash Flow (₹)	-15000	- 10000	15000	10000	5000
Probability	0.10	0.20	0.40	0.20	0.10

Project B

Cash Flow (₹)	- 10000	-4000	15000	5000	10000
Probability	0.10	0.15	0.40	0.25	0.10

Which project should be selected and why ?

22. A & Co. is contemplating whether to replace an existing machine or to spend money on overhauling it. A & Co. currently pays no taxes. The replacement machine costs ₹ 90,000 now and requires maintenance of ₹ 10,000 at the end of every year for eight years. At the end of eight years it would have a salvage value of ₹ 20,000 and would be sold. The existing machine requires increasing amounts of maintenance each year and its salvage value falls each year as follows:

Year	Maintenance ₹)	Salvage ₹)
Present	0	40,000
1	10,000	25,000
2	20,000	15,000
3	30,000	10,000
4	40,000	0

The opportunity cost of capital for A & Co. is 15%.

Required:

When should the company replace the machine?

(Notes: Present value of an annuity of Re. 1 per period for 8 years at interest rate of 15% : 4.4873; present value of Re. 1 to be received after 8 years at interest rate of 15% : 0.3269).

23. A company has an old machine having book value zero – which can be sold for ₹ 50,000. The company is thinking to choose one from following two alternatives:

- (i) To incur additional cost of ₹ 10,00,000 to upgrade the old existing machine.
- (ii) To replace old machine with a new machine costing ₹ 20,00,000 plus installation cost ₹ 50,000.

Both above proposals envisage useful life to be five years with salvage value to be nil.

The expected after tax profits for the above three alternatives are as under :

Year	Old existing Machine (₹)	Upgraded Machine (₹)	New Machine (₹)
1	5,00,000	5,50,000	6,00,000
2	5,40,000	5,90,000	6,40,000
3	5,80,000	6,10,000	6,90,000
4	6,20,000	6,50,000	7,40,000
5	6,60,000	7,00,000	8,00,000

The tax rate is 40 per cent.

The company follows straight line method of depreciation. Assume cost of capital to be 15 per cent.

P.V.F. of 15%, 5 = 0.870, 0.756, 0.658, 0.572 and 0.497. You are required to advise the company as to which alternative is to be adopted.

24. Company X is forced to choose between two machines A and B. The two machines are designed differently but have identical capacity and do exactly the same job. Machine A costs ₹ 1,50,000 and will last for 3 years. It costs ₹ 40,000 per year to run. Machine B is an 'economy' model costing only ₹ 1,00,000, but will last only for 2 years, and costs ₹ 60,000 per year to run. These are real cash flows. The costs are forecasted in rupees of constant purchasing power. Ignore tax. Opportunity cost of capital is 10 per cent. Which machine company X should buy?

25. Company Y is operating an elderly machine that is expected to produce a net cash inflow of ₹ 40,000 in the coming year and ₹ 40,000 next year. Current salvage value is ₹ 80,000 and next year's value is ₹ 70,000. The machine can be replaced now with a new machine, which costs ₹ 1,50,000, but is much more efficient and will provide a cash inflow of ₹ 80,000 a year for 3 years. Company Y wants to know whether it should replace the equipment now or wait a year with the clear understanding that the new machine is the best of the available alternatives and that it in turn be replaced at the optimal point. Ignore tax. Take opportunity cost of capital as 10 per cent. Advise with reasons.
26. A machine used on a production line must be replaced at least every four years. Costs incurred to run the machine according to its age are:

Age of the Machine (years)					
	0	1	2	3	4
Purchase price (in ₹)	60,000				
Maintenance (in ₹)		16,000	18,000	20,000	20,000
Repair (in ₹)		0	4,000	8,000	16,000
Scrap Value (in ₹)		32,000	24,000	16,000	8,000

Future replacement will be with identical machine with same cost. Revenue is unaffected by the age of the machine. Ignoring inflation and tax, determine the optimum replacement cycle. PV factors of the cost of capital of 15% for the respective four years are 0.8696, 0.7561, 0.6575 and 0.5718.

27. Trouble Free Solutions (TFS) is an authorized service center of a reputed domestic air conditioner manufacturing company. All complaints/service related matters of Air conditioner are attended by this service center. The service center employs a large number of mechanics, each of whom is provided with a motor bike to attend the complaints. Each mechanic travels approximately 40000 kms per annum. TFS decides to continue its present policy of always buying a new bike for its mechanics but wonders whether the present policy of replacing the bike every three year is optimal or not. It is believed that as new models are entering into market on yearly basis, it wishes to consider whether a replacement of either one year or two years would be better option than present three year period. The fleet of bike is due for replacement shortly in near future.

The purchase price of latest model bike is ₹ 55,000. Resale value of used bike at current prices in market is as follows:

Period	₹
1 Year old	35,000
2 Year old	21,000
3 Year old	9,000

Running and Maintenance expenses (excluding depreciation) are as follows:

Year	Road Taxes Insurance etc. (₹)	Petrol Repair Maintenance etc. (₹)
1	3,000	30,000
2	3,000	35,000
3	3,000	43,000

Using opportunity cost of capital as 10% you are required to determine optimal replacement period of bike.

ANSWERS/ SOLUTIONS

Answers to Theoretical Questions

1. This approach recognizes risk in capital budgeting analysis by adjusting estimated cash flows and employs risk free rate to discount the adjusted cash-flows. Under this method, the expected cash flows of the project are converted to equivalent riskless amounts. The greater the risk of an expected cash flow, the smaller the certainty equivalent values for receipts and longer the CE value for payment. This approach is superior to the risk adjusted discounted approach as it can measure risk more accurately.

This is yet another approach for dealing with risk in capital budgeting to reduce the forecasts of cash flows to some conservative levels. In certainty Equivalent approach we incorporate risk to adjust the cash flows of a proposal so as to reflect the risk element. The certainty Equivalent approach adjusts future cash flows rather than discount rates. This approach explicitly recognizes risk, but the procedure for reducing the forecasts of cash flows is implicit and likely to be inconsistent from one investment to another.

2. Sensitivity analysis is used in Capital budgeting for more precisely measuring the risk. It helps in assessing information as to how sensitive are the estimated parameters of the project such as cash flows, discount rate, and the project life to the estimation errors. Future being always uncertain and estimations are always subject to error, sensitivity analysis takes care of estimation errors by using a number of possible outcomes in evaluating a project. The

methodology adopted in sensitivity analysis is to evaluate a project by using a number of estimated cash flows so as to provide to the decision maker an insight into the variability of outcome. Thus, it is a technique of risk analysis which studies the responsiveness of a criterion of merit like NPV or IRR to variation in underlying factors like selling price, quantity sold, returns from an investment etc.

Sensitivity analysis answers questions like,

- (i) What happens to the present value (or some other criterion of merit) if flows are, say ₹ 50,000 than the expected ₹ 80,000?
- (ii) What will happen to NPV if the economic life of the project is only 3 years rather than expected 5 years?

Therefore, wherever there is an uncertainty, of whatever type, the sensitivity analysis plays a crucial role. However, it should not be viewed as the method to remove the risk or uncertainty, it is only a tool to analyse and measure the risk and uncertainty. In terms of capital budgeting the possible cash flows are based on three assumptions:

- (a) Cash flows may be worst (pessimistic)
- (b) Cash flows may be most likely.
- (c) Cash flows may be most optimistic.

Sensitivity analysis involves three steps

- (1) Identification of all those variables having an influence on the project's NPV or IRR.
- (2) Definition of the underlying quantitative relationship among the variables.
- (3) Analysis of the impact of the changes in each of the variables on the NPV of the project.

The decision maker, in sensitivity analysis always asks himself the question – what if?

3. Project Appraisal normally involves feasibility evaluation from technical, commercial, economic and financial aspects. It is generally an exercise in measurement and analysis of cash flows expected to occur over the life of the project. The project cash outflows usually occur initially and inflows come in the future.

During inflationary conditions, the project cost increases on all heads viz. labour, raw material, fixed assets such as equipments, plant and machinery, building material, remuneration of technicians and managerial personnel etc. Beside this, inflationary conditions

erode purchasing power of consumers and affect the demand pattern. Thus, not only cost of production but also the projected statement of profitability and cash flows are affected by the change in demand pattern. Even financial institutions and banks may revise their lending rates resulting in escalation in financing cost during inflationary conditions. Under such circumstances, project appraisal has to be done generally keeping in view the following guidelines which are usually followed by government agencies, banks and financial institutions.

- (i) It is always advisable to make provisions for cost escalation on all heads of cost, keeping in view the rate of inflation during likely period of delay in project implementation.
- (ii) The various sources of finance should be carefully scrutinized with reference to probable revision in the rate of interest by the lenders and the revision which could be affected in the interest-bearing securities to be issued. All these factors will push up the cost of funds for the organization.
- (iii) Adjustments should be made in profitability and cash flow projections to take care of the inflationary pressures affecting future projections.
- (iv) It is also advisable to examine the financial viability of the project at the revised rates and assess the same with reference to economic justification of the project. The appropriate measure for this aspect is the economic rate of return for the project which will equate the present value of capital expenditures to net cash flows over the life of the projects. The rate of return should be acceptable which also accommodates the rate of inflation per annum.
- (v) In an inflationary situation, projects having early payback periods should be preferred because projects with long payback period are riskier.

Under conditions of inflation, the project cost estimates that are relevant for a future date will suffer escalation. Inflationary conditions will tend to initiate the measurement of future cash flows. Either of the following two approaches may be used while appraising projects under such conditions:

- (a) Adjust each year's cash flows to an inflation index, recognizing selling price increases and cost increases annually; or
- (b) Adjust the 'Acceptance Rate' (cut-off) suitably retaining cash flow projections at current price levels.

An example of approach (ii) above can be as follows:

Normal Acceptance Rate : 15.0%

Expected Annual Inflation : 5.0%

Adjusted Discount Rate : 15.0×1.05 or 15.75%

It must be noted that measurement of inflation has no standard approach nor is easy. This makes the job of appraisal a difficult one under such conditions.

4. Please refer paragraph 4.3.3

Answers to the Practical Questions

1. (i) Expected NPV

(₹ in lakhs)

Year I			Year II			Year III		
CFAT	P	CF×P	CFAT	P	CF×P	CFAT	P	CF×P
14	0.1	1.4	15	0.1	1.5	18	0.2	3.6
18	0.2	3.6	20	0.3	6.0	25	0.5	12.5
25	0.4	10.0	32	0.4	12.8	35	0.2	7.0
40	0.3	12.0	45	0.2	9	48	0.1	4.8
	\bar{x} or $\bar{C}F$	<u>27.0</u>		\bar{x} or $\bar{C}F$	<u>29.3</u>		\bar{x} or $\bar{C}F$	<u>27.9</u>

NPV	PV factor @ 6%	Total PV
27	0.943	25.461
29.3	0.890	26.077
27.9	0.840	<u>23.436</u>
	PV of cash inflow	74.974
	Less: Cash outflow	<u>50.000</u>
	NPV	<u>24.974</u>

(ii) Possible deviation in the expected value

Year I				
X - \bar{X}	X - \bar{X}	$(X - \bar{X})^2$	P ₁	$(X - \bar{X})^2 P_1$
14 - 27	-13	169	0.1	16.9
18 - 27	-9	81	0.2	16.2

25 - 27	-2	4	0.4	1.6
40 - 27	13	169	0.3	50.7
				<u>85.4</u>

$$\sigma_1 = \sqrt{85.4} = 9.241$$

Year II				
X - \bar{X}	X - \bar{X}	(X - \bar{X}) ²	P ₂	(X - \bar{X}) ² × P ₂
15-29.3	-14.3	204.49	0.1	20.449
20-29.3	-9.3	86.49	0.3	25.947
32-29.3	2.7	7.29	0.4	2.916
45-29.3	15.7	246.49	0.2	49.298
				<u>98.61</u>

$$\sigma_2 = \sqrt{98.61} = 9.930$$

Year III				
X - \bar{X}	X - \bar{X}	(X - \bar{X}) ²	P ₃	(X - \bar{X}) ² × P ₃
18-27.9	-9.9	98.01	0.2	19.602
25-27.9	-2.9	8.41	0.5	4.205
35-27.9	7.1	50.41	0.2	10.082
48-27.9	20.1	404.01	0.1	<u>40.401</u>
				<u>74.29</u>

$$\sigma_3 = \sqrt{74.29} = 8.619$$

Standard deviation about the expected value:

$$\sigma = \sqrt{\frac{85.4}{(1.06)^2} + \frac{98.61}{(1.06)^4} + \frac{74.29}{(1.06)^6}} = 14.3696$$

- (iii) Standard deviation is a statistical measure of dispersion; it measures the deviation from a central number i.e. the mean.

In the context of capital budgeting decisions especially where we take up two or more projects giving somewhat similar mean cash flows, by calculating standard deviation in such cases, we can measure in each case the extent of variation. It can then be used to identify which of the projects is least risky in terms of variability of cash flows.

A project, which has a lower coefficient of variation will be preferred if sizes are heterogeneous.

Besides this, if we assume that probability distribution is approximately normal we are able to calculate the probability of a capital budgeting project generating a net present value less than or more than a specified amount.

2. Project A

Expected Net Cash flow (ENCF)

$$0.3 (6,00,000) + 0.4 (4,00,000) + 0.3 (2,00,000) = 4,00,000$$

$$\sigma^2 = 0.3 (6,00,000 - 4,00,000)^2 + 0.4 (4,00,000 - 4,00,000)^2 + 0.3 (2,00,000 - 4,00,000)^2$$

$$\sigma = \sqrt{24,00,00,00,00}$$

$$\sigma = 1,54,919.33$$

$$\text{Present Value of Expected Cash Inflows} = 4,00,000 \times 4.100 = 16,40,000$$

$$\text{NPV} = 16,40,000 - 5,00,000 = 11,40,000$$

Project B

$$\text{ENCF} = 0.3 (5,00,000) + 0.4 (4,00,000) + 0.3 (3,00,000) = 4,00,000$$

$$\sigma^2 = 0.3 (5,00,000 - 4,00,000)^2 + 0.4 (4,00,000 - 4,00,000)^2 + 0.3 (3,00,000 - 4,00,000)^2$$

$$\sigma = \sqrt{6,00,00,00,00}$$

$$\sigma = 77,459.66$$

$$\text{Present Value of Expected Cash Inflows} = 4,00,000 \times 4.100 = 16,40,000$$

$$\text{NPV} = 16,40,000 - 5,00,000 = 11,40,000$$

Recommendation: NPV in both projects being the same, the project should be decided on the basis of standard deviation and hence project 'B' should be accepted having lower standard deviation, means less risky.

3. (i) On the basis of standard deviation project X be chosen because it is less risky than Project Y having higher standard deviation.

$$(ii) \quad CV_x = \frac{SD}{ENPV} = \frac{90,000}{1,22,000} = 0.738$$

$$CV_y = \frac{1,20,000}{2,25,000} = 0.533$$

On the basis of Co-efficient of Variation (C.V.) Project X appears to be riskier and hence Y should be accepted.

- (iii) However, the NPV method in such conflicting situation is best because the NPV method is in compatibility of the objective of wealth maximisation in terms of time value.

4. Calculation of Variance and Standard Deviation

Project K

Expected Net Cash Flow

$$= (0.10 \times 11) + (0.20 \times 13) + (0.40 \times 15) + (0.20 \times 17) + (0.10 \times 19)$$

$$= 1.1 + 2.6 + 6 + 3.4 + 1.9 = 15$$

$$\sigma^2 = 0.10(11 - 15)^2 + 0.20(13 - 15)^2 + 0.40(15 - 15)^2 + 0.20(17 - 15)^2 + 0.10(19 - 15)^2$$

$$= 1.6 + 0.8 + 0 + 0.8 + 1.6 = 4.8$$

$$\sigma = \sqrt{4.8} = 2.19$$

Project S

Expected Net Cash Flow

$$= (0.10 \times 9) + (0.25 \times 13) + (0.30 \times 17) + (0.25 \times 21) + (0.10 \times 25)$$

$$= 0.9 + 3.25 + 5.1 + 5.25 + 2.5 = 17$$

$$\sigma^2 = 0.1(9 - 17)^2 + 0.25(13 - 17)^2 + 0.30(17 - 17)^2 + 0.25(21 - 17)^2 + 0.10(25 - 17)^2$$

$$= 6.4 + 4 + 0 + 4 + 6.4 = 20.8$$

$$\sigma = \sqrt{20.8} = 4.56$$

Calculation of Coefficient of Variation

$$\text{Coefficient of Variation} = \frac{\text{Standard Deviation}}{\text{Mean}}$$

$$\text{Project K} = \frac{2.19}{15} = 0.146$$

$$\text{Project S} = \frac{4.56}{17} = 0.268$$

Project S is riskier as it has higher Coefficient of Variation.

5. (a) Calculation of NPV of XY Co.:

Project X		Cash flow	PVF	PV
Year				
1	$(30 \times 0.3) + (50 \times 0.4) + (65 \times 0.3)$	48.5	0.909	44.09
2	$(30 \times 0.3) + (40 \times 0.4) + (55 \times 0.3)$	41.5	0.826	34.28
3	$(30 \times 0.3) + (40 \times 0.4) + (45 \times 0.3)$	38.5	0.751	<u>28.91</u>
				<u>107.28</u>
	NPV: $(107.28 - 70.00) =$			(+) <u>37.28</u>

Project Y (For 1-3 Years)

1-3	$(40 \times 0.2) + (45 \times 0.5) + (50 \times 0.3)$	45.5	2.487	<u>113.16</u>
	NPV $(113.16 - 80.00)$			(+) <u>33.16</u>

(b) Calculation of Standard deviation σ

As per Hiller's model

$$M = \sum_{i=0}^n (1+r)^{-1} M_i$$

$$\sigma^2 = \sum_{i=0}^n (1+r)^{-2i} \sigma_i^2$$

Hence

Project X

Year

$$1 \quad \sqrt{(30 - 48.5)^2 0.30 + (50 - 48.5)^2 0.40 + (65 - 48.5)^2 0.30} = \sqrt{185.25} = 13.61$$

$$2 \quad \sqrt{(30 - 41.5)^2 0.30 + (40 - 41.5)^2 0.40 + (55 - 41.5)^2 0.30} = \sqrt{95.25} = 9.76$$

$$3 \quad \sqrt{(30 - 38.5)^2 0.30 + (40 - 38.5)^2 0.40 + (45 - 38.5)^2 0.30} = \sqrt{35.25} = 5.94$$

Standard Deviation about the expected value

$$\begin{aligned}
 &= \sqrt{\frac{185.25}{(1+0.10)^2} + \frac{95.25}{(1+0.10)^4} + \frac{35.25}{(1+0.10)^6}} \\
 &= \sqrt{\frac{185.25}{1.21} + \frac{95.25}{1.4641} + \frac{35.25}{1.7716}} = \sqrt{153.10 + 65.06 + 19.90} \\
 &= \sqrt{238.06} = 15.43
 \end{aligned}$$

Project Y (For 1-3 Years)

$$\sqrt{(40 - 45.5)^2 0.20 + (45 - 45.5)^2 0.50 + (50 - 45.5)^2 0.30} = \sqrt{12.25} = 3.50$$

Standard Deviation about the expected value

$$\begin{aligned}
 &= \sqrt{\frac{12.25}{(1+0.10)^2} + \frac{12.25}{(1+0.10)^4} + \frac{12.25}{(1+0.10)^6}} \\
 &= \sqrt{\frac{12.25}{1.21} + \frac{12.25}{1.4641} + \frac{12.25}{1.7716}} = \sqrt{10.12 + 8.37 + 6.91} \\
 &= \sqrt{25.4} = 5.03
 \end{aligned}$$

Analysis: Project Y is less risky as its Standard Deviation is less than Project X.

6. (i) Statement showing computation of expected net present value of Projects A and B:

Project A			Project B		
NPV Estimate (₹)	Probability	Expected Value	NPV Estimate	Probability	Expected Value
15,000	0.2	3,000	15,000	0.1	1,500
12,000	0.3	3,600	12,000	0.4	4,800
6,000	0.3	1,800	6,000	0.4	2,400
3,000	0.2	600	3,000	0.1	300
	1.0	EV = 9,000		1.0	EV = 9,000

(ii) Computation of Standard deviation of each project

Project A

P	X	(X - EV)	P (X - EV) ²
0.2	15,000	6,000	72,00,000
0.3	12,000	3,000	27,00,000
0.3	6,000	- 3,000	27,00,000
0.2	3,000	- 6,000	<u>72,00,000</u>
			Variance = <u>1,98,00,000</u>

$$\text{Standard Deviation of Project A} = \sqrt{1,98,00,000} = ₹ 4,450$$

Project B

P	X	(X - EV)	P (X - EV) ²
0.1	15,000	6,000	36,00,000
0.4	12,000	3,000	36,00,000
0.4	6,000	- 3,000	36,00,000
0.1	3,000	- 6,000	<u>36,00,000</u>
			Variance = <u>1,44,00,000</u>

$$\text{Standard Deviation of Project A} = \sqrt{1,44,00,000} = ₹ 3,795$$

(iii) Computation of profitability of each project

Profitability index = Discount cash inflow / Initial outlay

$$\text{In case of Project A : PI} = \frac{9,000 + 36,000}{36,000} = \frac{45,000}{36,000} = 1.25$$

$$\text{In case of Project B : PI} = \frac{9,000 + 30,000}{30,000} = \frac{39,000}{30,000} = 1.30$$

- (iv)** Measurement of risk is made by the possible variation of outcomes around the expected value and the decision will be taken in view of the variation in the expected value where two projects have the same expected value, the decision will be the project which has smaller variation in expected value. In the selection of one of the two projects A and B, Project B is preferable because the possible profit which may occur is subject to less variation (or dispersion). Much higher risk is lying with project A.

7. (a) (i) **Expected cash flows:-**

Year		Net cash flows	P.V.	PV. @ 10%
0	(4,00,000 x 1) =	(-)4,00,000	1.000	(-)4,00,000
1 to 4	(1,00,000x0.3+1,10,000x0.5 + 1,20,000 x 0.2)	1,09,000	3.170	3,45,530
5	[1,09,000 + (20,000 x 0.3 + 50,000 x 0.5 + 60,000 x 0.2)]	1,52,000	0.621	94,392
	NPV =			39,922

(ii) **ENPV of the worst case**

$1,00,000 \times 3.790 = ₹ 3,79,000$ (Students may have 3.791 also the values will change accordingly)

$20,000 \times 0.621 = ₹ 12,420/-$

$ENPV = (-) 4,00,000 + 3,79,000 + 12,420 = (-) ₹ 8,580/-$

ENPV of the best case

$ENPV = (-) 4,00,000 + 1,20,000 \times 3.790 + 60,000 \times 0.621 = ₹ 92,060/-$

(iii) (a) Required probability = 0.3

(b) Required probability = $(0.3)^5 = 0.00243$

(iv) The base case NPV = $(-) 4,00,000 + (1,10,000 \times 3.79) + (50,000 \times 0.621)$

$= ₹ 47,950/-$

$ENPV = 0.30 \times (-) 8580 + 0.5 \times 47950 + 92060 \times 0.20 = ₹ 39,813/-$

Therefore,

$$\sigma_{ENPV} = \sqrt{0.3(-8580 - 39,813)^2 + 0.5(47950 - 39,813)^2 + 0.2(92,060 - 39,813)^2}$$

$$= ₹ 35,800/-$$

Therefore, $CV = 35,800/39,813 = 0.90$

(v) Risk adjusted out of cost of capital of X Ltd. = 10% - 1% = 9%.

NPV

Year	Expected net cash flow	PV @ 9%	
0	(-) 4,00,000	1.000	(-) 4,00,000
1 to 4	1,09,000	3.240	3,53,160
5	1,52,000	0.650	<u>98,800</u>
		ENPV =	<u>51,960</u>

Therefore, the project should be accepted.

8. The expected cash flows of the project are as follows:

Year	Pr = 0.1	Pr = 0.7	Pr = 0.2	Total
	₹	₹	₹	₹
0	-10,000	-70,000	-20,000	-1,00,000
1	2,000	21,000	8,000	31,000
2	2,000	21,000	8,000	31,000
3	2,000	21,000	8,000	31,000
4	2,000	21,000	8,000	31,000
5	2,000	21,000	8,000	31,000
5	0	14,000	6,000	20,000

- (i) NPV based on expected cash flows would be as follows:

$$\begin{aligned}
 &= -₹ 1,00,000 + \frac{₹ 31,000}{(1+0.20)^1} + \frac{₹ 31,000}{(1+0.20)^2} + \frac{₹ 31,000}{(1+0.20)^3} + \frac{₹ 31,000}{(1+0.20)^4} + \frac{₹ 31,000}{(1+0.20)^5} + \frac{₹ 20,000}{(1+0.20)^5} \\
 &= -₹ 1,00,000 + ₹ 25,833.33 + ₹ 21,527.78 + ₹ 17,939.81 + ₹ 14,949.85 + ₹ 12,458.20 \\
 &\quad + ₹ 8,037.55
 \end{aligned}$$

$$\text{NPV} = ₹ 746.52$$

- (ii) For the worst case, the cash flows from the cash flow column farthest on the left are used to calculate NPV

$$\begin{aligned}
 &= -₹ 100,000 + \frac{₹ 20,000}{(1+0.20)^1} + \frac{₹ 20,000}{(1+0.20)^2} + \frac{₹ 20,000}{(1+0.20)^3} + \frac{₹ 20,000}{(1+0.20)^4} + \frac{₹ 20,000}{(1+0.20)^5} \\
 &= -₹ 100,000 + ₹ 16,666.67 + ₹ 13,888.89 + ₹ 11,574.07 + ₹ 9,645.06 + ₹ 8037.76
 \end{aligned}$$

$$\text{NPV} = -₹ 40,187.76$$

For the best case, the cash flows from the cash flow column farthest on the right are used to calculate NPV

$$\begin{aligned}
 &= -₹ 100,000 + \frac{₹ 40,000}{(1+0.20)^1} + \frac{₹ 40,000}{(1+0.20)^2} + \frac{₹ 40,000}{(1+0.20)^3} + \frac{₹ 40,000}{(1+0.20)^4} + \frac{₹ 40,000}{(1+0.20)^5} + \frac{₹ 30,000}{(1+0.20)^5} \\
 &= -₹ 1,00,000 + ₹ 33,333.33 + ₹ 27,777.78 + ₹ 23,148.15 + ₹ 19,290.12 + ₹ 16,075.10 + ₹ 12,056.33
 \end{aligned}$$

$$\text{NPV} = ₹ 31,680.81$$

- (iii) If the cash flows are perfectly dependent, then the low cash flow in the first year will mean a low cash flow in every year. Thus, the possibility of the worst case occurring is the probability of getting ₹ 20,000 net cash flow in year 1 is 10%.

9. Calculation of NPV

$$\begin{aligned}
 \text{NPV} &= -10,00,000 + \frac{20,000 \times 20}{1.1} + \frac{30,000 \times 20}{1.21} + \frac{30,000 \times 20}{1.331} \\
 &= -₹ 10,00,000 + ₹ 3,63,636 + ₹ 4,95,868 + ₹ 4,50,789 \\
 &= ₹ 13,10,293 - ₹ 10,00,000 \\
 &= ₹ 3,10,293
 \end{aligned}$$

Measurement of sensitivity is as follows:

(a) Sales Price:-

Let the sale price/Unit be S so that the project would break even with 0 NPV.

$$\therefore 10,00,000 = \frac{20,000 \times (S - 40)}{1.1} + \frac{30,000 \times (S - 40)}{1.21} + \frac{30,000 \times (S - 40)}{1.331}$$

$$S - 40 = ₹ 10,00,000 / ₹ 65,514$$

$$S - 40 = ₹ 15.26$$

$$S = ₹ 55.26 \text{ which represents a fall of } (60 - 55.26) / 60$$

Or 0.079 or 7.9%

Alternative Method

$$\frac{10,00,000 \times 20}{13,10,293} = ₹ 15.26$$

$$\begin{aligned} S &= ₹ 40 + ₹ 15.26 \\ &= ₹ 55.26 \end{aligned}$$

Alternative Solution

If sale Price decreased by say 10%, then NPV (at Sale Price of ₹ 60 – ₹ 6 = ₹ 54)

$$NPV = -10,00,000 + \frac{20000 \times 14}{(1.1)^1} + \frac{30000 \times 14}{(1.1)^2} + \frac{30000 \times 14}{(1.1)^3}$$

$$\begin{aligned} &= -₹ 10,00,000 + ₹ 2,54,545 + ₹ 3,47,107 + ₹ 3,15,552 \\ &= -₹ 82,796 \end{aligned}$$

$$NPV \text{ decrease (\%)} = \frac{3,10,293 - (-82,796)}{3,10,293} \times 100 = 126.68\%$$

(b) Unit Cost:-

If sales price = ₹ 60 the cost price required to give a margin of ₹ 15.26 is (₹ 60 – ₹ 15.26) or ₹ 44.74 which would represent a rise of 11.85% i.e., $\left(\frac{44.74 - 40}{40} \times 100 \right)$

Alternative Solution

If unit cost increased by say 10%. The new NPV will be as follows:

$$\begin{aligned} NPV &= -10,00,000 + \frac{20000 \times 16}{(1.1)^1} + \frac{30000 \times 16}{(1.1)^2} + \frac{30000 \times 16}{(1.1)^3} \\ &= -₹ 10,00,000 + ₹ 2,90,909 + ₹ 3,96,694 + ₹ 3,60,631 \\ &= ₹ 48,234 \end{aligned}$$

$$NPV \text{ decrease (\%)} = \frac{3,10,293 - (48,234)}{3,10,293} \times 100 = 84.46\%$$

(c) Sales volume:-

The requisite percentage fall is:-

$$3,10,293 / 13,10,293 \times 100 = 23.68\%$$

Alternative Solution

If sale volume decreased by say 10%. The new NPV will be as follows:

$$\begin{aligned} \text{NPV} &= -10,00,000 + \frac{18000 \times 20}{(1.1)^1} + \frac{27000 \times 20}{(1.1)^2} + \frac{27000 \times 20}{(1.1)^3} \\ &= - ₹ 10,00,000 + ₹ 3,27,272 + ₹ 4,46,281 + ₹ 4,05,710 \\ &= ₹ 1,79,263 \end{aligned}$$

$$\text{NPV decrease (\%)} = \frac{3,10,293 - 1,79,263}{3,10,293} \times 100 = 42.22\%$$

- (d) Since PV of inflows remains at ₹13,10,293 the initial outlay must also be the same.

$$\therefore \text{Percentage rise} = 3,10,293/10,00,000 \times 100 = 31.03\%.$$

Alternative Solution

If initial outlay increased by say 10%. The new NPV will be as follows:

$$\begin{aligned} \text{NPV} &= - ₹ 11,00,000 + \frac{20000 \times 20}{(1.1)^1} + \frac{30000 \times 20}{(1.1)^2} + \frac{30000 \times 20}{(1.1)^3} \\ &= - ₹ 11,00,000 + ₹ 3,63,636 + ₹ 4,95,868 + ₹ 4,50,789 = ₹ 2,10,293 \\ \text{NPV decrease (\%)} &= \frac{3,10,293 - 2,10,293}{3,10,293} \times 100 = 32.22\% \end{aligned}$$

- (e) Present value for 1st two years.

$$\begin{aligned} &= - ₹ 10,00,000 + ₹ 4,00,000 \times 0.909 + ₹ 6,00,000 \times 0.826 \\ &= - ₹ 10,00,000 + ₹ 3,63,600 + ₹ 4,95,600 \\ &= - ₹ 10,00,000 + ₹ 8,59,200 \\ &= - ₹ 1,40,800 \end{aligned}$$

\therefore The project needs to run for some part of the third year so that the present value of return is ₹ 1,40,800. It can be computed as follows:

$$(i) 30,000 units $\times ₹ 20 \times 0.751 = ₹ 4,50,600$$$

$$(ii) \text{ Per day Production in (₹) assuming a year of 360 days} =$$

$$\frac{\text{₹ } 4,50,600}{360} = \text{₹ } 1,252$$

$$(iii) \text{ Days needed to recover ₹ } 1,40,800 = \frac{\text{₹ } 1,40,800}{\text{₹ } 1,252} = 112$$

Thus, if the project runs for 2 years and 112 days then break even would be achieved representing a fall of $\frac{(3 - 2.311)}{3} \times 100 = 22.97\%$.

10. CALCULATION OF NPV

	₹
PV of cash inflows ($\text{₹ } 45,000 \times 3.169$)	1,42,605
Initial Project Cost	<u>1,20,000</u>
NPV	<u>22,605</u>
If initial project cost is varied adversely by 10%*	
NPV (Revised) ($\text{₹ } 1,42,605 - \text{₹ } 1,32,000$)	₹ 10,605
Change in NPV ($\text{₹ } 22,605 - \text{₹ } 10,605$) / ₹ 22,605 i.e.	53.08 %
If annual cash inflow is varied adversely by 10%*	
Revised annual inflow	₹ 40,500
NPV (Revised) ($\text{₹ } 40,500 \times 3.169 - \text{₹ } 1,20,000$)	(+) ₹ 8,345
Change in NPV ($\text{₹ } 22,605 - \text{₹ } 8,345$) / ₹ 22,605	63.08 %
If cost of capital is varied adversely by 10%*	
NPV (Revised) ($\text{₹ } 45,000 \times 3.103 - \text{₹ } 1,20,000$)	(+) ₹ 19,635
Change in NPV ($\text{₹ } 22,605 - \text{₹ } 19,635$) / ₹ 22,605	13.14 %

Conclusion: Project is most sensitive to 'annual cash inflow'.

*Note: Students may please note that they may assume any other percentage rate other than 10 % say 15%, 20 % 25 % etc.

11. P.V. of Cash Flows

Year 1	Running Cost	₹ 4,000 x 0.917	= (₹ 3,668)
	Savings	₹ 12,000 x 0.917	= ₹ 11,004
Year 2	Running Cost	₹ 5,000 x 0.842	= (₹ 4,210)
	Savings	₹ 14,000 x 0.842	<u>= ₹ 11,788</u>
			₹ 14,914
Year 0	Less: P.V. of Cash Outflow	₹ 10,000 x 1	₹ 10,000
		NPV	<u>₹ 4,914</u>

Sensitivity Analysis

- (i) Increase of Plant Value by ₹ 4,914

$$\therefore \frac{4,914}{10,000} \times 100 = 49.14\%$$

- (ii) Increase in PV of Running Cost by ₹ 4,914

$$\frac{4,914}{3,668 + 4,210} = \frac{4,914}{7,878} \times 100 = 62.38\%$$

- (iii) Fall in PV of Saving by ₹ 4,914

$$\frac{4,914}{11,004 + 11,788} = \frac{4,914}{22,792} \times 100 = 21.56\%$$

Hence, savings factor is the most sensitive to affect the acceptability of the project as in comparison of other two factors a slight % change in this fact shall more affect the NPV than others.

Alternative Solution

P.V. of Cash Flows

Year 1	Running Cost	₹ 4,000 x 0.917	= (₹ 3,668)
	Savings	₹ 12,000 x 0.917	= ₹ 11,004
Year 2	Running Cost	₹ 5,000 x 0.842	= (₹ 4,210)
	Savings	₹ 14,000 x 0.842	= ₹ 11,788
			₹ 14,914
Year 0	Less: P.V. of Cash Outflow	₹ 10,000 x 1	₹ 10,000
		NPV	₹ 4,914

Sensitivity Analysis

- (i) If the initial project cost is varied adversely by say 10%*.

$$NPV (\text{Revised}) (\text{₹ } 4,914 - \text{₹ } 1,000) = \text{₹ } 3,914$$

$$\text{Change in NPV } \frac{\text{₹ } 4,914 - \text{₹ } 3,914}{\text{₹ } 4,914} = 20.35\%$$

- (ii) If Annual Running Cost is varied by say 10%*.

$$NPV (\text{Revised}) (\text{₹ } 4,914 - \text{₹ } 400 \times 0.917 - \text{₹ } 500 \times 0.842)$$

$$= ₹ 4,914 - ₹ 367 - ₹ 421 = ₹ 4,126$$

$$\text{Change in NPV } \frac{₹ 4,914 - ₹ 4,126}{₹ 4,914} = 16.04\%$$

- (iii) If Saving is varied by say 10%*.

$$\text{NPV (Revised)} (₹ 4,914 - ₹ 1,200 \times 0.917 - ₹ 1,400 \times 0.842)$$

$$= ₹ 4,914 - ₹ 1,100 - ₹ 1,179 = ₹ 2,635$$

$$\text{Change in NPV } \frac{4,914 - 2,635}{4,914} = 46.38\%$$

Hence, savings factor is the most sensitive to affect the acceptability of the project.

* Any percentage of variation other than 10% can also be assumed.

12. (i) Initial Investment

$$\text{IRR} = 16\% \text{ (Given)}$$

At IRR, NPV shall be zero, therefore

$$\begin{aligned} \text{Initial Cost of Investment} &= \text{PVAF (16%, 5)} \times \text{Cash Flow (Annual)} \\ &= 3.274 \times ₹ 57,500 \\ &= ₹ 1,88,255 \end{aligned}$$

(ii) Net Present Value (NPV)

$$\text{Let Cost of Capital be } X, \text{ then } \frac{16-X}{X} = 60\% \quad X = 10\%$$

Thus NPV of the project

$$\begin{aligned} &= \text{Annual Cash Flow} \times \text{PVAF (10%, 5)} - \text{Initial Investment} \\ &= ₹ 57,500 \times 3.791 - ₹ 1,88,255 \\ &= ₹ 2,17,982.50 - ₹ 1,88,255 = ₹ 29,727.50 \end{aligned}$$

(iii) Annual Fixed Cost

Let change in the Fixed Cost which makes NPV zero is X. Then,

$$₹ 29,727.50 - 3.791X = 0$$

$$\text{Thus } X = ₹ 7,841.60$$

Let original Fixed Cost be Y then,

$$Y \times 7.8416\% = ₹ 7,841.60$$

$$Y = ₹ 1,00,000$$

Thus Fixed Cost is equal to ₹ 1,00,000

(iv) Estimated Annual Units of Sales

$$\text{Selling Price per unit} = \frac{₹ 60}{100\% - 70\%} = ₹ 200$$

$$\frac{\text{Annual Cash Flow} + \text{Fixed Cost}}{\text{P/V Ratio}} = \text{Sales Value}$$

$$\frac{₹ 57,500 + ₹ 1,00,000}{0.70} = ₹ 2,25,000$$

$$\text{Sales in Units} = \frac{₹ 2,25,000}{₹ 200} = 1,125 \text{ units}$$

(v) Break Even Units

$$\frac{\text{Fixed Cost}}{\text{Contribution Per Unit}} = \frac{1,00,000}{140} = 714.285 \text{ units}$$

13. Calculation of NPV

$$= - ₹ 50,00,000 + [2,00,000 (₹ 30 - ₹ 16.50) - ₹ 10,00,000] \text{ PVIAF (12%,5)}$$

$$= - ₹ 50,00,000 + [2,00,000 (₹ 13.50) - ₹ 10,00,000] 3.605$$

$$= - ₹ 50,00,000 + ₹ 27,00,000 - ₹ 10,00,000] 3.605$$

$$= - ₹ 50,00,000 + ₹ 61,28,500 = ₹ 11,28,500$$

Measurement of Sensitivity Analysis

(a) Sales Price:-

Let the sale price/Unit be S so that the project would break even with 0 NPV.

$$\therefore ₹ 50,00,000 = [2,00,000 (S - ₹ 16.50) - ₹ 10,00,000] \text{ PVIAF (12%,5)}$$

$$₹ 50,00,000 = [2,00,000S - ₹ 33,00,000 - ₹ 10,00,000] 3.605$$

$$₹ 50,00,000 = [2,00,000S - ₹ 43,00,000] 3.605$$

$$\text{₹ } 13,86,963 = 2,00,000S - \text{₹ } 43,00,000$$

$$\text{₹ } 56,86,963 = 2,00,000S$$

$S = \text{₹ } 28.43$ which represents a fall of $(30 - 28.43)/30$ or 0.0523 or 5.23%

(b) Sales volume:-

Let V be the sale volume so that the project would break even with 0 NPV.

$$\text{₹ } 50,00,000 = [V (\text{₹ } 30 - \text{₹ } 16.50) - \text{₹ } 10,00,000] \text{ PVIAF (12%,5)}$$

$$\text{₹ } 50,00,000 = [V (\text{₹ } 13.50) - \text{₹ } 10,00,000] \text{ PVIAF (12%,5)}$$

$$\text{₹ } 50,00,000 = [\text{₹ } 13.50V - \text{₹ } 10,00,000] 3.605$$

$$\text{₹ } 13,86,963 = \text{₹ } 13.50V - \text{₹ } 10,00,000$$

$$\text{₹ } 23,86,963 = \text{₹ } 13.50V$$

$V = 1,76,812$ which represents a fall of $(2,00,000 - 1,76,812)/2,00,000$ or 0.1159 or 11.59%

(c) Variable Cost:

Let the variable cost be V so that the project would break even with 0 NPV.

$$\therefore \text{₹ } 50,00,000 = [2,00,000(\text{₹ } 30 - V) - \text{₹ } 10,00,000] \text{ PVIAF(12%,5)}$$

$$\text{₹ } 50,00,000 = [\text{₹ } 60,00,000 - 2,00,000 V - \text{₹ } 10,00,000] 3.605$$

$$\text{₹ } 50,00,000 = [\text{₹ } 50,00,000 - 2,00,000 V] 3.605$$

$$\text{₹ } 13,86,963 = \text{₹ } 50,00,000 - 2,00,000 V$$

$$\text{₹ } 36,13,037 = 2,00,000V$$

$V = \text{₹ } 18.07$ which represents a fall of $(18.07 - 16.50)/16.50$ or 0.0951 or 9.51%

(d) Expected Net Present Value

$$(1,75,000 X 0.30) + (2,00,000 X 0.60) + (2,25,000 X 0.10) = 1,95,000$$

$$\text{NPV} = [1,95,000 X \text{₹ } 13.50 - \text{₹ } 10,00,000] 3.605 - \text{₹ } 50,00,000 = \text{₹ } 8,85,163$$

Further NPV in worst and best cases will be as follows:

Worst Case:

$$[1,75,000 X \text{₹ } 13.50 - \text{₹ } 10,00,000] 3.605 - \text{₹ } 50,00,000 = - \text{₹ } 88,188$$

Best Case:

$$[2,25,000 \times ₹ 13.50 - ₹ 10,00,000] 3.605 - ₹ 50,00,000 = ₹ 23,45,188$$

Thus, there are 30% chances that the rise will be a negative NPV and 70% chances of positive NPV. Since acceptable level of risk of Unnat Ltd. is 20% and there are 30% chances of negative NPV hence project should not be accepted.

14. (i) Statement Showing the Net Present Value of Project M

Year end	Cash Flow (₹) (a)	C.E. (b)	Adjusted Cash flow (₹) (c) = (a) × (b)	Present value factor at 6% (d)	Total Present value (₹) (e) = (c) × (d)
1	4,50,000	0.8	3,60,000	0.943	3,39,480
2	5,00,000	0.7	3,50,000	0.890	3,11,500
3	5,00,000	0.5	2,50,000	0.840	2,10,000
<i>Less: Initial Investment</i>					8,60,980
<i>Net Present Value</i>					8,50,000
					10,980

Statement Showing the Net Present Value of Project N

Year end	Cash Flow (₹) (a)	C.E. (b)	Adjusted Cash flow (₹) (c) = (a) × (b)	Present value factor (d)	Total Present value (₹) (e) = (c) × (d)
1	4,50,000	0.9	4,05,000	0.943	3,81,915
2	4,50,000	0.8	3,60,000	0.890	3,20,400
3	5,00,000	0.7	3,50,000	0.840	2,94,000
<i>Less: Initial Investment</i>					9,96,315
<i>Net Present Value</i>					8,25,000
					1,71,315

Decision: Since the net present value of Project N is higher, so the project N should be accepted.

- (ii)** Certainty - Equivalent (C.E.) Co-efficient of Project M (2.0) is lower than Project N (2.4). This means Project M is riskier than Project N as "higher the riskiness of a cash flow, the lower will be the CE factor". If risk adjusted discount rate (RADR) method is used, Project M would be analysed with a higher rate.

RADR is based on the premise that riskiness of a proposal may be taken care of, by adjusting the discount rate. The cash flows from a more risky proposal should be discounted at a relatively higher discount rate as compared to other proposals whose cash flows are less risky. Any investor is basically risk averse. However, he may be ready to take risk provided he is rewarded for undertaking risk by higher returns. So, more risky the investment is, the greater would be the expected return. The expected return is expressed in terms of discount rate which is also the minimum required rate of return generated by a proposal if it is to be accepted. Therefore, there is a positive correlation between risk of a proposal and the discount rate.

15. Statement showing the determination of the risk adjusted net present value

Projects	Net cash outlays	Coefficient of variation	Risk adjusted discount rate	Annual cash inflow	PV factor 1-5 years	Discounted cash inflow	Net present value
	₹			₹		₹	₹
(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii) = (v) × (vi)	(viii) = (vii) – (ii)
X	2,10,000	1.20	16%	70,000	3.274	2,29,180	19,180
Y	1,20,000	0.80	14%	42,000	3.433	1,44,186	24,186
Z	1,00,000	0.40	12%	30,000	3.605	1,08,150	8,150

- 16. (i)** The risk free rate of interest and risk factor for each of the projects are given. The risk adjusted discount rate (RADR) for different projects can be found on the basis of CAPM as follows:

$$\text{Required Rate of Return} = I_{RF} + (k_o - I_{RF}) \text{ Risk Factor}$$

$$\text{For P-I : RADR} = 0.10 + (0.15 - 0.10) 1.80 = 19\%$$

$$\text{For P-II : RADR} = 0.10 + (0.15 - 0.10) 1.00 = 15\%$$

$$\text{For P-III : RADR} = 0.10 + (0.15 - 0.10) 0.60 = 13\%$$

- (ii)** The three projects can now be evaluated at 19%, 15% and 13% discount rate as follows:

Project P-I

Annual Inflows	₹ 6,00,000
PVAF (19 %, 4)	2.639
PV of Inflows (₹ 6,00,000 x 2.639)	₹ 15,83,400

Less: Cost of Investment	<u>₹ 15,00,000</u>
Net Present Value	<u>₹ 83,400</u>

Project P-II

Year	Cash Inflow (₹)	PVF (15%,n)	PV (₹)
1	6,00,000	0.870	5,22,000
2	4,00,000	0.756	3,02,400
3	5,00,000	0.658	3,29,000
4	2,00,000	0.572	<u>1,14,400</u>
Total Present Value			12,67,800
Less: Cost of Investment			<u>11,00,000</u>
Net Present Value			<u>1,67,800</u>

Project P-III

Year	Cash Inflow (₹)	PVF (13%,n)	PV (₹)
1	4,00,000	0.885	3,54,000
2	6,00,000	0.783	4,69,800
3	8,00,000	0.693	5,54,400
4	12,00,000	0.613	<u>7,35,600</u>
Total Present Value			21,13,800
Less: Cost of Investment			<u>19,00,000</u>
Net Present Value			<u>2,13,800</u>

Project P-III has highest NPV. So, it should be accepted by the firm

17. It is stated that the cash flows have been adjusted for inflation; hence they are “nominal”. The cost of capital or discount rate is “real”. In order to be compatible, the cash flows should be converted into “real flow”. This is done as below:

Year	Nominal cash flows	Adjusted Inflation* factor	Real cash flows	PVF @ 10%	PV of cash flows
0	(70)	—	(70)	1.000	(70)
1	30	0.952	28.56	0.909	25.96
2	40	0.907	36.28	0.826	29.97

3	30	0.864	25.92	0.751	<u>19.47</u>
				Total	75.40
<i>Less: Cash out flow</i>					<u>70.00</u>
<i>NPV (+)</i>					<u>5.40</u>

* $1/1.05; 1/(1.05)^2; 1/(1.05)^3;$

Advise: With positive NPV, the project is financially viable.

Alternatively, instead of converting cash flows into real terms, the discount rate can be converted into nominal rate. Result will be the same.

An alternative solution is presented herewith

Alternative solution:

Year	Nominal cash flows	PVF @ 15.50% adjusted by the inflation factor i.e. 5%*	PV of cash flows
1	30	0.866	25.98
2	40	0.749	29.96
3	30	0.649	<u>19.47</u>
		Cash inflow	75.41
		<i>Less: Cash out flow</i>	<u>70.00</u>
		Net present value	<u>5.41</u>

$$* \frac{0.909}{1.05} = 0.866, \frac{0.826}{1.1025} = 0.749, \frac{0.751}{1.1576} = 0.649$$

Advise: With positive NPV, the project is financially viable.

18. Here the given cash flows have to be adjusted for inflation. Alternatively, the discount rate can be converted into nominal rate, as follows:-

$$\text{Year 1} = \frac{0.909}{1.05} = 0.866; \quad \text{Year 2} = \frac{0.826}{(1.05)^2} \text{ or } \frac{0.826}{1.1025} = 0.749$$

$$\text{Year 3} = \frac{0.751}{(1.05)^3} = \frac{0.751}{1.1576} = 0.649$$

Year	Nominal Cash Flows (₹ in lakhs)	Adjusted PVF as above	PV of Cash Flows (₹ in lakhs)
1	30	0.866	25.98
2	40	0.749	29.96
3	30	0.649	<u>19.47</u>
	Cash Inflow		75.41
	Less: Cash Outflow		<u>72.00</u>
	Net Present Value		<u>3.41</u>

With positive NPV, the project is financially viable.

Alternative Solution

Assumption: The cost of capital given in the question is "Real".

Nominal cost of capital = $(1.10)(1.05) - 1 = 0.155 = 15.50\%$

DCF Analysis of the project

(₹ Lakhs)

	Period	PVF @15.50%	CF	PV
Investment	0	1	-72	-72.00
Operation	1	0.866	30	+25.98
---do---	2	0.750	40	+30.00
---do---	3	0.649	30	<u>+19.47</u>
NPV				<u>+3.45</u>

The proposal may be accepted as the NPV is positive.

19. (i) Inflation adjusted Revenues

Year	Revenues (₹)	Revenues (Inflation Adjusted) (₹)
1	10,00,000	$10,00,000(1.09) = 10,90,000$
2	13,00,000	$13,00,000(1.09)(1.08) = 15,30,360$
3	14,00,000	$14,00,000(1.09)(1.08)(1.06) = 17,46,965$

(ii) Inflation adjusted Costs

Year	Costs (₹)	Costs (Inflation Adjusted) (₹)
1	5,00,000	5,00,000(1.10) = 5,50,000
2	6,00,000	6,00,000(1.10)(1.09) = 7,19,400
3	6,50,000	6,50,000(1.10)(1.09)(1.07) = 8,33,905

(iii) Tax Benefit on Depreciation = ₹ 5,00,000 x 0.35 = ₹ 1,75,000

(iv) Net Profit after Tax

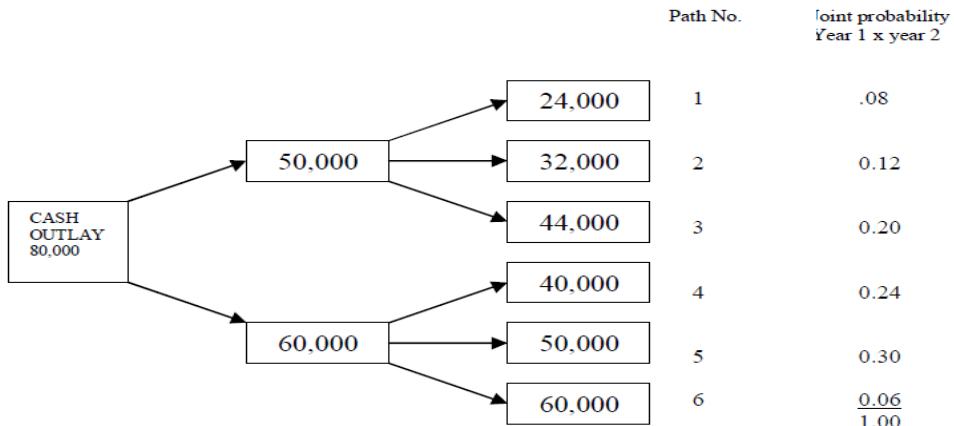
Year	Revenues (Inflation Adjusted) (₹) (1)	Costs (Inflation Adjusted) (₹) (2)	Net Profit (₹) (3) = (1) - (2)	Tax (₹) (4) = 35% of (3)	Profit after Tax (₹) (3) - (4)
1	10,90,000	5,50,000	5,40,000	1,89,000	3,51,000
2	15,30,360	7,19,400	8,10,960	2,83,836	5,27,124
3	17,46,965	8,33,905	9,13,060	3,19,571	5,93,489

(v) Present Value of Cash Inflows

Year	Net Profit after tax (₹)	Tax Benefit on Depreciation (₹)	Cash Inflow (₹)	PVF@ 14%	PV (₹)
1	3,51,000	1,75,000	5,26,000	0.877	4,61,302
2	5,27,124	1,75,000	7,02,124	0.769	5,39,933
3	5,93,489	1,75,000	7,68,489	0.675	5,18,730
					15,19,965

$$NPV = ₹ 15,19,965 - ₹ 15,00,000 = ₹ 19,965$$

20. (i) The decision tree diagram is presented in the chart, identifying various paths and outcomes, and the computation of various paths/outcomes and NPV of each path are presented in the following tables:



The Net Present Value (NPV) of each path at 10% discount rate is given below:

Path	Year 1 Cash Flows	Year 2 Cash Flows	Total Cash Inflows (PV) (₹)	Cash Inflows NPV	
	(₹)	(₹)		(₹)	(₹)
1	$50,000 \times .909 = 45,450$	$24,000 \times .826 = 19,824$	65,274	80,000	(-) 14,726
2	45,450	$32,000 \times .826 = 26,432$	71,882	80,000	(-) 8,118
3	45,450	$44,000 \times .826 = 36,344$	81,794	80,000	1,794
4	$60,000 \times .909 = 54,540$	$40,000 \times .826 = 33,040$	87,580	80,000	7,580
5	54,540	$50,000 \times .826 = 41,300$	95,840	80,000	15,840
6	54,540	$60,000 \times .826 = 49,560$	1,04,100	80,000	24,100

Statement showing Expected Net Present Value

z	NPV (₹)	Joint Probability	Expected NPV
1	-14,726	0.08	-1,178.08
2	-8,118	0.12	-974.16
3	1,794	0.20	358.80
4	7,580	0.24	1,819.20
5	15,840	0.30	4,752.00
6	24,100	0.06	<u>1,446.00</u>
			<u>6,223.76</u>

- (ii) If the worst outcome is realized the project will yield NPV of - ₹ 14,726. The probability of occurrence of this NPV is 8% and a loss of ₹ 1,178 (path 1).

- (iii) The best outcome will be path 6 when the NPV is at ₹ 24,100. The probability of occurrence of this NPV is 6% and a expected profit of ₹ 1,446.
- (iv) The project should be accepted because the expected NPV is positive at ₹ 6,223.76 based on joint probability.

21. Evaluation of project utilizes of Project A and Project B

Cash flow (in ₹)	Project A		
	Probability	Utility	Utility value
-15,000	0.10	-100	-10
-10,000	0.20	-60	-12
15,000	0.40	40	16
10,000	0.20	30	6
5,000	0.10	20	2
			<u>2</u>

Cash flow (in ₹)	Project B		
	Probability	Utility	Utility value
-10,000	0.10	-60	-6
-4,000	0.15	-3	-0.45
15,000	0.40	40	16
5,000	0.25	20	5
10,000	0.10	30	3
			<u>17.55</u>

Project B should be selected as its expected utility is more.

22. A & Co.

Equivalent cost of (EAC) of new machine

		₹
(i)	Cost of new machine now	90,000
	Add: PV of annual repairs @ ₹ 10,000 per annum for 8 years (₹ 10,000 × 4.4873)	<u>44,873</u>
		1,34,873
	Less: PV of salvage value at the end of 8 years (₹20,000×0.3269)	6,538
		<u>1,28,335</u>
	Equivalent annual cost (EAC) (₹ 1,28,335/4.4873)	<u>28,600</u>

PV of cost of replacing the old machine in each of 4 years with new machine

Scenario	Year	Cash Flow	PV @ 15%	PV
		(₹)		(₹)
Replace Immediately	0	(28,600) 40,000	1.00 1.00	(28,600) <u>40,000</u> <u>11,400</u>
Replace in one year	1	(28,600)	0.870	(24,882)
	1	(10,000)	0.870	(8,700)
	1	25,000	0.870	<u>21,750</u> <u>(11,832)</u>
Replace in two years	1	(10,000)	0.870	(8,700)
	2	(28,600)	0.756	(21,622)
	2	(20,000)	0.756	(15,120)
	2	15,000	0.756	<u>11,340</u> <u>(34,102)</u>
Replace in three years	1	(10,000)	0.870	(8,700)
	2	(20,000)	0.756	(15,120)
	3	(28,600)	0.658	(18,819)
	3	(30,000)	0.658	(19,740)
	3	10,000	0.658	<u>6,580</u> <u>(55,799)</u>
Replace in four years	1	(10,000)	0.870	(8,700)
	2	(20,000)	0.756	(15,120)
	3	(30,000)	0.658	(19,740)
	4	(28,600)	0.572	(16,359)
	4	(40,000)	0.572	<u>(22,880)</u> <u>(82,799)</u>

Advice: The company should replace the old machine immediately because the PV of cost of replacing the old machine with new machine is least.

Alternatively, optimal replacement period can also be computed using the following table:

Scenario	Year	Cashflow	PV at 15%	PV
Replace immediately	0	(40,000)	1	(40,000)
	1 to 4	28,600	2.855	81,652
				41,652
Replace after 1 year	1	10,000	0.870	8,696
	1	(25,000)	0.870	(21,739)
	2 to 4	28,600	1.985	56,783
				43,739
Replace after 2 years	1	10,000	0.870	8,696
	2	20,000	0.756	15,123
	2	(15,000)	0.756	(11,342)
	3 and 4	28,600	1.229	35,157
				47,633
Replace after 3 years	1	10,000	0.870	8,696
	2	20,000	0.756	15,123
	3	30,000	0.658	19,725
	3	(10,000)	0.658	(6,575)
	4	28,600	0.572	16,352
				53,321
Replace after 4 years	1	10,000	0.870	8,696
	2	20,000	0.756	15,123
	3	30,000	0.658	19,725
	4	40,000	0.572	22,870
				66,414

23.

(A)	Cash Outflow			₹
(i)	In case machine is upgraded:			
	Upgradation Cost			<u>10,00,000</u>
(ii)	In case new machine installed:			
	Cost			20,00,000
	Add: Installation cost			<u>50,000</u>
	Total Cost			20,50,000
	Less: Disposal of old machine			
	₹ 50,000 – 40% tax			<u>30,000</u>
	Total Cash Outflow			<u>20,20,000</u>

Working Note:

- (i) Depreciation – in case machine is upgraded
 $\text{₹ } 10,00,000 \div 5 = \text{₹ } 2,00,000$
- (ii) Depreciation – in case new machine is installed
 $\text{₹ } 20,50,000 \div 5 = \text{₹ } 4,10,000$
- (iii) Old existing machine – Book Value is zero. So, no depreciation.

(B) Cash Inflows after Taxes (CFAT)

Year	Old Existing Machine		Upgraded Machine		
	(i) EAT/CFAT ₹	(ii) EAT ₹	(iii) DEP ₹	(iv) CFAT ₹	= (iv)-(i) Incremental CFAT ₹
1	5,00,000	5,50,000	2,00,000	7,50,000	2,50,000
2	5,40,000	5,90,000	2,00,000	7,90,000	2,50,000
3	5,80,000	6,10,000	2,00,000	8,10,000	2,30,000
4	6,20,000	6,50,000	2,00,000	8,50,000	2,30,000
5	6,60,000	7,00,000	2,00,000	9,00,000	2,40,000

Cash Inflow after Taxes (CFAT)

Year	New Machine			
	(vi) EAT ₹	(vii) DEP ₹	(viii) CFAT ₹	(ix) = (viii) – (i) Incremental CFAT ₹
1	6,00,000	4,10,000	10,10,000	5,10,000
2	6,40,000	4,10,000	10,50,000	5,10,000
3	6,90,000	4,10,000	11,00,000	5,20,000
4	7,40,000	4,10,000	11,50,000	5,30,000
5	8,00,000	4,10,000	12,10,000	5,50,000

P.V. AT 15% - 5 Years – on Incremental CFAT

Year	Upgraded Machine			New Machine		
	Incremental CFAT ₹	PVF	Total P.V. ₹	Incremental CFAT	PVF	Total P.V. ₹
1	2,50,000	0.870	2,17,500	5,10,000	0.870	4,43,700
2	2,50,000	0.756	1,89,000	5,10,000	0.756	3,85,560
3	2,30,000	0.658	1,51,340	5,20,000	0.658	3,42,160
4	2,30,000	0.572	1,31,560	5,30,000	0.572	3,03,160
5.	2,40,000	0.497	1,19,280	5,50,000	0.497	2,73,350
Total P.V. of CFAT			8,08,680			17,47,930
Less: Cash Outflows			10,00,000			20,20,000*
N.P.V. =			-1,91,320			- 2,72,070

*Acquisition Cost (including installation cost) ₹ 20,50,000

Less: Salvage Value of existing machine net of Tax ₹ 30,000

₹ 20,20,000

As the NPV in both the new (alternative) proposals is negative, the company should continue with the existing old Machine.

24. Statement showing the evaluation of two machines

Machines	A	B
Purchase cost (₹): (i)	1,50,000	1,00,000
Life of machines (years)	3	2
Running cost of machine per year (₹): (ii)	40,000	60,000
Cumulative present value factor for 1-3 years @ 10% (iii)	2.486	—
Cumulative present value factor for 1-2 years @ 10% (iv)	—	1.735
Present value of running cost of machines (₹): (v)	99,440	1,04,100
	[(ii) × (iii)]	[(ii) × (iv)]
Cash outflow of machines (₹): (vi) = (i) + (v)	2,49,440	2,04,100
Equivalent present value of annual cash outflow	1,00,338	1,17,637
	[(vi) ÷ (iii)]	[(vi) ÷ (iv)]

Decision: Company X should buy machine A since its equivalent cash outflow is less than machine B.

25. Statement showing present value of cash inflow of new machine when it replaces elderly machine now

NPV of New Machine

PV of Cash Inflow (80000 x 2.486)	₹ 1,98,880
Less: Purchase Cost of New Machine	₹ 1,50,000
	₹ 48,880

Since NPV of New Machine is positive, it should be purchased.

Timing Decision

Replace Now

Current Realizable Value	₹ 80,000
NPV of New Machine	₹ 48,880
Total NPV	₹ 1,28,880

Replace after 1 Year

Cash Inflow for Year 1	₹ 40000
Realisable Value of Old Machine	₹ 70000

NPV of New Machine	₹ 48,880
Total NPV after 1 Year	₹ 1,58,880
PV of Total NPV (158880/1.1)	₹ 1,44,436

Advise: Since Total NPV is higher in case of Replacement after one year Machine should be replaced after 1 year.

26. Working Notes

First of all, we shall calculate cash flows for each replacement cycle as follows:

One Year Replacement Cycle

Year	Replacement Cost	Maintenance & Repair	Residual Value	Net cash Flow
0	(60,000)	-	-	(60,000)
1	-	(16,000)	32,000	16,000

Two Years Replacement Cycle

Year	Replacement Cost	Maintenance & Repair	Residual Value	Net cash Flow
0	(60,000)	-	-	(60,000)
1	-	(16,000)	-	(16,000)
2	-	(22,000)	24,000	2,000

Three Years Replacement Cycle

Year	Replacement Cost	Maintenance & Repair	Residual Value	Net cash Flow
0	(60,000)	-	-	(60,000)
1	-	(16,000)	-	(16,000)
2	-	(22,000)	-	(22,000)
3	-	(28,000)	16,000	(12,000)

Four Years Replacement Cycle

Year	Replacement Cost	Maintenance & Repair	Residual Value	Net cash Flow
0	(60,000)	-	-	(60,000)
1	-	(16,000)	-	(16,000)
2	-	(22,000)	-	(22,000)
3	-	(28,000)	-	(28,000)
4	-	(36,000)	8,000	(28,000)

Now we shall calculate NPV for each replacement cycles

		1 Year		2 Years		3 Years		4 Years	
Year	PVF@ 15%	Cash Flows	PV						
0	1	-60,000	-60,000	-60,000	-60,000	-60,000	-60,000	-60,000	-60,000
1	0.8696	16,000	13,914	-16,000	-13,914	-16,000	-13,914	-16,000	-13,914
2	0.7561	-	-	2,000	1,512	-22,000	-16,634	-22,000	-16,634
3	0.6575	-	-	-	0	-12,000	-7,890	-28,000	-18,410
4	0.5718	-	-	-	0		0	-28,000	-16,010
			-46,086		-72,402		-98,438		-1,24,968

Replacement Cycles		EAC (₹)	
1 Year		46,086	52,997
		0.8696	
2 Years		72,402	44,536
		1.6257	
3 Years		98,438	43,114
		2.2832	
4 Years		124,968	43,772
		2.855	

Since EAC is least in case of replacement cycle of 3 years hence machine should be replaced after every three years.

27. In this question the effect of increasing running cost and decreasing resale value have to be weighted upto against the purchase cost of bike. For this purpose, we shall compute Equivalent Annual Cost (EAC) of replacement in different years shall be computed and compared.

Year	Road Taxes (₹)	Petrol etc. (₹)	Total (₹)	PVF @10%	PV (₹)	Cumulative PV (₹)	PV of Resale Price (₹)	Net Outflow (₹)
1	3,000	30,000	33,000	0.909	29,997	29,997	31,815	(1,818)
2	3,000	35,000	38,000	0.826	31,388	61,385	17,346	44,039
3	3,000	43,000	46,000	0.751	34,546	95,931	6,759	89,172

Computation of EACs

Year*	Purchase Price of Bike (₹)	Net Outflow (₹)	Total Outflow (₹)	PVAF @ 10%	EAC [‡] (₹)
1	55,000	(1,818)	53,182	0.909	58,506
2	55,000	44,039	99,039	1.735	57,083
3	55,000	89,172	1,44,172	2.486	57,993

Thus, from above table it is clear that EAC is least in case of 2 years, hence bike should be replaced every two years.

* Assume these periods are the periods from which bike shall be kept in use.

‡ EAC is used to bring Cash Flows occurring for different periods at one point of Time.