PORTFOLIO MANAGEMENT

LEARNING OUTCOMES

After going through the chapter student shall be able to understand

❑ Activities in Portfolio Management

❑ Objectives of Portfolio Management

❑ Phases of Portfolio Management

    (1)   Security Analysis

    (2)   Portfolio Analysis

    (3)   Portfolio Selection

    (4)   Portfolio Revision

    (5)   Portfolio Evaluation

❑ Portfolio Theories

    (1)   Traditional Approach

    (2)   Modern Approach (Markowitz Model or Risk-Return Optimization)

❑ Risk Analysis

   (1)   Types of Risk

    (2)   Diversion of Risk

    (3)   Risk & Return

    (4)   Portfolio Analysis

❑ Markowitz Model of Risk-Return Optimization

❑ Capital Market Theory

❑ Sharpe Index Model (Single Index Model)

❑ Capital Asset Pricing Model (CAPM)

❑ Arbitrage Pricing Theory Model (APT)

❑ Portfolio Evaluation Methods

❑ Sharpe's Optimal Portfolio

❑ Formulation of Portfolio Strategy

❑ Portfolio Revision and Rebalancing

❑ Asset Allocation Strategies

❑ Fixed Income Portfolio

❑ Alternative Investment Avenues
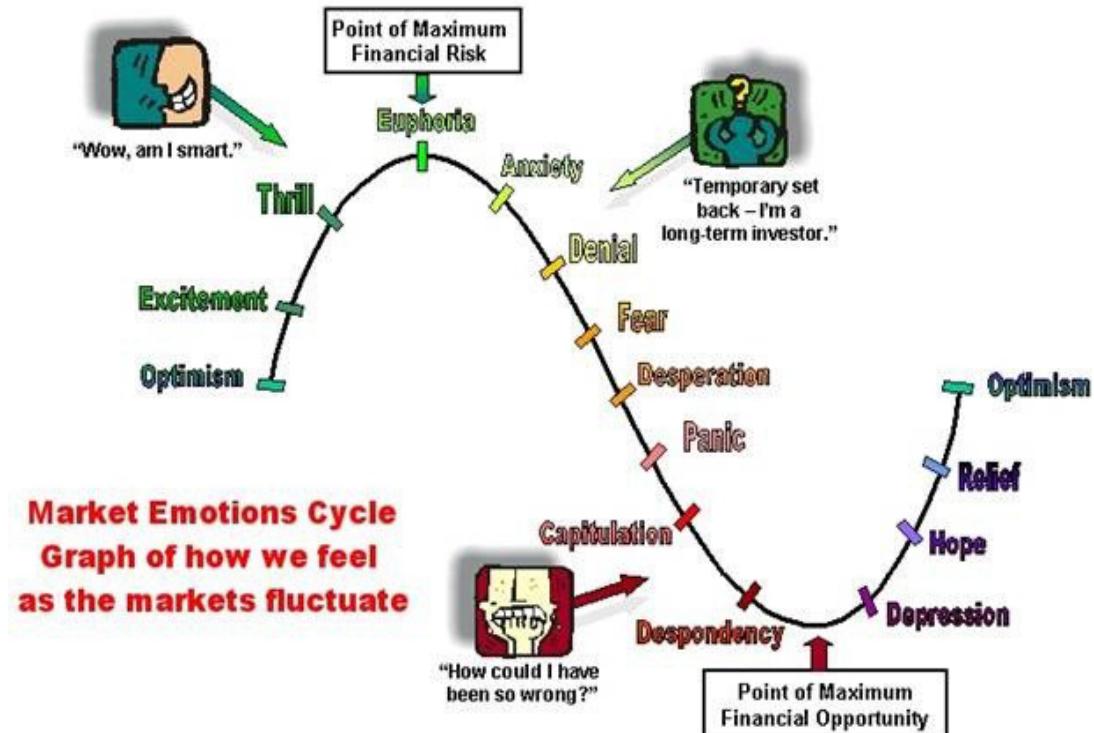
1. INTRODUCTION

Investment in the securities such as bonds, debentures and shares etc. is lucrative as well as exciting for the investors. Though investment in these securities may be rewarding, it is also fraught with risk. Therefore, investment in these securities requires a good amount of scientific and analytical skills. As per the famous principle of not putting all eggs in the same basket, an investor never invests his entire investable funds in one security. He invests in a well diversified portfolio of a number of securities which will optimize the overall risk-return profile. Investment in a portfolio can reduce risk without diluting the returns. An investor, who is expert in portfolio analysis, may be able to generate trading profits on a sustained basis.

Every investment is characterized by return and risk. The concept of risk is intuitively understood by investors. In general, it refers to the possibility of the rate of return from a security or a portfolio of securities deviating from the corresponding expected/average rate and can be measured by the standard deviation/variance of the rate of return.

How different type of Investors react in different situations

Source: www.missiassaugahsale.com

1.1   Activities in Portfolio Management

The following three major activities are involved in the formation of an Optimal Portfolio suitable for any given investor:

(a)   Selection of securities.

(b)   Construction of all Feasible Portfolios with the help of the selected securities.

(c)   Deciding the weights/proportions of the different constituent securities in the portfolio so that it is an Optimal Portfolio for the concerned investor.

The activities are directed to achieve an Optimal Portfolio of investments commensurate with the risk appetite of the investor.

1.2 Objectives of Portfolio Management

Some of the important objectives of portfolio management are:

(i)   *Security/Safety of Principal Amount*: Security not only involves keeping the principal sum intact but also its purchasing power.

(ii)   *Stability of Income*: To facilitate planning more accurately and systematically the reinvestment or consumption of income.

(iii) *Capital Growth*: It can be attained by reinvesting in growth securities or through purchase of growth securities.

(iv)   *Marketability i.e. the case with which a security can be bought or sold*: This is essential for providing flexibility to investment portfolio.

(v)   *Liquidity i.e. nearness to money*: It is desirable for the investor so as to take advantage of attractive opportunities upcoming in the market.

(vi)   *Diversification*: The basic objective of building a portfolio is to reduce the risk of loss of capital and/or income by investing in various types of securities and over a wide range of industries.

(vii) *Favourable Tax Status*: The effective yield an investor gets from his investment depends on tax to which it is subjected to. By minimising the tax burden, yield can be effectively improved.

2. PHASES OF PORTFOLIO MANAGEMENT

Portfolio management is a process and broadly it involves following five phases and each phase is an integral part of the whole process and the success of portfolio management depends upon the efficiency in carrying out each of these phases.

2.1 Security Analysis

The securities available to an investor for investment are numerous in number and of various types. The securities are normally classified on the basis of ownership of securities such as equity shares, preference shares, debentures and bonds, In recent times a number of new securities with innovative features are available in the market e.g. Convertible Debentures, Deep Discount Bonds, Zero Coupon Bonds, Flexi Bonds, Floating Rate Bonds, Global Depository Receipts, Euro-currency Bonds, etc. are some examples of these new securities. Among this vast group of securities, an

investor has to choose those ones which he considers worthwhile to be included in his investment portfolio. This requires a detailed analysis of the all securities available for making investment.

Security analysis constitutes the initial phase of the portfolio formation process and consists of examining the risk-return characteristics of individual securities and also the correlation among them. A simple strategy in securities investment is to buy under-priced securities and sell overpriced securities. But the basic problem is how to identify under-priced and overpriced securities and this is what security analysis is all about.

As discussed in the chapter of Security Analysis, there are two alternative approaches to analyse any security viz. Fundamental Analysis and Technical Analysis. They are based on different premises and follow different techniques. Fundamental analysis, the older of the two approaches, concentrates on the fundamental factors affecting the company such as

❑ the EPS of the company,

❑ the dividend pay-out ratio,

❑ the competition faced by the company,

❑ the market share, quality of management, etc.

❑ fundamental factors affecting the industry to which the company belongs.

The Fundamental Analyst compares this intrinsic value (true worth of a security based on its fundamentals) with the current market price. If the current market price is higher than the intrinsic value, the share is said to be overpriced and vice versa. This mispricing of securities gives an opportunity to the investor to acquire the share or sell off the share profitably. An intelligent investor would buy those securities which are under-priced and sell those securities which are overpriced. Thus, it can be said that fundamental analysis helps to identify fundamentally strong companies whose shares are worthy to be included in the investor's portfolio.

The second approach to security analysis is 'Technical Analysis'. As per this approach the share price movements are systematic and exhibit certain consistent patterns. Therefore, properly studied past movements in the prices of shares help to identify trends and patterns in security prices and efforts are made to predict the future price movements by looking at the patterns of the immediate past. Thus, Technical Analyst concentrates more on price movements and ignores the fundamentals of the shares.

In order to construct well diversified portfolios, so that Unsystematic Risk can be eliminated or substantially mitigated, an investor would like to select securities across diverse industry sectors which should not have strong positive correlation among themselves.

The Random Walk Theory holds that the share price movements are random and not systematic. Consequently, neither Fundamental Analysis nor Technical Analysis is of value in generating trading gains on a sustained basis. The Efficient Market Hypothesis (EMH) does not subscribe to the belief that it is possible to book gains in the long term on a sustained basis from trading in the stock market. Markets, though becoming increasingly efficient everywhere with the passage of time, are never perfectly efficient. So, there are opportunities all the time although their durations are decreasing and only the smart investors can look forward to booking gains consistently out of stock market deals.

2.2   Portfolio Analysis

Once the securities for investment have been identified, the next step is to combine these to form a suitable portfolio. Each such portfolio has its own specific risk-return characteristics which are not just the aggregates of the characteristics of the individual securities constituting it. The return and risk of each portfolio can be computed mathematically based on the Risk-Return profiles for the constituent securities and the pair-wise correlations among them.

From any chosen set of securities, an indefinitely large number of portfolios can be constructed by varying the fractions of the total investable resources allocated to each one of them. All such portfolios that can be constructed out of the set of chosen securities are termed as Feasible Portfolios. Detailed discussion on Risk- Return concept has been covered later in this chapter.

2.3   Portfolio Selection

The goal of a rational investor is to identify the Efficient Portfolios out of the whole set of Feasible Portfolios mentioned above and then to zero in on the Optimal Portfolio suiting his risk appetite. An Efficient Portfolio has the highest return among all Feasible Portfolios having same or lower Risk or has the lowest Risk among all Feasible Portfolios having same or higher Return. Harry Markowitz's portfolio theory (Modern Portfolio Theory) outlines the methodology for locating the Optimal Portfolio for an investor out of efficient portfolios. Detailed discussion on Markowitz's Portfolio Theory has been covered later in this chapter.

2.4   Portfolio Revision

Once an optimal portfolio has been constructed, it becomes necessary for the investor to constantly monitor the portfolio to ensure that it does not lose it optimality. Since the economy and financial markets are dynamic in nature, changes take place in these variables almost on a daily basis and securities which were once attractive may cease to be so with the passage of time. New securities with expectations of high returns and low risk may emerge. In light of these developments in the

market, the investor now has to revise his portfolio. This revision leads to addition (purchase) of some new securities and deletion (sale) of some of the existing securities from the portfolio. The nature of securities and their proportion in the portfolio changes as a result of the revision.

This portfolio revision may also be necessitated by some investor-related changes such as availability of additional funds for investment, change in risk appetite, need of cash for other alternative use, etc.

Portfolio revision is not a casual process to be taken lightly and needs to be carried out with care, scientifically and objectively so as to ensure the optimality of the revised portfolio. Hence, in the entire process of portfolio management, portfolio revision is as important as portfolio analysis and selection.

2.5 Portfolio Evaluation

This process is concerned with assessing the performance of the portfolio over a selected period of time in terms of return and risk and it involves quantitative measurement of actual return realized and the risk borne by the portfolio over the period of investment. The objective of constructing a portfolio and revising it periodically is to maintain its optimal risk-return characteristics. Various types of alternative measures of performance evaluation have been developed for use by investors and portfolio managers.

This step provides a mechanism for identifying weaknesses in the investment process and for improving these deficient areas.

It should however be noted that the portfolio management process is an ongoing process. It starts with security analysis, proceeds to portfolio construction, and continues with portfolio -revision and ends with portfolio evaluation. Superior performance is achieved through continual refinement of portfolio management skill. Detailed discussion on the techniques of Portfolio Evaluation has been covered later in this chapter.

3. PORTFOLIO THEORIES

Portfolio theory forms the basis for portfolio management. Portfolio management deals with the selection of securities and their continuous shifting in the portfolio to optimise returns to suit the objectives of an investor. This, however, requires financial expertise in selecting the right mix of securities in changing market conditions to get the best out of the stock market. In India as well as in a number of Western countries, portfolio management service has assumed the role of a specialised service and a number of professional investment bankers/fund managers compete

aggressively to provide the best options to high net-worth clients, who have little time to manage their own investments. The idea is catching on with the growth of the capital market and an increasing number of people want to earn profits by investing their hard-earned savings in a planned manner.

A portfolio theory guides investors about the method of selecting and combining securities that will provide the highest expected rate of return for any given degree of risk or that will expose the investor to the lowest degree of risk for a given expected rate of return. Portfolio theory can be discussed under the following heads:

3.1 Traditional Approach

The traditional approach to portfolio management concerns itself with the investor, definition of portfolio objectives, investment strategy, diversification and selection of individual investment as detailed below:

(i) Investor's study includes an insight into his –

    (a) age, health, responsibilities, other assets, portfolio needs;

    (b) need for income, capital maintenance, liquidity;

    (c) attitude towards risk; and

    (d) taxation status;

(ii) Portfolio objectives are defined with reference to maximising the investors' wealth which is subject to risk. The higher the level of risk borne, the more the expected returns.

(iii) Investment strategy covers examining a number of aspects including:

    (a) Balancing fixed interest securities against equities;

    (b) Balancing high dividend payout companies against high earning growth companies as required by investor;

    (c) Finding the income of the growth portfolio;

    (d) Balancing income tax payable against capital gains tax;

    (e) Balancing transaction cost against capital gains from rapid switching; and

    (f) Retaining some liquidity to seize upon bargains.

(iv) Diversification reduces volatility of returns and risks and thus adequate equity diversification is sought. Balancing of equities against fixed interest bearing securities is also sought.

(v)   Selection of individual investments is made on the basis of the following principles:

(a)   Methods for selecting sound investments by calculating the true or intrinsic value of a share and comparing that value with the current market value (i.e. by following the fundamental analysis) or trying to predict future share prices from past price movements (i.e., following the technical analysis);

(b)   Expert advice is sought besides study of published accounts to predict intrinsic value;

(c)   Inside information is sought and relied upon to move to diversified growth companies, switch quickly to winners than loser companies;

(d)   Newspaper tipsters about good track record of companies are followed closely;

(e)   Companies with good asset backing, dividend growth, good earning record, high quality management with appropriate dividend paying policies and leverage policies are traced out constantly for making selection of portfolio holdings.

In India, most of the share and stock brokers follow the above traditional approach for selecting a portfolio for their clients.

The Traditional Approach suggests that one should not put all money in one basket, instead an investor should diversify by investing in different securities and assets. As long as an investor invests in different assets and securities, he shall get the advantage of diversification. Markowitz questioned this wisdom of the Traditional Approach and proved that putting money in particular kinds of securities or assets will give the investor advantage of diversification. Therefore, one should not go blindly picking up securities and assets to make portfolio.

3.2   Modern Approach (Markowitz Model or Risk-Return Optimization)

Originally developed by Harry Markowitz in the early 1950's, Portfolio Theory - sometimes referred to as Modern Portfolio Theory - provides a logical/mathematical framework in which investors can optimise their risk and return. The central plank of the theory is that diversification through portfolio formation by adding securities whose returns are having low correlation or negative correlation into portfolio can reduce risk, and second, one can get higher return by taking higher risk.

Harry Markowitz is regarded as the father of Modern Portfolio Theory. According to him, investors are mainly concerned with two properties of an asset: risk and return. The essence of his theory is that risk of an individual asset hardly matters to an investor. What really matters is the contribution it makes to the investor's overall risk. By turning his principle into a useful technique for selecting the right portfolio from a range of different assets, he developed the 'Mean Variance Analysis' in 1952.

We shall discuss this theory in detail later in this chapter.

4.   RISK ANALYSIS

Before proceeding further it will be better if the concept of risk and return is discussed. A person makes an investment in the expectation of getting some return in the future, but the future is uncertain and so is the uncertain future return. It is this uncertainty associated with the returns from an investment that introduces risk for an investor.
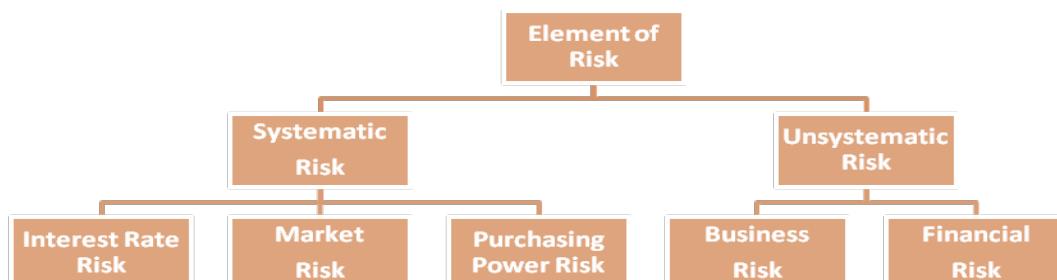
It is important here to distinguish between the expected return and the realized return from an investment. The expected future return is what an investor expects to get from his investment. On the other hand, the realized return is what an investor actually obtains from his investment at the end of the investment period. The investor makes the investment decision based on the expected return from the investment. However, the actual return realized from the investment may not correspond to the expected return. This possible variation of the actual return from the expected return is termed as risk. If actual realizations correspond to expectations exactly, there would be no risk. Risk arises where there is a probability of variation between expectations and realizations with regard to an investment.

Thus, risk arises from the variability in returns. An investment whose returns are fairly stable is considered to be a low-risk investment, whereas an investment whose returns fluctuate significantly is considered to be a highly risky investment. Government securities whose returns are fairly stable and which are free from default are considered to possess low risk whereas equity shares whose returns are likely to fluctuate widely around their mean are considered risky investments.

The essence of risk in an investment is the variation in its returns. This variation in returns is caused by a number of factors. These factors which produce variations in the returns from an investment constitute different types of risk.

4.1   Types of Risk

Let us consider the risk in holding equity securities. The elements of risk may be broadly classified into two groups as shown in the following diagram.

The first group i.e. systematic risk comprises factors that are external to a company (macro in nature) and affect a large number of securities simultaneously. These are mostly uncontrollable in nature. The second group i.e. unsystematic risk includes those factors which are internal to companies (micro in nature) and affect only those particular companies. These are controllable to a great extent.

The total variability in returns of a security is due to the total risk of that security. Hence,

Total risk = Total Systematic risk + Total Unsystematic risk

4.1.1 Systematic Risk

Due to dynamic nature of an economy, the changes occur in the economic, political and social conditions constantly. These changes have an influence on the performance of companies and thereby on their stock prices but in varying degrees. For example, economic and political instability adversely affects all industries and companies. When an economy moves into recession, corporate profits will shift downwards, and stock prices of most companies may decline. Thus, the impact of economic, political and social changes is system-wide and that portion of total variability in security returns caused by such macro level factors is referred to as systematic risk. Systematic risk can be further subdivided into interest rate risk, market risk and purchasing power risk.

(i) **Interest Rate Risk:** This arises due to variability in the interest rates from time to time and its impact on security prices. A change in the interest rates establishes an inverse relationship with the price of security i.e. price of securities tends to move inversely with change in rate of interest, long term securities show greater variability in the price with respect to interest rate changes than short term securities.

(ii) **Purchasing Power Risk:** It is also known as inflation risk, as it also emanates from the very fact that inflation affects the purchasing power adversely. Nominal return contains both the real return component and an inflation premium in a transaction involving risk of the above type to compensate for inflation over an investment holding period. Inflation rates vary over time and investors are caught unaware when rate of inflation changes unexpectedly causing erosion in the value of realised rate of return and expected return.

Purchasing power risk is more in inflationary conditions especially in respect of bonds and fixed income securities. It is not desirable to invest in such securities during inflationary periods. Purchasing power risk is however, less in flexible income securities like equity shares or common stock where rise in dividend income off-sets increase in the rate of inflation and provides advantage of capital gains.

(iii) **Market risk:** This is a type of systematic risk that affects prices of a share that moves up or down consistently for some time periods in line with other shares in the market. A general rise in share prices is referred to as a bullish trend, whereas a general fall in share prices is referred to as a bearish trend. In other words, the share market moves between the bullish phase and the bearish

phase. The market movements can be easily seen in the movement of share price indices such as the BSE Sensitive Index, BSE National Index, NSE Index etc.

4.1.2 Unsystematic Risk
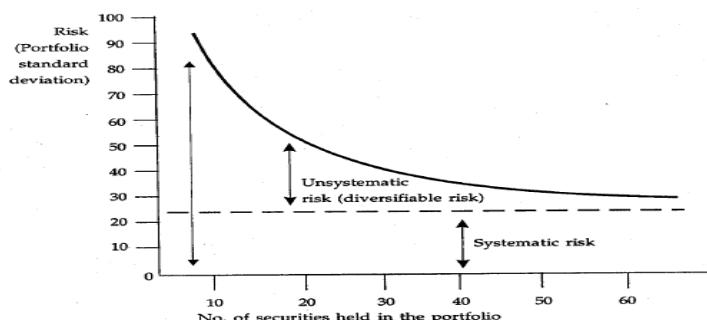
Also called 'idiosyncratic risk' the return from a security of any company may vary because of certain factors particular to that company. Variability in returns of the security on account of these factors (micro in nature), is known as unsystematic risk. It should be noted that this risk is in addition to the systematic risk affecting all the companies.   Unsystematic risk can be further subdivided into business risk and financial risk.

(i)   **Business Risk***:* Business risk emanates variability in the operating profits of a company – higher the variability in the operating profits of a company, higher is the business risk. Such a risk can be measured using operating leverage.

(ii)   **Financial Risk***:* It arises due to presence of debt in the capital structure of the company. It is also known as leveraged risk and expressed in terms of debt-equity ratio. Excess of debt vis-à-vis equity in the capital structure indicates that the company is highly geared and hence, has higher financial risk. Although a leveraged company's earnings per share may be more but dependence on borrowings exposes it to the risk of winding-up for its inability to honour its commitments towards lenders/creditors. This risk is known as leveraged or financial risk of which investors should be aware of and portfolio managers should be very careful.

4.2   Diversion of Risk

As discussed above, the total risk of an individual security consists of two risks systematic risk and unsystematic risk. It should be noted that by combining many securities in a portfolio the unsystematic risk can be avoided or diversified which is attached to any particular security. The following diagram depicts how the risk can be reduced with the increase in the number of securities.

From the above diagram it can be seen that total risk is reducing with the increase in the number of securities in the portfolio. However, ultimately when the size of the portfolio reaches certain limit, it will contain only the systematic risk.

4.3 Risk & Return

It is very common that an intelligent investor would attempt to anticipate the kind of risk that he/she is likely to face and would also attempt to estimate the extent of risk associated with different investment proposals. In other words an attempt is made by him/her to measure or quantify the risk of each investment under consideration before making the final selection. Thus quantification of risk is necessary for analysis of any investment.

As risk is attached with return its risk cannot be measured without reference to return. The return, in turn, depends on the cash inflows to be received from the investment. Let us take an example of purchase of a share. With an investment in an equity share, an investor expects to receive future dividends declared by the company. In addition, he expects to receive capital gain in the form of difference between the selling price and purchase price, when the share is finally sold.

Suppose a share of X Ltd. is currently selling at ₹ 12.00. An investor who is interested in the share anticipates that the company will pay a dividend of ₹ 0.50 in the next year. Moreover, he expects to sell the share at ₹ 17.50 after one year. The expected return from the investment in share will be as follows:

$$R = \frac{\text{Forecasted dividend} + \text{Forecasted end of the period stock price}}{\text{Initial investment}} - 1$$

$$R = \frac{₹\ 0.50 + ₹\ 17.50}{₹\ 12.00} - 1 = 0.5 \text{ or } 50 \text{ per cent}$$

It is important to note that here the investor expects to get a return of 50 per cent in the future, which is uncertain. It might be possible that the dividend declared by the company may turn out to be either more or less than the figure anticipated by the investor. Similarly, the selling price of the share may be less than the price expected by the investor at the time of investment. It may sometimes be even more. Hence, there is a possibility that the future return may be more than 50 per cent or less than 50 per cent. Since the future is uncertain the investor has to consider the probability of several other possible returns. The expected returns may be 20 per cent, 30 per cent, 50 per cent, 60 per cent or 70 per cent. The investor now has to assign the probability of occurrence of these possible alternative returns as given below:

Possible returns (in per cent) X_i	Probability of occurrence $p(X_i)$
20	0.20
30	0.20

50	0.40
60	0.10
70	0.10

The above table gives the probability distribution of possible returns from an investment in shares. Such distribution can be developed by the investor with the help of analysis of past data and modifying it appropriately for the changes he expects to occur in a future period of time.

With the help of available probability distribution two statistical measures one expected return and the other risk of the investment can be calculated.

4.3.1 Expected Return

a.   In case the data is given with probabilities: The expected return of the investment is the probability weighted average of all the possible returns. If the possible returns are denoted by Xi and the related probabilities are p(Xi) the expected return may be represented as $\overline{X}$ and can be calculated as:

$$\overline{X} = \sum_{i=1}^{n} x_i \, p(X_i)$$

It is the sum of the products of possible returns with their respective probabilities.

The expected return of the share in the example given above can be calculated as shown below:

Example

Calculation of Expected Return

Possible returns X_i	Probability $p(X_i)$	$X_i \, p(X_i)$
0.20	0.20	0.04
0.30	0.20	0.06
0.40	0.40	0.16
0.50	0.10	0.05
0.60	0.10	0.06
	$\sum_{i=1}^{n} x_i \, p(X_i)$	0.37

Hence the expected return is 0.37 i.e. 37%

b.   In case the historical data is given:

$$\bar{x} = \sum_{i=1}^{n} \frac{x_1 + x_2 + x_3 \ldots \ldots x_i}{n}$$

i.e. Simple average or Arithmetic Mean of the historical data series

Example

Suppose the total return from shares of X Ltd. for a period of last five years is as follows:

Year	Total Return % (x)
1	9.50
2	7.00
3	11.00
4	-6.00
5	2.50
	$\sum x_i = 24.00$

The Average Return of the share will be = $\dfrac{24.00}{5}$ = 4.80

4.3.2 Risk

As risk is attached with every return hence calculation of only expected return is not sufficient for decision making. Therefore risk aspect should also be considered along with the expected return. The most popular measure of risk is the variance or standard deviation of the probability distribution of possible returns.

Risk can be defined as deviation of actual/possible return from expected return i.e. Mean return.

Variance of each security is generally denoted by σ^2 and is calculated by using the following formula:

a.   In case the data is given with probabilities:

$$\sigma^2 = \sum_{i=1}^{n} [(X_i - \bar{X})^2 \, p(X_i)]$$

Continuing our earlier example the following table provides calculations required to calculate the risk i.e. Variance or Standard Deviation (SD).

Possible returns X_i	Probability $p(X_i)$	Deviation $(X_i - \overline{X})$	Deviation squared $(X_i - \overline{X})^2$	Product $(X_i - \overline{X})^2 p(X_j)$
0.20	0.20	-0.17	0.0289	0.00578
0.30	0.20	-0.07	0.0049	0.00098
0.40	0.40	0.03	0.0009	0.00036
0.50	0.10	0.13	0.0169	0.00169
0.60	0.10	0.23	0.0529	0.00529
			Var (σ^2)	0.0141

Variance = 0.0141

Standard Deviation of the return will be the positive square root of the variance and is generally represented by σ. Accordingly, the standard deviation of return in the above example will be $\sqrt{0.0141}$ = 0.1187 i.e. 11.87%.

b.   In case the historical data is given:

$$\sigma^2 = \sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n}$$

Thus, keeping the above formulae same, instead of multiplying with probabilities, we need to divide it by no of observations.

Continuing our earlier example the following table provides calculations required to calculate the risk i.e. Variance or Standard Deviation (SD).

Year	Total Return % (x)	Deviation $(x_i - \overline{x})$	Deviation squared $(x_i - \overline{x})^2$
1	9.50	4.70	22.09
2	7.00	2.20	4.84
3	11.00	6.20	38.44
4	-6.00	-10.80	116.64
5	2.50	-2.30	5.29
	$\sum x_i$ = 24.00		187.30
Mean	4.80		

Variance = $\dfrac{187.30}{5}$ = 37.46

Standard Deviation of the return will be the positive square root of the variance and is generally represented by σ. Accordingly, the standard deviation of return in the above example will be $\sqrt{37.46}$ = 6.12 i.e. 6.12%.

The basic purpose to calculate the variance and standard deviation is to measure the extent of variability of possible returns from the expected return. Several other measures such as range, semi-variance and mean absolute deviation can also be used to measure risk, but standard deviation has been the most popularly accepted measure.

For example, if a security has a return of 11% and Standard Deviation i.e. risk of 3%, then it can be concluded that as per the given conditions, the return's range can be expected from 8% to 14% i.e. 11% - 3% would be the lower range and 11% + 3% would be the upper range.

The method described above is widely used for assessing risk.

The standard deviation or variance, however, provides a measure of the total risk associated with a security. As we know, the total risk comprises two components, namely systematic risk and unsystematic risk. Unsystematic risk is the risk specific or unique to a company. Unsystematic risk associated with the security of a particular company can be eliminated/reduced by combining it with another security having negative correlation. This process is known as diversification of unsystematic risk. As a means of diversification the investment is spread over a group of securities with different characteristics. This collection of diverse securities is called a Portfolio.

As unsystematic risk can be reduced or eliminated through diversification, it is not very important for an investor to consider. The risk that is relevant in investment decisions is the systematic risk because it is not diversifiable. Hence, the main interest of the investor lies in the measurement of systematic risk of a security.

4.3.3 Measurement of Systematic Risk

As discussed earlier, systematic risk is the variability in security returns caused by changes in the economy or the market and all securities are affected by such changes to some extent. Some securities exhibit greater variability in response to market changes and some may exhibit less response. Securities that are more sensitive to changes in factors are said to have higher systematic risk. The average effect of a change in the economy can be represented by the change in the stock market index. The systematic risk of a security can be measured by relating that security's variability vis-à-vis variability in the stock market index. A higher variability would indicate higher systematic risk and vice versa.

The systematic risk of a security is measured by a statistical measure which is called Beta (β). The main input data required for the calculation of beta of any security are the historical data of returns of the individual security and corresponding return of a representative market return (stock market index). There are two statistical methods i.e. correlation method and the regression method, which can be used for the calculation of Beta.

4.3.3.1 Correlation Method : Correlation measures the extent to which two variables are related. In respect to Beta, it means how market returns and security returns are related. Using this method beta (β) can be calculated from the historical data of returns by the following formula:

$$\beta_i = \frac{r_{im}\sigma_i}{\sigma_m}$$

Where

r_{im} = Correlation coefficient between the returns of the stock i and the returns of the market index.

σ_i = Standard deviation *of* returns of stock *i*

σ_m = Standard deviation of returns of the market index.

4.3.3.2 Regression Method : The regression model is based on the postulation that there exists a linear relationship between a dependent variable and an independent variable. The model helps to calculate the values of two constants, namely alfa (*α*) and beta (*β*). β measures the change in the dependent variable in response to unit change in the independent variable, while *α* measures the value *of* the dependent variable even when the independent variable has zero value. The formula of the regression equation is as follows:

$$Y = α + βX$$

where

Y = Dependent variable

X = Independent variable

α and β are constants.

$$α = Y - βX$$

The formula used for the calculation of α and β are given below.

$$\beta = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2}$$

where

n = Number of items.

Y = Dependent variable scores.

X = Independent variable scores.

For the purpose of calculation of β, the return of the individual security is taken as the dependent variable and the return of the market index is taken as the independent variable.

Here it is very important to note that a security can have betas that are positive, negative or zero.

- Positive Beta- indicates that security's return is dependent on the market return and moves in the direction in which market moves.

- Negative Beta- indicates that security's return is dependent on the market return but moves in the opposite direction in which market moves.

- Zero Beta- indicates that security's return is independent of the market return.

Further as beta measures the volatility of a security's returns relative to the market, the larger the beta, the more volatile the security.

- A beta of 1.0 indicates that a security has risk as that of the market.

- A stock with beta greater than 1.0 has above average risk i.e. its returns would be more volatile than the market returns. For example, when market returns move up by 6%, a stock with beta of 2 would find its returns moving up by 12% (i.e. 6% x 2). Similarly, decline in market returns by 6% would produce a decline of 12% (i.e. 6% x 2) in the return of that security.

- A stock with beta less than 1.0 would have below market risk. Variability in its returns would be less than the market variability.

Beta is calculated from historical data of returns to measure the systematic risk of a security. It is a historical measure of systematic risk. In using this beta for investment decision making, the investor is assuming that the relationship between the security variability and market variability will continue to remain the same in future also.

4.4 Portfolio Analysis

Till now we have discussed the risk and return of a single security. Let us now discuss the return and risk of a portfolio of securities.

4.4.1 Portfolio Return

For a portfolio analysis an investor first needs to specify the list of securities eligible for selection or inclusion in the portfolio. Then he has to generate the risk-return expectations for these securities. The expected return for the portfolio is expressed as the mean of its rates of return over the time horizon under consideration and risk for the portfolio is the variance or standard deviation of these rates of return around the mean return.

The expected return of a portfolio of assets is simply the weighted average of the returns of the individual securities constituting the portfolio. The weights to be applied for calculation of the portfolio return are the fractions of the portfolio invested in such securities.

Let us consider a portfolio of two equity shares A and B with expected returns of 16 per cent and 22 per cent respectively.

The formula for the calculation of expected portfolio return may be expressed as shown below:

$$\bar{r}_p = \sum_{i=1}^{n} x_i \bar{r}_i$$

$\bar{r}_p$ = Expected return of the portfolio.

X_i = Proportion of funds invested in security i

$\bar{r}_i$ = Expected return of security i.

n = Number of securities in the portfolio.

If 40 per cent of the total funds is invested in share A and the remaining 60 per cent in share B, then the expected portfolio return will be:

Return on the portfolio = (0.40 x 16) + (0.60 x 22) = 19.6 per cent

4.4.2 Portfolio Risk

As discussed earlier, the variance of return and standard deviation of return are statistical measures that are used for measuring risk in investment. The variance of a portfolio can be written down as the sum of 2 terms, one containing the aggregate of the weighted variances of the constituent securities and the other containing the weighted co-variances among different pairs of securities.

Covariance (a statistical measure) between two securities or two portfolios or a security and a portfolio indicates how the rates of return for the two concerned entities behave relative to each other.

The covariance between two securities A and B may be calculated using the following formula:

$$COV_{AB} = \frac{\sum [R_A - \overline{R_A}][R_B - \overline{R_B}]}{N}$$

where

COV_{AB} = Covariance between A and B

R_A = Return of security A

R_B = Return of security B

$\overline{R_A}$ = Expected or mean return of security A

$\overline{R_B}$ = Expected or mean return of security B

N = Number of observations.

The calculation of covariance can be understood with the help of following table:

Calculation of Covariance

Year	R_X	Deviation $R_X - \overline{R_x}$	R_Y	Deviation $R_Y - \overline{R_Y}$	$[R_x - \overline{R_x}][R_y - \overline{R_y}]$
1	11	-4	18	5	-20
2	13	-2	14	1	-2
3	17	2	11	-2	-4
4	19	4	9	-4	-16
	$\overline{R_x}$ = 15		$\overline{R_y}$ = 13		-42

$$Cov_{xy} = \frac{\sum_{i=1}^{n} [R_x - \overline{R_x}][R_y - \overline{R_y}]}{n} = \frac{-42}{4} = -10.5$$

From the above table it can be seen that the covariance is a measure of how returns of two securities move together. In case the returns of the two securities move in the same direction consistently the covariance is said to be positive (+). Contrarily, if the returns of the two securities move in opposite directions consistently the covariance would be negative (-). If the movements of returns are independent of each other, covariance would be close to zero (0).

The coefficient of correlation is expressed as:

$$r_{AB} = \frac{Cov_{AB}}{\sigma_A \sigma_B}$$

where

r_{AB} = Coefficient of correlation between A and B

Cov_{AB} = Covariance between A and B

σ_A = Standard deviation of A

σ_B = Standard deviation of B

It may be noted on the basis of above formula the covariance can be expressed as the product of correlation between the securities and the standard deviation of each of the securities as shown below:

$$Cov_{AB} = \sigma_A \, \sigma_B \, r_{AB}$$

It is very important to note that the correlation coefficients may range from -1 to 1.

- A value of -1 indicates perfect negative correlation between the two securities' returns.

- A value of +1 indicates a perfect positive correlation between them.

- A value of zero indicates that the returns are independent.

The calculation of the variance (or risk) of a portfolio is not simply a weighted average of the variances of the individual securities in the portfolio as in the calculation of the return of portfolio. The variance of a portfolio with only two securities in it can be calculated with the following formula.

$$\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \left(r_{12} \sigma_1 \sigma_2 \right)$$

where

σ_p^2 = Portfolio variance.

x_1 = Proportion of funds invested in the first security.

x_2 = Proportion of funds invested in the second security ($x_1 + x_2 = 1$).

σ_1^2 = Variance of first security.

σ_2^2 = Variance of second security.

σ_1 = Standard deviation of first security.

σ_2 = Standard deviation of second security.

r_{12} = Correlation coefficient between the returns of the first and second securities.

As the standard deviation is the square root of the variance the portfolio standard deviation can be obtained by taking the square root of portfolio variance.

Let us take an example to understand the calculation of portfolio variance and portfolio standard deviation. Two securities A and B generate the following sets of expected returns, standard deviations and correlation coefficient:

	A	B
$\overline{r}$ =	20%	25%
σ =	50%	30%
r_{ab}=		-0.60

Now suppose a portfolio is constructed with 40 per cent of funds invested in A and the remaining 60 per cent of funds in B (i.e. P = 0.4A + 0.6B).

Using the formula of portfolio return the expected return of the portfolio will be:

R_P= (0.40 x 20) + (0.60 x 25) = 23%

And the Variance and Standard Deviation of the portfolio will be:

Variance

$\sigma_p{}^2$ = (0.40)^2 (50)^2 + (0.60)^2 (30)^2 + 2(0.40)(0.60)(- 0.60)(50)(30) = 400 + 324 - 432 = 292

Standard deviation

$\sigma_p = \sqrt{292}$ = 17.09 per cent.

The return and risk of a portfolio depends on following two sets of factors:

(a) Returns and risks of individual securities and the covariance between securities forming the portfolio

(b) Proportion of investment in each of securities.

As the first set of factors is parametric in nature for the investor in the sense that he has no control over the returns, risks and co-variances of individual securities. The second set of factors is choice factor or variable for the investors in the sense that they can choose the proportions of each security in the portfolio.

4.4.3 Reduction or Dilution of Portfolio Risk through Diversification

The process of combining more than one security in to a portfolio is known as diversification. The main purpose of this diversification is to reduce the total risk by eliminating or substantially mitigating the unsystematic risk, without sacrificing portfolio return. As shown in the example mentioned above, diversification has helped to reduce risk. The portfolio standard deviation of 17.09% is lower than the standard deviation of either of the two securities taken separately which were 50% and 30% respectively. Incidentally, such risk reduction is possible even when the two constituent securities are uncorrelated. In case, however, these have the maximum positive correlation between them, no reduction of risk can be achieved.

In order to understand the mechanism and power of diversification, it is necessary to consider the impact of covariance or correlation on portfolio risk more closely. We shall discuss following three cases taking two securities in the portfolio:

(a)   Securities' returns are perfectly positively correlated,

(b)   Securities' returns are perfectly negatively correlated, and

(c)   Securities' returns are not correlated i.e. they are independent.

4.4.3.1 Perfectly Positively Correlated : In case two securities returns are perfectly positively correlated the correlation coefficient between these securities will be +1 and the returns of these securities then move up or down together.

The variance of such portfolio can be calculated by using the following formula:

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2r_{12}\sigma_1\sigma_2$$

As r_{12} = 1, this may be rewritten as:

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2$$

or

$$\sigma_p^2 = \left(x_1\sigma_1 + x_2\sigma_2\right)^2$$

Hence Standard Deviation will become

$$\sigma_p = x_1\sigma_1 + x_2\sigma_2$$

In other words this is simply the weighted average of the standard deviations of the individual securities.

Taking the above example we shall now calculate the portfolio standard deviation when correlation coefficient is +1.

Standard deviation of security A = 40%

Standard deviation of security B = 25%

Proportion of investment in A = 0.4

Proportion of investment in B = 0.6

Correlation coefficient = +1.0

Portfolio standard deviation maybe calculated as:

σ_p = (0.4) (40) + (0.6) (25) = 31

Thus it can be seen that the portfolio standard deviation will lie between the standard deviations of the two individual securities. It will vary between 40 and 25 as the proportion of investment in each security changes.

Now suppose, if the proportion of investment in A and B are changed to 0.75 and 0.25 respectively; portfolio standard deviation of the portfolio will become:

$$\sigma_p = (0.75)\ (40) + (0.25)\ (25) = 36.25$$

It is important to note that when the security returns are perfectly positively correlated, diversification provides only risk averaging and no risk reduction because the portfolio risk cannot be reduced below the individual security risk. Hence,   reduction of risk is not achieved when the constituent securities' returns are perfectly positively correlated.

4.4.3.2 Perfectly Negatively Correlated : When two securities' returns are perfectly negatively correlated, two returns always move in exactly opposite directions and correlation coefficient between them becomes -1. The variance of such negatively correlated portfolio may be calculated as:

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 - 2x_1x_2\left(r_{12}\sigma_1\sigma_2\right)$$

As r_{12} = -1, this may be rewritten as:

$$\sigma_p^2 = \left(x_1\sigma_1 - x_2\sigma_2\right)^2$$

Hence Standard Deviation will become

$$\sigma_p = x_1\sigma_1 - x_2\sigma_2$$

Taking the above example we shall now calculate the portfolio standard deviation when correlation coefficient is -1.

$$\sigma_p = (0.4)(40) - (0.6)(25) = 1$$

Thus, from above it can be seen that the portfolio risk has become very low in comparison of risk of individual securities. By changing the weights it can even be reduced to zero. For example, if the proportion of investment in A and B are 0.3846 and 0.6154 respectively, portfolio standard deviation becomes:

$$= (0.3846)(40) - (0.6154)(25) = 0$$

Although in above example the portfolio contains two risky assets, the portfolio has no risk at all. Thus, the portfolio may become entirely risk-free when security returns are perfectly negatively correlated. Therefore, diversification can substantially reduce or even eliminate risk when securities are perfectly negatively correlated, . However, in real life it is very rare to find securities that are perfectly negatively correlated.

4.4.3.3 Returns are uncorrelated or independent : When the returns of two securities are entirely uncorrelated, the coefficient of correlation of these two securities would be zero and the formula for portfolio variance will be as follows:

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2r_{12}\sigma_1\sigma_2$$

As $r_{12} = 0$, this may be rewritten as:

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2$$

Hence Standard Deviation will become

$$\sigma_p = \sqrt{x_1^2\sigma_1^2 + x_2^2\sigma_2^2}$$

Taking the above example we shall now calculate the portfolio standard deviation when correlation coefficient is 0.

$$\sigma_p = \sqrt{(0.4)^2(40)^2 + (0.6)^2(25)^2}$$

$$\sigma_p = \sqrt{256 + 225}$$

$$\sigma_p = 21.93$$

Thus, it can be observed that the portfolio standard deviation is less than the standard deviations of individual securities in the portfolio. Therefore, when security returns are uncorrelated, diversification can reduce risk .

We may now tabulate the portfolio standard deviations of our illustrative portfolio having two securities A and B, for different values of correlation coefficients between them. The proportion of investments in A and B are 0.4 and 0.6 respectively. The individual standard deviations of A and B are 40 and 25 respectively.

Portfolio Standard Deviations

Correlation coefficient	Portfolio Standard Deviation
1.00	31
0.60	27.73
0	21.93
-0.60	13.89
-1.00	1.00

Summarily it can be concluded that diversification reduces risk in all cases except when the security returns are perfectly positively correlated. With the decline of correlation coefficient from +1 to -1, the portfolio standard deviation also declines. But the risk reduction is greater when the security returns are negatively correlated.

4.4.4 Portfolio with more than two securities

So far we have considered a portfolio with only two securities. The benefits from diversification increase as more and more securities with less than perfectly positively correlated returns are included in the portfolio. As the number of securities added to a portfolio increases, the standard deviation of the portfolio becomes smaller and smaller. Hence, an investor can make the portfolio risk arbitrarily small by including a large number of securities with negative or zero correlation in the portfolio.

But, in reality, securities rarely show negative or even zero correlation. Typically, securities show some positive correlation, that is above zero but less than the perfectly positive value (+1). As a result, diversification (that is, adding securities to a portfolio) results in some reduction in total portfolio risk but not in complete elimination of risk. Moreover, the effects of diversification are exhausted fairly rapidly. That is, *most* of the reduction in portfolio standard deviation occurs by the time the portfolio size increases to 25 or 30 securities. Adding securities beyond this size brings about only marginal reduction in portfolio standard deviation.

Adding securities to a portfolio reduces risk because securities are not perfectly positively correlated. But the effects of diversification are exhausted rapidly because the securities are still positively correlated to each other though not perfectly correlated. Had they been negatively correlated, the portfolio risk would have continued to decline as portfolio size increased. Thus, in practice, the benefits of diversification are limited.

4.4.5 Calculation of Return and Risk of Portfolio with more than two securities

The expected return of a portfolio is the weighted average of the returns of individual securities in the portfolio, the weights being the proportion of investment in each security. The formula for calculation of expected portfolio return is the same for a portfolio with two securities and for portfolios with more than two securities. The formula is:

$$\overline{r_p} = \sum_{i=1}^{n} x_i \overline{r_i}$$

Where

$\overline{r_p}$ = Expected return of portfolio.

x_i = Proportion of funds invested in each security.

$\overline{r_i}$ = Expected return of each security.

n = Number of securities in the portfolio.

Let us consider a portfolio with four securities having the following characteristics:

Security	Returns (per cent)	Proportion of investment
P	11	0.3
Q	16	0.2
R	22	0.1
S	20	0.4

The expected return of this portfolio may be calculated using the formula:

$\overline{r_p}$ = (0.3)(11) + (0.2)(16) + (0.1)(22) + (0.4)(20) = 16.7 per cent

The portfolio variance and standard deviation depend on the proportion of investment in each security as also the variance and covariance of each security included in the portfolio.

Let us take the following example to understand how we can compute the risk of multiple asset portfolio.

Security	x_i	σ_i	Correlation Coefficient
X	0.25	16	X and Y = 0.7
Y	0.35	7	X and Z = 0.3
Z	0.40	9	Y and Z = 0.4

It may be noted that correlation coefficient between X and X, Y and Y, Z and Z is 1.

A convenient way to obtain the result is to set up the data required for calculation in the form of a variance-covariance matrix.

As per data given in the example, the first cell in the first row of the matrix represents X and X the second cell in the first row represents securities X and Y, and so on. The variance or covariance in each cell has to be multiplied by the weights of the respective securities represented by that cell. These weights are available in the matrix at the left side of the row and the top of the column containing the cell.

This process may be started from the first cell in the first row and continued for all the cells till the last cell of the last row is reached as shown below:

Weights →		0.25	0.35	0.40
↓		X	Y	Z
0.25	X	1 x 16 x 16	0.7 x 16 x 7	0.3 x 16 x 9
0.35	Y	0.7 x 7 x 16	1 x 7 x 7	0.4 x 7 x 9
0.40	Z	0.3 x 9 x 16	0.4 x 9 x 7	1 x 9 x 9

Once the variance-covariance matrix is set up, the computation of portfolio variance is a comparatively simple operation. Each cell in the matrix represents a pair of two securities.

When all these products are summed up, the resulting figure is the portfolio variance. The square root *of* this figure gives the portfolio standard deviation.

Thus the variance *of* the portfolio given in the example above can now be calculated.

σ_p^2 = (0.25 x 0.25 x 1 x 16 x 16) + (0.25 x 0.35 x 0.7 x 16 x 7) + (0.25 x 0.40 x 0.3 x 16 x 9) + (0.35 x 0.25 x 0.7 x 7 x 16) + (0.35 x 0.35 x 1 x 7 x 7) + (0.35 x 0.40 x 0.4 x7 x 9) + (0.40 x 0.25 x 0.3 x 9 x 16) + (0.40 x 0.35 x 0.4 x 9 x 7) + (0.40 x 0.40 x 1 x 9 x 9)

= 16 + 6.86 + 4.32 + 6.86 + 6.0025 + 3.528 + 4.32 + 3.528 + 12.96 = 64.3785

The portfolio standard deviation is:

$\sigma_p = \sqrt{64.3785} = 8.0236$

Hence, the formula for computing Portfolio Variance may also be stated as follows:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j r_{ij} \sigma_i \sigma_j$$

Or

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}$$

where

σ_p^2 = Portfolio variance.

x_i = Proportion of funds invested in security i (the first of a pair of securities).

x_j = Proportion of funds invested in security j (the second of a pair of securities).

σ_i = Standard Deviation of security i

σ_j = Standard Deviation of security j

r_{ij} = The co-efficient of correlation between the pair of securities i and j

σ_{ij} = The covariance between the pair of securities i and j

n = Total number of securities in the portfolio.

Thus from above discussion it can be said that a portfolio is a combination of assets. *From* a given set of *'n'* securities, any number of portfolios can be created. These portfolios may comprise of *two* securities, three securities, all the way up to *'n'* securities. A portfolio may contain the same securities as another portfolio but with different weights. A new portfolios can be created either by changing the securities in the portfolio or by changing the proportion of investment in the existing securities.

Thus summarily it can be concluded that each portfolio is characterized by its expected return and risk. Determination of expected return and risk (variance or standard deviation) of each portfolio that can be used to create a set of selected securities which is the first step in portfolio management and called portfolio analysis.

5. MARKOWITZ MODEL OF RISK-RETURN OPTIMIZATION

Unlike the CAPM, the Optimal Portfolio as per Markowitz Theory is investor specific. The portfolio selection problem can be divided into two stages:

(1)   finding the mean-variance efficient portfolios and

(2)   selecting one such portfolio.

Investors do not like risk and the greater the riskiness of returns on an investment, the greater will be the returns expected by investors. There is a trade-off between risk and return which must be reflected in the required rates of return on investment opportunities. The standard deviation (or variance) of return measures the total risk of an investment. It is not necessary for an investor to accept the total risk of an individual security. Investors can diversify to reduce risk. As number of holdings approach larger, a good deal of total risk is removed by diversification.

5.1   Assumptions of the Model

It is a common phenomenon that the diversification of investments in the portfolio leads to reduction in variance of the return, even for the same level of expected return. This model has taken into account risks associated with investments - using variance or standard deviation of the return. This model is based on the following assumptions. :

(i)   The return on an investment adequately summarises the outcome of the investment.

(ii)   The investors can visualise a probability distribution of rates of return.

(iii)   The investors' risk estimates are proportional to the variance of return they perceive for a security or portfolio.

(iv)   Investors base their investment decisions on two criteria i.e. expected return and variance of return.

(v)   Investors are assumed to be rational in so far as they would prefer greater returns to lesser ones given equal or smaller risk and are risk averse. Risk aversion in this context means merely that, as between two investments with equal expected returns, the investment with the smaller risk would be preferred.

(vi)   Though 'Expected Return' could be a suitable measure of monetary inflows like NPV but yield in different scenarios is the most commonly used measure of return. The standard deviation of return is referred to as deviation of yields from its expected value or return.

5.2   Efficient Frontier

Markowitz has formalised the risk return relationship and developed the concept of efficient frontier using the Mean-Variance Dominance Principle. For selection of a portfolio, comparison between

combinations of portfolios is essential. As a rule, a portfolio is dominating another portfolio in terms of mean and variance if there is another portfolio with:

(a)   A lower expected value of return and same or higher standard deviation (risk).

(b)   The same or higher standard deviation (risk) but a lower expected return.

Markowitz has defined the diversification as the process of combining assets that are less than perfectly positively correlated in order to reduce portfolio risk without sacrificing any portfolio returns. If an investors' portfolio is not efficient he may:

(i)   Increase the expected value of return without increasing the risk.

(ii)   Decrease the risk without decreasing the expected value of return, or

(iii)   Obtain some combination of increase of expected return and decrease risk.

This is possible by switching to a portfolio on the efficient frontier.
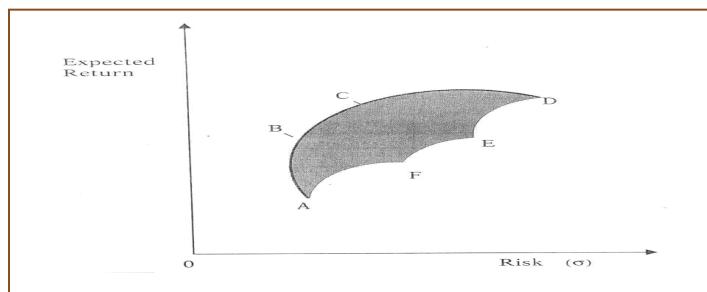
Fig. 1: Markowitz Efficient Frontier

If all the investments are plotted on the risk-return space, individual securities would be dominated by portfolios, and the efficient frontier would be containing all Efficient Portfolios (An Efficient Portfolio has the highest return among all portfolios with identical risk and the lowest risk among all portfolios with identical return). Fig – 1 depicts the boundary of possible investments in securities, A, B, C, D, E and F; and B, C, D, are lying on the efficient frontier.

The best combination of expected value of return and risk (standard deviation) depends upon the investors' utility function. The individual investor will want to hold that portfolio of securities which places him on the highest indifference curve, choosing from the set of available portfolios. The dark line at the top of the set is the line of efficient combinations, or the efficient frontier. The optimal portfolio for an investor lies at the point where the indifference curve for the concerned investor touches the efficient frontier. This point reflects the risk level acceptable to the investor in order to achieve a desired

return and provide maximum return for the bearable level of risk. The concept of efficient frontier and the location of the optimal portfolio are explained with help of Fig-2.
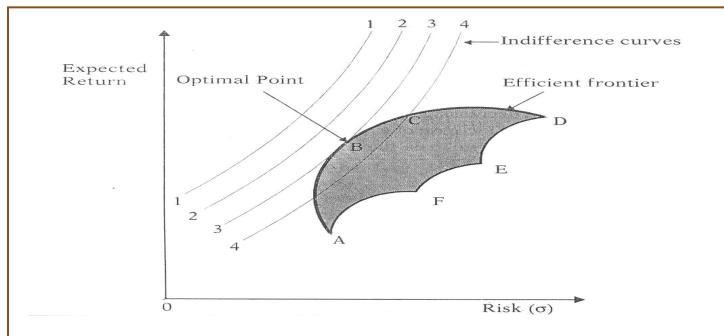
Fig. 2 : Optimal Investment under Markowitz Model

In Fig-2 A, B, C, D, E and F define the boundary of all possible investments out of which investments in B, C and D are the efficient portfolios lying on the efficient frontier. The attractiveness of the investment proposals lying on the efficient frontier depends on the investors' attitude to risk. At point B, the level of risk and return is at optimum level. The returns are highest at point D, but simultaneously it carries higher risk than any other investment.
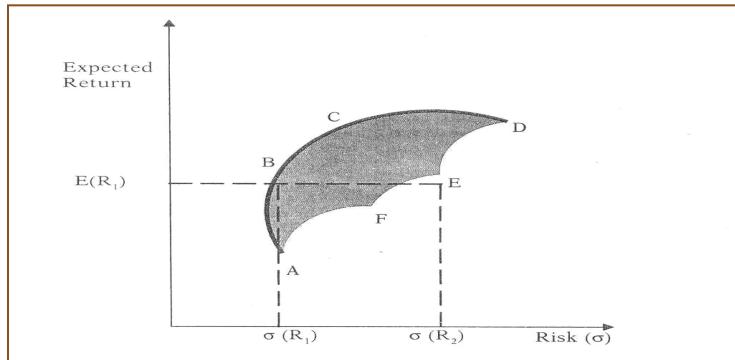
Fig.3 : Selection of Portfolios

The shaded area in Fig-3 represents all attainable or feasible portfolios, that is all the combinations of risk and expected return which may be achieved with the available securities. The efficient frontier contains all possible efficient portfolios and any point on the frontier dominates any point to the right of it or below it.

Consider the portfolios represented by points B and E. B and E promise the same expected return E (R_1) but the risk associated with B is σ (R_1) whereas the associated with E is σ (R_2). Investors, therefore, prefer portfolios on the efficient frontier rather than interior portfolios given the assumption

of risk aversion; obviously, point A on the frontier represents the portfolio with the least possible risk, whilst D represents the portfolio with the highest possible rate of return with highest risk.

The investor has to select a portfolio from the set of efficient portfolios lying on the efficient frontier. This will depend upon his risk-return preference. As different investors have different preferences, the optimal portfolio of securities will vary from one investor to another.

6. CAPITAL MARKET THEORY

The above figure 2 portrays the relationship between risk and return for efficient portfolio graphically. Point B represents the market portfolio and if a line tangent to this point is drawn and extended upto y-axis the point at which it will touch will be the riskless rate of interest. This is shown in Fig 4.
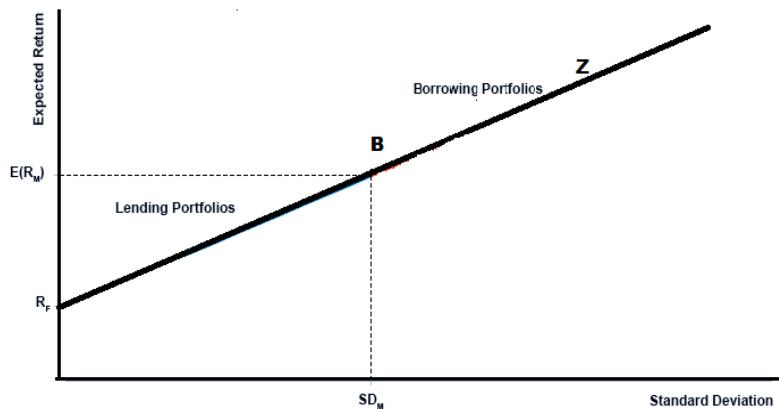
Fig.4 : Selection of Portfolios

Preferred investment strategies plot along line R_fBZ, representing alternative combinations of risk and return obtainable by combining the market portfolio with borrowing or lending. This is known as the Capital Market Line (CML). Portfolio lying on line from R_f to B shall be lending portfolio as it will involve some investment in risk-free securities and some investment in market portfolio. Portfolios lying from B to Z will be borrowing portfolio as it will be an investment in market portfolio by borrowing the same amount.

The slope of the capital market line can be regarded as the reward per unit of risk borne and it is computed as follows:

$$\text{Slope} = \frac{R_M - R_f}{\sigma_M}$$

Where R_M = Market Return

R_f= Risk Free Rate of Return

σ_M = Standard Deviation of Market

From the Capital Market Line the expected return of a portfolio can be found as follows:

$$E(R) = R_f + \frac{R_M - R_f}{\sigma_M} \times \sigma_P$$

Where σ_P = Standard Deviation of Portfolio

7. SINGLE INDEX MODEL (SHARPE INDEX MODEL)

This model assumes that co-movement between stocks is due to change or movement in the market index. Casual observation of the stock prices over a period of time reveals that most of the stock prices move with the market index. When the Sensex increases, stock prices also tend to increase and vice-versa. This indicates that some underlying factors affect the market index as well as the stock prices. Stock prices are related to the market index and this relationship could be used to estimate the return on stock. Towards this purpose, the following equation can be used:

$$R_i = \alpha_i + \beta_i R_m + \in_i$$

Where,

R_i = expected return on security i

α_i = intercept of the straight line or alpha co-efficient

β_i = slope of straight line or beta co-efficient

R_m = the rate of return on market index

$\in_i$ = error term or return expected on account of unsystematic risk

According to the equation, the return of a stock can be divided into two components, the return due to the market and the return independent of the market. β_i indicates the sensitiveness of the stock return to the changes in the market return. For example, βi of 1.5 means that the stock return is expected to increase by 1.5% when the market index return increases by 1% and vice-versa. Likewise, β_i of 0.5 expresses that the individual stock return would change by 0.5 per cent when there is a change of 1 per cent in the market return. β_i of 1 indicates that the market return and the security return are moving in tandem. The estimates of β_i and α_i are obtained from regression analysis.

The single index model is based on the assumption that stocks vary together because of the common movement in the stock market and there are no effects beyond the market (i.e. any fundamental factor effects) that accounts for the stocks co-movement. The expected return, standard deviation and co-variance of the single index model represent the joint movement of securities. The mean return is:

$$R_i = \alpha_i + \beta_i \, R_m + \in_i$$

The variance of security's return:

$$\sigma^2 = \beta^2_i \, \sigma^2_m + \sigma^2_{\in i}$$

The covariance of returns between securities i and j is:

$$\sigma_{ij} = \beta_i \beta_j \, \sigma^2_m$$

The variance of the security has two components namely, systematic risk or market risk and unsystematic risk or unique risk. The variance explained by the index is referred to as systematic risk. The unexplained variance is called Residual Variance or Unsystematic Risk.

The systematic risk can be calculated by using following formula:

Systematic risk = β^2_i × variance of market index

$$= \beta^2_i \, \sigma^2_m$$

Unsystematic risk = Total variance - Systematic risk.

$$\in_i^2 \ = \ \sigma_i^2 \ \text{- Systematic risk.}$$

Thus, the total risk = Systematic risk + Unsystematic risk.

$$= \beta_i^2 \ \sigma_m^2 + \in_i^2.$$

From this, the portfolio variance can be derived

$$\sigma_p^{\,2} = \left[\left(\sum_{i=1}^{N} X_i \beta_i \right)^2 \sigma_m^{\,2} \right] + \left[\left(\sum_{i=1}^{N} X_i^2 \in_i^2 \right) \right]$$

Where,

$\sigma_p^{\,2}$ = variance of portfolio

$\sigma_m^{\,2}$ = expected variance of index

$\in_i^2$ = variation in security's return not related to the market index

x_i = the portion of stock i in the portfolio.

β_i = Beta of stock i in the portfolio

Likewise expected return on the portfolio also can be estimated. For each security α_i and β_i should be estimated.

$$R_P = \sum_{i=1}^{N} x_i (\alpha_i + \beta_i\ R_m)$$

β_i = Value of the beta for security i

x_i = Proportion of the investment on security i

α_i = Value of alpha for security i

N = The number of securities in the portfolio

Portfolio return is the weighted average of the estimated return for each security in the portfolio. The weights are the respective stocks' proportions in the portfolio.

A portfolio's alpha value is the weighted average of the alpha values for its component securities using the proportion of the investment in a security as weight.

$$\alpha_P = \sum_{i=1}^{N} x_i \alpha_i$$

α_P = Value of the alpha for the portfolio

Similarly, a portfolio's beta value is the weighted average of the beta values of its component stocks using relative share of them in the portfolio as weights.

$$\beta_P = \sum_{i=1}^{N} x_i \beta_i$$

Where,

β_P = Value of the beta for the portfolio.

Illustration 1

The following details are given for X and Y companies' stocks and the Bombay Sensex for a period of one year. Calculate the systematic and unsystematic risk for the companies' stocks. If equal amount of money is allocated for the stocks what would be the portfolio risk?

	X Stock	Y Stock	Sensex
Average return	0.15	0.25	0.06
Variance of return	6.30	5.86	2.25
β	0.71	0.685	
Correlation Co-efficient	0.424		
Co-efficient of determination (r^2)	0.18		

Solution

The co-efficient of determination (r^2) i.e. square of Coefficient of Correlation gives the percentage of the variation in the security's return that is explained by the variation of the market index return. In the X company stock return, 18 per cent of variation is explained by the variation of the index and 82 per cent is not explained by the index.

According to Sharpe, the variance explained by the index is the systematic risk. The unexplained variance or the residual variance is the unsystematic risk.

Company X:

Systematic risk $\quad = \beta_i^2 \times$ Variance of market index

$\qquad = (0.71)^2 \times 2.25 = 1.134$

Unsystematic risk($\in_i^2$) $= $ Total variance of security return - Systematic Risk

$\qquad = 6.30 - 1.134$

$\qquad = 5.166$

or

$\qquad = $ Variance of Security Return $(1-r^2)$

$\qquad = 6.30 \times (1-0.18) = 6.3 \times 0.82 = 5.166$

Total risk $\qquad = \beta_i^2 \times \sigma_m^2 + \in_i^2$

$\qquad = $ Systematic Risk + Unsystematic Risk

$\qquad = 1.134 + 5.166 = 6.30$

Company Y:

Systematic risk $\quad = \beta_i^2 \times \sigma_m^2$

$$= (0.685)^2 \times 2.25 = 1.056$$

Unsystematic risk $\quad$ = Total variance of the security return - systematic risk.

$$= 5.86 - 1.056 = 4.804$$

Portfolio Risk

$$\sigma_p{}^2 = \left[\left(\sum_{i=1}^{N} X_i \beta_i \right)^2 \sigma_m{}^2 \right] + \left[\left(\sum_{i=1}^{N} X_i{}^2 \in_i{}^2 \right) \right]$$

$$= [(0.5 \times 0.71 + 0.5 \times 0.685)^2 \ 2.25] + [(0.5)^2(5.166) + (0.5)^2(4.804)]$$

$$= [(0.355 + 0.3425)^2 \ 2.25] + [(1.292 + 1.201)]$$

$$= 1.0946 + 2.493 = 3.5876$$

8. CAPITAL ASSET PRICING MODEL (CAPM)

The CAPM distinguishes between risk of holding a single asset and holding a portfolio of assets. There is a trade-off between risk and return. Modern portfolio theory concentrates on risk and stresses on risk management rather than on return management. Risk may be security risk involving danger of loss of return from an investment in a single financial or capital asset. Security risk differs from portfolio risk, which is the probability of loss from investment in a portfolio of assets. Portfolio risk is comprised of unsystematic risk and systematic risk. Unsystematic risks can be averted through diversification and is related to random variables. Systematic risk is market related component of portfolio risk. It is commonly measured by regression coefficient Beta or the Beta coefficient. Low Beta reflects low risk and high Beta reflects high risk.

As the unsystematic risk can be diversified by building a portfolio, the relevant risk for determining the prices of securities is the non-diversifiable component of the total risk. As mentioned earlier, it can be measured by using Beta (β) a statistical parameter which measures the market sensitivity of returns. The beta for the market is equal to 1.0. Beta explains the systematic relationship between the return on a security and the return on the market by using a simple linear regression equation. The return on a security is taken as a dependent variable and the return on market is taken as independent variable then $R_j = R_f + \beta (R_m - R_f)$. The beta parameter ($\beta$) in this William Sharpe model represents the slope of the above regression relationship and measures the sensitivity or responsiveness of the security returns to the general market returns. The portfolio beta is merely the weighted average of the betas of individual securities included in the portfolio.

Portfolio Beta (β_P) = $\sum$ Proportion of Security × Beta for Security.

CAPM provides a conceptual framework for evaluating any investment decision where capital is committed with a goal of producing future returns. CAPM is based on certain assumptions to provide conceptual framework for evaluating risk and return. Some of the important assumptions are discussed below:

(i) *Efficient market:* It is the first assumption of CAPM. Efficient market refers to the existence of competitive market where financial securities and capital assets are bought and sold with full information of risk and return available to all participants. In an efficient market, the price of individual assets will reflect a real or intrinsic value of a share as the market prices will adjust quickly to any new situation, John J. Hampton has remarked in "Financial decision making" that although efficient capital market is not much relevant to capital budgeting decisions, but CAPM would be useful to evaluate capital budgeting proposal because the company can compare risk and return to be obtained by investment in machinery with risk and return from investment in securities.

(ii) *Rational investment goals:* Investors desire higher return for any acceptable level of risk or the lowest risk for any desired level of return. Such a rational choice is made on logical and consistent ranking of proposals in order of preference for higher good to lower good and this is the scale of the marginal efficiency of capital. Beside, transactive preferences and certainty equivalents are other parameters of rational choice.

(iii) Risk aversion in efficient market is adhered to although at times risk seeking behaviour is adopted for gains.

(iv) CAPM assumes that all assets are divisible and liquid assets.

(v) Investors are able to borrow freely at a risk less rate of interest i.e. borrowings can fetch equal return by investing in safe Government securities.

(vi) Securities can be exchanged without payment of brokerage, commissions or taxes and without any transaction cost.

(vii) Securities or capital assets face no bankruptcy or insolvency.

(viii) CAPM assumes that the Capital Market is in equilibrium.

Based on above assumptions the CAPM is developed with the main goal to formulate the return required by investors from a single investment or a portfolio of assets. The required rate of return is defined as the minimum expected return needed so that investors will purchase and hold an asset.

Risk and return relationship in this model stipulates higher return for higher level of risk and *vice versa*. However, there may be exception to this general rule where markets are not efficient.

Three aspects are worth consideration:

(a)   Stock market is not concerned with diversifiable risk

(b)   It is not concerned with an investor having a diversified portfolio

(c)   Compensation paid is restricted to non-diversifiable risk.

Thus, an investor has to look into the non-diversifiable portion of risk on one side and returns on the other side. To establish a link between the two, the required return one expects to get for a given level of risk has been mandated by the Capital Asset Pricing Model.

If the risk free investment R_f is 5%, an investor can earn this return of 5% by investing in risk free investment. Again if the stock market earns a rate of return R_m which is 15% then an investor investing in stocks constituting the stock market index will earn also 15%. Thus, the excess return earned over and above the risk free return is called the market risk premium ($R_m – R_f$) i.e. (15% - 5%) = 10% which is the reward for undertaking risk. So, if an investment is as risky as the stock market, the risk premium to be earned is 10%.

If an investment is 30% riskier than the stock market, it would carry risk premium i.e. 30% more than the risk premium of the stock market i.e. 10% + 30% of 10% = 10% + 3% = 13%. β identifies how much more risky is an investment with reference to the stock market. Hence the risk premium that a stock should earn is β times the risk premium from the market [$β × (R_m – R_f)$]. The total return from an investment is the risk free rate of return plus the risk premium. So the required return from a stock would be $R_j = R_f + [β × (R_m – R_f)]$. In the above example 5% + 1.3 × (15% - 5%) = 18%

The risk premium on a stock varies in direct proportion to its Beta. If the market risk premium is 6% and β of a stock is 1.2 then the risk premium for that stock shall be 7.2% (6% × 1.2)

Illustration 2

A company's beta is 1.40. The market return is 14% and the risk free rate is 10%.

(i)   What is the expected return of the company's stock based on CAPM.

(ii)   If the risk premium on the market goes up by 2.50% points, what would be the revised expected return on this stock?

Solution

(i)   Computation of expected return based on CAPM

$R_j = R_f + β (R_m – R_f)$ = 10% + 1.40 (14% - 10%) = 10% + 5.6% = 15.6%

(ii)   Computation of expected return if the market risk premium goes up by 2.50% points

The return from the market goes up by 2.50% i.e. 14% + 2.50% = 16.50%

Expected Return based on CAPM is given by

R_j = 10% + 1.40 (16.5% - 10%) = 10% + 1.40 × 6.5% = 10% + 9.10% = 19.10%

A graphical representation of CAPM is the Security Market Line, (SML). This line indicates the rate of return required to compensate at a given level of risk. Plotting required return on Y axis and Beta on the X-axis we get an upward sloping line which is given by (R_m – R_f), the risk premium.

The higher the Beta value of a security, higher would be the risk premium relative to the market. This upward sloping line is called the Security Market Line. It measures the relationship between systematic risk and return.
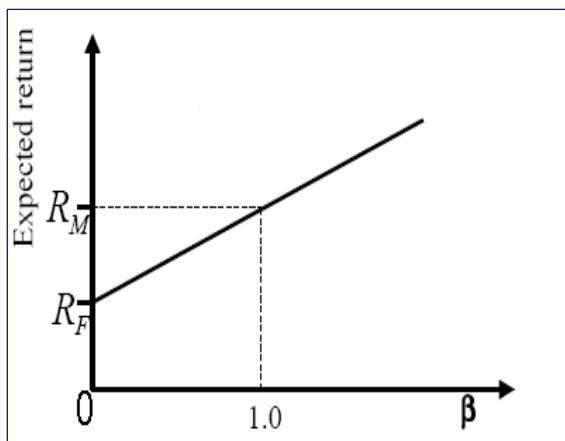
Illustration 3

The risk premium for the market is 10%. Assuming Beta values of Security K are 0, 0.25, 0.42, 1.00 and 1.67. Compute the risk premium on Security K.

Solution

Market Risk Premium is 10%

β Value of K	Risk Premium of K
0.00	0%
0.25	2.50%
0.42	4.20%
1.00	10.00%
1.67	16.70%

Illustration 4

Treasury Bills give a return of 5%. Market Return is 13%

(i)   What is the market risk premium

(ii)   Compute the β Value and required returns for the following combination of investments.

Treasury Bill	100	70	30	0
Market	0	30	70	100

Solution

Risk Premium $R_m - R_f$ = 13% - 5% = 8%

β is the weighted average investing in portfolio consisting of market β = 1 and treasury bills (β = 0)

Portfolio	Treasury Bills: Market	β	$R_j = R_f + \beta \times (R_m - R_f)$
1	100:0	0	5% + 0(13% - 5%) = 5%
2	70:30	0.7(0) + 0.3(1) = 0.3	5% + 0.3(13% - 5%) = 7.40%
3	30:70	0.3(0) + 0.7(1) = 0.7	5% + 0.7(13% - 5%) = 10.60%
4	0:100	1	5% + 1.0(13% - 5%) = 13%

8.1 Risk free Rate of Return

In CAPM, there is only one risk free rate. It presumes that the returns on a security include both dividend payments and capital appreciation. These require to be factored in judging the value of Beta and in computing the required rate of return.

Illustration 5

Pearl Ltd. expects that considering the current market prices, the equity shareholders as per Moderate Approach, should get a return of at least 15.50% while the current return on the market is 12%. RBI has closed the latest auction for ₹ 2500 crores of 182 day bills for the lowest bid of 4.3% although there were bidders at a higher rate of 4.6% also for lots of less than ₹ 10 crores. What is Pearl Ltd's Beta?

Solution

Determining Risk free rate: Two risk free rates are given. The aggressive approach would be to consider 4.6% while the conservative approach would be to take 4.3%. If we take the moderate value then the simple average of the two i.e. 4.45% would be considered

Application of CAPM

$$R_j = R_f + \beta (R_m - R_f)$$

$$15.50\% = 4.45\% + \beta (12\% - 4.45\%)$$

$$\beta = \frac{15.50\% - 4.45\%}{12\% - 4.45\%} = \frac{11.05}{7.55}$$

$$= 1.464$$

Illustration 6

The following information is available with respect of Jaykay Ltd.

Year	Jay Kay Limited		Market		Return on Govt. Bonds
	Average Share Price (₹)	DPS (₹)	Average Index	Dividend Yield (%)	
2002	242	20	1812	4	6
2003	279	25	1950	5	5
2004	305	30	2258	6	4
2005	322	35	2220	7	5

Compute Beta Value of the company as at the end of 2005. What is your observation?

Solution

(1) Computation of Beta Value

Calculation of Returns

$$\text{Returns} = \frac{D_1 + (P_1 - P_0)}{P_0} \times 100$$

Year	*Returns*
2002 – 2003	$\dfrac{25 + (279 - 242)}{242} \times 100 = 25.62\%$
2003 – 2004	$\dfrac{30 + (305 - 279)}{279} \times 100 = 20.07\%$
2004 – 2005	$\dfrac{35 + (322 - 305)}{305} \times 100 = 17.05\%$

Calculation of Returns from market Index

Year	% of Index Appreciation	Dividend Yield %	Total Return %
2002 − 2003	$\frac{1950 - 1812}{1812} \times 100 = 7.62\%$	5%	12.62%
2003 − 2004	$\frac{2258 - 1950}{1950} \times 100 = 15.79\%$	6%	21.79%
2004 − 2005	$\frac{2220 - 2258}{2258} \times 100 = (-)1.68\%$	7%	5.32%

Computation of Beta

Year	X	Y	XY	Y²
2002-2003	25.62	12.62	323.32	159.26
2003-2004	20.07	21.79	437.33	474.80
2004-2005	17.05	5.32	90.71	28.30
	62.74	39.73	851.36	662.36

$$\overline{X} = \frac{62.74}{3} = 20.91, \ \overline{Y} = \frac{39.73}{3} = 13.24$$

$$\beta = \frac{\sum XY - n \ \overline{X} \ \overline{Y}}{\sum Y^2 - n(\overline{Y})^2}$$

$$= \frac{851.36 \ - \ 3(20.91)(13.24)}{662.36 \ - \ 3(13.24)^2}$$

$$= \frac{851.36 \ - \ 830.55}{662.36 \ - \ 525.89} = \frac{20.81}{136.47} = 0.15$$

(2) Observation: Based on the beta value of the company and the analysis of returns, it can be concluded that changes in the return on share of Jaykay Ltd. are less affected by changes in market returns.

8.2 Under Valued and Over Valued Stocks

The CAPM model can be practically used to buy, sell or hold stocks. CAPM provides the required rate of return on a stock after considering the risk involved in an investment. Based on current market price or any other judgmental factors (benchmark) one can identify as to what would be the expected return over a period of time. By comparing the required return as per CAPM with the expected return the following investment decisions are available:

(a)   **When required return as per CAPM < Actual Return – Buy:** This is due to the stock being undervalued i.e. the stock gives more return than what it should give.

(b)   **When required return as per CAPM > Actual Return – Sell:** This is due to the stock being overvalued i.e. the stock gives less return than what it should give.

(c)   **When required return as per CAPM = Actual Return – Hold:** This is due to the stock being correctly valued i.e. the stock gives same return than what it should give.

From another angle, if the current market price is considered as a basis of CAPM then:

(i)   Actual Market Price < Market Price using CAPM, stock is undervalued

(ii)   Actual market Price > Market Price using CAPM, stock is overvalued

(iii)   Actual market Price = Market Price using CAPM, stock is correctly valued.

Illustration 7

Information related to an investment is as follows:

Risk free rate   10%

Market Return   15%

Beta   1.2

(i)   What would be the return from this investment?

(ii)   If the projected return is 18%, is the investment rightly valued?

(iii)   What is your strategy?

Solution

(i)   Required rate of Return as per CAPM is given by

$R_j = R_f + \beta (R_m - R_f)$

= 10 + 1.2 (15-10) = 16%

(ii)   Since projected return is 18%, the stock is not rightly valued rather undervalued as return as per CAPM less than Projected Return.

(iii)   Had this Project Return is considered as expected return, the decision should be to BUY the share.

Illustration 8

The expected returns and Beta of three stocks are given below

Stock	A	B	C
Expected Return (%)	18	11	15
Beta Factor	1.7	0.6	1.2

If the risk free rate is 9% and the expected rate of return on the market portfolio is 14% which of the above stocks are over, under or correctly valued in the market? What shall be the strategy?

Solution

Required Rate of Return is given by

$$R_j = R_f + \beta\ (R_m\text{-}R_f)$$

For Stock A, $R_j = 9 + 1.7\ (14 - 9) = 17.50\%$

Stock B, $R_j = 9 + 0.6\ (14\text{-}9) = 12.00\%$

Stock C, $R_j = 9 + 1.2\ (14\text{-}9) = 15.00\%$

Required Return %	Expected Return %	Valuation	Decision
17.50%	18.00%	Under Valued	Buy
12.00%	11.00%	Over Valued	Sell
15.00%	15.00%	Correctly Valued	Hold

8.3 Advantages and Limitations of CAPM

The advantages of CAPM can be listed as:

(i)   *Risk Adjusted Return:* It provides a reasonable basis for estimating the required return on an investment which has risk inbuilt into it. Hence, it can also be used as Risk Adjusted Discount Rate in Capital Budgeting.

(ii)   *No Dividend Company*: It is useful in computing the cost of equity of a company which does not declare dividend.

There are certain limitations of CAPM as well, which are discussed as follows:

(a)   *Reliability of Beta*: Statistically reliable Beta might not exist for shares of many firms. It may not be possible to determine the cost of equity of all firms using CAPM. All shortcomings that apply to Beta value applies to CAPM too.

(b)   *Other Risks*: By emphasising on systematic risk only, the unsystematic risk that are of importance too does not find place in a diversified portfolio.

(c)   *Information Available*: It is extremely difficult to obtain important information on risk free interest rate and expected return on market portfolio because depending upon the situations there can be multiple risk free rates and market being volatile its return varies from time to time.

9.   ARBITRAGE PRICING THEORY MODEL (APT)

Arbitrage pricing theory (APT) is used as an alternative to Capital Assets Pricing Model (CAPM). While the CAPM formula helps to calculate the market's expected return, APT uses the risky asset's expected return and the risk premium of a number of macroeconomic factors.

In a simplistic way, if a particular asset, say a stock, has its major influencers factors then the stocks' return would be calculated by using the Arbitrage Pricing Theory (APT) in the following manner:

(a)   Calculate the risk premium for these risk factors (beta for the risk factor 1 – interest rate, and beta of the risk factor 2 – sector growth rate; etc. Conceptually risk premium is compensation over and above risk free rate of return that an investor expects/ requires for bearing that risk.

(b)   Adding the risk free rate of return.

Thus, the formula for APT is represented as –

$$E(R_i) = R_f + \lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3\beta_3 \ldots\ldots\ldots\ldots\lambda_n\beta_n$$

Where,

R_f = Risk Free Rate

λ_n = nth factor price or risk premium

β_n = Sensitivity of the Factor n

The above formula provides the expected return in efficient market when equilibrium is reached.

Illustration 9

With the help of following data determine the return on the security X.

Factor	Risk Premium associated with the Factor	β_i
Market	4%	1.3
Growth Rate of GDP	1%	0.3
Inflation	-4%	0.2

Risk Free Rate of Return is 8%.

Solution

Expected Return = $R_f + \lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3\beta_3$

$\qquad\qquad$ = 8% + 1.3 x 4% + 0.3 x 1% + 0.2 x (-4%)

$\qquad\qquad$ = 8% + 5.2% + 0.3% - 0.8%

$\qquad\qquad$ = 12.7%

As mentioned earlier while CAPM concentrates on one factor (market risk) in its Model, APT does not specifically requires any particular type of factor to be concentrated upon. Though Stephan Ross identified change in the following factors:

❖   Inflation

❖   Level of Industrial Production

❖   Risk Premium

❖   Term Structure of Interest Rates

Further according to Ross, if no surprise happens to these macro-economic factors then actual returns shall be equal to expected. In case, if any unanticipated changes happens in these factors, then formula of APT shall be as follows:

$E(R) = R_f + \beta_1 (EV_1 - AV_1) + \beta_2 (EV_2 - AV_2) + \ldots\ldots \beta_n (EV_n - AV_n)$

Where

$(EV_n - AV_n)$ = Surprise Factor due to change in the Value of Factor

R_f = Risk Free Rate of Return

β_n = Sensitivity of corresponding Macro-economic factor

10. PORTFOLIO EVALUATION METHODS

Following three ratios are used to evaluate the portfolio:

10.1 Sharpe Ratio

Sharpe Ratio measures the Risk Premium per unit of Total Risk for a security or a portfolio of securities. The formula is as follows:

$$S = \frac{R_i - R_f}{\sigma_i}$$

Where,

R_i = Return on Security/Portfolio

R_f = Risk Free Rate of Return

σ_i = Standard Deviation of Return of Security/Portfolio

S = Sharpe Ratio

Example

Let's assume that we look at a one year period of time where an index fund earned 11%

Treasury bills earned 6%. The standard deviation of the index fund was 20%

Therefore S = (0.11 –0.06)/.20 = 25%

The Sharpe ratio is an appropriate measure of performance for an overall portfolio particularly when it is compared to another portfolio, or another index such as the S&P 500, Small Cap index, etc.

That said however, it is not often provided in most rating services.

Example

Consider two Portfolios A and B. Let return of A be 30% and that of B be 25%. On the outset, it appears that A has performed better than B. Let us now incorporate the risk factor and find out the Sharpe ratios for the portfolios. Let risk of A and B be 11% and 5% respectively. This means that the standard deviation of returns - or the volatility of returns of A is much higher than that of B.

If risk free rate is assumed to be 8%,

Sharpe ratio for portfolio A = (30% - 8%)/11% = 2 and

Sharpe ratio for portfolio B = (25% – 8%)/5% = 3.4

Higher the Sharpe Ratio, better is the portfolio on a risk adjusted return metric. Hence, our primary judgment based solely on returns was erroneous. Portfolio B provides better risk adjusted returns than Portfolio A and hence is the preferred investment. Producing healthy returns with low volatility is generally preferred by most investors to high returns with high volatility. Sharpe ratio is a good tool to use to determine a portfolio that is suitable to such investors.

10.2 Treynor Ratio

This ratio is same as Sharpe ratio with only difference that it measures the Risk Premium per unit of Systematic Risk (β) for a security or a portfolio of securities. The formula is as follows:

$$T = \frac{R_i - R_f}{\beta_i}$$

Where,

R_i = Return on Security/Portfolio

R_f = Risk Free Rate of Return

β_i = Beta of Security or Portfolio

T = Treynor Ratio

Treynor ratio is based on the premise that unsystematic or specific risk can be diversified and hence, only incorporates the systematic risk (beta) to gauge the portfolio's performance. It measures the returns earned in excess of those that could have been earned on a riskless investment per unit of market risk assumed.

In above example if beta of Portfolio A and B are 1.5 and 1.1 respectively,

Treynor ratio for Portfolio A= (30%-8%)/1.5=14.67%

Treynor ratio for Portfolio B= (25%-8%)/1.1= 15.45%

The results are in line with that of the Sharpe ratio results.

Both Sharpe ratio and Treynor ratio measure risk adjusted returns. The difference lies in how risk is defined in either case. In Sharpe ratio, risk is determined as the degree of volatility in returns - the variability in month-on-month or period-on-period returns - which is expressed through the standard deviation of the stream of returns you are considering.

In Treynor ratio, you look at the beta of the portfolio or security - the degree of "momentum" that has been built into the portfolio by the fund manager in order to derive his excess returns. High momentum - or high beta (where beta is > 1) implies that the portfolio will move faster (up as well as down) than the market.

While Sharpe ratio measures total risk (as the degree of volatility in returns captures all elements of risk - systematic as well as unsystematic), the Treynor ratio captures only the systematic risk in its computation.

When one has to evaluate the funds which are sector specific, Sharpe ratio would be more meaningful. This is due to the fact that unsystematic risk would be present in sector specific funds. Hence, a true measure of evaluation would be to judge the returns based on the total risk.

On the contrary, if we consider diversified equity funds, the element of unsystematic risk would be very negligible as these funds are expected to be well diversified by virtue of their nature. Hence, Treynor ratio would be more apt here.

It is widely found that both ratios usually give similar rankings. This is based on the fact that most of the portfolios are fully diversified. To summarize, we can say that when the fund is not fully diversified, Sharpe ratio would be a better measure of performance and when the portfolio is fully diversified, Treynor ratio would better justify the performance of a fund.

Example: In 2019-20 where Fidelity Magellan had earned about 18%. Many bond funds had earned 13%. Which is better? In absolute numbers, 18% beats 13%. But if we then state that the bond funds had about half the market risk, now which is better? We don't even need to do the formula for that analysis. But that is missing in almost all reviews by all brokers. For clarification, we do not suggest they put all the money into either one- just that they need to be aware of the implications.

10.3 Jensen Alpha

This is the difference between a portfolio's actual return and those that could have been made on a benchmark portfolio with the same risk- i.e. Beta. It measures the ability of active management to increase returns above those that are purely a reward for bearing market risk. However, caveat applies it will only produce meaningful results if it is used to compare two portfolios which have similar betas. The formula is given as follows:

Alpha = Actual Return on portfolio – Expected Return on portfolio.

Assume Two Portfolios

	A	B	Market Return
Return	12	14	12
Beta	0.7	1.2	1.0

Risk Free Rate = 9%

The return expected as per CAPM

= Risk Free Return + Beta portfolio (Return of Market - Risk Free Return)

Hence, the expected return of Portfolio A using CAPM

= 0 .09 + 0.7 (0.12 - 0.09) = 0.09 + 0.021 = 0.111

Alpha　　= Return of Portfolio - Expected Return

　　　　= 0.12 - 0.111 = 0.009

As long as "apples are compared to apples"- in other words a computer sector fund A to computer sector fund B - it is a viable number. But if taken out of context, it loses meaning. Alphas are found in many rating services but are not always developed the same way- so you can't compare an alpha from one service to another. However, we have usually found that their relative position in the particular rating service is to be viable. Short-term alphas are not valid. Minimum time frames are one year- three year is more preferable.

Alpha can also be used to assess the performance of a Portfolio or a Fund Manager. While a positive alpha indicates that the Fund Manager is doing well, a negative alpha indicates that Portfolio Manager is not doing well in comparison of market return.

11. SHARPE'S OPTIMAL PORTFOLIO

William Sharpe has developed a simplified variant of Markowitz model that reduces substantially its data and computational requirements.

This model is based on desirability of an investor for excess return of risk free rate of return to beta. Accordingly, the ranking of securities shall be based on the Sharpe Ratio and unique cut off point C^* discussed below.

The steps for finding out the stocks to be included in the optimal portfolio are given below:

(a)　Find out the "excess return to beta" ratio for each stock under consideration i.e. Treynor's ratio.

(b)　Rank them from the highest to the lowest.

(c)　Proceed to calculate C_i for all the stocks according to the ranked order using the following formula:

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{N} \frac{\left(R_i - R_f\right)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{N} \frac{\beta_i^2}{\sigma_{ei}^2}}$$

Where,

$\sigma_m^2 =$ variance of the market index

$\sigma_{\in i}^2 =$ variance of a stock's movement that is not associated with the movement of market index i.e. stock's unsystematic risk.

(d) Compute the cut-off point which the highest value of C_i and is taken as C*. The stock whose excess-return to risk ratio is above the cut-off ratio are selected and all whose ratios are below are rejected. The main reason for this selection is that since securities are ranked from highest excess return to Beta to lowest, and if particular security belongs to optional portfolio all higher ranked securities also belong to optimal portfolio.

(e) Once we came to know which securities are to be included in the optimum portfolio, we shall calculate the percent to be invested in each security by using the following formula:

$$X_i^o = \frac{Z_i}{\sum\limits_{j=1}^{N} Z_i}$$

where

$$Z_i = \frac{B_i}{\sigma_{ei}^2}\left(\frac{R_i - R_0}{B_i} - C^*\right)$$

The first portion determines the weight each stock and total comes to 1 to ensure that all funds are invested and second portion determines the relative investment in each security.

12. FORMULATION OF PORTFOLIO STRATEGY

Two broad choices are required for the formulation of an appropriate Portfolio Strategy. They are Active Portfolio Strategy and Passive Portfolio Strategy.

12.1 Active Portfolio Strategy (APS)

An APS is followed by most investment professionals and aggressive investors who strive to earn superior return after adjustment for risk. The vast majority of funds (or schemes) available in India follow an "active" investment approach, wherein fund managers of "active" funds spend a great deal of time on researching individual companies, gathering extensive data about financial performance, business strategies and management characteristics. In other words, "active" fund managers try to identify and invest in stocks of those companies that they think will produce better returns and beat the overall market (or Index).

There are four principles of an active strategy. These are:

(a) Market Timing : This involves departing from the normal i.e. strategy for long run asset mix to reflect assessment of the prospect of various assets in the near future. Market timing is based on an explicit or implicit forecast of general market movement. A variety of tools are employed for market timing analysis namely business cycle analysis, moving average analysis, advance-decline analysis, Econometric models. The forecast for the general market movement derived with the help of one or more of these tools is tempted by the subjective judgment of the investors. In most cases investors may go largely by their market sense. Those who reveal the fluctuation in the market may be tempted to play the game of market timing but few will succeed in this game. Further an investment manager has to forecast the market correctly and 75% of the time he is only able to break even after taking into account the cost of errors and cost of transactions. According to Fisher Black, the market is just as well as on an average when the investor is out of the market as it does when he is in. So, he loses money relative to a single buy and sale strategy by being out of the market part of the time.

(b) Sector Rotation: Sector or group rotation may apply to both stock and bond component of the portfolio. It is used more compulsorily with respect to strategy. The components of the portfolio are used when it involves shifting. The weighting for various industry sectors is based on their asset outlook. If one thinks that steel and pharmaceutical would do well as compared to other sectors in the forthcoming period he may overweigh the sector relative to their position in the market portfolio, with the result that his portfolio will be tilted more towards these sectors in comparison to the market portfolio.

With respect to bond portfolio sector rotation it implies a shift in the composition of the bond portfolio in terms of quality as reflected in credit rating, coupon rate, term of maturity etc. If one anticipates a rise in the interest rate one may shift from long term bonds to medium and short term. A long term bond is more sensitive to interest rate variation compared to a short term bond.

(c) Security Selection: Security selection involves a search for under-priced security. If one has to resort to active stock selection he may employ fundamental / technical analysis to identify stocks which seems to promise superior return and concentrate the stock components of portfolio on them. Such stock will be over weighted relative to their position in the market portfolio. Like wise stock which are perceived to be unattractive will be under weighted relative to their position in the market portfolio.

As far as bonds are concerned security selection calls for choosing bonds which offer the highest yields to maturity and at a given level of risk.

(d) *Use of Specialised Investment Concept:* To achieve superior return, one has to employ a specialised concept/philosophy particularly with respect to investment in stocks. The concept which have been exploited successfully are growth stock, neglected or out of favour stocks, asset stocks, technology stocks and cyclical stocks.

The advantage of cultivating a specialized investment concept is that it helps to:

(i)   Focus one's effort on a certain kind of investment that reflects one's ability and talent.

(ii)   Avoid the distraction of perusing other alternatives.

(iii)   Master an approach or style through sustained practice and continual self criticism.

The greatest disadvantage of focusing exclusively on a specialized concept is that it may become obsolete. The changes in the market risk may cast a shadow over the validity of the basic premise underlying the investor philosophy.

12.2 Passive Portfolio Strategy

Active strategy was based on the premise that the capital market is characterized by efficiency which can be exploited by resorting to market timing or sector rotation or security selection or use of special concept or some combination of these basis.

Passive strategy, on the other hand, rests on the tenet that the capital market is fairly efficient with respect to the available information. Hence they search for superior return. Basically, passive strategy involves adhering to two guidelines. They are:

(a)   Create a well-diversified portfolio at a predetermined level of risk.

(b)   Continue to hold the portfolio relatively unchanged over a period unless it becomes inadequately diversified or inconsistent with the investor risk return preference.

A fund which is *passively* managed is called index funds. An Index fund is a mutual fund scheme that invests in the securities of the target Index in the same proportion or weightage. Though it is designed to provide returns that closely track the benchmark Index, an Index Fund carries all the risks normally associated with the type of asset the fund holds. So, when the overall stock market rises/falls, you can expect the price of shares in the index fund to rise/fall, too. In short, an index fund does not mitigate market risks. Indexing merely ensures that your returns will not stray far from the returns on the Index that the fund mimics. In other words, an index fund is a fund whose daily returns are the same as the daily returns obtained from an index. Thus, it is passively managed in the sense that an index fund manager invests in a portfolio which is exactly the same as the portfolio which makes up an index. For instance, the NSE-50 index (Nifty) is a market index which is made

up of 50 companies. A Nifty index fund has all its money invested in the Nifty fifty companies, held in the same weights of the companies which are held in the index.

12.3 Selection of Securities

There are certain criteria which must be kept in mind while selecting securities. The selection criteria for both bonds and equity shares are given as following:

12.3.1 Selection of Bonds

Bonds are fixed income avenues. The following factors have to be evaluated in selecting fixed income avenues:

(a)   *Yield to maturity*: The yield to maturity for a fixed income avenues represent the rate of return earned by the investor, if he invests in the fixed income avenues and holds it till its maturity.

(b)   *Risk of Default*: To assess such risk on a bond, one has to look at the credit rating of the bond. If no credit rating is available relevant financial ratios of the firm have to be examined such as debt equity, interest coverage, earning power etc and the general prospect of the industry to which the firm belongs have to be assessed.

(c)   *Tax Shield:* In the past, several fixed income avenues offers tax shields but at present only a few of them do so.

(d)   *Liquidity:* If the fixed income avenues can be converted wholly or substantially into cash at a fairly short notice it possesses a liquidity of a high order.

12.3.2 Selection of Stock (Equity Share)

Three approaches are applied for selection of equity shares- Technical analysis, Fundamental analysis and Random selection analysis.

(a)   Technical analysis looks at price behaviours and volume data to determine whether the share price will move up or down or remain trend less.

(b)   Fundamental analysis focuses on fundamental factors like earning level, growth prospects and risk exposure to establish intrinsic value of a share. The recommendation to buy hold or sell is based on comparison of intrinsic value and prevailing market price.

(c)   Random selection analysis is based on the premise that the market is efficient and security is properly priced.

Levels of Market Efficiency and Approach to Security Selection

Approach / Levels of Efficiency	Technical Analysis	Fundaments Analysis	Random Selection
1) Inefficiency	Best	Poor	Poor
2) Weak form efficiency	Poor	Best	Poor
3) Semi-strong efficiency	Poor	Good	Fair
4) Strong Form efficiency	Poor	Fair	Best
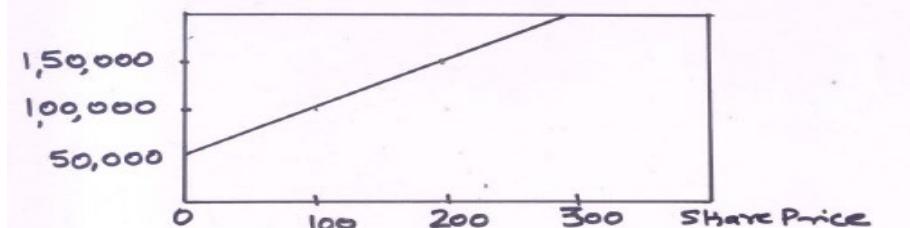
13. PORTFOLIO REVISION AND REBALANCING

It means the value of portfolio as well as its composition. The relative proportion of bond and stocks may change as stock and bonds fluctuate in response to such changes. Portfolio rebalancing is necessary. There are three policies of portfolio rebalancing- Buy and hold policy, Constant mix policy, and Constant Proportion Portfolio Insurance (CPPI) policy. These policies have different pay off under varying market conditions. Under all these policies portfolio consists of investment in stock and bonds.

(a) Buy and Hold Policy: Sometime this policy is also called 'do nothing policy' as under this strategy no balancing is required and therefore investor maintain an exposure to stocks and therefore linearly related to the value of stock in general.

Under this strategy investors set a limit (floor) below which he does not wish the value of portfolio should go. Therefore, he invests an amount equal to floor value in non-fluctuating assets (Bonds). Since the value of portfolio is linearly related to value of stocks the pay-off diagram is a straight line. This can be better understood with the help of an example. Suppose a portfolio consisting of Debt/ Bonds for ₹ 50,000 and ₹ 50,000 in equity shares currently priced at ₹ 100 per share. If price of the share moves from ₹ 100 to ₹ 200 the value of portfolio shall become ₹ 1,50,000. The pay-off diagram is shown in figure below i.e. a straight line:

This policy is suitable for the investor whose risk tolerance is positively related to portfolio and stock market return but drops to zero of below floor value.

Concluding, it can be said that following are main features of this policy:
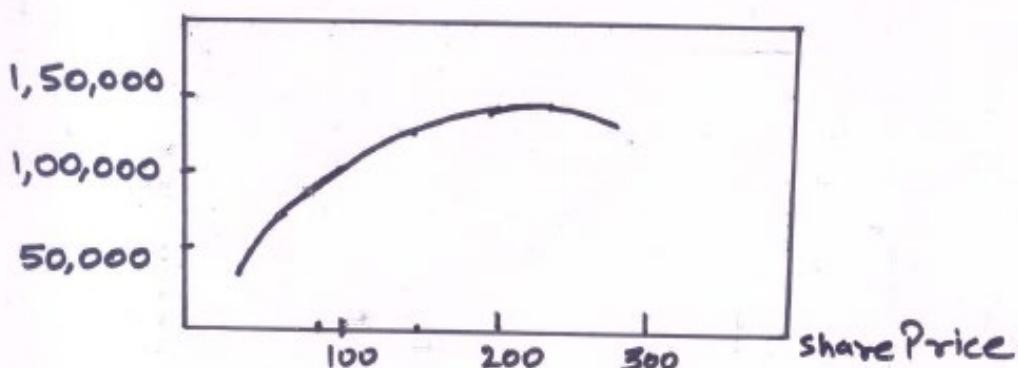
(a)　The value of portfolio is positively related and linearly dependent on the value of the stock.

(b)　The value of portfolio cannot fall below the floor value i.e. investment in Bonds.

(c)　This policy performs better if initial percentage is higher in stock and stock outperform the bond. Reverse will happen if stock under perform in comparison of bond or their prices goes down.

(b)　Constant Mix Policy:　Contrary to above policy this policy is a 'Do Something Policy'. Under this policy investor maintains an exposure to stock at a constant percentage of total portfolio. This strategy involves periodic rebalancing to required (desired) proportion by purchasing and selling stocks as and when their prices goes down and up respectively. In other words this plan specifies that value of aggressive portfolio to the value of conservative portfolio will be held constant at a pre-determined ratio. However, it is important to this action is taken only there is change in the prices of share at a predetermined percentage.

For example if an investor decided his portfolio shall consist of 60% in equity shares and balance 40% in bonds on upward or downward of 10% in share prices he will strike a balance.

In such situation if the price of share goes down by 10% or more, he will sell the bonds and invest money in equities so that the proportion among the portfolio i.e. 60:40 remains the same. According if the prices of share goes up by 10% or more he will sell equity shares and shall in bonds so that the ratio remains the same i.e. 60:40. This strategy is suitable for the investor whose tolerance varies proportionally with the level of wealth and such investor holds equity at all levels.

The pay-off diagram of this policy shall be as follows:

Accordingly, it gives a concave pay off, tends to do well in flat but fluctuating market.

Continuing above example let us how investor shall rebalance his portfolio (50 : 50) under different scenarios as follows:

(a)   If price decreases

Share Price		Value of Shares	Value of Bonds	Total	Stock to Bond Switching	Bond to Stock Switching
100	Starting Level	50,000	50,000	1,00,000	-	-
80	Before Rebalancing	40,000	50,000	90,000	-	-
	After Rebalancing	45,000	45,000	90,000	-	5,000
60	Before Rebalancing	33,750	45,000	78,750	-	-
	After Rebalancing	39,360	39,390	78,750	-	5,610

(b)   If price increases

Share Price		Value of Shares	Value of Bonds	Total	Stock to Bond Switching	Bond to Stock Switching
100	Starting Level	50,000	50,000	1,00,000	-	-
150	Before Rebalancing	75,000	50,000	1,25,000	-	-
	After Rebalancing	62,400	62,600	1,25,000	12,600	-
200	Before Rebalancing	83,200	62,600	1,45,800	-	-
	After Rebalancing	72,800	73,000	1,45,800	10,400	-

(c)   Constant Proportion Insurance Policy : Under this strategy investor sets a limit below which he does not wish his asset to fall called 'Floor Value', which is invested in some non-fluctuating assets such as Treasury Bills, Bonds etc. The value of portfolio under this strategy shall not fall below this specified floor under normal market conditions. This strategy performs well especially in bull market as the value of shares purchased as cushion increases. In contrast in bearish market losses are avoided by sale of shares. It should however be noted that this strategy performs very poorly in the market hurt by sharp reversals. The following equation is used to determine equity allocation:

Target Investment in Shares = Multiplier (Portfolio Value – Floor Value)
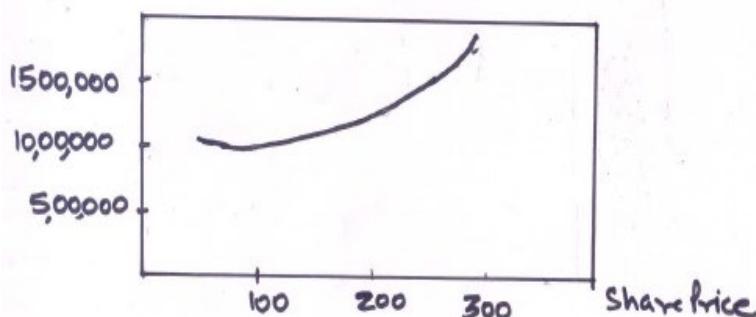
Multiplier is a fixed constant whose value shall be more than 1.

The pay-off under this strategy can be understood better with the help of an example. Suppose wealth of Mr. A is ₹ 10,00,000, a floor value of ₹ 7,50,000 and a multiplier of 2. Since the initial cushion (difference between Portfolio Value and Floor) is ₹ 2,50,000, the initial investment in the share shall be ₹ 5,00,000 (double of the initial cushion). Accordingly, initial portfolio mix shall consist of ₹ 5,00,000 in shares and balance ₹ 5,00,000 in Bonds.

Situation 1: Suppose stock market rises from 100 to 150. The value of shares of Mr. A's holding shall rise from ₹ 5,00,000 to ₹ 7,50,000 and value of portfolio shall jump to ₹ 12,50,000 and value of cushion to ₹ 7,50,000. Since the CPPI Policy requires the component of shares should go up to ₹ 10,00,000. This will necessitate the selling of bonds amounting ₹ 2,50,000 and re-investing proceeds in shares.

Situation 2: If stock market falls from 100 to 80, the value of shares of portfolio falls from ₹ 5,00,000 to ₹ 4,00,000 resulting in reduction of value of portfolio to ₹ 9,00,000 and cushion to ₹ 1,50,000. Since as per CPPI the share component should be ₹ 3,00,000 (₹ 1,50,000 x 2), hence shares of ₹ 1,00,000 should be sold and invest in Bonds.

Thus from above it is clear that as per CPPI sell the shares as their prices fall and buy them as their prices rise. This policy is contrary to the Constant Mix Policy and hence pay-off of CPPI shall be convex as shown below:

(d) **Comparative Evaluation**

Basis	Buy & Hold Policy	Constant Mix Policy	Constant Proportion Portfolio Insurance
Pay-off Line	Straight	Concave	Convex
Protection in Down/Up Markets	Definitely poor in Down market	Not much in Down market but relatively poor in Up market	Good in Down market and performs well in Up market

Performance in flat market	Performs between Constant and CPPI	Tend to do well in flat market.	Performs poorly.

14. ASSET ALLOCATION STRATEGIES

Many portfolios containing equities also contain other asset categories, so the management factors are not limited to equities. There are four asset allocation strategies:

(a)   Integrated Asset Allocation: Under this strategy, capital market conditions and investor objectives and constraints are examined and the allocation that best serves the investor's needs while incorporating the capital market forecast is determined.

(b)   Strategic Asset Allocation: Under this strategy, optimal portfolio mixes based on returns, risk and co-variances is generated using historical information and adjusted periodically to restore target allocation within the context of the investor's objectives and constraints.

(c)   Tactical Asset Allocation: Under this strategy, investor's risk tolerance is assumed constant and the asset allocation is changed based on expectations about capital market conditions.

(d)   Insured Asset Allocation: Under this strategy, risk exposure for changing portfolio values (wealth) is adjusted; more value means more ability to take risk.

15. FIXED INCOME PORTFOLIO

Fixed Income Portfolio is same as equity portfolio with difference that it consist of fixed income securities such as bonds, debentures, money market instruments etc. Since, it mainly consists of bonds, it is also called Bond Portfolio.

15.1 Fixed Income Portfolio Process

Just like other portfolios, following five steps are involved in fixed income portfolio.

1.   Setting up objective

2.   Drafting guideline for investment policy

3.   Selection of Portfolio Strategy - Active and Passive

4.   Selection of securities and other assets

5.   Evaluation of performance with benchmark

15.2 Calculation of Return on Fixed Income Portfolio

First and foremost step in evaluation of performance of a portfolio is calculation of return. Although there can be many types of measuring returns as per requirements but some of the commonly used measures are :

(i)   Arithmetic Average Rate of Return

(ii)   Time Weighted Rate of Return

(iii)   Rupee Weighted Rate of Return

(iv)   Annualized Return

15.3 Fixed Income Portfolio Management Strategies

There are two strategies

(i)   Passive Strategy

(ii)   Active Strategy

15.3.1 Passive Strategy

As mentioned earlier Passive Strategy is based on the premise that securities are fairly priced commensurate with the level of risk. Though investor does not try to outperform the market but it does not imply they remain totally inactive. Common strategies applied by passive investors of fixed income portfolios are as follows:

(i)   Buy and Hold Strategy: This technique is do nothing technique and investor continues with initial selection and do not attempt to churn bond portfolio to increase return or reduce the level of risk.

However, sometime to control the interest rate risk, the investor may set the duration of fixed income portfolio equal to benchmarked index.

(ii)   Indexation Strategy: This strategy involves replication of a predetermined benchmark well known bond index as closely as possible.

(iii)   Immunization: This strategy cannot exactly be termed as purely passive strategy but a hybrid strategy. This strategy is more popular among pension funds. Since pension funds promised to pay fixed amount to retiring people in the form of annuities any inverse movement in interest may threaten fund's ability to meet their liability timely. By building an immunized portfolio the interest rate risk can be avoided.

(iv) Matching Cash Flows: Another stable approach to immunize the portfolio is Cash Flow Matching. This approach involves buying of Zero Coupon Bonds to meet the promised payment out of the proceeds realized.

15.3.2 Active Strategy

As mentioned earlier Active Strategy is usually adopted to outperform the market. Following are some of active strategies:

(1) Forecasting Returns and Interest Rates: This strategy involves the estimation of return on basis of change in interest rates. Since interest rate and bond values are inversely related if portfolio manager is expecting a fall in interest rate of bonds he/she should buy with longer maturity period. On the contrary, if he/she expected a fall in interest then he/she should sell bonds with longer period.

Based on short term yield movement following three strategies can be adopted:

(a) Bullet Strategy: This strategy involves concentration of investment in one particular bond. This type of strategy is suitable for meeting the fund after a point of time such as meeting education expenses of children etc. For example, if 100% of fund meant for investing in bonds is invested in 5-years Bond.

(b) Barbell Strategy: As the name suggests this strategy involves investing equal amount in short term and long term bonds. For example, half of fund meant for investment in bonds is invested in 1-year Bond and balance half in 10-year Bonds.

(c) Ladder Strategy: This strategy involves investment of equal amount in bonds with different maturity periods. For example if 20% of fund meant for investment in bonds is invested in Bonds of periods ranging from 1 year to 5 years.

Further estimation of interest ratio is a daunting task, and quite difficult to ascertain. There are several models available to forecast the expected interest rates which are based on:

(i)   Inflation

(ii)   Past Trends

(iii)   Multi Factor Analysis

It should be noted that these models can be used as estimates only, as it is difficult to calculate the accurate changes.

There is one another technique of estimating expected change in interest rate called 'Horizon Analysis'. This technique requires that analyst should select a particular holding period and then

predict yield curve at the end of that period as with a given period of maturity, a bond yield curve of a selected period can be estimated and its end price can also be calculated.

(2) Bond Swaps: This strategy involves regularly monitoring bond process to identify mispricing and try to exploit this situation. Some of the popular swap techniques are as follows:

(a) Pure Yield Pickup Swap - This strategy involves switch from a lower yield bond to a higher yield bonds of almost identical quantity and maturity. This strategy is suitable for portfolio manager who is willing to assume interest rate risk as in switching from short term bond to long term bonds to earn higher rate of interest, he may suffer a capital loss.

(b) Substitution Swap - This swapping involves swapping with similar type of bonds in terms of coupon rate, maturity period, credit rating, liquidity and call provision but with different prices. This type of differences exits due to temporary imbalance in the market. The risk a portfolio manager carries if some features of swapped bonds may not be truly identical to the swapped one.

(c) International Spread Swap – In this swap portfolio manager is of the belief that yield spreads between two sectors is temporarily out of line and he tries to take benefit of this mismatch. Since the spread depends on many factor and a portfolio manager can anticipate appropriate strategy and can profit from these expected differentials.

(d) Tax Swap – This is based on taking tax advantage by selling existing bond whose price decreased at capital loss and set it off against capital gain in other securities and buying another security which has features like that of disposed one.

(3) Interest Rate Swap: Interest Rate Swap is another technique that is used by Portfolio Manager. This technique has been discussed in details in the chapter on Interest Rate Risk Management.

16. ALTERNATIVE INVESTMENT AVENUES

Plainly speaking, Alternative Investments (AIs) are Investments other than traditional investments (stock, bond and cash).

Features of Alternative Investments

Though here may be many features of Alternative Investment but following are some common features.

(i) High Fees – Being a specific nature product the transaction fees are quite on higher side.

(ii) Limited Historical Rate – The data for historic return and risk is verity limited where data for equity market for more than 100 years in available.

(iii)   *Illiquidity* – The liquidity of Alternative Investment is not good as next buyer may not be easily available due to limited market.

(iv)   *Less Transparency* – The level of transparency is not adequate due to limited public information available.

(v)   *Extensive Research Required* – Due to limited availability of market information extensive analysis is required by the Portfolio Managers.

(vi)   *Leveraged Buying* – Generally investment in alternative investments is highly leveraged.

Over the time various types of AIs have been evolved but some of the important AIs are as follows:

1.   Real Estates

2.   Gold

3.   Private Equity

4.   REITs

5.   Hedge Funds

6.   Exchange Traded Funds

7.   Mutual Funds

8.   Commodities

9.   Distressed Securities

Since, some of the above terms have been covered under the respective chapter in this study, we shall cover other terms hereunder.

16.1 Real Estates

As opposed to financial claims in the form of paper or a dematerialized mode, real estate is a tangible form of assets which can be seen or touched. Real Assets consists of land, buildings, offices, warehouses, shops etc.

Although real investment is like any other investment but it has some special features as every country has their own laws and paper works which makes investment in foreign properties less attractive. However, in recent time due to globalization investment in foreign real estate has been increased.

16.1.1 Valuation Approaches

Comparing to financial instrument the valuation of Real Estate is quite complex as number of transactions or dealings comparing to financial instruments are very small.

Following are some characteristics that make valuation of Real Estate quite complex:

(i)   Inefficient market: Information may not be freely available as in case of financial securities.

(ii)   Illiquidity: Real Estates are not as liquid as financial instruments.

(iii)   Comparison: Real estates are only approximately comparable to other properties.

(iv)   High Transaction cost: In comparison to financial instruments, the transaction and management cost of Real Estate is quite high.

(v)   No Organized market: There is no such organized exchange or market as for equity shares and bonds.

16.1.2 Valuation of Real Estates

Generally, following four approaches are used in valuation of Real estates:

(1)   Sales Comparison Approach – It is like Price Earning Multiplier as in case of equity shares. Benchmark value of similar type of property can be used to value Real Estate.

(2)   Income Approach – This approach is like value of Perpetual Debenture or Irredeemable Preference Shares. In this approach the perpetual cash flow of potential net income (after deducting expense) is discounted at market required rate of return.

(3)   Cost Approach – In this approach, the cost is estimated to replace the building in its present form plus estimated value of land. However, adjustment of other factors such as good location, neighbourhood is also made in it.

(4)   Discounted After Tax Cash Flow Approach – In comparison to NPV technique, PV of expected inflows at required rate of return is reduced by amount of investment.

16.2 Gold

Being a real asset Gold is an attractive alternative form of investment by various categories of investors. Gold has been a very popular source of investment since a long time especially among Indians. The most common avenue of making investment in the gold has been buying the jewellery by most of the households. However, this form of investment in gold suffers from a serious limitation of making charges because jeweller charge them both at the time of selling and buying back.

Hence with the passage of time other forms have been evolved some of which are as follows:

(a) Gold Bars: An alternative to investment in jewellery, investors can buy gold coins/ bar of different denominations. However, similar to jewellery this form of investment suffers from the limitation of cost of physical storage.

(b) Sovereign Gold Bonds (SGBs): The SGB offers a superior alternative to holding gold in physical form. SGBs are government securities denominated in grams of gold. They are substitutes for holding physical gold. Investors have to pay the issue price in cash and the bonds will be redeemed in cash on maturity. The Bond is issued by Reserve Bank on behalf of Government of India. The quantity of gold for which the investor pays is protected, since he receives the ongoing market price at the time of redemption/ premature redemption. The risks and costs of storage are eliminated. Investors are assured of the market value of gold at the time of maturity and periodical interest. SGB is free from issues like making charges and purity in the case of gold in jewellery form. The bonds are held in the books of the RBI or in demat form eliminating risk of loss of scrip etc.

(c) Gold Exchange Traded Funds (ETFs): Gold ETFs can be considered as an investment avenue which is a hybrid of flexibility of stock investment and the simplicity of gold investments. Like any other company stock, they can be bought and sold continuously at market prices on Stock Exchanges.

Prices of Gold ETFs are based on gold prices and investment of fund amount is made in gold bullion. Further because of its direct gold pricing, there is a complete transparency on the holdings of an ETF. Compared to physical gold investments due to its unique structure and creation mechanism, the ETFs have much lower expenses.

(d) E-gold: Started in 2010 in India, E-gold is offered by the National Spot Exchange Ltd (NSEL). It can be bought by setting up a trading account with an authorized participant with NSEL. Each unit of e-gold is equivalent to one gram of physical gold and is held in the Demat account (different from holding and transacting in equities). Like Gold ETFs, e-gold units are fully backed by an equivalent quantity of gold kept with the custodian and have less storage cost compared to physical gold. These units can be traded on the exchange.

16.3 Distressed securities

It is purchasing the securities of companies that are in or near bankruptcy. Since these securities are available at very low price, the main purpose of buying such securities is to make efforts to revive the sick company. Further, these securities are suitable for those investors who cannot participate in the market and those who wants to avoid due diligence.

Now, question arises how profit can be earned from distressed securities. It can be by taking Long Position in Debt and Short Position in Equity. Now let us see how investor can earn arbitrage profit.

(i)   In case company's condition improves because of priority, the investor will get his interest payment which shall be more than the dividend on his short position in equity shares.

(ii)   If company's condition further deteriorates the value of both share and debenture goes down. He will make good profit from his short position.

Risks Analysis of Investment in Distressed Securities : On the face, investment in distressed securities appears to be a good proposition but following types of risks are needed to be analyzed.

(i)   Liquidity Risk – These securities may be saleable in the market.

(ii)   Event Risk – Any event that particularly effect the company not economy as a whole

(iii)   Market Risk – This is another type of risk though it is not important.

(iv)   Human Risk – The judge's decision on the company in distress also play a big role.

TEST YOUR KNOWLEDGE

Theoretical Questions

1.   Write short note on factors affecting decision of investment in fixed income securities.

2.   Briefly explain the objectives of "Portfolio Management".

3.   Discuss the Capital Asset Pricing Model (CAPM) and its relevant assumptions.

Practical Questions

1.   A stock costing ₹ 120 pays no dividends. The possible prices that the stock might sell for at the end of the year with the respective probabilities are:

Price	Probability
115	0.1
120	0.1
125	0.2
130	0.3
135	0.2
140	0.1

Required:

(i)   Calculate the expected return.

(ii)   Calculate the Standard deviation of returns.

2.   Following information is available in respect of expected dividend, market price and market condition after one year.

Market condition	Probability	Market Price	Dividend per share
		₹	₹
Good	0.25	115	9
Normal	0.50	107	5
Bad	0.25	97	3

The existing market price of equity share is ₹ 106 (F.V. ₹ 1), which is cum 10% bonus debenture of ₹ 6 each, per share. M/s. X Finance Company Ltd. had offered the buy-back of debentures at face value.

Find out the expected return and variability of returns of the equity shares if buyback offer is accepted by the investor.

And also advise-Whether to accept buy-back offer?

3. Mr. A is interested to invest ₹ 1,00,000 in the securities market. He selected two securities B and D for this purpose. The risk return profile of these securities are as follows :

Security	Risk (σ)	Expected Return (ER)
B	10%	12%
D	18%	20%

Co-efficient of correlation between B and D is 0.15.

You are required to calculate the portfolio return of the following portfolios of B and D to be considered by A for his investment.

(i)   100 percent investment in B only;

(ii)   50 percent of the fund in B and the rest 50 percent in D;

(iii)   75 percent of the fund in B and the rest 25 percent in D; and

(iv)   100 percent investment in D only.

Also indicate that which portfolio is best for him from risk as well as return point of view?

4. Consider the following information on two stocks, A and B :

Year	Return on A (%)	Return on B (%)
2006	10	12
2007	16	18

You are required to determine:

(i)　The expected return on a portfolio containing A and B in the proportion of 40% and 60% respectively.

(ii)　The Standard Deviation of return from each of the two stocks.

(iii)　The covariance of returns from the two stocks.

(iv)　Correlation coefficient between the returns of the two stocks.

(v)　The risk of a portfolio containing A and B in the proportion of 40% and 60%.

5.　Following is the data regarding six securities:

	A	B	C	D	E	F
Return (%)	8	8	12	4	9	8
Risk (Standard deviation)	4	5	12	4	5	6

(i)　Assuming three will have to be selected, state which ones will be picked.

(ii)　Assuming perfect correlation, show whether it is preferable to invest 75% in A and 25% in C or to invest 100% in E.

6.　The historical rates of return of two securities over the past ten years are given. Calculate the Covariance and the Correlation coefficient of the two securities:

Years:	1	2	3	4	5	6	7	8	9	10
Security 1: (Return per cent)	12	8	7	14	16	15	18	20	16	22
Security 2: (Return per cent)	20	22	24	18	15	20	24	25	22	20

7.　An investor has decided to invest to invest ₹ 1,00,000 in the shares of two companies, namely, ABC and XYZ. The projections of returns from the shares of the two companies along with their probabilities are as follows:

Probability	ABC(%)	XYZ(%)
0.20	12	16
0.25	14	10
0.25	-7	28
0.30	28	-2

You are required to

(i)   Comment on return and risk of investment in individual shares.

(ii)   Compare the risk and return of these two shares with a Portfolio of these shares in equal proportions.

(iii)   Find out the proportion of each of the above shares to formulate a minimum risk portfolio.

8.   The following information are available with respect of Krishna Ltd.

Year	Krishna Ltd. Average share price	Dividend per Share	Average Market Index	Dividend Yield	Return on Govt. bonds
	₹	₹			
2012	245	20	2013	4%	7%
2013	253	22	2130	5%	6%
2014	310	25	2350	6%	6%
2015	330	30	2580	7%	6%

Compute Beta Value of the Krishna Ltd. at the end of 2015 and state your observation.

9.   The distribution of return of security 'F' and the market portfolio 'P' is given below:

Probability   Return %

                          F   P

    0.30   30   -10

    0.40   20   20

    0.30   0   30

You are required to calculate the expected return of security 'F' and the market portfolio 'P', the covariance between the market portfolio and security and beta for the security.

10. Given below is information of market rates of Returns and Data from two Companies A and B:

	Year 2007	Year 2008	Year 2009
Market (%)	12.0	11.0	9.0
Company A (%)	13.0	11.5	9.8
Company B (%)	11.0	10.5	9.5

You are required to determine the beta coefficients of the Shares of Company A and Company B.

11. The returns on stock A and market portfolio for a period of 6 years are as follows:

Year	Return on A (%)	Return on market portfolio (%)
1	12	8
2	15	12
3	11	11
4	2	-4
5	10	9.5
6	-12	-2

You are required to determine:

(i) Characteristic line for stock A

(ii) The systematic and unsystematic risk of stock A.

12. The rates of return on the security of Company X and market portfolio for 10 periods are given below:

Period	Return of Security X (%)	Return on Market Portfolio (%)
1	20	22
2	22	20
3	25	18
4	21	16
5	18	20
6	−5	8
7	17	−6
8	19	5
9	−7	6
10	20	11

(i) What is the beta of Security X?

(ii) What is the characteristic line for Security X?

13. Expected returns on two stocks for particular market returns are given in the following table:

Market Return	Aggressive	Defensive
7%	4%	9%
25%	40%	18%

You are required to calculate:

(a) The Betas of the two stocks.

(b)   Expected return of each stock, if the market return is equally likely to be 7% or 25%.

(c)   The Security Market Line (SML), if the risk free rate is 7.5% and market return is equally likely to be 7% or 25%.

(d)   The Alphas of the two stocks.

14.   A study by a Mutual fund has revealed the following data in respect of three securities:

Security	σ (%)	Correlation with Index, Pm
A	20	0.60
B	18	0.95
C	12	0.75

The standard deviation of market portfolio (BSE Sensex) is observed to be 15%.

(i)   What is the sensitivity of returns of each stock with respect to the market?

(ii)   What are the covariances among the various stocks?

(iii)   What would be the risk of portfolio consisting of all the three stocks equally?

(iv)   What is the beta of the portfolio consisting of equal investment in each stock?

(v)   What is the total, systematic and unsystematic risk of the portfolio in (iv)?

15.   Mr. X owns a portfolio with the following characteristics:

	Security A	Security B	Risk Free security
Factor 1 sensitivity	0.80	1.50	0
Factor 2 sensitivity	0.60	1.20	0
Expected Return	15%	20%	10%

It is assumed that security returns are generated by a two factor model.

(i)   If Mr. X has ₹ 1,00,000 to invest and sells short ₹ 50,000 of security B and purchases ₹ 1,50,000 of security A what is the sensitivity of Mr. X's portfolio to the two factors?

(ii)   If Mr. X borrows ₹ 1,00,000 at the risk free rate and invests the amount he borrows along with the original amount of ₹ 1,00,000 in security A and B in the same proportion as described in part (i), what is the sensitivity of the portfolio to the two factors?

(iii)   What is the expected return premium of factor 2?

16.   Mr. Tempest has the following portfolio of four shares:

Name	Beta	Investment ₹ Lac.
Oxy Rin Ltd.	0.45	0.80
Boxed Ltd.	0.35	1.50
Square Ltd.	1.15	2.25
Ellipse Ltd.	1.85	4.50

The risk-free rate of return is 7% and the market rate of return is 14%.

Required.

(i)   Determine the portfolio return.   (ii)   Calculate the portfolio Beta.

17.   Mr. Abhishek is interested in investing ₹ 2,00,000 for which he is considering following three alternatives:

(i)   Invest ₹ 2,00,000 in Mutual Fund X (MFX)

(ii)   Invest ₹ 2,00,000 in Mutual Fund Y (MFY)

(iii)   Invest ₹ 1,20,000 in Mutual Fund X (MFX) and ₹ 80,000 in Mutual Fund Y (MFY)

Average annual return earned by MFX and MFY is 15% and 14% respectively. Risk free rate of return is 10% and market rate of return is 12%.

Covariance of returns of MFX, MFY and market portfolio Mix are as follow:

	MFX	MFY	Mix
MFX	4.800	4.300	3.370
MFY	4.300	4.250	2.800
Mix	3.370	2.800	3.100

You are required to calculate:

(i)   variance of return from MFX, MFY and market return,

(ii)   portfolio return, beta, portfolio variance and portfolio standard deviation,

(iii)   expected return, systematic risk and unsystematic risk; and

(iv)   Sharpe ratio, Treynor ratio and Alpha of MFX, MFY and Portfolio Mix.

18.   Amal Ltd. has been maintaining a growth rate of 12% in dividends. The company has paid dividend @ ₹ 3 per share. The rate of return on market portfolio is 15% and the risk-free rate of return in the market has been observed as10%. The beta co-efficient of the company's share is 1.2.

You are required to calculate the expected rate of return on the company's shares as per CAPM model and the equilibrium price per share by dividend growth model.

19.   The following information is available in respect of Security X

Equilibrium Return	15%
Market Return	15%
7% Treasury Bond Trading at	$140
Covariance of Market Return and Security Return	225%
Coefficient of Correlation	0.75

You are required to determine the Standard Deviation of Market Return and Security Return.

20.   Assuming that shares of ABC Ltd. and XYZ Ltd. are correctly priced according to Capital Asset Pricing Model. The expected return from and Beta of these shares are as follows:

Share	Beta	Expected return
ABC	1.2	19.8%
XYZ	0.9	17.1%

You are required to derive Security Market Line.

21.   A Ltd. has an expected return of 22% and Standard deviation of 40%. B Ltd. has an expected return of 24% and Standard deviation of 38%. A Ltd. has a beta of 0.86 and B Ltd. has a beta of 1.24. The correlation coefficient between the return of A Ltd. and B Ltd. is 0.72. The Standard deviation of the market return is 20%. Suggest:

(i)   Is investing in B Ltd. better than investing in A Ltd.?

(ii)   If you invest 30% in B Ltd. and 70% in A Ltd., what is your expected rate of return and portfolio Standard deviation?

(iii)   What is the market portfolios expected rate of return and how much is the risk-free rate?

(iv)   What is the beta of Portfolio if A Ltd.'s weight is 70% and B Ltd.'s weight is 30%?

22.   XYZ Ltd. has substantial cash flow and until the surplus funds are utilised to meet the future capital expenditure, likely to happen after several months, are invested in a portfolio of short-term equity investments, details for which are given below:

Investment	No. of shares	Beta	Market price per share ₹	Expected dividend yield
I	60,000	1.16	4.29	19.50%
II	80,000	2.28	2.92	24.00%
III	1,00,000	0.90	2.17	17.50%
IV	1,25,000	1.50	3.14	26.00%

The current market return is 19% and the risk free rate is 11%.

Required to:

(i) Calculate the risk of XYZ's short-term investment portfolio relative to that of the market;

(ii) Whether XYZ should change the composition of its portfolio.

23. A company has a choice of investments between several different equity oriented mutual funds. The company has an amount of ₹1 crore to invest. The details of the mutual funds are as follows:

Mutual Fund	Beta
A	1.6
B	1.0
C	0.9
D	2.0
E	0.6

Required:

(i) If the company invests 20% of its investment in each of the first two mutual funds and an equal amount in the mutual funds C, D and E, what is the beta of the portfolio?

(ii) If the company invests 15% of its investment in C, 15% in A, 10% in E and the balance in equal amount in the other two mutual funds, what is the beta of the portfolio?

(iii) If the expected return of market portfolio is 12% at a beta factor of 1.0, what will be the portfolios expected return in both the situations given above?

24. Suppose that economy A is growing rapidly and you are managing a global equity fund and so far you have invested only in developed-country stocks only. Now you have decided to add stocks of economy A to your portfolio. The table below shows the expected rates of

return, standard deviations, and correlation coefficients (all estimates are for aggregate stock market of developed countries and stock market of Economy A).

	Developed Country Stocks	Stocks of Economy A
Expected rate of return (annualized percentage)	10	15
Risk [Annualized Standard Deviation (%)]	16	30
Correlation Coefficient (ρ)	0.30	

Assuming the risk-free interest rate to be 3%, you are required to determine:

(a)   What percentage of your portfolio should you allocate to stocks of Economy A if you want to increase the expected rate of return on your portfolio by 0.5%?

(b)   What will be the standard deviation of your portfolio assuming that stocks of Economy A are included in the portfolio as calculated above?

(c)   Also show how well the Fund will be compensated for the risk undertaken due to inclusion of stocks of Economy A in the portfolio comparing with investment in developed country's stocks only.

25.   Mr. FedUp wants to invest an amount of ₹ 520 lakhs and had approached his Portfolio Manager. The Portfolio Manager had advised Mr. FedUp to invest in the following manner:

Security	Moderate	Better	Good	Very Good	Best
Amount (in ₹ Lakhs)	60	80	100	120	160
Beta	0.5	1.00	0.80	1.20	1.50

You are required to advise Mr. FedUp in regard to the following, using Capital Asset Pricing Methodology:

(i)   Expected return on the portfolio, if the Government Securities are at 8% and the NIFTY is yielding 10%.

(ii)   Advisability of replacing Security 'Better' with NIFTY.

26.   Your client is holding the following securities:

Particulars of Securities	Cost	Dividends/Interest	Market price	Beta
	₹	₹	₹	
Equity Shares:				
Gold Ltd.	10,000	1,725	9,800	0.6
Silver Ltd.	15,000	1,000	16,200	0.8
Bronze Ltd.	14,000	700	20,000	0.6
GOI Bonds	36,000	3,600	34,500	0.01

Average return of the portfolio is 15.7%, calculate:

(i)   Expected rate of return in each, using the Capital Asset Pricing Model (CAPM).

(ii)   Risk free rate of return.

27.   A holds the following portfolio:

Share/Bond	Beta	Initial Price	Dividends	Market Price at end of year
		₹	₹	₹
Epsilon Ltd.	0.8	25	2	50
Sigma Ltd.	0.7	35	2	60
Omega Ltd.	0.5	45	2	135
GOI Bonds	0.01	1,000	140	1,005

Calculate:

(i)   The expected rate of return of each security using Capital Asset Pricing Method (CAPM)

(ii)   The average return of his portfolio.

Risk-free return is 14%.

28.   Your client is holding the following securities:

Particulars of Securities	Cost ₹	Dividends ₹	Market Price ₹	BETA
Equity Shares:				
Co. X	8,000	800	8,200	0.8
Co. Y	10,000	800	10,500	0.7
Co. Z	16,000	800	22,000	0.5
PSU Bonds	34,000	3,400	32,300	0.2

Assuming a Risk-free rate of 15%, calculate:

–   Expected rate of return in each, using the Capital Asset Pricing Model (CAPM).

–   Simple Average return of the portfolio.

29.   An investor is holding 1,000 shares of Fatlass Company. Presently the rate of dividend being paid by the company is ₹ 2 per share and the share is being sold at ₹ 25 per share in the market. However, several factors are likely to be changed during the course of the year as indicated below:

	Existing	Revised
Risk free rate	12%	10%
Market risk premium	6%	4%
Beta value	1.4	1.25
Expected growth rate	5%	9%

In view of the above factors whether the investor should buy, hold or sell the shares? And why?

30.   An investor is holding 5,000 shares of X Ltd. Current year dividend is ₹ 3 share. Market price of the share is ₹ 40 each. The investor is concerned about several factors which are likely to change during the next financial year as indicated below:

	Current Year	Next Year
Dividend paid /anticipated per share (₹)	3	2.5
Risk free rate	12%	10%
Market Risk Premium	5%	4%
Beta Value	1.3	1.4
Expected growth	9%	7%

In view of the above, advise whether the investor should buy, hold or sell the shares.

31.   An investor has two portfolios known to be on minimum variance set for a population of three securities A, B and C having below mentioned weights:

	WA	WB	WC
Portfolio X	0.30	0.40	0.30
Portfolio Y	0.20	0.50	0.30

It is supposed that there are no restrictions on short sales.

(i)   What would be the weight for each stock for a portfolio constructed by investing ₹ 5,000 in portfolio X and ₹ 3,000 in portfolio Y?.

(ii)   Suppose the investor invests ₹ 4,000 out of ₹ 8,000 in security A. How he will allocate the balance between security B and C to ensure that his portfolio is on minimum variance set?

32.   X Co., Ltd., invested on 1.4.2009 in certain equity shares as below:

Name of Co.	No. of shares	Cost (₹)
M Ltd.	1,000 (₹ 100 each)	2,00,000
N Ltd.	500 (₹ 10 each)	1,50,000

In September, 2009, 10% dividend was paid out by M Ltd. and in October, 2009, 30% dividend paid out by N Ltd. On 31.3.2010 market quotations showed a value of ₹ 220 and ₹ 290 per share for M Ltd. and N Ltd. respectively.

On 1.4.2010, investment advisors indicate (a) that the dividends from M Ltd. and N Ltd. for the year ending 31.3.2011 are likely to be 20% and 35%, respectively and (b) that the probabilities of market quotations on 31.3.2011 are as below:

Probability factor	Price/share of M Ltd.	Price/share of N Ltd.
0.2	220	290
0.5	250	310
0.3	280	330

You are required to:

(i)   Calculate the average return from the portfolio for the year ended 31.3.2010;

(ii)   Calculate the expected average return from the portfolio for the year 2010-11; and

(iii)   Advise X Co. Ltd., of the comparative risk in the two investments by calculating the standard deviation in each case.

33.   An investor holds two stocks A and B. An analyst prepared ex-ante probability distribution for the possible economic scenarios and the conditional returns for two stocks and the market index as shown below:

Economic scenario	Probability	Conditional Returns %		
		A	B	Market
Growth	0.40	25	20	18
Stagnation	0.30	10	15	13
Recession	0.30	-5	-8	-3

The risk free rate during the next year is expected to be around 11%. Determine whether the investor should liquidate his holdings in stocks A and B or on the contrary make fresh investments in them. CAPM assumptions are holding true.

34. Following are the details of a portfolio consisting of three shares:

Share	Portfolio weight	Beta	Expected return in %	Total variance
A	0.20	0.40	14	0.015
B	0.50	0.50	15	0.025
C	0.30	1.10	21	0.100

Standard Deviation of Market Portfolio Returns = 10%

You are given the following additional data:

Covariance (A, B) = 0.030

Covariance (A, C) = 0.020

Covariance (B, C) = 0.040

Calculate the following:

(i)   The Portfolio Beta

(ii)   Residual variance of each of the three shares

(iii) Portfolio variance using Sharpe Index Model

(iv)   Portfolio variance (on the basis of modern portfolio theory given by Markowitz)

35. Ramesh wants to invest in stock market. He has got the following information about individual securities:

Security	Expected Return	Beta	σ^2_{ci}
A	15	1.5	40
B	12	2	20
C	10	2.5	30
D	09	1	10
E	08	1.2	20
F	14	1.5	30

Market index variance is 10 percent and the risk free rate of return is 7%. What should be the optimum portfolio assuming no short sales?

36. A Portfolio Manager (PM) has the following four stocks in his portfolio:

Security	No. of Shares	Market Price per share (₹)	β
VSL	10,000	50	0.9
CSL	5,000	20	1.0
SML	8,000	25	1.5
APL	2,000	200	1.2

Compute the following:

(i) Portfolio beta.

(ii) If the PM seeks to reduce the beta to 0.8, how much risk free investment should he bring in?

(iii) If the PM seeks to increase the beta to 1.2, how much risk free investment should he bring in?

37. A has portfolio having following features:

Security	β	Random Error σ_{ei}	Weight
L	1.60	7	0.25
M	1.15	11	0.30
N	1.40	3	0.25
K	1.00	9	0.20

You are required to find out the risk of the portfolio if the standard deviation of the market index (σ_m) is 18%.

38. Mr. Tamarind intends to invest in equity shares of a company the value of which depends upon various parameters as mentioned below:

Factor	Beta	Expected value in %	Actual value in %
GNP	1.20	7.70	7.70
Inflation	1.75	5.50	7.00
Interest rate	1.30	7.75	9.00
Stock market index	1.70	10.00	12.00
Industrial production	1.00	7.00	7.50

If the risk free rate of interest be 9.25%, how much is the return of the share under Arbitrage Pricing Theory?

39. The total market value of the equity share of O.R.E. Company is ₹ 60,00,000 and the total value of the debt is ₹ 40,00,000. The treasurer estimate that the beta of the stock is currently 1.5 and that the expected risk premium on the market is 10 per cent. The treasury bill rate is 8 per cent.

    Required:

    (i)   What is the beta of the Company's existing portfolio of assets?

    (ii)   Estimate the Company's Cost of capital and the discount rate for an expansion of the company's present business.

40. Mr. Nirmal Kumar has categorized all the available stock in the market into the following types:

    (i)   Small cap growth stocks

    (ii)   Small cap value stocks

    (iii)   Large cap growth stocks

    (iv)   Large cap value stocks

    Mr. Nirmal Kumar also estimated the weights of the above categories of stocks in the market index. Further, the sensitivity of returns on these categories of stocks to the three important factor are estimated to be:

Category of Stocks	Weight in the Market Index	Factor I (Beta)	Factor II (Book Price)	Factor III (Inflation)
Small cap growth	25%	0.80	1.39	1.35
Small cap value	10%	0.90	0.75	1.25
Large cap growth	50%	1.165	2.75	8.65
Large cap value	15%	0.85	2.05	6.75
Risk Premium		6.85%	-3.5%	0.65%

The rate of return on treasury bonds is 4.5%

Required:

(a)   Using Arbitrage Pricing Theory, determine the expected return on the market index.

(b)   Using Capital Asset Pricing Model (CAPM), determine the expected return on the market index.

(c)   Mr. Nirmal Kumar wants to construct a portfolio constituting only the 'small cap value' and 'large cap growth' stocks. If the target beta for the desired portfolio is 1, determine the composition of his portfolio.

41.   The following are the data on five mutual funds:

Fund	Return	Standard Deviation	Beta
A	15	7	1.25
B	18	10	0.75
C	14	5	1.40
D	12	6	0.98
E	16	9	1.50

You are required to compute Reward to Volatility Ratio and rank these portfolio using:

♦   Sharpe method and

♦   Treynor's method

assuming the risk free rate is 6%.

42.   Five portfolios experienced the following results during a 7- year period:

Portfolio	Average Annual Return (R_p) (%)	Standard Deviation (S_p)	Correlation with the market returns (r)
A	19.0	2.5	0.840
B	15.0	2.0	0.540
C	15.0	0.8	0.975
D	17.5	2.0	0.750
E	17.1	1.8	0.600
Market Risk (σ_m)		1.2	
Market rate of Return (R_m)	14.0		
Risk-free Rate (R_f)	9.0		

Rank the portfolios using (a) Sharpe's method, (b) Treynor's method and (c) Jensen's Alpha

ANSWERS/ SOLUTIONS

Answers to Theoretical Questions

1.   Please refer paragraph 12.3.

2.   Please refer paragraph 1.2.

3.   Please refer paragraph 8.

Answers to the Practical Questions

1.   (i)   Here, the probable returns have to be calculated using the formula

$$R = \frac{D}{P_0} + \frac{P_1 - P_0}{P_0}$$

Calculation of Probable Returns

Possible prices (P₁) ₹	P₁-P₀ ₹	[(P₁-P₀)/ P₀] x 100 Return (per cent)
115	-5	-4.17
120	0	0.00
125	5	4.17
130	10	8.33
135	15	12.50
140	20	16.67

Alternatively, it can be calculated as follows:

Calculation of Expected Returns

Possible return X_i	Probability $p(X_i)$	Product $X_1\text{-}p(X_i)$
-4.17	0.1	-0.417
0.00	0.1	0.000
4.17	0.2	0.834
8.33	0.3	2.499
12.50	0.2	2.500
16.67	0.1	1.667
		X = 7.083

Expected return X = 7.083 per

Alternatively, it can also be calculated as follows:

Expected Price = 115 x 0.1 + 120 x 0.1 + 125 x 0.2 + 130 x 0.3 + 135 x 0.2 + 140 x 0.1 = 128.50

Return = $\dfrac{128.50 - 120}{120} \times 100 = 7.0833\%$

(ii)   **Calculation of Standard Deviation of Returns**

Probable return X_i	Probability $p(X_i)$	Deviation $(X_i - X)$	Deviation squared $(X_i - X)^2$	Product $(X_i - X)^2 p(X_i)$
-4.17	0.1	-11.253	126.63	12.66
0.00	0.1	-7.083	50.17	5.017
4.17	0.2	-2.913	8.49	1.698
8.33	0.3	1.247	1.56	0.467
12.50	0.2	5.417	29.34	5.869
16.67	0.1	9.587	91.91	9.191
				σ^2 = 34.902

Variance, σ^2 = 34.902 per cent

Standard deviation, $\sigma = \sqrt{34.902}$ = 5.908 per cent

2.   **The Expected Return of the equity share may be found as follows:**

Market Condition	Probability	Total Return	Cost (*)	Net Return
Good	0.25	₹ 124	₹ 100	₹ 24
Normal	0.50	₹ 112	₹ 100	₹ 12
Bad	0.25	₹ 100	₹ 100	₹ 0

Expected Return = $(24 \times 0.25) + (12 \times 0.50) + (0 \times 0.25) = 12 = \left(\dfrac{12}{100}\right) \times 100 = 12\%$

The variability of return can be calculated in terms of standard deviation.

V SD $= 0.25 (24 - 12)^2 + 0.50 (12 - 12)^2 + 0.25 (0 - 12)^2$

       $= 0.25 (12)^2 + 0.50 (0)^2 + 0.25 (-12)^2$

       $= 36 + 0 + 36$

SD $= \sqrt{72}$

SD $= 8.485$ or say 8.49

(*) The present market price of the share is ₹ 106 cum bonus 10% debenture of ₹ 6 each; hence the net cost is ₹ 100.

M/s X Finance company has offered the buyback of debenture at face value. There is reasonable 10% rate of interest compared to expected return 12% from the market. Considering the dividend rate and market price the creditworthiness of the company seems to be very good. The decision regarding buy-back should be taken considering the maturity period and opportunity in the market. Normally, if the maturity period is low say up to 1 year better to wait otherwise to opt buy back option.

3. We have $E_p = W_1E_1 + W_3E_3 + \ldots\ldots W_nE_n$

and for standard deviation $\sigma^2_p = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} w_iw_j\sigma_{ij}$

$$\sigma^2_p = \sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} w_iw_j\rho_{ij}\,\sigma_i\,\sigma_j$$

Two asset portfolio

$\sigma^2_p = w^2_1\sigma^2_1 + w^2_2\sigma^2_2 + 2\,w_1w_2\sigma_1\sigma_2\rho_{12}$

Substituting the respective values we get,

(i) All funds invested in B

$Ep = 12\%$

$\sigma_p = 10\%$

(ii) 50% of funds in each of B & D

$Ep = 0.50X12\%+0.50X20\%=16\%$

$\sigma^2_p = (0.50)^2(10\%)^2 + (0.50)^2(18\%)^2 +2(0.50)(0.50)(0.15)(10\%)(18\%)$

$\sigma^2_p = 25 + 81 + 13.5 = 119.50$

$\sigma_p = 10.93\%$

(iii) 75% in B and 25% in D

$Ep = 0.75\%X12\%+0.25\%X20=14\%$

$\sigma^2_p = (0.75)^2(10\%)^2 + (0.25)^2(18\%)^2 + 2(0.75)(0.25)(0.15)(10\%)(18\%)$

$\sigma^2_p = 56.25 + 20.25 + 10.125 = 86.625$

$\sigma_p = 9.31\%$

(iv)   All funds in D

Ep = 20%

σ_p = 18.0%

Portfolio	(i)	(ii)	(iii)	(iv)
Return	12	16	14	20
σ	10	10.93	9.31	18

In the terms of return, we see that portfolio (iv) is the best portfolio. In terms of risk we see that portfolio (iii) is the best portfolio.

4.   (i)   Expected return of the portfolio A and B

E (A) = (10 + 16) / 2 = 13%

E (B) = (12 + 18) / 2 = 15%

Rp $= \sum_{i-l}^{N} X_i R_i = 0.4(13) + 0.6(15) = 14.2\%$

(ii)   Stock A:

Variance = 0.5 (10 – 13)² + 0.5 (16 – 13) ² = 9

Standard deviation = $\sqrt{9}$ = 3%

Stock B:

Variance = 0.5 (12 – 15) ² + 0.5 (18 – 15) ² = 9

Standard deviation = 3%

(iii)   Covariance of stocks A and B

Cov$_{AB}$ = 0.5 (10 – 13) (12 – 15) + 0.5 (16 – 13) (18 – 15) = 9

(iv)   Correlation of coefficient

r$_{AB}$ = $\dfrac{\text{Cov}_{AB}}{\sigma_A \sigma_B} = \dfrac{9}{3 \times 3} = 1$

(v)   Portfolio Risk

$$\sigma_P = \sqrt{X^2_A \sigma^2_A + X^2_B \sigma^2_B + 2X_A X_B (\sigma_A \sigma_B \sigma_{AB})}$$

$$= \sqrt{(0.4)^2 (3)^2 + (0.6)^2 (3)^2 + 2(0.4)(0.6)(3)(3)(1)}$$

$$= \sqrt{1.44 + 3.24 + 4.32} = 3\%$$

5.   **(i)**   Security A has a return of 8% for a risk of 4, whereas B and F have a higher risk for the same return. Hence, among them A dominates.

For the same degree of risk 4, security D has only a return of 4%. Hence, D is also dominated by A.

Securities C and E remain in reckoning as they have a higher return though with higher degree of risk.

Hence, the ones to be selected are A, C & E.

(ii)   The average values for A and C for a proportion of 3 : 1 will be :

Risk $= \dfrac{(3 \times 4) + (1 \times 12)}{4} = 6\%$

Return $= \dfrac{(3 \times 8) + (1 \times 12)}{4} = 9\%$

Therefore:	75% A	E
	25% C	_
Risk	6	5
Return	9%	9%

For the same 9% return the risk is lower in E. Hence, E will be preferable.

6.   **Calculation of Covariance**

Year	R_1	Deviation $(R_1 - \overline{R_1})$	Deviation $(R_1 - \overline{R_1})^2$	R_2	Deviation $(R_2 - \overline{R_2})$	Deviation $(R_2 - \overline{R_2})^2$	Product of deviations
1	12	-2.8	7.84	20	-1	1	2.8
2	8	-6.8	46.24	22	1	1	-6.8
3	7	-7.8	60.84	24	3	9	-23.4
4	14	-0.8	0.64	18	-3	9	2.4
5	16	1.2	1.44	15	-6	36	-7.2

6	15	0.2	0.04	20	-1	1	-0.2
7	18	3.2	10.24	24	3	9	9.6
8	20	5.2	27.04	25	4	16	20.8
9	16	1.2	1.44	22	1	1	1.2
10	22	7.2	51.84	20	-1	1	-7.2
$\overline{R_1} = \dfrac{148}{10} = 14.8$			Σ=207.60		$\overline{R_2} = \dfrac{210}{10} = 21$	Σ=84.00	

Covariance $= \dfrac{\sum\limits_{i=1}^{N} [R_1 - \overline{R_1}][R_2 - \overline{R_2}]}{N} = -8/10 = -0.8$

Standard Deviation of Security 1

$$\sigma_1 = \sqrt{\frac{(R_1 - \overline{R_1})^2}{N}}$$

$$\sigma_1 = \sqrt{\frac{207.60}{10}} = \sqrt{20.76}$$

$$\sigma_1 = 4.56$$

Standard Deviation of Security 2

$$\sigma_2 = \sqrt{\frac{(R_2 - \overline{R_2})^2}{N}}$$

$$\sigma_2 = \sqrt{\frac{84}{10}} = \sqrt{8.40}$$

$$\sigma_2 = 2.90$$

Alternatively, Standard Deviation of securities can also be calculated as follows:

Calculation of Standard Deviation

Year	R_1	R_1^2	R_2	R_2^2
1	12	144	20	400
2	8	64	22	484
3	7	49	24	576
4	14	196	18	324
5	16	256	15	225
6	15	225	20	400

7	18	324	24	576
8	20	400	25	625
9	16	256	22	484
10	22	484	20	400
	148	2398	210	4494

Standard deviation of security 1:

$$\sigma_1 = \sqrt{\frac{N\sum R_1^2 - (\sum R_1)^2}{N^2}}$$

$$= \sqrt{\frac{(10\times 2398) - (148)^2}{10\times 10}} = \sqrt{\frac{23980 - 21904}{100}}$$

$$= \sqrt{20.76} = 4.56$$

Standard deviation of security 2:

$$\sigma_2 = \sqrt{\frac{N\sum R_2^2 - (\sum R_2)^2}{N^2}}$$

$$= \sqrt{\frac{(10\times 4494) - (210)^2}{10\times 10}} = \sqrt{\frac{44940 - 44100}{100}}$$

$$= \sqrt{\frac{840}{100}} = \sqrt{8.4} = 2.90$$

Correlation Coefficient

$$r_{12} = \frac{Cov}{\sigma_1 \sigma_2} = \frac{-0.8}{4.56\times 2.90} = \frac{-0.8}{13.22} = -0.0605$$

7.   (i)

Probability	ABC (%)	XYZ (%)	1X2 (%)	1X3 (%)
(1)	(2)	(3)	(4)	(5)
0.20	12	16	2.40	3.2
0.25	14	10	3.50	2.5
0.25	-7	28	-1.75	7.0

0.30	28	-2	8.40	-0.6
	Average return		12.55	12.1

Hence the expected return from ABC = 12.55% and XYZ is 12.1%

Probability	(ABC-$\overline{ABC}$)	(ABC-$\overline{ABC}$)²	1X3	(XYZ-$\overline{XYZ}$)	(XYZ-$\overline{XYZ}$)²	(1)X(6)
(1)	(2)	(3)	(4)	(5)	(6)	
0.20	-0.55	0.3025	0.06	3.9	15.21	3.04
0.25	1.45	2.1025	0.53	-2.1	4.41	1.10
0.25	-19.55	382.2025	95.55	15.9	252.81	63.20
0.30	15.45	238.7025	71.61	-14.1	198.81	59.64
			167.75			126.98

σ^2_{ABC} = 167.75(%)² ; σ_{ABC} = 12.95%

σ^2_{XYZ} = 126.98(%)² ; σ_{XYZ} = 11.27%

(ii) In order to find risk of portfolio of two shares, the covariance between the two is necessary here.

Probability	(ABC-$\overline{ABC}$)	(XYZ-$\overline{XYZ}$)	2X3	1X4
(1)	(2)	(3)	(4)	(5)
0.20	-0.55	3.9	-2.145	-0.429
0.25	1.45	-2.1	-3.045	-0.761
0.25	-19.55	15.9	-310.845	-77.71
0.30	15.45	-14.1	-217.845	-65.35
				-144.25

σ^2_P = (0.5² x 167.75) + (0.5² x 126.98) + 2 x (-144.25) x 0.5 x 0.5

σ^2_P = 41.9375 + 31.745 – 72.125

σ^2_P = 1.5575 or 1.56(%)

$\sigma_P = \sqrt{1.56}$ = 1.25%

E (R_p) = (0.5 x 12.55) + (0.5 x 12.1) = 12.325%

Hence, the return is 12.325% with the risk of 1.25% for the portfolio. Thus, the portfolio results in the reduction of risk by the combination of two shares.

(iii) For constructing the minimum risk portfolio the condition to be satisfied is

$$X_{ABC} = \frac{\sigma_X^2 - r_{AX}\sigma_A\sigma_X}{\sigma_A^2 + \sigma_X^2 - 2r_{AX}\sigma_A\sigma_X} \text{ or } = \frac{\sigma_X^2 - Cov._{AX}}{\sigma_A^2 + \sigma_X^2 - 2Cov._{AX}}$$

σ_X = Std. Deviation of XYZ

σ_A = Std. Deviation of ABC

r_{AX}= Coefficient of Correlation between XYZ and ABC

$Cov._{AX}$ = Covariance between XYZ and ABC.

Therefore,

$$\% \text{ ABC} = \frac{126.98 - (-144.25)}{126.98 + 167.75 - [2 \times (-144.25)]} = \frac{271.23}{583.23} = 0.46 \text{ or } 46\%$$

% ABC = 46%, XYZ = 54%

(1 – 0.46) =0.54

8.   (i)   Computation of Beta Value

Calculation of Returns

$$\text{Returns} = \frac{D_1 + (P_1 - P_0)}{P_0} \times 100$$

Year	Returns
2012 – 13	$\frac{22 + (253 - 245)}{245} \times 100 = 12.24\%$
2013 – 14	$\frac{25 + (310 - 253)}{253} \times 100 = 32.41\%$
2014 – 15	$\frac{30 + (330 - 310)}{310} \times 100 = 16.13\%$

Calculation of Returns from market Index

Year	% of Index Appreciation	Dividend Yield %	Total Return %
2012–13	$\frac{(2130 - 2013)}{2013} \times 100 = 5.81\%$	5%	10.81%

2013–14	$\dfrac{(2350 - 2130)}{2130} \times 100 = 10.33\%$	6%	16.33%
2014–15	$\dfrac{(2580 - 2350)}{2350} \times 100 = 9.79\%$	7%	16.79%

Computation of Beta

Year	Krishna Ltd. (X)	Market Index (Y)	XY	Y²
2012–13	12.24%	10.81%	132.31	116.86
2013–14	32.41%	16.33%	529.25	266.67
2014–15	16.13%	16.79%	270.82	281.90
Total	60.78%	43.93%	932.38	665.43

Average Return of Krishna Ltd. = $\dfrac{60.78}{3}$ = 20.26%

Average Market Return = $\dfrac{43.93}{3}$ = 14.64%

Beta (β) = $\dfrac{\sum XY - n\overline{X}\,\overline{Y}}{\sum Y^2 - n(\overline{Y})^2}$ = $\dfrac{932.38 - 3 \times 20.26 \times 14.64}{665.43 - 3(14.64)^2}$ = 1.897

(ii) Observation

	Expected Return (%)	Actual Return (%)	Action
2012 – 13	6%+ 1.897(10.81% - 6%) = 15.12%	12.24%	Sell
2013 – 14	6%+ 1.897(16.33% - 6%) = 25.60%	32.41%	Buy
2014 – 15	6%+ 1.897(16.79% - 6%) = 26.47%	16.13%	Sell

9. Security F

Prob(P)	R_f	$P \times R_f$	Deviations of F $(R_f - ER_f)$	(Deviation)² of F	(Deviations)² P_X
0.3	30	9	13	169	50.7
0.4	20	8	3	9	3.6
0.3	0	0	-17	289	86.7
		ER_f=17			Var_f =141

STDEV $\sigma_f = \sqrt{141}$ = 11.87

Market Portfolio, P

R_M %	P_M	Exp. Return $R_M \times P_M$	Dev. of P (R_M-ER_M)	(Dev. of P)²	(DeV.)² P_M	(Deviation of F) x (Deviation of P)	Dev. of F x Dev. of P) x P
-10	0.3	-3	-24	576	172.8	-312	-93.6
20	0.4	8	6	36	14.4	18	7.2
30	0.3	9	16	256	76.8	-272	-81.6
		ER_M=14			Var $_M$=264		=Co Var P_M
					σ_M=16.25		=- 168

Beta= $\dfrac{\text{Co Var } P_M}{\sigma_M^2} = \dfrac{-168}{264} = -.636$

10.   **Company A:**

Year	Return % (Ra)	Market return % (Rm)	Deviation R(a)	Deviation Rm	D Ra × DRm	Rm²
1	13.0	12.0	1.57	1.33	2.09	1.77
2	11.5	11.0	0.07	0.33	0.02	0.11
3	9.8	9.0	−1.63	−1.67	2.72	2.79
	34.3	32.0			4.83	4.67

Average Ra = 11.43

Average Rm = 10.67

Covariance = $\dfrac{\sum (R_m - \bar{R}_m)(R_a - \bar{R}_a)}{N}$

Covariance = $\dfrac{4.83}{3} = 1.61$

Variance $(\sigma_m{}^2)$ = $\dfrac{\sum (R_m - \bar{R}_m)^2}{N}$

= $\dfrac{4.67}{3} = 1.557$

$\beta = \dfrac{1.61}{1.557} = 1.03$

Company B:

Year	Return % (Rb)	Market return % (Rm)	Deviation R(b)	Deviation Rm	D Rb × D Rm	Rm²
1	11.0	12.0	0.67	1.33	0.89	1.77
2	10.5	11.0	0.17	0.33	0.06	0.11
3	9.5	9.0	−0.83	−1.67	1.39	2.79
	31.0	32.0			2.34	4.67

Average Rb = 10.33

Average Rm = 10.67

$$\text{Covariance} = \frac{\sum (R_m - \bar{R}_m)(R_b - \bar{R}_b)}{N}$$

$$\text{Covariance} = \frac{2.34}{3} = 0.78$$

$$\text{Variance } (\sigma_m{}^2) = \frac{\sum (R_m - \bar{R}_m)^2}{N}$$

$$= \frac{4.67}{3} = 1.557$$

$$\beta = \frac{0.78}{1.557} = 0.50$$

11. Characteristic line is given by

$$\alpha + \beta \, Rm$$

$$\beta i = \frac{\Sigma xy - n\bar{x}\,\bar{y}}{\Sigma x^2 - n(\bar{x})^2}$$

$$\alpha i = \bar{y} - \beta \bar{x}$$

Return on A (Y)	Return on market (X)	xy	x²	(x-$\bar{x}$)	(x-$\bar{x}$)²	(y-$\bar{y}$)	(y-$\bar{y}$)²
12	8	96	64	2.25	5.06	5.67	32.15
15	12	180	144	6.25	39.06	8.67	75.17

11	11	121	121	5.25	27.56	4.67	21.81
2	-4	-8	16	-9.75	95.06	-4.33	18.75
10	9.5	95	90.25	3.75	14.06	3.67	13.47
-12	-2	24	4	-7.75	60.06	-18.33	335.99
38	34.5	508	439.25		240.86		497.34

$$\bar{y} = {38}/{6} = 6.33$$

$$\bar{x} = {34.5}/{6} = 5.75$$

$$\beta = \frac{\Sigma xy - n\bar{x}\bar{y}}{\Sigma x^2 - n(\bar{x})^2} = \frac{508 - 6(5.75)(6.33)}{439.25 - 6(5.75)^2} = \frac{508 - 218.385}{439.25 - 198.375}$$

$$= \frac{289.615}{240.875} = 1.202$$

$$\alpha = \bar{y} - \beta\,\bar{x} = 6.33 - 1.202\,(5.75) = -0.58$$

Hence the characteristic line is -0.58 + 1.202 (R_m)

Total Risk of Market = $\sigma_{m^2} = \frac{\Sigma(x-\bar{x})^2}{n} = \frac{240.86}{6} = 40.14(\%)$

Total Risk of Stock = $\frac{497.34}{6} = 82.89\,(\%)$

Systematic Risk = $\beta i^2\,\sigma_2 = (1.202)^2 \times 40.14 = 57.99(\%)$

Unsystematic Risk is   = Total Risk – Systematic Risk

                        = 82.89 - 57.99 = 24.90(%)

12.   (i)

Period	R_X	R_M	$R_X - \bar{R}_X$	$R_M - \bar{R}_M$	$\left(R_X - \bar{R}_X\right)\left(R_M - \bar{R}_M\right)$	$\left(R_M - \bar{R}_M\right)^2$
1	20	22	5	10	50	100
2	22	20	7	8	56	64
3	25	18	10	6	60	36
4	21	16	6	4	24	16
5	18	20	3	8	24	64
6	-5	8	-20	-4	80	16
7	17	-6	2	-18	-36	324

8	19	5	4	-7	-28	49
9	-7	6	-22	-6	132	36
10	20	11	5	-1	-5	1
	150	120			357	706
	ΣR_X	ΣR_M			$\sum (R_X - \bar{R}_X)(R_M - \bar{R}_M)$	$\sum (R_M - \bar{R}_M)^2$

$\bar{R}_X = 15 \quad \bar{R}_M = 12$

$$\sigma^2{}_M = \frac{\sum \left(R_M - \bar{R}_M\right)^2}{n} = \frac{706}{10} = 70.60$$

$$Cov_{X\,M} = \frac{\sum \left(R_X - \bar{R}_X\right)\left(R_M - \bar{R}_M\right)}{n} = \frac{357}{10} = 35.70$$

$$Beta_x = \frac{Cov_{XM}}{\sigma^2{}_M} = \frac{35.70}{70.60} = 0.505$$

Alternative Solution

Period	X	Y	Y²	XY
1	20	22	484	440
2	22	20	400	440
3	25	18	324	450
4	21	16	256	336
5	18	20	400	360
6	-5	8	64	-40
7	17	-6	36	-102
8	19	5	25	95
9	-7	6	36	-42
10	20	11	121	220
	150	120	2146	2157
	$\bar{X} = 15$	$\bar{Y} = 12$		

$$= \frac{\Sigma XY - n\overline{X}\,\overline{Y}}{\Sigma X^2 - n(\overline{X})^2}$$

$$= \frac{2157 - 10 \times 15 \times 12}{2146 - 10 \times 12 \times 12} = \frac{357}{706} = 0.506$$

(ii) $\overline{R}_X = 15$ $\overline{R}_M = 12$

$y = \alpha + \beta x$

$15 = \alpha + 0.505 \times 12$

Alpha (α) $= 15 - (0.505 \times 12) = 8.94\%$

Characteristic line for security X $= \alpha + \beta \times R_M$

Where, R_M = Expected return on Market Index

$\therefore$ Characteristic line for security X $= 8.94 + 0.505\,R_M$

13.   **(a)**   The Betas of two stocks:

Aggressive stock   -   (40% - 4%)/(25% - 7%) = 2

Defensive stock   -   (18% - 9%)/(25% - 7%) = 0.50

Alternatively, it can also be solved by using the Characteristic Line Relationship as follows:

$R_s = \alpha + \beta R_m$

Where

       α = Alpha

       β = Beta

       R_m = Market Return

For Aggressive Stock

$4\% = \alpha + \beta(7\%)$

$40\% = \alpha + \beta(25\%)$

$36\% = \beta(18\%)$

$\beta = 2$

For Defensive Stock

$9\% = \alpha + \beta(7\%)$

$18\% = \alpha + \beta(25\%)$

$9\% = \beta(18\%)$

$\beta = 0.50$

(b) Expected returns of the two stocks:-

Aggressive stock   - $0.5 \times 4\% + 0.5 \times 40\% = 22\%$

Defensive stock   - $0.5 \times 9\% + 0.5 \times 18\% = 13.5\%$

(c) Expected return of market portfolio $= 0.5 \times 7\% + 0.5\% \times 25\% = 16\%$

∴ Market risk prem. $= 16\% - 7.5\% = 8.5\%$

∴ SML is, required return $= 7.5\% + \beta_i\ 8.5\%$

(d) $R_s = \alpha + \beta R_m$

For Aggressive Stock

$22\% = \alpha_A + 2(16\%)$

$\alpha_A = -10\%$

For Defensive Stock

$13.5\% = \alpha_D + 0.50(16\%)$

$\alpha_D = 5.5\%$

14. **(i)** Sensitivity of each stock with market is given by its beta.

Standard deviation of market Index = 15%

Variance of market Index = 0.0225

Beta of stocks = $\sigma_i\ r/\sigma_m$

$A = 20 \times 0.60/15 = 0.80$

$B = 18 \times 0.95/15 = 1.14$

$C = 12 \times 0.75/15 = 0.60$

(ii) Covariance between any 2 stocks = $\beta_1\beta_2\ \sigma_m^2$

Covariance matrix

Stock/Beta	0.80	1.14	0.60
A	400.000	205.200	108.000
B	205.200	324.000	153.900
C	108.000	153.900	144.000

(iii) Total risk of the equally weighted portfolio (Variance) = $400(1/3)^2 + 324(1/3)^2 + 144(1/3)^2 + 2(205.20)(1/3)^2 + 2(108.0)(1/3)^2 + 2(153.900)(1/3)^2 = 200.244$

(iv) β of equally weighted portfolio $= \beta_p = \sum \beta_i/N = \dfrac{0.80+1.14+0.60}{3}$

$= 0.8467$

(v) Systematic Risk $\beta_P^2 \sigma_m^2 = (0.8467)^2 (15)^2 = 161.302$

Unsystematic Risk = Total Risk – Systematic Risk

= 200.244 – 161.302 = 38.942

15. (i) Mr. X's position in the two securities are +1.50 in security A and -0.5 in security B. Hence the portfolio sensitivities to the two factors:-

b prop. 1 =1.50 x 0.80 + (-0.50 x 1.50) = 0.45

b prop. 2 = 1.50 x 0.60 + (-0.50 x 1.20) = 0.30

(ii) Mr. X's revised position:-

Security A ₹ 3,00,000 / ₹ 1,00,000 = 3

Security B -₹ 1,00,000 / ₹ 1,00,000 = -1

Risk free asset -₹ 100000 / ₹ 100000 = -1

b prop. 1 = 3.0 x 0.80 + (-1 x 1.50) + (- 1 x 0) = 0.90

b prop. 2 = 3.0 x 0.60 + (-1 x 1.20) + (-1 x 0) = 0.60

(iii) Expected Return = Risk Free Rate of Return + Risk Premium

Let λ_1 and λ_2 are the Value Factor 1 and Factor 2 respectively.

Accordingly

$15 = 10 + 0.80\ \lambda_1 + 0.60\ \lambda_2$

$20 = 10 + 1.50\ \lambda_1 + 1.20\ \lambda_2$

On solving equation, the value of $\lambda_1 = 0$, and risk premium of factor 2 for Securities A & B shall be as follows:

Using Security A's Return

Total Return = 15% = 10% + 0.60 λ_2

Risk Premium (λ_2) = 5%/ 0.60 = 8.33%

Alternatively using Security B's Return

Total Return = 20% = 10 + 1.20 λ_2

Risk Premium = 10%/ 1.20 = 8.33%

16. Market Risk Premium (A) = 14% – 7% = 7%

Share	Beta	Risk Premium (Beta x A) %	Risk Free Return %	Return %	Return ₹
Oxy Rin Ltd.	0.45	3.15	7	10.15	8,120
Boxed Ltd.	0.35	2.45	7	9.45	14,175
Square Ltd.	1.15	8.05	7	15.05	33,863
Ellipse Ltd.	1.85	12.95	7	19.95	89,775
Total Return					1,45,933

Total Investment ₹ 9,05,000

(i) Portfolio Return = $\dfrac{₹\,1,45,933}{₹\,9,05,000} \times 100$ = 16.13%

(ii) Portfolio Beta

Portfolio Return = Risk Free Rate + Risk Premium x β = 16.13%

7% + 7β = 16.13%

β = 1.30

Alternative Approach

First we shall compute Portfolio Beta using the weighted average method as follows:

$$\text{Beta}_P = 0.45 \times \frac{0.80}{9.05} + 0.35 \times \frac{1.50}{9.05} + 1.15 \times \frac{2.25}{9.05} + 1.85 \times \frac{4.50}{9.05}$$

$$= 0.45 \times 0.0884 + 0.35 \times 0.1657 + 1.15 \times 0.2486 + 1.85 \times 0.4972$$

$$= 0.0398 + 0.058 + 0.2859 + 0.9198 = 1.3035$$

Accordingly,

(i)   Portfolio Return using CAPM formula will be as follows:

$$R_P = R_F + \beta_P(R_M - R_F)$$

$$= 7\% + 1.3035(14\% - 7\%) = 7\% + 1.3035(7\%)$$

$$= 7\% + 9.1245\% = 16.1245\%$$

(ii)   Portfolio Beta

As calculated above 1.3035

17.   (i)   Variance of Returns

$$\text{Cor}_{i,j} = \frac{\text{Cov}(i,j)}{\sigma_i \sigma_j}$$

Accordingly, for MFX

$$1 = \frac{\text{Cov}(X,X)}{\sigma_X \sigma_X}$$

$$\sigma_X^2 = 4.800$$

Accordingly, for MFY

$$1 = \frac{\text{Cov}(Y,Y)}{\sigma_Y \sigma_Y}$$

$$\sigma_Y^2 = 4.250$$

Accordingly, for Market Return

$$1 = \frac{\text{Cov}(M,M)}{\sigma_M \sigma_M}$$

$$\sigma_M^2 = 3.100$$

Alternatively, by referring diagonally the given Table these values can identified as follows:

Variance$_X$ = 4.800

Variance$_Y$ = 4.250

Variance$_M$ = 3.100

(ii) Portfolio return, beta, variance and standard deviation

Weight of MFX in portfolio = $\dfrac{1,20,000}{2,00,000}$ =0.60

Weight of MFY in portfolio = $\dfrac{80,000}{2,00,000}$ =0.40

Accordingly Portfolio Return

0.60 × 15% + 0.40 × 14% = 14.60%

Beta of each Fund

$$\beta = \dfrac{Cov\left(Fund, Market\right)}{Variance\, of\, Market}$$

$$\beta_X = \dfrac{3.370}{3.100} = 1.087$$

$$\beta_Y = \dfrac{2.800}{3.100} = 0.903$$

Portfolio Beta

0.60 x 1.087 + 0.40 x 0.903 = 1.013

Portfolio Variance

$$\sigma^2_{XY} = w^2_X \sigma^2_X + w^2_Y \sigma^2_Y + 2\, w_X w_Y Cov_{X,Y}$$

$$= (0.60)^2 (4.800) + (0.40)^2 (4.250) + 2(0.60)\,(0.40)\,(4.300)$$

= 4.472

Or Portfolio Standard Deviation

$$\sigma_{XY} = \sqrt{4.472} = 2.115$$

(iii) Expected Return, Systematic and Unsystematic Risk of Portfolio

Portfolio Return = 10% + 1.0134(12% - 10%) = 12.03%

MF X Return = 10% + 1.087(12% - 10%) = 12.17%

MF Y Return = 10% + 0.903(12% - 10%) = 11.81%

Systematic Risk = $\beta^2 \sigma^2$

Accordingly,

Systematic Risk of MFX = $(1.087)^2$ x 3.10 = 3.663

Systematic Risk of MFY = $(0.903)^2$ x 3.10 = 2.528

Systematic Risk of Portfolio = $(1.013)^2$ x 3.10 = 3.181

Unsystematic Risk = Total Risk – Systematic Risk

Accordingly,

Unsystematic Risk of MFX = 4.80 – 3.663 = 1.137

Unsystematic Risk of MFY = 4.250 – 2.528 = 1.722

Unsystematic Risk of Portfolio = 4.472 – 3.181 = 1.291

(iv) Sharpe and Treynor Ratios and Alpha

Sharpe Ratio

MFX = $\dfrac{15\% - 10\%}{\sqrt{4.800}}$ = 2.282

MFY = $\dfrac{14\% - 10\%}{\sqrt{4.250}}$ = 1.94

Portfolio = $\dfrac{14.6\% - 10\%}{2.115}$ = 2.175

Treynor Ratio

MFX = $\dfrac{15\% - 10\%}{1.087}$ = 4.60

MFY = $\dfrac{14\% - 10\%}{0.903}$ = 4.43

Portfolio = $\dfrac{14.6\% - 10\%}{1.0134}$ = 4.54

Alpha

MFX = 15% - 12.17% = 2.83%

MFY = 14% - 11.81% = 2.19%

Portfolio = 14.6% - 12.03% = 2.57%

18. Capital Asset Pricing Model (CAPM) formula for calculation of expected rate of return is

$E_R = R_f + \beta (R_m - R_f)$

E_R = Expected Return

β = Beta of Security

R_m = Market Return

R_f = Risk free Rate

= 10 + [1.2 (15 – 10)]

= 10 + 1.2 (5)

= 10 + 6 = 16% or 0.16

Applying dividend growth mode for the calculation of per share equilibrium price:-

$E_R = \dfrac{D_1}{P_0} + g$

or $0.16 = \dfrac{3(1.12)}{P_0} + 0.12$ or $0.16 - 0.12 = \dfrac{3.36}{P_0}$

or $0.04 \, P_0 = 3.36$ or $P_0 = \dfrac{3.36}{0.04} = ₹ \, 84$

Therefore, equilibrium price per share will be ₹ 84.

19. First we shall compute the β of Security X.

Risk Free Rate = $\dfrac{\text{Coupon Payment}}{\text{Current Market Price}} = \dfrac{7}{140} = 5\%$

Assuming equilibrium return to be equal to CAPM return then:

$15\% = R_f + \beta_X (R_m - R_f)$

$15\% = 5\% + \beta_X (15\% - 5\%)$

$\beta_X = 1$

or it can also be computed as follows:

$$\frac{R_m}{R_s} = \frac{15\%}{15\%} = 1$$

(i)   Standard Deviation of Market Return

$$\beta_m = \frac{Cov_{X,m}}{\sigma_m^2} = \frac{225\%}{\sigma_m^2} = 1$$

$$\sigma_m^2 = 225$$

$$\sigma_m = \sqrt{225} = 15\%$$

(ii)   Standard Deviation of Security Return

$$\beta_X = \frac{\sigma_X}{\sigma_m} \times \rho_{Xm} = \frac{\sigma_X}{15} \times 0.75 = 1$$

$$\sigma_X = \frac{15}{0.75} = 20\%$$

20.   CAPM = $R_f + \beta (R_m - R_f)$

Accordingly

$R_{ABC} = R_f + 1.2 (R_m - R_f) = 19.8$

$R_{XYZ} = R_f + 0.9 (R_m - R_f) = 17.1$

$19.8 = R_f + 1.2 (R_m - R_f)$ ------(1)

$17.1 = R_f + 0.9 (R_m - R_f)$ ------(2)

Deduct (2) from (1)

$2.7 = 0.3 (R_m - R_f)$

$R_m - R_f = 9$

$R_f = R_m - 9$

Substituting in equation (1)

$19.8 = (R_m - 9) + 1.2 (R_m - R_m + 9)$

$19.8 = R_m - 9 + 10.8$

$19.8 = R_m + 1.8$

Then $R_m = 18\%$ and $R_f = 9\%$

Security Market Line

= $R_f + \beta$ (Market Risk Premium)

= $9\% + \beta \times 9\%$

21. **(i)** A Ltd. has lower return and higher risk than B Ltd. Hence, investing in B Ltd. is better than in A Ltd. because the return is higher and the risk is lower. However, investing in both will yield diversification advantage.

(ii) $r_{AB} = 0.22 \times 0.7 + 0.24 \times 0.3 = 0.226$ i.e. 22.6%

$\sigma^2_{AB} = 0.40^2 \times 0.7^2 + 0.38^2 \times 0.3^2 + 2 \times 0.7 \times 0.3 \times 0.72 \times 0.40 \times 0.38 = 0.1374$

$\sigma_{AB} = \sqrt{\sigma^2_{AB}} = \sqrt{0.1374} = 0.37 = 37\%$

* Answer = 37.06% is also correct and variation may occur due to approximation.

(iii) This risk-free rate will be the same for A and B Ltd. Their rates of return are given as follows:

$r_A = 22 = r_f + (r_m - r_f)\, 0.86$

$r_B = 24 = r_f + (r_m - r_f)\, 1.24$

$r_A - r_B = -2 = (r_m - r_f)\,(-0.38)$

$r_m - r_f = -2/-0.38 = 5.26\%$

$r_A = 22 = r_f + (5.26)\, 0.86$

$r_f = 17.48\%$

Or

$r_B = 24 = r_f + (5.26)\, 1.24$

$r_f = 17.48\%$

$r_m - 17.48 = 5.26$

$r_m = 22.74\%$

(iv) $\beta_{AB} = \beta_A \times W_A + \beta_B \times W_B$

$= 0.86 \times 0.7 + 1.24 \times 0.3 = 0.974$

22. **(i)** **Computation of Beta of Portfolio**

Investment	No. of shares	Market Price	Market Value	Dividend Yield	Dividend	Composition	β	Weighted β
I	60,000	4.29	2,57,400	19.50%	50,193	0.2339	1.16	0.27
II	80,000	2.92	2,33,600	24.00%	56,064	0.2123	2.28	0.48
III	1,00,000	2.17	2,17,000	17.50%	37,975	0.1972	0.90	0.18
IV	1,25,000	3.14	3,92,500	26.00%	1,02,050	0.3566	1.50	0.53
			11,00,500		2,46,282	1.0000		1.46

Return of the Portfolio $\dfrac{2,46,282}{11,00,500} = 0.2238$

Beta of Port Folio   1.46

Market Risk implicit

$0.2238 = 0.11 + β× (0.19 – 0.11)$

Or, $0.08 β + 0.11 = 0.2238$

$β = \dfrac{0.2238 - 0.11}{0.08} = 1.42$

Market β implicit is 1.42 while the port folio β is 1.46. Thus the portfolio is marginally risky compared to the market.

(ii)   The decision regarding change of composition may be taken by comparing the dividend yield (given) and the expected return as per CAPM as follows:

Expected return R_s as per CAPM is:

R_s	=	$I_{RF} + (R_M – I_{RF})β$
For investment I R_s	=	$I_{RF} + (R_M – I_{RF})β$
	=	.11 + (.19 - .11) 1.16
	=	20.28%
For investment II, R_s	=	.11 + (.19 - .11) 2.28 = 29.24%
For investment III, R_s	=	.11 + (.19 - .11) .90
	=	18.20%
For investment IV, R_s	=	.11 + (.19 - .11) 1.50
	=	23%

Comparison of dividend yield with the expected return R_s shows that the dividend yields of investment I, II and III are less than the corresponding R_s,. So, these investments are over-priced and should be sold by the investor. However, in case of investment IV, the dividend yield is more than the corresponding R_s, so, XYZ Ltd. should increase its proportion.

23.   With 20% investment in each MF Portfolio Beta is the weighted average of the Betas of various securities calculated as below:

(i)

Investment	Beta (β)	Investment (₹ Lacs)	Weighted Investment
A	1.6	20	32
B	1.0	20	20
C	0.9	20	18
D	2.0	20	40
E	0.6	20	12
		100	122
Weighted *Beta (β)* = 1.22			

(ii) With varied percentages of investments portfolio beta is calculated as follows:

Investment	Beta (β)	Investment (₹ Lacs)	Weighted Investment
A	1.6	15	24
B	1.0	30	30
C	0.9	15	13.5
D	2.0	30	60
E	0.6	10	6
		100	133.5
Weighted *Beta (β)* = 1.335			

(iii) Expected return of the portfolio with pattern of investment as in case (i)

= 12% × 1.22 i.e. 14.64%

Expected Return with pattern of investment as in case (ii) = 12% × 1.335 i.e., 16.02%.

24. (a) Let the weight of stocks of Economy A be expressed as w, then

$(1- w) \times 10.0 + w \times 15.0 = 10.5$

i.e. w = 0.1 or 10%.

(b) Variance of portfolio shall be:

$(0.9)^2 (0.16)^2 + (0.1)^2 (0.30)^2 + 2(0.9) (0.1) (0.16) (0.30) (0.30) = 0.02423$

Standard deviation is $(0.02423)^{1/2} = 0.15565$ or 15.6%.

(c) The Sharpe ratio will improve by approximately 0.04, as shown below:

$$\text{Sharpe Ratio} = \frac{\text{Expected Return - Risk Free Rate of Return}}{\text{Standard Deviation}}$$

Investment only in developed countries: $\dfrac{10 - 3}{16} = 0.437$

With inclusion of stocks of Economy A: $\dfrac{10.5 - 3}{15.6} = 0.481$

25. **(i)** Computation of Expected Return from Portfolio

Security	Beta (β)	Expected Return (r) as per CAPM	Amount (₹ Lakhs)	Weights (w)	wr
Moderate	0.50	8%+0.50(10% - 8%) = 9%	60	0.115	1.035
Better	1.00	8%+1.00(10% - 8%) = 10%	80	0.154	1.540
Good	0.80	8%+0.80 (10% - 8%) = 9.60%	100	0.192	1.843
Very Good	1.20	8%+1.20(10% - 8%) = 10.40%	120	0.231	2.402
Best	1.50	8%+1.50(10% - 8%) = 11%	160	0.308	3.388
Total			520	1	10.208

Thus Expected Return from Portfolio 10.208% say 10.21%.

Alternatively, it can be computed as follows:

Average β = 0.50 x $\dfrac{60}{520}$ + 1.00 x $\dfrac{80}{520}$ + 0.80 x $\dfrac{100}{520}$ + 1.20 x $\dfrac{120}{520}$ + 1.50 x $\dfrac{160}{520}$ = 1.104

As per CAPM

= 0.08 + 1.104(0.10 – 0.08) = 0.10208 i.e. 10.208%

(ii) As computed above the expected return from Better is 10% same as return from Nifty, hence there will be no difference even if the replacement of security is made. The main logic behind this neutrality is that the beta of security 'Better' is 1 which clearly indicates that this security shall yield same return as market return.

26.

Particulars of Securities	Cost ₹	Dividend	Capital gain
Gold Ltd.	10,000	1,725	−200
Silver Ltd.	15,000	1,000	1,200
Bronz Ltd.	14,000	700	6,000

GOI Bonds	36,000	3,600	−1,500
Total	75,000	7,025	5,500

Expected rate of return on market portfolio

$$\frac{\text{Dividend Earned} + \text{Capital appreciation}}{\text{Initial investment}} \times 100$$

$$= \frac{₹\ 7,025 + ₹\ 5,500}{₹\ 75,000} \times 100 = 16.7\%$$

Risk free return

$$\text{Average of Betas} = \frac{0.6 + 0.8 + 0.6 + 0.01}{4} \qquad = \text{Average of Betas* = 0.50}$$

Average return = Risk free return + Average Betas (Expected return – Risk free return)

15.7 = Risk free return + 0.50 (16.7 – Risk free return)

Risk free return = 14.7%

* Alternatively, it can also be calculated through Weighted Average Beta.

Expected Rate of Return for each security is

Rate of Return	= Rf + B (Rm – Rf)
Gold Ltd.	= 14.7 + 0.6 (16.7 – 14.7)= 15.90%
Silver Ltd.	= 14.7 + 0.8 (16.7 – 14.7) = 16.30%
Bronz Ltd.	= 14.7 + 0.6 (16.7 – 14.7) = 15.90%
GOI Bonds	= 14.7 + 0.01 (16.7 – 14.7) = 14.72%

* Alternatively, it can also be computed as follows:

Particulars of Securities (1)	Cost ₹ (2)	Market price ₹ (3)	Dividend ₹ (4)	Capital Gain ₹ (5)	Beta (6)	(3) x (6)
Gold Ltd.	10,000	9,800	1,725	−200	0.6	5,880
Silver Ltd.	15,000	16,200	1,000	1,200	0.8	12,960
Bronz Ltd.	14,000	20,000	700	6,000	0.6	12,000
GOI Bonds	36,000	34,500	3,600	−1,500	0.01	345
Total	75,000	80,500	7,025	5,500		31,185

β of the Portfolio = $\dfrac{31,185}{80,500}$ = 0.387

Using GOI Bond we can compute Risk Free Rate of Return as follows:

R_f = $\dfrac{(34,500 - 36,000) + 3,600}{36,000}$ = 0.0583 i.e. 5.83%

Now we can calculate Market Return (R_m) using average return of the portfolio as follows:

R_p = R_f + β(R_m - R_f)

15.7% = 5.83% + 0.387(R_m − 5.83%)

27. (i) Expected rate of return

	Total Investments	*Dividends*	*Capital Gains*
Epsilon Ltd.	25	2	25
Sigma Ltd.	35	2	25
Omega Ltd.	45	2	90
GOI Bonds	1,000	140	5
	1,105	146	145

Expected Return on market portfolio= $\dfrac{146 + 145}{1105} = 26.33\%$

CAPM   E(Rp) = RF + β [E(RM) – RF]

Epsilon Ltd	14+0.8 [26.33-14] =	14+9.86	=	23.86%
Sigma Ltd.	14+0.7 [26.33-14] =	14+8.63	=	22.63%
Omega Ltd.	14+0.5 [26.33-14] =	14+6.17	=	20.17%
GOI Bonds	14+0.01 [26.33-14] =	14+0.12	=	14.12%

(ii) Average Return of Portfolio

$\dfrac{23.86 + 22.63 + 20.17 + 14.12}{4} = \dfrac{80.78}{4} = 20.20\%$

Alternatively, $\dfrac{0.8 + 0.7 + 0.5 + 0.01}{4} = \dfrac{2.01}{4} = 0.5025$

14 + 0.5025 (26.33 - 14) = 14+ 6.20 = 20.20%

28. Calculation of expected return on market portfolio (R_m)

Investment	Cost (₹)	Dividends (₹)	Capital Gains (₹)
Shares X	8,000	800	200
Shares Y	10,000	800	500
Shares Z	16,000	800	6,000
PSU Bonds	34,000	3,400	−1,700
	68,000	5,800	5,000

$$R_m = \frac{5,800 + 5,000}{68,000} \times 100 = 15.88\%$$

Calculation of expected rate of return on individual security:

Security

Shares X	15 + 0.8 (15.88 – 15.0)	= 15.70%
Shares Y	15 + 0.7 (15.88 – 15.0)	= 15.62%
Shares Z	15 + 0.5 (15.88 – 15.0)	= 15.44%
PSU Bonds	15 + 0.2 (15.88 – 15.0)	= 15.18%

Calculation of the Average Return of the Portfolio:

$$= \frac{15.70 + 15.62 + 15.44 + 15.18}{4} = 15.49\%.$$

29. On the basis of existing and revised factors, rate of return and price of share is to be calculated.

Existing rate of return

= R_f + Beta (R_m – R_f) = 12% + 1.4 (6%) = 20.4%

Revised rate of return

= 10% + 1.25 (4%) = 15%

Price of share (original)

$$P_o = \frac{D (1 + g)}{K_e - g} = \frac{2 (1.05)}{0.204 - 0.05} = \frac{2.10}{0.154} = ₹ 13.63$$

Price of share (Revised)

$$P_o = \frac{2(1.09)}{0.15 - 0.09} = \frac{2.18}{0.06} = ₹\ 36.33$$

In case of existing market price of ₹ 25 per share, rate of return (20.4%) and possible equilibrium price of share at ₹ 13.63, this share needs to be sold because the share is overpriced (₹ 25 – 13.63) by ₹ 11.37. However, under the changed scenario where growth of dividend has been revised at 9% and the return though decreased at 15% but the possible price of share is to be at ₹ 36.33 and therefore, in order to expect price appreciation to ₹ 36.33 the investor should hold the shares, if other things remain the same

30. On the basis of existing and revised factors, rate of return and price of share is to be calculated.

Existing rate of return

$= R_f + Beta\ (R_m – R_f)$

= 12% + 1.3 (5%) = 18.5%

Revised rate of return

= 10% + 1.4 (4%) = 15.60%

Price of share (original)

$$P_o = \frac{D(1+g)}{K_e - g} = \frac{3(1.09)}{0.185 - 0.09} = \frac{3.27}{0.095} = ₹\ 34.42$$

Price of share (Revised)

$$P_o = \frac{2.50(1.07)}{0.156 - 0.07} = \frac{2.675}{0.086} = ₹\ 31.10$$

Market price of share of ₹ 40 is higher in comparison to current equilibrium price of ₹ 34.42 and revised equity price of ₹ 31.10. Under this situation investor should sell the share.

31. (i) Investment committed to each security would be:-

	<u>A</u> (₹)	<u>B</u> (₹)	<u>C</u> (₹)	<u>Total</u> (₹)
Portfolio X	1,500	2,000	1,500	5,000
Portfolio Y	600	1,500	900	3,000
Combined Portfolio	2,100	3,500	2,400	8,000
∴ Stock weights	0.26	0.44	0.30	

(ii)   The equation of critical line takes the following form:-

WB = a + bWA

Substituting the values of WA & WB from portfolio X and Y in above equation, we get

0.40 = a + 0.30b, and

0.50 = a + 0.20b

Solving above equation we obtain the slope and intercept, a = 0.70 and b= -1 and thus, the critical line is

WB = 0.70 – WA

If half of the funds is invested in security A then,

WB = 0.70 – 0.50   = 0.20

Since WA + WB + WC = 1

WC = 1 - 0.50 – 0.20   = 0.30

∴ Allocation of funds to security B   = 0.20 x 8,000 = ₹ 1,600, and

Security C   = 0.30 x 8,000 = ₹ 2,400

32.   Workings:

Calculation of return on portfolio for 2009-10	(Calculation in ₹ / share)		
	M	**N**	
Dividend received during the year	10	3	
Capital gain/loss by 31.03.10			
Market value by 31.03.10	220	290	
Cost of investment	200	300	
Gain/loss	20	(-)10	
Yield	30	(-)7	
Cost	200	300	
% return	15%	(-)2.33%	
Weight in the portfolio	57	43	
Weighted average return			7.55%
Calculation of estimated return for 2010-11			

Expected dividend	20	3.5
Capital gain by 31.03.11		
(220x0.2)+ (250x0.5)+(280x0.3) – 220=(253-220)	33	-
(290x0.2)+(310x0.5)+(330x0.3) – 290= (312 – 290)	-	22
Yield	53	25.5
*Market Value 01.04.10	220	290
% return	24.09%	8.79%
*Weight in portfolio (1,000x220): (500x290)	60.3	39.7
Weighted average (Expected) return		18.02%
(*The market value on 31.03.10 is used as the base for calculating yield for 10-11)		

(i)   Average Return from Portfolio for the year ended 31.03.2010 is 7.55%.

(ii) Expected Average Return from portfolio for the year 2010-11 is 18.02%

(iii)   **Calculation of Standard Deviation**

M Ltd.

Exp. market value	Exp. gain	Exp. div.	Exp Yield (1)	Prob. Factor (2)	(1) X(2)	Dev. $(P_M - \overline{P_M})$	Square of dev. (3)	(2) X (3)
220	0	20	20	0.2	4	-33	1089	217.80
250	30	20	50	0.5	25	-3	9	4.50
280	60	20	80	0.3	24	27	729	218.70
					53			$\sigma^2_M =$ 441.00

Standard Deviation (σ_M)   21

N Ltd.

Exp. market value	Exp. gain	Exp. div.	Exp Yield (1)	Prob. Factor (2)	(1) X(2)	Dev. $(P_N - \overline{P_N})$	Square of dev. (3)	(2) X (3)
290	0	3.5	3.5	0.2	0.7	-22	484	96.80
310	20	3.5	23.5	0.5	11.75	-2	4	2.00
330	40	3.5	43.5	0.3	13.05	18	324	97.20
					25.5			$\sigma^2_N = 196.00$

Standard Deviation (σ_N)   14

Alternatively based on return in percentage terms Standard Deviation can also be computed as follows:

M Ltd.

Exp. market value	Exp. gain	Exp. div.	Exp Return (1)	Prob. Factor (2)	(1) X(2)	Dev. $(P_M - \overline{P_M})$	Square of dev. (3)	(2) X (3)
220	0	20	9.09	0.2	1.82	-15.01	225.30	45.06
250	30	20	22.73	0.5	11.37	-1.37	1.88	0.94
280	60	20	36.36	0.3	10.91	12.26	150.31	45.09
					24.10			σ^2_M = 91.09

Standard Deviation (σ_M)   9.54%

N Ltd.

Exp. market value	Exp. gain	Exp. div.	Exp Return (1)	Prob. Factor (2)	(1) X (2)	Dev. $(P_N - \overline{P_N})$	Square of dev. (3)	(2) X (3)
290	0	3.5	1.21	0.2	0.24	-7.58	57.46	11.49
310	20	3.5	8.10	0.5	4.05	-0.69	0.48	0.24
330	40	3.5	15.00	0.3	4.50	6.21	38.56	11.57
					8.79			σ^2_N = 23.30

Standard Deviation (σ_N)   4.83%

Share of company M Ltd. is more risky as the S.D. is more than company N Ltd.

33.   Expected Return on stock A = E (A) = $\sum\limits_{i=G,S,R} P_i A_i$

(G,S & R, denotes Growth, Stagnation and Recession)

(0.40)(25) + 0.30(10)+ 0.30(-5) = 11.5%

Expected Return on 'B'

(0.40×20) + (0.30×15) +0.30× (-8)=10.1%

Expected Return on Market index

(0.40 × 18) + (0.30 × 13) + 0.30 × (-3) =10.2%

Variance of Market index

$(18 - 10.2)^2 (0.40) + (13 - 10.2)^2 (0.30) + (-3 - 10.2)^2 (0.30)$

= 24.34 + 2.35 + 52.27 = 78.96%

Covariance of stock A and Market Index M

$\text{Cov. (AM)} = \sum_{i=G,S,R} ([A_i - E(A)] [M_i - E(M)]P$

(25 -11.5) (18 - 10.2)(0.40) + (10 - 11.5) (13 - 10.2) (0.30) + (-5-11.5) (-3-10.2)(0.30)

= 42.12 + (-1.26) + 65.34=106.20

Covariance of stock B and Market index M

(20-10.1) (18-10.2)(0.40)+(15-10.1)(13-10.2)(0.30) + (-8-10.1)(-3-10.2)(0.30)= 30.89 + 4.12 + 71.67=106.68

Beta for stock A = $\dfrac{\text{CoV(AM)}}{\text{VAR(M)}} = \dfrac{106.20}{78.96} = 1.345$

Beta for Stock B = $\dfrac{\text{CoV(BM)}}{\text{VarM}} = \dfrac{106.68}{78.96} = 1.351$

Required Return for A

$R(A) = R_f + \beta (M - R_f)$

11% + 1.345(10.2 - 11) % = 9.924%

Required Return for B

11% + 1.351 (10.2 – 11) % = 9.92%

Alpha for Stock A

E (A) – R (A) i.e. 11.5 % – 9.924% = 1.576%

Alpha for Stock B

E (B) – R (B) i.e. 10.1% - 9.92% = 0.18%

Since stock A and B both have positive Alpha, therefore, they are underpriced. The investor should make fresh investment in them.

34.　**(i)**　Portfolio Beta

0.20 x 0.40 + 0.50 x 0.50 + 0.30 x 1.10 = 0.66

(ii)　Residual Variance

To determine Residual Variance first of all we shall compute the Systematic Risk as follows:

$\beta_A^2 \times \sigma_M^2$ = (0.40)²(0.01) = 0.0016

$\beta_B^2 \times \sigma_M^2$ = (0.50)²(0.01) = 0.0025

$\beta_C^2 \times \sigma_M^2$ = (1.10)²(0.01) = 0.0121

Residual Variance

A　0.015 – 0.0016 = 0.0134

B　0.025 – 0.0025 = 0.0225

C　0.100 – 0.0121 = 0.0879

(iii)　Portfolio variance using Sharpe Index Model

Systematic Variance of Portfolio = (0.10)² x (0.66)² = 0.004356

Unsystematic Variance of Portfolio = 0.0134 x (0.20)² + 0.0225 x (0.50)² + 0.0879 x (0.30)² = 0.014072

Total Variance = 0.004356 + 0.014072 = 0.018428

(iv)　Portfolio variance on the basis of Markowitz Theory

= (w$_A$ x w$_A$xσ_A^2) + (w$_A$ x w$_B$xCov$_{AB}$) + (w$_A$ x w$_C$xCov$_{AC}$) + (w$_B$ x w$_A$xCov$_{AB}$) + (w$_B$ x w$_B$x σ_B^2) + (w$_B$ x w$_C$xCov$_{BC}$) + (w$_C$ x w$_A$xCov$_{CA}$) + (w$_C$ x w$_B$xCov$_{CB}$) + (w$_C$ x w$_C$x σ_C^2)

= (0.20 x 0.20 x 0.015) + (0.20 x 0.50 x 0.030) + (0.20 x 0.30 x 0.020) + (0.20 x 0.50 x 0.030) + (0.50 x 0.50 x 0.025) + (0.50 x 0.30 x 0.040) + (0.30 x 0.20 x 0.020) + (0.30 x 0.50 x 0.040) + (0.30 x 0.30 x 0.10)

= 0.0006 + 0.0030 + 0.0012 + 0.0030 + 0.00625 + 0.0060 + 0.0012 + 0.0060 + 0.0090

= 0.0363

35. Securities need to be ranked on the basis of excess return to beta ratio from highest to the lowest.

Security	R_i	β_i	$R_i - R_f$	$\dfrac{R_i - R_f}{\beta_i}$
A	15	1.5	8	5.33
B	12	2	5	2.5
C	10	2.5	3	1.2
D	9	1	2	2
E	8	1.2	1	0.83
F	14	1.5	7	4.67

Ranked Table:

Security	$R_i - R_f$	β_i	σ^2_{ei}	$\dfrac{(R_i - R_f)\times\beta_i}{\sigma^2_{ei}}$	$\displaystyle\sum_{e=i}^{N}\dfrac{(R_i - R_f)\times\beta_i}{\sigma^2_{ei}}$	$\dfrac{\beta_i^2}{\sigma^2_{ei}}$	$\displaystyle\sum_{e=i}^{N}\dfrac{\beta_i^2}{\sigma^2_{ei}}$	C_i
A	8	1.5	40	0.30	0.30	0.056	0.056	1.923
F	7	1.5	30	0.35	0.65	0.075	0.131	2.814
B	5	2	20	0.50	1.15	0.20	0.331	2.668
D	2	1	10	0.20	1.35	0.10	0.431	2.542
C	3	2.5	30	0.25	1.60	0.208	0.639	2.165
E	1	1.2	20	0.06	1.66	0.072	0.711	2.047

$C_A = 10 \times 0.30 / [1 + (10 \times 0.056)] = 1.923$

$C_F = 10 \times 0.65 / [1 + (10 \times 0.131)] = 2.814$

$C_B = 10 \times 1.15 / [1 + (10 \times 0.331)] = 2.668$

$C_D = 10 \times 1.35 / [1 + (10 \times 0.431)] = 2.542$

$C_C = 10 \times 1.60 / [1 + (10 \times 0.639)] = 2.165$

$C_E = 10 \times 1.66 / [1 + (10 \times 0.7111)] = 2.047$

Cut off point is 2.814

$$Z_i = \frac{\beta_i}{\sigma^2_{ei}}\left[\left(\left[\frac{\left(R_i - R_f\right)}{\beta_i} - C\right]\right)\right]$$

$$Z_A = \frac{1.5}{40}(5.33 - 2.814) = 0.09435$$

$$Z_F = \frac{1.5}{30}(4.67 - 2.814) = 0.0928$$

$$X_A = 0.09435/[0.09435 + 0.0928] = 50.41\%$$

$$X_F = 0.0928/[0.09435 + 0.0928] = 49.59\%$$

Funds to be invested in security A & F are 50.41% and 49.59% respectively.

36.   (i)

Security	No. of shares (1)	Market Price of Per Share (2)	(1) × (2)	% to total (w)	ß (x)	wx
VSL	10000	50	500000	0.4167	0.9	0.375
CSL	5000	20	100000	0.0833	1	0.083
SML	8000	25	200000	0.1667	1.5	0.250
APL	2000	200	400000	0.3333	1.2	0.400
			1200000	1		1.108

Portfolio beta   1.108

(ii)   Required Beta   0.8

It should become   (0.8 / 1.108)   72.2 % of present portfolio

If ₹ 12,00,000 is 72.20%, the total portfolio should be

₹ 12,00,000 × 100/72.20 or   ₹ 16,62,050

Additional investment in zero risk should be (₹ 16,62,050 – ₹ 12,00,000) = ₹ 4,62,050

Revised Portfolio will be

Security	No. of shares (1)	Market Price of Per Share (2)	(1) × (2)	% to total (w)	ß (x)	wx
VSL	10000	50	500000	0.3008	0.9	0.271
CSL	5000	20	100000	0.0602	1	0.060
SML	8000	25	200000	0.1203	1.5	0.180
APL	2000	200	400000	0.2407	1.2	0.289

Risk free asset	46205	10	462050	0.2780	0	0
			1662050	1		0.800

(iii)   To increase Beta to   1.2

Required beta   1.2

It should become 1.2 / 1.108   108.30% of present beta

If 1200000 is 108.30%, the total portfolio should be

1200000 × 100/108.30 or   1108033 say 1108030

Additional investment should be (-) 91967 i.e. Divest ₹ 91970 of Risk Free Asset

Revised Portfolio will be

Security	No. of shares (1)	Market Price of Per Share (2)	(1) × (2)	% to total (w)	ß (x)	wx
VSL	10000	50	500000	0.4513	0.9	0.406
CSL	5000	20	100000	0.0903	1	0.090
SML	8000	25	200000	0.1805	1.5	0.271
APL	2000	200	400000	0.3610	1.2	0.433
Risk free asset	-9197	10	-91970	-0.0830	0	0
			1108030	1		1.20

Portfolio beta   1.20

Alternative Approach

(ii)   Let x be the amount of Risk-Free Asset to be acquired, then

Security	(1) × (2)	ß (x)	wx
VSL	500000	0.9	450000
CSL	100000	1	100000
SML	200000	1.5	300000
APL	400000	1.2	480000
Risk free asset	x	0	0
	1200000 + x		1330000

Accordingly,

$$\frac{13,30,000}{12,00,000 + x} = 0.8$$

x = 462500 i.e. value of Risk Free Asset to be purchased to decrease beta of portfolio to 0.8.

(iii) Similarly let y the amount of Risk Free Assets to be divest, then

$$\frac{13,30,000}{12,00,000 + y} = 1.20$$

y = -91,667 i.e. value of Risk Free Asset to be divested to increase beta of portfolio to 1.20.

37. $\beta_p = \sum_{i=1}^{4} x_i \beta_i$

= 1.60 x 0.25 + 1.15 x 0.30 + 1.40 x 0.25 + 1.00 x 0.20

= 0.4 + 0.345 + 0.35 + 0.20 = 1.295

The Standard Deviation (Risk) of the portfolio is

= $[(1.295)^2(18)^2+(0.25)^2(7)^2+(0.30)^2(11)^2+(0.25)^2(3)^2+(0.20)^2(9)^2)]^{\frac{1}{2}}$

= $[543.36 + 3.0625 + 10.89 + 0.5625 + 3.24]^{\frac{1}{2}}$ = $[561.115]^{\frac{1}{2}}$ = 23.69%

Alternative Answer

The variance of Security's Return

$\sigma^2 = \beta_i^2 \sigma^2_m + \sigma^2_{\varepsilon i}$

Accordingly, variance of various securities

		σ^2	Weight(w)	$\sigma^2 Xw$
L	$(1.60)^2 (18)^2 + 7^2 =$	878.44	0.25	219.61
M	$(1.15)^2 (18)^2 + 11^2 =$	549.49	0.30	164.85
N	$(1.40)^2 (18)^2 + 3^2 =$	644.04	0.25	161.01
K	$(1.00)^2 (18)^2 + 9^2 =$	405.00	0.20	81
	Variance			626.47

SD = $\sqrt{626.47}$ = 25.03

38.   Return of the stock under APT

Factor	Actual value in %	Expected value in %	Difference	Beta	Diff. x Beta
GNP	7.70	7.70	0.00	1.20	0.00
Inflation	7.00	5.50	1.50	1.75	2.63
Interest rate	9.00	7.75	1.25	1.30	1.63
Stock index	12.00	10.00	2.00	1.70	3.40
Ind. Production	7.50	7.00	0.50	1.00	0.50
					8.16
Risk free rate in %					9.25
Return under APT					17.41

39.   (i) $\beta_{asset} = \beta_{equity} \times \dfrac{V_E}{V_0} + B_{debt} \times \dfrac{V_D}{V_0}$

Note: Since β_{debt} is not given it is assumed that company debt capital is virtually riskless.

If company's debt capital is riskless than above relationship become:

Here $\beta_{equity} = 1.5$; $\beta_{asset} = \beta_{equity} \dfrac{V_E}{V_0}$

As $\beta_{debt} = 0$

$V_E = ₹ 60$ lakhs.

$V_D = ₹ 40$ lakhs.

$V_0 = ₹ 100$ lakhs.

$\beta_{asset} = 1.5 \times \dfrac{60 \text{ lakhs}}{100 \text{ lakhs}}$

$= 0.9$

(ii)   (a)   If only equity is used to finance the expansion, the Cost of Capital for discounting company's expansion of existing business shall be computed as follows:

Company's cost of equity = $R_f + \beta_A \times$ Market Risk premium

Where R_f = Risk free rate of return

β_A = Beta of company assets

Therefore, company's cost of equity = 8% + 0.9 × 10 = 17% and overall cost of capital shall be 17%.

(b) Alternatively, if funds expansion are raised for in same proportion as exiting capital structure, then cost of capital shall be computed as follows:

Cost of Equity = 8% + 1.5 x 10 = 23%

Cost of Debt = 8%

WACC (Cost of Capital) = $23\% \times \dfrac{3}{5} + 8\% \times \dfrac{2}{5} = 17\%$

40. **(a)** **Method I**

Stock's return

Small cap growth = 4.5 + 0.80 x 6.85 + 1.39 x (-3.5) + 1.35 x 0.65 = 5.9925%

Small cap value = 4.5 + 0.90 x 6.85 + 0.75 x (-3.5) + 1.25 x 0.65 = 8.8525%

Large cap growth = 4.5 + 1.165 x 6.85 + 2.75 x (-3.5) + 8.65 x 0.65 = 8.478%

Large cap value = 4.5 + 0.85 x 6.85 + 2.05 x (-3.5) + 6.75 x 0.65 = 7.535%

Expected return on market index

0.25 x 5.9925 + 0.10 x 8.8525 + 0.50 x 8.478 + 0.15 x 7.535 = 7.7526%

Method II

Expected return on the market index

= 4.5% + [0.1x0.9 + 0.25x0.8 + 0.15x0.85 + 0.50x1.165] x 6.85 + [(0.75 x 0.10 + 1.39 x 0.25 + 2.05 x 0.15 + 2.75 x 0.5)] x (-3.5) + [{1.25 x 0.10 + 1.35 x 0.25 + 6.75 x 0.15 + 8.65 x 0.50)] x 0.65

= 4.5 + 6.85 + (-7.3675) + 3.77 = 7.7525%.

(b) Using CAPM,

Small cap growth   = 4.5 + 6.85 x 0.80 = 9.98%

Small cap value = 4.5 + 6.85 x 0.90 = 10.665%

Large cap growth   = 4.5 + 6.85 x 1.165 = 12.48%

Large cap value = 4.5 + 6.85 x 0.85 = 10.3225%

Expected return on market index

= 0.25 x 9.98 + 0.10 x 10.665 + 0.50 x 12.45 + 0.15 x 10.3225 = 11.33%

(c)   Let us assume that Mr. Nirmal will invest X_1% in small cap value stock and X_2% in large cap growth stock

$X_1 + X_2 = 1$

$0.90 X_1 + 1.165 X_2 = 1$

$0.90 X_1 + 1.165(1 - X_1) = 1$

$0.90 X_1 + 1.165 - 1.165 X_1 = 1$

$0.165 = 0.265 X_1$

$\dfrac{0.165}{0.265} = X_1$

$0.623 = X_1, X_2 = 0.377$

62.3% in small cap value

37.7% in large cap growth.

41.   Sharpe Ratio $S = (R_p - R_f)/\sigma_p$

Treynor Ratio $T = (R_p - R_f)/\beta_p$

Where,

R_p =   Return on Fund

R_f =   Risk-free rate

σ_p =   Standard deviation of Fund

β_p =   Beta of Fund

Reward to Variability (Sharpe Ratio)

Mutual Fund	R_p	R_f	$R_p - R_f$	σ_p	Reward to Variability	Ranking
A	15	6	9	7	1.285	2
B	18	6	12	10	1.20	3
C	14	6	8	5	1.60	1
D	12	6	6	6	1.00	5
E	16	6	10	9	1.11	4

Reward to Volatility (Treynor Ratio)

Mutual Fund	R_p	R_f	$R_p - R_f$	β_p	Reward to Volatility	Ranking
A	15	6	9	1.25	7.2	2
B	18	6	12	0.75	16	1
C	14	6	8	1.40	5.71	5
D	12	6	6	0.98	6.12	4
E	16	6	10	1.50	6.67	3

42. Let portfolio standard deviation be σ_p

Market Standard Deviation = σ_m

Coefficient of correlation = r

Portfolio beta (β_p) = $\dfrac{\sigma_p r}{\sigma_m}$

Required portfolio return (R_p) = $R_f + \beta_p (R_m - R_f)$

Portfolio	Beta	Return from the portfolio (R_p) (%)
A	1.75	17.75
B	0.90	13.50
C	0.65	12.25
D	1.25	15.25
E	0.90	13.50

Portfolio	Sharpe Method		Treynor Method		Jensen's Alpha	
	Ratio	Rank	Ratio	Rank	Ratio	Rank
A	4.00	IV	5.71	V	1.25	V
B	3.00	V	6.67	IV	1.50	IV
C	7.50	I	9.23	I	2.75	II
D	4.25	III	6.80	III	2.25	III
E	4.50	II	9.00	II	3.60	I