



# SEQUENCE AND SERIES-ARITHMETIC AND GEOMETRIC PROGRESSIONS



## LEARNING OBJECTIVES

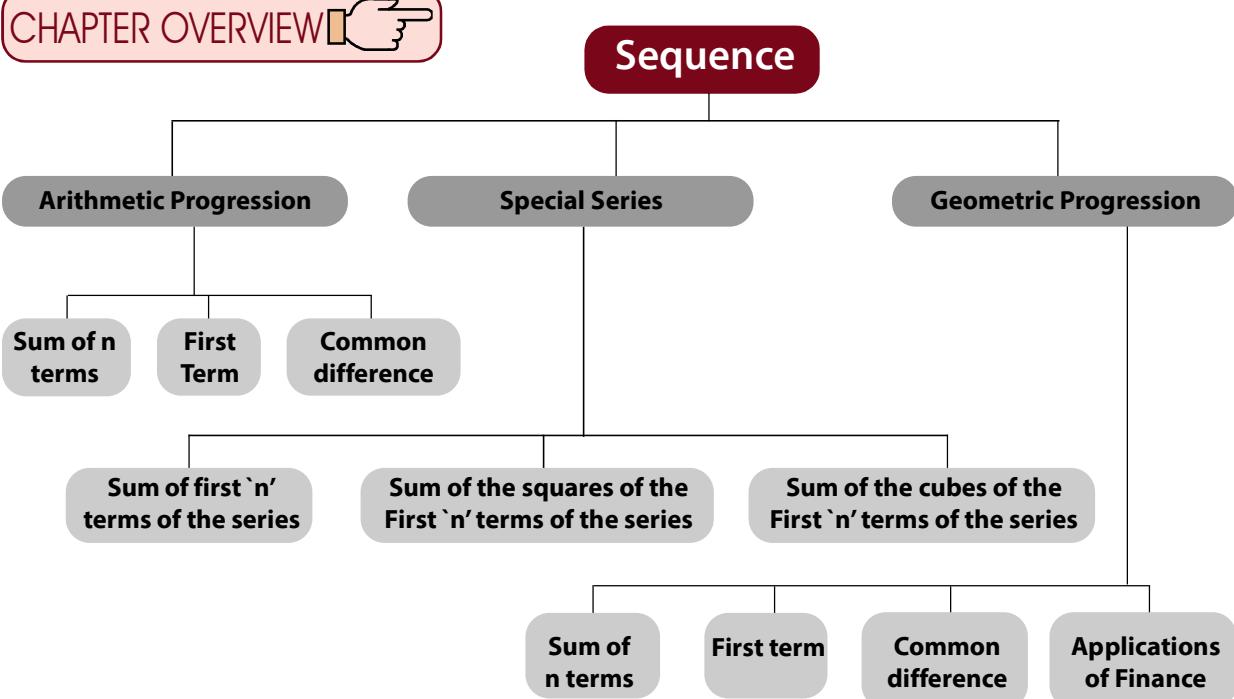
Often students will come across a sequence of numbers which are having a common difference, i.e., difference between the two consecutive pairs are the same. Also another very common sequence of numbers which are having common ratio, i.e., ratio of two consecutive pairs are the same. Could you guess what these special type of sequences are termed in mathematics?

Read this chapter to understand that these two special type of sequences are called Arithmetic Progression and Geometric Progression respectively. Further learn how to find out an element of these special sequences and how to find sum of these sequences.

These sequences will be useful for understanding various formulae of accounting and finance.

The topics of sequence, series, A.P. G.P. find useful applications in commercial problems among others; viz., to find interest earned through compound interest, depreciations after certain amount of time and total sum earned on recurring deposits, etc.

## CHAPTER OVERVIEW





## 6.1 SEQUENCE

Let us consider the following collection of numbers-

- (1) 28, 2, 25, 27, \_\_\_\_\_
- (2) 2, 7, 11, 19, 31, 51, \_\_\_\_\_
- (3) 1, 2, 3, 4, 5, 6, \_\_\_\_\_
- (4) 20, 18, 16, 14, 12, 10, \_\_\_\_\_

In (1) the nos. are not arranged in a particular order. In (2) the nos. are in ascending order but they do not obey any rule or law. It is, therefore, not possible to indicate the number next to 51.

In (3) we find that by adding 1 to any number, we get the next one. Here the number next to 6 is  $6 + 1 = 7$ .

In (4) if we subtract 2 from any number we get the nos. that follows. Here the number next to 10 is  $10 - 2 = 8$ .

Under these circumstances, we say, the numbers in the collections (1) and (2) do not form sequences whereas the numbers in the collections (3) & (4) form sequences.

Thus a sequence may be defined as follows:—

**An ordered collection of numbers  $a_1, a_2, a_3, a_4, \dots, a_n, \dots$  is a sequence if according to some definite rule or law, there is a definite value of  $a_n$ , called the term or element of the sequence, corresponding to any value of the natural number n.**

Clearly,  $a_1$  is the 1st term of the sequence,  $a_2$  is the 2nd term, ..... ,  $a_n$  is the nth term.

In the nth term  $a_n$ , by putting  $n = 1, 2, 3, \dots$  successively, we get  $a_1, a_2, a_3, a_4, \dots$

Thus it is clear that the nth term of a sequence is a function of the positive integer n. The nth term is also called the general term of the sequence. To specify a sequence, nth term must be known, otherwise it may lead to confusion. A sequence may be finite or infinite.

If the number of elements in a sequence is finite, the sequence is called *finite sequence*; while if the number of elements is unending, the sequence is *infinite*.

A finite sequence  $a_1, a_2, a_3, a_4, \dots, a_n$  is denoted by  $\{a_i\}_{i=1}^n$  and an infinite sequence  $a_1, a_2, a_3, a_4, \dots, a_n, \dots$  is denoted by  $\{a_n\}_{n=1}^\infty$  or simply by  $\{a_n\}$  where  $a_n$  is the nth element of the sequence.

**Example :**

- 1) The sequence  $\{1/n\}$  is  $1, 1/2, 1/3, 1/4, \dots$
- 2) The sequence  $\{(-1)^n n\}$  is  $-1, 2, -3, 4, -5, \dots$
- 3) The sequence  $\{n\}$  is  $1, 2, 3, \dots$
- 4) The sequence  $\{n / (n+1)\}$  is  $1/2, 2/3, 3/4, 4/5, \dots$
- 5) A sequence of even positive integers is  $2, 4, 6, \dots$
- 6) A sequence of odd positive integers is  $1, 3, 5, 7, \dots$

All the above are infinite sequences.

**Example:**

- 1) A sequence of even positive integers within 12 i.e., is 2, 4, 6, 8, 10.
- 2) A sequence of odd positive integers within 11 i.e., is 1, 3, 5, 7, 9.

All the above are finite sequences.



## 6.2 SERIES

An expression of the form  $a_1 + a_2 + a_3 + \dots + a_n + \dots$  which is the sum of the elements of the sequence  $\{a_n\}$  is called a *series*. If the series contains a finite number of elements, it is called a *finite series*, otherwise called an *infinite series*.

If  $S_n = u_1 + u_2 + u_3 + u_4 + \dots + u_n$ , then  $S_n$  is called the sum to n terms (or the sum of the first n terms) of the series and the term sum is denoted by the Greek letter  $\Sigma$ .

Thus,  $S_n = \sum_{r=1}^n u_r$  or simply by  $\Sigma u_n$ .



### ILLUSTRATIONS:

- (i)  $1 + 3 + 5 + 7 + \dots$  is a series in which 1st term = 1, 2nd term = 3, and so on.
- (ii)  $2 - 4 + 8 - 16 + \dots$  is also a series in which 1st term = 2, 2nd term = -4, and so on.



## 6.3 ARITHMETIC PROGRESSION (A.P.)

A sequence  $a_1, a_2, a_3, \dots, a_n$  is called an Arithmetic Progression (A.P.) when  $a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1}$ . That means A. P. is a sequence in which each term is obtained by adding a constant  $d$  to the preceding term. This constant 'd' is called the *common difference* of the A.P. If 3 numbers  $a, b, c$  are in A.P., we say

$b - a = c - b$  or  $a + c = 2b$ ;  $b$  is called the arithmetic mean between  $a$  and  $c$ .

**Example:** 1) 2, 5, 8, 11, 14, 17, .... is an A.P. in which  $d = 3$  is the common difference.

2) 15, 13, 11, 9, 7, 5, 3, 1, -1, is an A.P. in which -2 is the common difference.

**Solution:** In (1) 2nd term = 5, 1st term = 2, 3rd term = 8,

so 2nd term - 1st term =  $5 - 2 = 3$ , 3rd term - 2nd term =  $8 - 5 = 3$

Here the difference between a term and the preceding term is same that is always constant. This constant is called common difference.

Now in general an A.P. series can be written as

$a, a + d, a + 2d, a + 3d, a + 4d, \dots$

where 'a' is the 1<sup>st</sup> term and 'd' is the common difference.

Thus 1<sup>st</sup> term ( $t_1$ ) =  $a = a + (1 - 1)d$

2<sup>nd</sup> term ( $t_2$ ) =  $a + d = a + (2 - 1)d$

3<sup>rd</sup> term ( $t_3$ ) =  $a + 2d = a + (3 - 1)d$

$$4^{\text{th}} \text{ term } (t_4) = a + 3d = a + (4 - 1)d$$

.....

$$n^{\text{th}} \text{ term } (t_n) = a + (n - 1)d, \text{ where } n \text{ is the position number of the term.}$$

Using this formula we can get

$$50^{\text{th}} \text{ term } (= t_{50}) = a + (50 - 1)d = a + 49d$$

**Example 1:** Find the 7th term of the A.P. 8, 5, 2, -1, -4, ....

**Solution:** Here  $a = 8, d = 5 - 8 = -3$

$$\begin{aligned} \text{Now } t_7 &= 8 + (7 - 1)d \\ &= 8 + (7 - 1)(-3) \\ &= 8 + 6(-3) \\ &= 8 - 18 \\ &= -10 \end{aligned}$$

**Example 2:** Which term of the AP  $\frac{3}{\sqrt{7}}, \frac{4}{\sqrt{7}}, \frac{5}{\sqrt{7}}, \dots$  is  $\frac{17}{\sqrt{7}}$ ?

$$\text{Solution: } a = \frac{3}{\sqrt{7}}, d = \frac{4}{\sqrt{7}} - \frac{3}{\sqrt{7}} = \frac{1}{\sqrt{7}}, t_n = \frac{17}{\sqrt{7}}$$

We may write

$$\frac{17}{\sqrt{7}} = \frac{3}{\sqrt{7}} + (n - 1) \times \frac{1}{\sqrt{7}}$$

$$\text{or, } 17 = 3 + (n - 1)$$

$$\text{or, } n = 17 - 2 = 15$$

Hence, 15<sup>th</sup> term of the A.P. is  $\frac{17}{\sqrt{7}}$ .

**Example 3:** If 5<sup>th</sup> and 12<sup>th</sup> terms of an A.P. are 14 and 35 respectively, find the A.P.

**Solution:** Let  $a$  be the first term &  $d$  be the common difference of A.P.

$$t_5 = a + 4d = 14$$

$$t_{12} = a + 11d = 35$$

On solving the above two equations,

$$7d = 21 \Rightarrow d = 3$$

$$\text{and } a = 14 - (4 \times 3) = 14 - 12 = 2$$

Hence, the required A.P. is 2, 5, 8, 11, 14,.....

**Example 4:** Divide 69 into three parts which are in A.P. and are such that the product of the first two parts is 483.

**Solution:** Given that the three parts are in A.P., let the three parts which are in A.P. be  $a - d$ ,  $a$ ,  $a + d$ .

$$\text{Thus } a - d + a + a + d = 69$$

$$\text{or } 3a = 69$$

$$\text{or } a = 23$$

So the three parts are  $23 - d$ ,  $23$ ,  $23 + d$

Since the product of first two parts is 483, therefore, we have

$$23(23 - d) = 483$$

$$\text{or } 23 - d = 483 / 23 = 21$$

$$\text{or } d = 23 - 21 = 2$$

Hence, the three parts which are in A.P. are

$$23 - 2 = 21, 23, 23 + 2 = 25$$

Hence the three parts are 21, 23, 25.

**Example 5:** Find the arithmetic mean between 4 and 10.

**Solution:** We know that the A.M. of  $a$  &  $b$  is  $= (a + b) / 2$

Hence, The A. M between 4 & 10  $= (4 + 10) / 2 = 7$

**Example 6:** Insert 4 arithmetic means between 4 and 324.

$$4, -, -, -, 324$$

**Solution:** Here  $a = 4$ ,  $d = ?$ ,  $n = 2 + 4 = 6$ ,  $t_n = 324$

$$\text{Now } t_n = a + (n - 1)d$$

$$\text{or } 324 = 4 + (6 - 1)d$$

$$\text{or } 320 = 5d \text{ i.e., } d = 320 / 5 = 64$$

$$\text{So the } 1^{\text{st}} \text{ AM} = 4 + 64 = 68$$

$$2^{\text{nd}} \text{ AM} = 68 + 64 = 132$$

$$3^{\text{rd}} \text{ AM} = 132 + 64 = 196$$

$$4^{\text{th}} \text{ AM} = 196 + 64 = 260$$

### Sum of the first $n$ terms

Let  $S$  be the Sum,  $a$  be the  $1^{\text{st}}$  term and  $\ell$  the last term of an A.P. If the number of term is  $n$ , then  $t_n = \ell$ . Let  $d$  be the common difference of the A.P.

$$\text{Now } S = a + (a + d) + (a + 2d) + \dots + (\ell - 2d) + (\ell - d) + \ell$$

$$\text{Again } S = \ell + (\ell - d) + (\ell - 2d) + \dots + (a + 2d) + (a + d) + a$$

On adding the above, we have

$$\begin{aligned} 2S &= (a + \ell) + (a + \ell) + (a + \ell) + \dots + (a + \ell) \\ &= n(a + \ell) \end{aligned}$$

or  $S = n(a + \ell) / 2$

**Note:** The above formula may be used to determine the sum of n terms of an A.P. when the first term a and the last term is given.

Now  $\ell = t_n = a + (n - 1)d$

$$\therefore S = \frac{n\{a + a + (n - 1)d\}}{2}$$

or  $S = \frac{n}{2}\{2a + (n - 1)d\}$

**Note:** The above formula may be used when the first term a, common difference d and the number of terms of an A.P. are given.

### Sum of 1<sup>st</sup> n natural or counting numbers

$$S = 1 + 2 + 3 + \dots + (n - 2) + (n - 1) + n$$

Again  $S = n + (n - 1) + (n - 2) + \dots + 3 + 2 + 1$

On adding the above, we get

$$2S = (n + 1) + (n + 1) + \dots \text{ to } n \text{ terms}$$

or  $2S = n(n + 1)$

$$S = n(n + 1)/2$$

Then Sum of first n natural number is  $n(n + 1) / 2$

$$\text{i.e. } 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}.$$

### Sum of 1<sup>st</sup> n odd number

$$S = 1 + 3 + 5 + \dots + (2n - 1)$$

Sum of first n odd number

$$S = 1 + 3 + 5 + \dots + (2n - 1)$$

Since  $S = n\{2a + (n - 1)d\} / 2$ , we find

$$S = \frac{n}{2}\{2.1 + (n - 1)2\} = \frac{n}{2}(2n) = n^2$$

or  $S = n^2$

Then sum of first, n odd numbers is  $n^2$ , i.e.  $1 + 3 + 5 + \dots + (2n - 1) = n^2$

### Sum of the Squares of the first n natural nos.

Let  $S = 1^2 + 2^2 + 3^2 + \dots + n^2$

$$\text{We know } m^3 - (m - 1)^3 = 3m^2 - 3m + 1$$

We put  $m = 1, 2, 3, \dots, n$

$$\begin{aligned} 1^3 - 0 &= 3.1^2 - 3.1 + 1 \\ 2^3 - 1^3 &= 3.2^2 - 3.2 + 1 \\ 3^3 - 2^3 &= 3.3^2 - 3.3 + 1 \\ \dots & \\ + n^3 - (n-1)^3 &= 3n^2 - 3.n + 1 \end{aligned}$$

Adding both sides term by term,

$$\begin{aligned} n^3 &= 3S - 3n(n+1)/2 + n \\ \text{or } 2n^3 &= 6S - 3n^2 - 3n + 2n \\ \text{or } 6S &= 2n^3 + 3n^2 + n \\ \text{or } 6S &= n(2n^2 + 3n + 1) \\ \text{or } 6S &= n(n+1)(2n+1) \end{aligned}$$

Thus sum of the squares of the first n natural numbers is  $\frac{n(n+1)(2n+1)}{6}$

$$\text{i.e. } 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Similarly, sum of the cubes of first  $n$  natural numbers can be found out as  $\left\{ \frac{n(n+1)}{2} \right\}^2$  by taking the identity

$$m^4 - (m-1)^4 = 4m^3 - 6m^2 + 4m - 1 \text{ and putting } m = 1, 2, 3, \dots, n.$$

Thus

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left\{ \frac{n(n+1)}{2} \right\}^2$$

 ... EXERCISE 6 (A)

Choose the most appropriate option ( a ), ( b ), ( c ) or ( d ).

4. The sum to  $\infty$  of the series  $-5, 25, -125, 625, \dots$  can be written as
- (a)  $\sum_{k=1}^{\infty} (-5)^k$       (b)  $\sum_{k=1}^{\infty} 5^k$       (c)  $\sum_{k=1}^{\infty} -5^k$       (d) none of these
5. The first three terms of sequence when nth term  $t_n$  is  $n^2 - 2n$  are
- (a)  $-1, 0, 3$       (b)  $1, 0, 2$       (c)  $-1, 0, -3$       (d) none of these
6. Which term of the progression  $-1, -3, -5, \dots$  is  $-39$
- (a) 21<sup>st</sup>      (b) 20<sup>th</sup>      (c) 19<sup>th</sup>      (d) none of these
7. The value of  $x$  such that  $8x + 4, 6x - 2, 2x + 7$  will form an AP is
- (a) 15      (b) 2      (c)  $15/2$       (d) none of the these
8. The  $m^{\text{th}}$  term of an A. P. is  $n$  and  $n^{\text{th}}$  term is  $m$ . The  $r^{\text{th}}$  term of it is
- (a)  $m + n + r$       (b)  $n + m - 2r$       (c)  $m + n + r/2$       (d)  $m + n - r$
9. The number of the terms of the series  $10 + 9\frac{2}{3} + 9\frac{1}{3} + 9 + \dots$  will amount to 155 is
- (a) 30      (b) 31      (c) 32      (d) none of these
10. The  $n^{\text{th}}$  term of the series whose sum to  $n$  terms is  $5n^2 + 2n$  is
- (a)  $3n - 10$       (b)  $10n - 2$       (c)  $10n - 3$       (d) none of these
11. The 20<sup>th</sup> term of the progression  $1, 4, 7, 10, \dots$  is
- (a) 58      (b) 52      (c) 50      (d) none of these
12. The last term of the series  $5, 7, 9, \dots$  to 21 terms is
- (a) 44      (b) 43      (c) 45      (d) none of these
13. The last term of the A.P.  $0.6, 1.2, 1.8, \dots$  to 13 terms is
- (a) 8.7      (b) 7.8      (c) 7.7      (d) none of these
14. The sum of the series  $9, 5, 1, \dots$  to 100 terms is
- (a)  $-18,900$       (b)  $18,900$       (c)  $19,900$       (d) none of these
15. The two arithmetic means between  $-6$  and  $14$  is
- (a)  $2/3, 1/3$       (b)  $2/3, 7\frac{1}{3}$       (c)  $-2/3, -7\frac{1}{3}$       (d) none of these
16. The sum of three integers in AP is  $15$  and their product is  $80$ . The integers are
- (a)  $2, 8, 5$       (b)  $8, 2, 5$       (c)  $2, 5, 8$       (d)  $8, 5, 2$
17. The sum of  $n$  terms of an AP is  $3n^2 + 5n$ . The series is
- (a)  $8, 14, 20, 26$       (b)  $8, 22, 42, 68$       (c)  $22, 68, 114, \dots$       (d) none of these
18. The number of numbers between  $74$  and  $25,556$  divisible by  $5$  is
- (a) 5,090      (b) 5,097      (c) 5,095      (d) none of these
19. The  $p^{\text{th}}$  term of an AP is  $(3p - 1)/6$ . The sum of the first  $n$  terms of the AP is
- (a)  $n(3n + 1)$       (b)  $n(3n + 1)/12$       (c)  $n/12(3n - 1)$       (d) none of these
20. The arithmetic mean between  $33$  and  $77$  is
- (a) 50      (b) 45      (c) 55      (d) none of these

21. The 4 arithmetic means between -2 and 23 are  
 (a) 3, 13, 8, 18      (b) 18, 3, 8, 13      (c) 3, 8, 13, 18      (d) none of these
22. The first term of an A.P is 14 and the sums of the first five terms and the first ten terms are equal in magnitude but opposite in sign. The 3<sup>rd</sup> term of the AP is  
 (a)  $6\frac{4}{11}$       (b) 6      (c) 4/11      (d) none of these
23. The sum of a certain number of terms of an AP series -8, -6, -4, ..... is 52. The number of terms is  
 (a) 12      (b) 13      (c) 11      (d) none of these
24. The first and the last term of an AP are -4 and 146. The sum of the terms is 7171. The number of terms is  
 (a) 101      (b) 100      (c) 99      (d) none of these
25. The sum of the series  $3\frac{1}{2} + 7 + 10\frac{1}{2} + 14 + \dots$  to 17 terms is  
 (a) 530      (b) 535      (c) 535  $\frac{1}{2}$       (d) none of these



## 6.4 GEOMETRIC PROGRESSION (G.P.)

If in a sequence of terms each term is constant multiple of the proceeding term, then the sequence is called a Geometric Progression (G.P.). The constant multiplier is called the *common ratio*.

- Examples:**
- 1) In 5, 15, 45, 135,.... common ratio is  $15/5 = 3$
  - 2) In  $1, 1/2, 1/4, 1/9 \dots$  common ratio is  $(1/2)/1 = 1/2$
  - 3) In  $2, -6, 18, -54, \dots$  common ratio is  $(-6)/2 = -3$

**Illustrations:** Consider the following series :-

(i)  $1 + 4 + 16 + 64 + \dots$

Here second term / first term =  $4/1 = 4$ ; third term / second term =  $16/4 = 4$

fourth term/third term =  $64/16 = 4$  and so on.

Thus, we find that, in the entire series, the ratio of any term and the term preceding it, is a constant.

(ii)  $1/3 - 1/9 + 1/27 - 1/81 + \dots$

Here second term / 1<sup>st</sup> term =  $(-1/9)/(1/3) = -1/3$

third term / second term =  $(1/27)/(-1/9) = -1/3$

fourth term / third term =  $(-1/81)/(1/27) = -1/3$  and so on.

Here also, in the entire series, the ratio of any term and the term preceding one is constant.

The above mentioned series are known as **Geometric Series**.

Let us consider the sequence  $a, ar, ar^2, ar^3, \dots$

1<sup>st</sup> term =  $a$ , 2<sup>nd</sup> term =  $ar = ar^{2-1}$ , 3<sup>rd</sup> term =  $ar^2 = ar^{3-1}$ , 4<sup>th</sup> term =  $ar^3 = ar^{4-1}, \dots$

Similarly      nth term of GP  $t_n = ar^{n-1}$

$$\text{Thus, common ratio} = \frac{\text{Any term}}{\text{Preceding term}} = \frac{t_n}{t_{n-1}}$$

$$= ar^{n-1}/ar^{n-2} = r$$

Thus, general term of a G.P is given by  $ar^{n-1}$  and the general form of G.P. is  
 $a + ar + ar^2 + ar^3 + \dots \dots \dots$

$$\text{For example, } r = \frac{t_2}{t_1} = \frac{ar}{a} = r$$

$$\text{So } r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \dots$$

**Example 1:** If  $a, ar, ar^2, ar^3, \dots$  be in G.P. Find the common ratio.

**Solution:** 1<sup>st</sup> term =  $a$ , 2<sup>nd</sup> term =  $ar$

Ratio of any term to its preceding term =  $ar/a = r$  = common ratio.

**Example 2:** Which term of the progression  $1, 2, 4, 8, \dots$  is 256?

**Solution:**       $a = 1, r = 2/1 = 2, n = ? t_n = 256$

$$t_n = ar^{n-1}$$

or       $256 = 1 \times 2^{n-1}$  i.e.,  $2^8 = 2^{n-1}$  or,  $n - 1 = 8$  i.e.,  $n = 9$

Thus 9<sup>th</sup> term of the G. P. is 256



## 6.5 GEOMETRIC MEAN

If  $a, b, c$  are in G.P we get  $b/a = c/b \Rightarrow b^2 = ac$ ,  $b$  is called the geometric mean between  $a$  and  $c$

**Example 1:** Insert 3 geometric means between  $1/9$  and  $9$ .

**Solution:**       $1/9, -, -, -, 9$

$$a = 1/9, r = ?, n = 2 + 3 = 5, t_n = 9$$

we know       $t_n = ar^{n-1}$

$$\text{or } 1/9 \times r^{5-1} = 9$$

$$\text{or } r^4 = 81 = 3^4 \Rightarrow r = 3$$

$$\text{Thus } 1^{\text{st}} \text{ G. M} = 1/9 \times 3 = 1/3$$

$$2^{\text{nd}} \text{ G. M} = 1/3 \times 3 = 1$$

$$3^{\text{rd}} \text{ G. M} = 1 \times 3 = 3$$

**Example 2:** Find the G.P where 4<sup>th</sup> term is 8 and 8<sup>th</sup> term is 128/625

**Solution:** Let  $a$  be the 1<sup>st</sup> term and  $r$  be the common ratio.

By the question  $t_4 = 8$  and  $t_8 = 128/625$

So  $ar^3 = 8$  and  $ar^7 = 128/625$

Therefore  $ar^7 / ar^3 = \frac{128}{625 \cdot 8} \Rightarrow r^4 = 16 / 625 = (\pm 2/5)^4 \Rightarrow r = 2/5 \text{ and } -2/5$

Now  $ar^3 = 8 \Rightarrow a \times (2/5)^3 = 8 \Rightarrow a = 125$

Thus the G. P is

125, 50, 20, 8, 16/5, .....

When  $r = -2/5$ ,  $a = -125$  and the G.P is -125, 50, -20, 8, -16/5, .....

Finally, the G.P. is 125, 50, 20, 8, 16/5, .....

or, -125, 50, -20, 8, -16/5, .....

### Sum of first n terms of a G P

Let  $a$  be the first term and  $r$  be the common ratio. So the first  $n$  terms are  $a, ar, ar^2, \dots, ar^{n-1}$ .

If  $S$  be the sum of  $n$  terms,

$$S_n = a + ar + ar^2 + \dots + ar^{n-1} \quad (i)$$

$$\text{Now } rS_n = ar + ar^2 + \dots + ar^{n-1} + ar^n \quad (ii)$$

Subtracting (i) from (ii)

$$S_n - rS_n = a - ar^n$$

$$\text{or } S_n(1 - r) = a(1 - r^n)$$

$$\text{or } S_n = a(1 - r^n) / (1 - r) \text{ when } r < 1$$

$$S_n = a(r^n - 1) / (r - 1) \text{ when } r > 1$$

If  $r = 1$ , then  $S_n = a + a + a + \dots$  to  $n$  terms

$$= na$$

If the  $n$ th term of the G. P be 1 then  $\ell = ar^{n-1}$

$$\text{Therefore, } S_n = (ar^n - a) / (r - 1) = (a r^{n-1} r - a) / (r - 1) = \frac{\ell r - a}{r - 1}$$

So, when the last term of the G. P is known, we use this formula.

### Sum of infinite geometric series

$$S = a(1 - r^n) / (1 - r) \text{ when } r < 1$$

$$= a(1 - 1/R^n) / (1 - 1/R) \text{ (since } r < 1, \text{ we take } r = 1/R).$$

If  $n \rightarrow \infty$ ,  $1/R^n \rightarrow 0$

$$\text{Thus } S_{\infty} = \frac{a}{1-r}, \quad r < 1$$

i.e. Sum of G.P. upto infinity is  $\frac{a}{1-r}$ , where  $r < 1$

$$\text{Also, } S_{\infty} = \frac{a}{1-r}, \quad \text{if } -1 < r < 1.$$

**Example 1:** Find the sum of  $1 + 2 + 4 + 8 + \dots$  to 8 terms.,

**Solution:** Here  $a = 1, r = 2/1 = 2, n = 8$

Let  $S = 1 + 2 + 4 + 8 + \dots$  to 8 terms

$$= 1 (2^8 - 1) / (2 - 1) = 2^8 - 1 = 255$$

**Example 2:** Find the sum to  $n$  terms of  $6 + 27 + 128 + 629 + \dots$

**Solution:** Required Sum =  $(5 + 1) + (5^2 + 2) + (5^3 + 3) + (5^4 + 4) + \dots$  to  $n$  terms

$$= (5 + 5^2 + 5^3 + \dots + 5^n) + (1 + 2 + 3 + \dots + n \text{ terms})$$

$$= \{5(5^n - 1) / (5 - 1)\} + \{n(n + 1) / 2\}$$

$$= \{5(5^n - 1) / 4\} + \{n(n + 1) / 2\}$$

**Example 3:** Find the sum to  $n$  terms of the series

$$3 + 33 + 333 + \dots$$

**Solution:** Let  $S$  denote the required sum.

$$\begin{aligned} \text{i.e. } S &= 3 + 33 + 333 + \dots \text{ to } n \text{ terms} \\ &= 3(1 + 11 + 111 + \dots \text{ to } n \text{ terms}) \end{aligned}$$

$$= \frac{3}{9} (9 + 99 + 999 + \dots \text{ to } n \text{ terms})$$

$$= \frac{3}{9} \{(10 - 1) + (10^2 - 1) + (10^3 - 1) + \dots + (10^n - 1)\}$$

$$= \frac{3}{9} \{(10 + 10^2 + 10^3 + \dots + 10^n) - n\}$$

$$= \frac{3}{9} \{10(1 + 10 + 10^2 + \dots + 10^{n-1}) - n\}$$

$$= \frac{3}{9} [\{10(10^n - 1) / (10 - 1)\} - n]$$

$$= \frac{3}{81} (10^{n+1} - 10 - 9n)$$

$$= \frac{1}{27} (10^{n+1} - 9n - 10)$$

**Example 4:** Find the sum of n terms of the series  $0.7 + 0.77 + 0.777 + \dots$  to n terms

**Solution:** Let S denote the required sum.

$$\begin{aligned} \text{i.e. } S &= 0.7 + 0.77 + 0.777 + \dots \text{ to } n \text{ terms} \\ &= 7(0.1 + 0.11 + 0.111 + \dots \text{ to } n \text{ terms}) \end{aligned}$$

$$= \frac{7}{9} (0.9 + 0.99 + 0.999 + \dots \text{ to } n \text{ terms})$$

$$= \frac{7}{9} \{(1 - 1/10) + (1 - 1/10^2) + (1 - 1/10^3) + \dots + (1 - 1/10^n)\}$$

$$= \frac{7}{9} \left\{ n - \frac{1}{10} (1 + 1/10 + 1/10^2 + \dots + 1/10^{n-1}) \right\}$$

$$\text{So } S = \frac{7}{9} \left\{ n - \frac{1}{10} (1 - 1/10^n) / (1 - 1/10) \right\}$$

$$= \frac{7}{9} \left\{ n - (1 - 10^{-n}) / 9 \right\}$$

$$= \frac{7}{81} \{9n - 1 + 10^{-n}\}$$

**Example 5:** Evaluate  $0.\dot{2}1\dot{7}\dot{5}$  using the sum of an infinite geometric series.

**Solution:**  $0.\dot{2}1\dot{7}\dot{5} = 0.2175757575 \dots$

$$\begin{aligned} 0.\dot{2}1\dot{7}\dot{5} &= 0.21 + 0.0075 + 0.000075 + \dots \\ &= 0.21 + 75 (1 + 1/10^2 + 1/10^4 + \dots) / 10^4 \\ &= 0.21 + 75 \{1 / (1 - 1/10^2) / 10^4\} \\ &= 0.21 + (75/10^4) \times 10^2 / 99 \\ &= 21/100 + (3/4) \times (1/99) \\ &= 21/100 + 1/132 \\ &= (693 + 25) / 3300 = 718 / 3300 = 359 / 1650 \end{aligned}$$

**Example 6:** Find three numbers in G. P whose sum is 19 and product is 216.

**Solution:** Let the 3 numbers be  $a/r, a, ar$ .

According to the question  $a/r \times a \times ar = 216$

$$\text{or } a^3 = 6^3 \Rightarrow a = 6$$

So the numbers are  $6/r, 6, 6r$

Again  $6/r + 6 + 6r = 19$

or  $6/r + 6r = 13$

or  $6 + 6r^2 = 13r$

or  $6r^2 - 13r + 6 = 0$

or  $6r^2 - 4r - 9r + 6 = 0$

or  $2r(3r - 2) - 3(3r - 2) = 2$

or  $(3r - 2)(2r - 3) = 0 \quad \text{or, } r = 2/3, 3/2$

So the numbers are

$$6/(2/3), 6, 6 \times (2/3) = 9, 6, 4$$

$$\text{or } 6/(3/2), 6, 6 \times (3/2) = 4, 6, 9$$

 EXERCISE 6 (B)

**Choose the most appropriate option (a), (b), (c) or (d)**

11. The second term of a G P is 24 and the fifth term is 81. The series is  
 (a) 16, 36, 24, 54,..      (b) 24, 36, 53,...      (c) 16, 24, 36, 54,..      (d) none of these
12. The sum of 3 numbers of a G P is 39 and their product is 729. The numbers are  
 (a) 3, 27, 9      (b) 9, 3, 27      (c) 3, 9, 27      (d) none of these
13. In a G. P, the product of the first three terms is  $27/8$ . The middle term is  
 (a)  $3/2$       (b)  $2/3$       (c)  $2/5$       (d) none of these
14. If you save 1 paise today, 2 paise the next day 4 paise the succeeding day and so on, then your total savings in two weeks will be  
 (a) ₹ 163      (b) ₹ 183      (c) ₹ 163.83      (d) none of these
15. Sum of n terms of the series  $4 + 44 + 444 + \dots$  is  
 (a)  $4/9 \{ 10/9 ( 10^n - 1 ) - n \}$       (b)  $10/9 ( 10^n - 1 ) - n$   
 (c)  $4/9 ( 10^n - 1 ) - n$       (d) none of these
16. Sum of n terms of the series  $0.1 + 0.11 + 0.111 + \dots$  is  
 (a)  $1/9 \{ n - ( 1 - ( 0.1 )^n ) \}$       (b)  $1/9 \{ n - ( 1 - ( 0.1 )^n ) / 9 \}$   
 (c)  $n - 1 - ( 0.1 )^n / 9$       (d) none of these
17. The sum of the first 20 terms of a G. P is 244 times the sum of its first 10 terms. The common ratio is  
 (a)  $\pm\sqrt{3}$       (b)  $\pm 3$       (c)  $\sqrt{3}$       (d) none of these
18. Sum of the series  $1 + 3 + 9 + 27 + \dots$  is 364. The number of terms is  
 (a) 5      (b) 6      (c) 11      (d) none of these
19. The product of 3 numbers in G P is 729 and the sum of squares is 819. The numbers are  
 (a) 9, 3, 27      (b) 27, 3, 9      (c) 3, 9, 27      (d) none of these
20. The sum of the series  $1 + 2 + 4 + 8 + \dots$  to n term  
 (a)  $2^n - 1$       (b)  $2n - 1$       (c)  $1/2^n - 1$       (d) none of these
21. The sum of the infinite GP  $14, -2, +2/7, -2/49, \dots$  is  
 (a)  $4\frac{1}{12}$       (b)  $12\frac{1}{4}$       (c) 12      (d) none of these
22. The sum of the infinite G. P.  $1 - 1/3 + 1/9 - 1/27 + \dots$  is  
 (a) 0.33      (b) 0.57      (c) 0.75      (d) none of these
23. The number of terms to be taken so that  $1 + 2 + 4 + 8 + \dots$  will be 8191 is  
 (a) 10      (b) 13      (c) 12      (d) none of these
24. Four geometric means between 4 and 972 are  
 (a) 12, 36, 108, 324      (b) 12, 24, 108, 320      (c) 10, 36, 108, 320      (d) none of these

### ILLUSTRATIONS:

- (I) A person is employed in a company at ₹ 3000 per month and he would get an increase of ₹ 100 per year. Find the total amount which he receives in 25 years and the monthly salary in the last year.



### SOLUTION:

He gets in the 1<sup>st</sup> year at the Rate of 3000 per month;  
 In the 2<sup>nd</sup> year he gets at the rate of ₹ 3100 per month;  
 In the 3<sup>rd</sup> year at the rate of ₹ 3200 per month so on.  
 In the last year the monthly salary will be  
 $\text{₹} \{3000 + (25 - 1) \times 100\} = \text{₹} 5400$

$$\begin{aligned}\text{Total amount} &= \text{₹} 12 (3000 + 3100 + 3200 + \dots + 5400) \left[ \text{Use } S_n = \frac{n}{2}(a+l) \right] \\ &= \text{₹} 12 \times 25/2 (3000 + 5400) \\ &= \text{₹} 150 \times 8400 \\ &= \text{₹} 12,60,000\end{aligned}$$

- (II) A person borrows ₹ 8,000 at 2.76% Simple Interest per annum. The principal and the interest are to be paid in the 10 monthly instalments. If each instalment is double the preceding one, find the value of the first and the last instalment.



### SOLUTION:

Interest to be paid =  $2.76 \times 10 \times 8000 / 100 \times 12 = \text{₹} 184$   
 Total amount to be paid in 10 monthly instalment is ₹ (8000 + 184) = ₹ 8184  
 The instalments form a G P with common ratio 2 and so ₹ 8184 =  $a (2^{10} - 1) / (2 - 1)$ ,  
 $a = 1^{\text{st}}$  instalment  
 Here  $a = \text{₹} 8184 / 1023 = \text{₹} 8$   
 The last instalment =  $ar^{10-1} = 8 \times 2^9 = 8 \times 512 = \text{₹} 4096$



### SUMMARY

- ◆ **Sequence:** An ordered collection of numbers  $a_1, a_2, a_3, a_4, \dots, a_n, \dots$  is a sequence if according to some definite rule or law, there is a definite value of  $a_n$ , called the term or element of the sequence, corresponding to any value of the natural number n.
- ◆ An expression of the form  $a_1 + a_2 + a_3 + \dots + a_n + \dots$  which is the sum of the elements of the sequence  $\{a_n\}$  is called a *series*. If the series contains a finite number of elements, it is called a *finite series*, otherwise called an *infinite series*.
- ◆ **Arithmetic Progression:** A sequence  $a_1, a_2, a_3, \dots, a_n$  is called an Arithmetic Progression (A.P.) when  $a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1}$ . That means A. P. is a sequence in which each term is obtained by adding a constant d to the preceding term. This constant 'd' is called the *common difference* of the A.P. If 3 numbers a, b, c are in A.P., we say  
 $b - a = c - b$  or  $a + c = 2b$ ; b is called the arithmetic mean between a and c.  
 $n^{\text{th}}$  term ( $t_n$ ) =  $a + (n - 1)d$ ,

Where  $a$  = First Term

$d$  = Common difference =  $t_n - t_{n-1}$

Sum of  $n$  terms of AP=

$$S = \frac{n}{2} [2a + (n - 1)d]$$

- ◆ Sum of the first  $n$  terms : Sum of 1st  $n$  natural or counting numbers

$$S = n(n + 1)/2$$

**Sum of 1st  $n$  odd numbers :**  $S = n^2$

Sum of the Squares of the first,  $n$  natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

**sum of the cubes of the first  $n$  natural numbers is**

$$\left\{ \frac{n(n+1)}{2} \right\}^2$$

- ◆ **Geometric Progression (G.P).** If in a sequence of terms each term is constant multiple of the proceeding term, then the sequence is called a Geometric Progression (G.P). The constant multiplier is called the *common ratio*

$$= \frac{\text{Any term}}{\text{Preceding term}} = \frac{t_n}{t_{n-1}}$$

$$= ar^{n-1}/ar^{n-2} = r$$

- ◆ Sum of first  $n$  terms of a G P:

$$S_n = a(1 - r^n) / (1 - r) \text{ when } r < 1$$

$$S_n = a(r^n - 1) / (r - 1) \text{ when } r > 1$$

Sum of infinite geometric series

$$S_\infty = \frac{a}{1 - r}, \quad r < 1$$

- ◆ A.M. of  $a$  &  $b$  is  $= (a + b) / 2$
- ◆ If  $a, b, c$  are in G.P we get  $b/a = c/b \Rightarrow b^2 = ac$ ,  $b$  is called the geometric mean between  $a$  and  $c$



## EXERCISE 6 (C)

Choose the most appropriate option (a), (b), (c) or (d).

1. Three numbers are in AP and their sum is 21. If 1, 5, 15 are added to them respectively, they form a G. P. The numbers are  
 (a) 5, 7, 9      (b) 9, 5, 7      (c) 7, 5, 9      (d) none of these
2. The sum of  $1 + 1/3 + 1/3^2 + 1/3^3 + \dots + 1/3^{n-1}$  is  
 (a)  $2/3$       (b)  $3/2$       (c)  $4/5$       (d) none of these
3. The sum of the infinite series  $1 + 2/3 + 4/9 + \dots$  is  
 (a)  $1/3$       (b) 3      (c)  $2/3$       (d) none of these
4. The sum of the first two terms of a G.P. is  $5/3$  and the sum to infinity of the series is 3. The common ratio is  
 (a)  $1/3$       (b)  $2/3$       (c)  $-2/3$       (d) none of these
5. If p, q and r are in A.P. and x, y, z are in G.P. then  $x^{q-r} \cdot y^{r-p} \cdot z^{p-q}$  is equal to  
 (a) 0      (b) -1      (c) 1      (d) none of these
6. The sum of three numbers in G.P. is 70. If the two extremes by multiplied each by 4 and the mean by 5, the products are in AP. The numbers are  
 (a) 12, 18, 40      (b) 10, 20, 40      (c) 40, 20, 10      (d) none of these
7. The sum of 3 numbers in A.P. is 15. If 1, 4 and 19 be added to them respectively, the results are in G. P. The numbers are  
 (a) 26, 5, -16      (b) 2, 5, 8      (c) 5, 8, 2      (d) none of these
8. Given x, y, z are in G.P. and  $x^p = y^q = z^r$ , then  $1/p, 1/q, 1/r$  are in  
 (a) A.P.      (b) G.P.      (c) Both A.P. and G.P. (d) none of these
9. If the terms  $2x, (x+10)$  and  $(3x+2)$  be in A.P., the value of x is  
 (a) 7      (b) 10      (c) 6      (d) none of these
10. If A be the A.M. of two positive unequal quantities x and y and G be their G. M, then  
 (a)  $A < G$       (b)  $A > G$       (c)  $A \geq G$       (d)  $A \leq G$
11. The A.M. of two positive numbers is 40 and their G. M. is 24. The numbers are  
 (a) (72, 8)      (b) (70, 10)      (c) (60, 20)      (d) none of these
12. Three numbers are in A.P. and their sum is 15. If 8, 6, 4 be added to them respectively, the numbers are in G.P. The numbers are  
 (a) 2, 6, 7      (b) 4, 6, 5      (c) 3, 5, 7      (d) none of these
13. The sum of four numbers in G. P. is 60 and the A.M. of the first and the last is 18. The numbers are  
 (a) 4, 8, 16, 32      (b) 4, 16, 8, 32      (c) 16, 8, 4, 20      (d) none of these
14. A sum of ₹ 6240 is paid off in 30 instalments such that each instalment is ₹ 10 more than the preceding installment. The value of the 1<sup>st</sup> instalment is  
 (a) ₹ 36      (b) ₹ 30      (c) ₹ 60      (d) none of these
15. The sum of  $1.03 + (1.03)^2 + (1.03)^3 + \dots$  to n terms is  
 (a)  $103 \{(1.03)^n - 1\}$       (b)  $103/3 \{(1.03)^n - 1\}$       (c)  $(1.03)^n - 1$       (d) none of these

16. If  $x, y, z$  are in A.P. and  $x, y, (z + 1)$  are in G.P. then  
 (a)  $(x - z)^2 = 4x$       (b)  $z^2 = (x - y)$       (c)  $z = x - y$       (d) none of these
17. The numbers  $x, 8, y$  are in G.P. and the numbers  $x, y, -8$  are in A.P. The value of  $x$  and  $y$  are  
 (a)  $(-8, -8)$       (b)  $(16, 4)$       (c)  $(8, 8)$       (d) none of these
18. The  $n$ th term of the series  $16, 8, 4, \dots$  in  $1/2^{17}$ . The value of  $n$  is  
 (a) 20      (b) 21      (c) 22      (d) none of these
19. The sum of  $n$  terms of a G.P. whose first term is 1 and the common ratio is  $1/2$ , is equal to  
 $1 - \frac{1}{2^n}$ . The value of  $n$  is  
 (a) 7      (b) 8      (c) 6      (d) none of these
20. If  $t_4$  of a G.P. is  $x$ ,  $t_{10} = y$  and  $t_{16} = z$ . Then  
 (a)  $x^2 = yz$       (b)  $z^2 = xy$       (c)  $y^2 = zx$       (d) none of these
21. If  $x, y, z$  are in G.P., then  
 (a)  $y^2 = xz$       (b)  $y(z^2 + x^2) = x(z^2 + y^2)$       (c)  $2y = x+z$       (d) none of these
22. The sum of all odd numbers between 200 and 300 is  
 (a) 11,600      (b) 12,490      (c) 12,500      (d) 24,750
23. The sum of all natural numbers between 500 and 1000 which are divisible by 13, is  
 (a) 28,405      (b) 24,805      (c) 28,540      (d) none of these
24. If unity is added to the sum of any number of terms of the A.P. 3, 5, 7, 9, ..., the resulting sum is  
 (a) 'a' perfect cube      (b) 'a' perfect square      (c) 'a' number      (d) none of these
25. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 or 5 is  
 (a) 10,200      (b) 15,200      (c) 16,200      (d) none of these
26. The sum of all natural numbers from 100 to 300 which are exactly divisible by 4 and 5 is  
 (a) 2,200      (b) 2,000      (c) 2,220      (d) none of these
27. A person pays ₹ 975 by monthly instalment each less than the former by ₹ 5. The first instalment is ₹ 100. The time by which the entire amount will be paid is  
 (a) 10 months      (b) 15 months      (c) 14 months      (d) none of these
28. A person saved ₹ 16,500 in ten years. In each year after the first year he saved ₹ 100 more than he did in the preceding year. The amount of money he saved in the 1<sup>st</sup> year was  
 (a) ₹ 1000      (b) ₹ 1500      (c) ₹ 1200      (d) none of these
29. At 10% C.I. p.a., a sum of money accumulate to ₹ 9625 in 5 years. The sum invested initially is  
 (a) ₹ 5976.37      (b) ₹ 5970      (c) ₹ 5975      (d) ₹ 5370.96
30. The population of a country was 55 crores in 2005 and is growing at 2% p.a C.I. the population in the year 2015 is estimated as  
 (a) 5705      (b) 6005      (c) 6700      (d) none of these

## ANSWERS

### Exercise 6 (A)

1. (b) 2. (a) 3. (a) 4. (a) 5. (a) 6. (b) 7. (c) 8. (d)  
 9. (a), (b) 10. (c) 11. (a) 12. (c) 13. (b) 14. (a) 15. (b) 16. (c), (d)  
 17. (a) 18. (b) 19. (b) 20. (c) 21. (c) 22. (a) 23. (b) 24. (a)  
 25. (c)

### Exercise 6 (B)

1. (a) 2. (b) 3. (c) 4. (c) 5. (a) 6. (b) 7. (c) 8. (a)  
 9. (d) 10. (a) 11. (c) 12. (c) 13. (a) 14. (c) 15. (a) 16. (b)  
 17. (a) 18. (b) 19. (c) 20. (a) 21. (b) 22. (c) 23. (b) 24. (a)

### Exercise 6 (C)

1. (a) 2. (d) 3. (b) 4. (b), (c) 5. (c) 6. (b), (c) 7. (a), (b) 8. (a)  
 9. (c) 10. (b) 11. (a) 12. (c) 13. (a) 14. (d) 15. (b) 16. (a)  
 17. (a), (b) 18. (c) 19. (b) 20. (c) 21. (a) 22. (c) 23. (a) 24. (b)  
 25. (c) 26. (a) 27. (b) 28. (c) 29. (a) 30. (d)

## ADDITIONAL QUESTION BANK

- If  $a, b, c$  are in A.P. as well as in G.P. then –
 

(a) They are also in H.P. (Harmonic Progression) (b) Their reciprocals are in A.P.  
 (c) Both (a) and (b) are true (d) Both (a) and (b) are false
- If  $a, b, c$  be respectively  $p^{\text{th}}$ ,  $q^{\text{th}}$  and  $r^{\text{th}}$  terms of an A.P. the value of  $a(q-r)+b(r-p)+c(p-q)$  is \_\_\_\_\_.
- If the  $p^{\text{th}}$  term of an A.P. is  $q$  and the  $q^{\text{th}}$  term is  $p$  the value of the  $r^{\text{th}}$  term is \_\_\_\_\_.
 

(a) 0 (b) 1 (c) -1 (d) None
- If the  $p^{\text{th}}$  term of an A.P. is  $q$  and the  $q^{\text{th}}$  term is  $p$  the value of the  $(p+q)^{\text{th}}$  term is \_\_\_\_\_.
 

(a) 0 (b) 1 (c) -1 (d) None
- The sum of first  $n$  natural number is \_\_\_\_\_.
 

(a)  $(n/2)(n+1)$  (b)  $(n/6)(n+1)(2n+1)$  (c)  $[(n/2)(n+1)]^2$  (d) None

6. The sum of square of first  $n$  natural number is \_\_\_\_\_.  
 (a)  $(n/2)(n+1)$       (b)  $(n/6)(n+1)(2n+1)$       (c)  $[(n/2)(n+1)]^2$       (d) None
7. The sum of cubes of first  $n$  natural number is \_\_\_\_\_.  
 (a)  $(n/2)(n+1)$       (b)  $(n/6)(n+1)(2n+1)$       (c)  $[(n/2)(n+1)]^2$       (d) None
8. The sum of a series in A.P. is 72 the first term is 17 and the common difference -2. the number of terms is \_\_\_\_\_.  
 (a) 6      (b) 12      (c) 6 or 12      (d) None
9. Find the sum to  $n$  terms of  $(1-1/n) + (1-2/n) + (1-3/n) + \dots$   
 (a)  $\frac{1}{2}(n-1)$       (b)  $\frac{1}{2}(n+1)$       (c)  $(n-1)$       (d)  $(n+1)$
10. If  $S_n$  the sum of first  $n$  terms in a series is given by  $2n^2 + 3n$  the series is in \_\_\_\_\_.  
 (a) A.P.      (b) G.P.      (c) H.P.      (d) None
11. The sum of all natural numbers between 200 and 400 which are divisible by 7 is \_\_\_\_\_.  
 (a) 7,730      (b) 8,729      (c) 7,729      (d) 8,730
12. The sum of natural numbers upto 200 excluding those divisible by 5 is \_\_\_\_\_.  
 (a) 20,100      (b) 4,100      (c) 16,000      (d) None
13. If  $a, b, c$  be the sums of  $p, q, r$  terms respectively of an A.P. the value of  $(a/p)(q-r)+(b/q)(r-p)+(c/r)(p-q)$  is \_\_\_\_\_.  
 (a) 0      (b) 1      (c) -1      (d) None
14. If  $S_1, S_2, S_3$  be the respectively the sum of terms of  $n, 2n, 3n$  an A.P. the value of  $S_3/(S_2-S_1)$  is given by \_\_\_\_\_.  
 (a) 1      (b) 2      (c) 3      (d) None
15. The sum of  $n$  terms of two A.P.s are in the ratio of  $(7n-5)/(5n+17)$ . Then the \_\_\_\_\_ term of the two series are equal.  
 (a) 12      (b) 6      (c) 3      (d) None
16. Find three numbers in A.P. whose sum is 6 and the product is -24  
 (a) -2, 2, 6      (b) -1, 1, 3      (c) 1, 3, 5      (d) 1, 4, 7
17. Find three numbers in A.P. whose sum is 6 and the sum of whose square is 44.  
 (a) -2, 2, 6      (b) -1, 1, 3      (c) 1, 3, 5      (d) 1, 4, 7

18. Find three numbers in A.P. whose sum is 6 and the sum of their cubes is 232.
- (a) -2, 2, 6      (b) -1, 1, 3      (c) 1, 3, 5      (d) 1, 4, 7
19. Divide 12.50 into five parts in A.P. such that the first part and the last part are in the ratio of 2:3
- (a) 2, 2.25, 2.5, 2.75, 3      (b) -2, -2.25, -2.5, -2.75, -3  
 (c) 4, 4.5, 5, 5.5, 6      (d) -4, -4.5, -5, -5.5, -6
20. If  $a, b, c$  are in A.P. then the value of  $(a^3 + 4b^3 + c^3)/[b(a^2 + c^2)]$  is
- (a) 1      (b) 2      (c) 3      (d) None
21. If  $a, b, c$  are in A.P. then the value of  $(a^2 + 4ac + c^2)/(ab + bc + ca)$  is
- (a) 1      (b) 2      (c) 3      (d) None
22. If  $a, b, c$  are in A.P. then  $(a/bc)(b+c), (b/ca)(c+a), (c/ab)(a+b)$  are in \_\_\_\_\_.
- (a) A.P.      (b) G.P.      (c) H.P.      (d) None
23. If  $a, b, c$  are in A.P. then  $a^2(b+c), b^2(c+a), c^2(a+b)$  are in \_\_\_\_\_.
- (a) A.P.      (b) G.P.      (c) H.P.      (d) None
24. If  $(b+c)^{-1}, (c+a)^{-1}, (a+b)^{-1}$  are in A.P. then  $a^2, b^2, c^2$  are in \_\_\_\_\_.
- (a) A.P.      (b) G.P.      (c) H.P.      (d) None
25. If  $a^2, b^2, c^2$  are in A.P. then  $(b+c), (c+a), (a+b)$  are in \_\_\_\_\_.
- (a) A.P.      (b) G.P.      (c) H.P.      (d) None
26. If  $a^2, b^2, c^2$  are in A.P. then  $a/(b+c), b/(c+a), c/(a+b)$  are in \_\_\_\_\_.
- (a) A.P.      (b) G.P.      (c) H.P.      (d) None
27. If  $(b+c-a)/a, (c+a-b)/b, (a+b-c)/c$  are in A.P. then  $a, b, c$  are in \_\_\_\_\_.
- (a) A.P.      (b) G.P.      (c) H.P.      (d) None
28. If  $(b-c)^2, (c-a)^2, (a-b)^2$  are in A.P. then  $(b-c), (c-a), (a-b)$  are in \_\_\_\_\_.
- (a) A.P.      (b) G.P.      (c) H.P.      (d) None

29. If  $a$ ,  $b$ ,  $c$  are in A.P. then  $(b + c)$ ,  $(c + a)$ ,  $(a + b)$  are in \_\_\_\_\_.  
(a) A.P.      (b) G.P.      (c) H.P.      (d) None

30. Find the number which should be added to the sum of any number of terms of the A.P. 3, 5, 7, 9, 11 ..... resulting in a perfect square.  
(a) -1      (b) 0      (c) 1      (d) None

31. The sum of  $n$  terms of an A.P. is  $2n^2 + 3n$ . Find the  $n^{\text{th}}$  term.  
(a)  $4n + 1$       (b)  $4n - 1$       (c)  $2n + 1$       (d)  $2n - 1$

32. The  $p^{\text{th}}$  term of an A.P. is  $1/q$  and the  $q^{\text{th}}$  term is  $1/p$ . The sum of the  $(pq)^{\text{th}}$  term is \_\_\_\_\_.  
(a)  $\frac{1}{2}(pq+1)$       (b)  $\frac{1}{2}(pq-1)$       (c)  $pq+1$       (d)  $pq-1$

33. The sum of  $p$  terms of an A.P. is  $q$  and the sum of  $q$  terms is  $p$ . The sum of  $p + q$  terms is \_\_\_\_\_.  
(a)  $-(p + q)$       (b)  $p + q$       (c)  $(p - q)^2$       (d)  $p^2 - q^2$

34. If  $S_1$ ,  $S_2$ ,  $S_3$  be the sums of  $n$  terms of three A.P.s the first term of each being unity and the respective common differences 1, 2, 3 then  $(S_1 + S_3) / S_2$  is \_\_\_\_\_.  
(a) 1      (b) 2      (c) -1      (d) None

35. The sum of all natural numbers between 500 and 1000, which are divisible by 13, is \_\_\_\_\_.  
(a) 28,400      (b) 28,405      (c) 28,410      (d) None

36. The sum of all natural numbers from 100 and 300, which are divisible by 4, is \_\_\_\_\_.  
(a) 10,200      (b) 30,000      (c) 8,200      (d) 2,200

37. The sum of all natural numbers from 100 to 300 excluding those, which are divisible by 4, is \_\_\_\_\_.  
(a) 10,200      (b) 30,000      (c) 8,200      (d) 2,200

38. The sum of all natural numbers from 100 to 300, which are divisible by 5, is \_\_\_\_\_.  
(a) 10,200      (b) 30,000      (c) 8,200      (d) 2,200

39. The sum of all natural numbers from 100 to 300, which are divisible by 4 and 5, is \_\_\_\_\_.  
(a) 10,200      (b) 30,000      (c) 8,200      (d) 2,200

40. The sum of all natural numbers from 100 to 300, which are divisible by 4 or 5, is \_\_\_\_\_.  
(a) 10,200      (b) 8,200      (c) 2,200      (d) 16,200



53. The sum of  $n$  terms of  $(x+y)^2, (x^2+y^2), (x-y)^2, \dots$  is  
 (a)  $(x+y)^2 - 2(n-1)xy$       (b)  $n(x+y)^2 - n(n-1)xy$       (c) both the above      (d) None
54. The sum of  $n$  terms of  $(1/n)(n-1), (1/n)(n-2), (1/n)(n-3), \dots$  is  
 (a) 0      (b)  $(1/2)(n-1)$       (c)  $(1/2)(n+1)$       (d) None
55. The sum of  $n$  terms of 1.4, 3.7, 5.10, ... is  
 (a)  $(n/2)(4n^2+5n-1)$       (b)  $n(4n^2+5n-1)$       (c)  $(n/2)(4n^2-5n-1)$       (d) None
56. The sum of  $n$  terms of  $1^2, 3^2, 5^2, 7^2, \dots$  is  
 (a)  $(n/3)(4n^2-1)$       (b)  $(n/2)(4n^2-1)$       (c)  $(n/3)(4n^2+1)$       (d) None
57. The sum of  $n$  terms of 1, (1 + 2), (1 + 2 + 3), ... is  
 (a)  $(n/3)(n+1)(n-2)$       (b)  $(n/3)(n+1)(n+2)$       (c)  $n(n+1)(n+2)$       (d) None
58. The sum of  $n$  terms of the series  $1^2/1 + (1^2+2^2)/2 + (1^2+2^2+3^2)/3 + \dots$  is  
 (a)  $(n/36)(4n^2+15n+17)$       (b)  $(n/12)(4n^2+15n+17)$   
 (c)  $(n/12)(4n^2+15n+17)$       (d) None
59. The sum of  $n$  terms of the series 2.4.6 + 4.6.8 + 6.8.10 + ... is  
 (a)  $2n(n^3+6n^2+11n+6)$       (b)  $2n(n^3-6n^2+11n-6)$   
 (c)  $n(n^3+6n^2+11n+6)$       (d)  $n(n^3+6n^2+11n-6)$
60. The sum of  $n$  terms of the series  $1.3^2+4.4^2+7.5^2+10.6^2+\dots$  is  
 (a)  $(n/12)(n+1)(9n^2+49n+44)-8n$       (b)  $(n/12)(n+1)(9n^2+49n+44)+8n$   
 (c)  $(n/6)(2n+1)(9n^2+49n+44)-8n$       (d) None
61. The sum of  $n$  terms of the series 4 + 6 + 9 + 13, ... is  
 (a)  $(n/6)(n^2+3n+20)$       (b)  $(n/6)(n+1)(n+2)$       (c)  $(n/3)(n+1)(n+2)$       (d) None
62. The sum to  $n$  terms of the series 11, 23, 59, 167, ... is  
 (a)  $3^{n+1}+5n-3$       (b)  $3^{n+1}+5n+3$       (c)  $3^n+5n-3$       (d) None
63. The sum of  $n$  terms of the series  $1/(4.9)+1/(9.14)+1/(14.19)+1/(19.24)+\dots$  is  
 (a)  $(n/4)(5n+4)^{-1}$       (b)  $(n/4)(5n+4)$       (c)  $(n/4)(5n-4)^{-1}$       (d) None
64. The sum of  $n$  terms of the series 1 + 3 + 5 + ... is  
 (a)  $n^2$       (b)  $2n^2$       (c)  $n^2/2$       (d) None

65. The sum of  $n$  terms of the series  $2 + 6 + 10 + \dots$  is  
 (a)  $2n^2$       (b)  $n^2$       (c)  $n^2/2$       (d)  $4n^2$
66. The sum of  $n$  terms of the series  $1.2 + 2.3 + 3.4 + \dots$  is  
 (a)  $(n/3)(n+1)(n+2)$       (b)  $(n/2)(n+1)(n+2)$       (c)  $(n/3)(n+1)(n-2)$  (D) None
67. The sum of  $n$  terms of the series  $1.2.3 + 2.3.4 + 3.4.5 + \dots$  is  
 (a)  $(n/4)(n+1)(n+2)(n+3)$       (b)  $(n/3)(n+1)(n+2)(n+3)$   
 (c)  $(n/2)(n+1)(n+2)(n+3)$       (d) None
68. The sum of  $n$  terms of the series  $1.2+3.2^2+5.2^3+7.2^4+\dots$  is  
 (a)  $(n-1)2^{n+2}-2^{n+1}+6$       (b)  $(n+1)2^{n+2}-2^{n+1}+6$       (c)  $(n-1)2^{n+2}-2^{n+1}-6$  (d) None
69. The sum of  $n$  terms of the series  $1/(3.8)+1/(8.13)+1/(13.18)+\dots$  is  
 (a)  $(n/3)(5n+3)^{-1}$       (b)  $(n/2)(5n+3)^{-1}$       (c)  $(n/2)(5n-3)^{-1}$       (d) None
70. The sum of  $n$  terms of the series  $1/1+1/(1+2)+1/(1+2+3)+\dots$  is  
 (a)  $2n(n+1)^{-1}$       (b)  $n(n+1)$       (c)  $2n(n-1)^{-1}$       (d) None
71. The sum of  $n$  terms of the series  $2^2+5^2+8^2+\dots$  is  
 (a)  $(n/2)(6n^2+3n-1)$       (b)  $(n/2)(6n^2-3n-1)$   
 (c)  $(n/2)(6n^2+3n+1)$       (d) None
72. The sum of  $n$  terms of the series  $1^2+3^2+5^2+\dots$  is  
 (a)  $\frac{n}{3}(4n^2-1)$       (b)  $n^2(2n^2+1)$       (c)  $n(2n-1)$       (d)  $n(2n+1)$
73. The sum of  $n$  terms of the series  $1.4 + 3.7 + 5.10 + \dots$  is  
 (a)  $\frac{n}{3}(4n^2+5n+5)$       (b)  $(n/2)(5n^2+4n-1)$   
 (c)  $(n/2)(4n^2+5n+1)$       (d) None
74. The sum of  $n$  terms of the series  $2.3^2+5.4^2+8.5^2+\dots$  is  
 (a)  $(n/12)(9n^3+62n^2+123n+22)$       (b)  $(n/12)(9n^3-62n^2+123n-22)$   
 (c)  $(n/6)(9n^3+62n^2+123n+22)$       (d) None
75. The sum of  $n$  terms of the series  $1 + (1 + 3) + (1 + 3 + 5) + \dots$  is  
 (a)  $(n/6)(n+1)(2n+1)$       (b)  $(n/6)(n+1)(n+2)$       (c)  $(n/3)(n+1)(2n+1)$  (d) None

76. The sum of  $n$  terms of the series  $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$  is  
 (a)  $(n/12)(n+1)^2(n+2)$       (b)  $(n/12)(n-1)^2(n+2)$       (c)  $(n/12)(n^2 - 1)(n+2)$       (d) None
77. The sum of  $n$  terms of the series  $1 + (1+1/3) + (1+1/3+1/3^2) + \dots$  is  
 (a)  $(3/2)(1-3^{-n})$       (b)  $(3/2)[n-(1/2)(1-3^{-n})]$       (c) Both      (d) None
78. The sum of  $n$  terms of the series  $n.1 + (n-1).2 + (n-2).3 + \dots$  is  
 (a)  $(n/6)(n+1)(n+2)$       (b)  $(n/3)(n+1)(n+2)$       (c)  $(n/2)(n+1)(n+2)$       (d) None
79. The sum of  $n$  terms of the series  $1 + 5 + 12 + 22 + \dots$  is  
 (a)  $(n^2/2)(n+1)$       (b)  $n^2(n+1)$       (c)  $(n^2/2)(n-1)$       (d) None
80. The sum of  $n$  terms of the series  $4 + 14 + 30 + 52 + 80 + \dots$  is  
 (a)  $n(n+1)^2$       (b)  $n(n-1)^2$       (c)  $n(n^2 - 1)$       (d) None
81. The sum of  $n$  terms of the series  $3 + 6 + 11 + 20 + 37 + \dots$  is  
 (a)  $2^{n+1} + (n/2)(n+1)-2$       (b)  $2^{n+1} + (n/2)(n+1)-1$       (c)  $2^{n+1} + (n/2)(n-1)-2$       (d) None
82. The  $n^{\text{th}}$  terms of the series is  $1/(4.7) + 1/(7.10) + 1/(10.13) + \dots$  is  
 (a)  $(1/3)[(3n+1)^{-1} - (3n+4)^{-1}]$       (b)  $(1/3)[(3n-1)^{-1} - (3n+4)^{-1}]$   
 (c)  $(1/3)[(3n+1)^{-1} - (3n-4)^{-1}]$       (d) None
83. In question No.(82) the sum of the series upto  $n$  is  
 (a)  $(n/4)(3n+4)^{-1}$       (b)  $(n/4)(3n-4)^{-1}$       (c)  $(n/2)(3n+4)^{-1}$       (d) None
84. The sum of  $n$  terms of the series  $1^2/1 + (1^2 + 2^2)/(1+2) + (1^2 + 2^2 + 3^2)/(1+2+3) + \dots$  is  
 (a)  $(n/3)(n+2)$       (b)  $(n/3)(n+1)$       (c)  $(n/3)(n+3)$       (d) None
85. The sum of  $n$  terms of the series  $1^3/1 + (1^3 + 2^3)/2 + (1^3 + 2^3 + 3^3)/3 + \dots$  is  
 (a)  $(n/48)(n+1)(n+2)(3n+5)$       (b)  $(n/24)(n+1)(n+2)(3n+5)$   
 (c)  $(n/48)(n+1)(n+2)(5n+3)$       (d) None



97. If  $a, b, c$  are in A.P. and  $x, y, z$  in G.P. then the value of  $x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$  is \_\_\_\_\_  
 (a) 0 (b) 1 (c) -1 (d) None

98. If  $a, b, c$  are in A.P. and  $x, y, z$  in G.P. then the value of  $(x^b \cdot y^c \cdot z^a) \div (x^c \cdot y^a \cdot z^b)$  is \_\_\_\_\_  
 (a) 0 (b) 1 (c) -1 (d) None

99. The sum of  $n$  terms of the series  $7 + 77 + 777 + \dots$  is  
 (a)  $(7/9)[(1/9)(10^{n+1}-10)-n]$  (b)  $(9/10)[(1/9)(10^{n+1}-10)-n]$   
 (c)  $(10/9)[(1/9)(10^{n+1}-10)-n]$  (d) None

100. The least value of  $n$  for which the sum of  $n$  terms of the series  $1 + 3 + 3^2 + \dots$  is greater than 7000 is \_\_\_\_\_.  
 (a) 9 (b) 10 (c) 8 (d) 7

101. If ' $S'$  be the sum, ' $P'$  the product and ' $R'$  the sum of the reciprocals of  $n$  terms in a G.P. then ' $P'$  is the \_\_\_\_\_ of  $S^n$  and  $R^{-n}$ .  
 (a) Arithmetic Mean (b) Geometric Mean (c) Harmonic Mean (d) None

102. Sum upto  $\infty$  of the series  $8+4\sqrt{2}+4+\dots$  is  
 (a)  $8(2+\sqrt{2})$  (b)  $8(2-\sqrt{2})$  (c)  $4(2+\sqrt{2})$  (d)  $4(2-\sqrt{2})$

103. Sum upto  $\infty$  of the series  $1/2+1/3^2+1/2^3+1/3^4+1/2^5+1/3^6+\dots$  is  
 (a)  $19/24$  (b)  $24/19$  (c)  $5/24$  (d) None

104. If  $1+a+a^2+\dots=\infty=x$  and  $1+b+b^2+\dots=\infty=y$  then  $1+ab+a^2b^2+\dots=\infty$  is given by \_\_\_\_\_.  
 (a)  $(xy)/(x+y-1)$  (b)  $(xy)/(x-y-1)$  (c)  $(xy)/(x+y+1)$  (d) None

105. If the sum of three numbers in G.P. is 35 and their product is 1000 the numbers are \_\_\_\_\_.  
 (a) 20, 10, 5 (b) 5, 10, 20 (c) both (d) None

106. If the sum of three numbers in G.P. is 21 and the sum of their squares is 189 the numbers are \_\_\_\_\_.  
 (a) 3, 6, 12 (b) 12, 6, 3 (c) both (d) None

107. If  $a, b, c$  are in G.P. then the value of  $a(b^2+c^2)-c(a^2+b^2)$  is \_\_\_\_\_.  
 (a) 0 (b) 1 (c) -1 (d) None







141. The sum upto infinity of the series  $(\sqrt{2}+1)+1+(\sqrt{2}-1)+\dots$  is

- (a)  $(1/2)(4+3\sqrt{2})$       (b)  $(1/2)(4-3\sqrt{2})$       (c)  $4+3\sqrt{2}$       (d) None

142. The sum upto infinity of the series  $(1+2^{-2})+(2^{-1}+2^{-4})+(2^{-2}+2^{-6})+\dots$  is

- (a)  $7/3$       (b)  $3/7$       (c)  $4/7$       (d) None

143. The sum upto infinity of the series  $4/7-5/7^2+4/7^3-5/7^4+\dots$  is

- (a)  $23/48$       (b)  $25/48$       (c)  $1/2$       (d) None

144. If the sum of infinite terms in a G.P. is 2 and the sum of their squares is  $4/3$  the series is

- (a)  $1, 1/2, 1/4 \dots$       (b)  $1, -1/2, 1/4 \dots$       (c)  $-1, -1/2, -1/4 \dots$       (d) None

145. The infinite G.P. with first term  $1/4$  and sum  $1/3$  is

- (a)  $1/4, 1/16, 1/64 \dots$       (b)  $1/4, -1/16, 1/64 \dots$       (c)  $1/4, 1/8, 1/16 \dots$       (d) None

146. If the first term of a G.P. exceeds the second term by 2 and the sum to infinity is 50 the series is \_\_\_\_\_.

- (a)  $10, 8, 32/5 \dots$       (b)  $10, 8, 5/2 \dots$       (c)  $10, 10/3, 10/9 \dots$       (d) None

147. Three numbers in G.P. with their sum 130 and their product 27,000 are \_\_\_\_\_.

- (a)  $10, 30, 90 \dots$       (b)  $90, 30, 10 \dots$       (c) both      (d) None

148. Three numbers in G.P. with their sum  $13/3$  and sum of their squares  $91/9$  are \_\_\_\_\_.

- (a)  $1/3, 1, 3$       (b)  $3, 1, 1/3$       (c) both      (d) None

149. Find five numbers in G.P. such that their product is 32 and the product of the last two is 108.

- (a)  $2/9, 2/3, 2, 6, 18$       (b)  $18, 6, 2, 2/3, 2/9$       (c) both      (d) None

150. If the continued product of three numbers in G.P. is 27 and the sum of their products in pairs is 39 the numbers are \_\_\_\_\_.

- (a)  $1, 3, 9$       (b)  $9, 3, 1$       (c) both      (d) None

151. The numbers  $x, 8, y$  are in G.P. and the numbers  $x, y, -8$  are in A.P. The values of  $x, y$  are \_\_\_\_\_.

- (a)  $16, 4$       (b)  $4, 16$       (c) both      (d) None

## ANSWERS

- |                 |         |         |          |          |
|-----------------|---------|---------|----------|----------|
| 1. (c)          | 31. (a) | 61. (a) | 91. (a)  | 121. (a) |
| 2. (a)          | 32. (a) | 62. (a) | 92. (a)  | 122. (a) |
| 3. (b)          | 33. (a) | 63. (a) | 93. (a)  | 123. (a) |
| 4. (a)          | 34. (b) | 64. (a) | 94. (c)  | 124. (a) |
| 5. (a)          | 35. (b) | 65. (a) | 95. (c)  | 125. (a) |
| 6. (b)          | 36. (a) | 66. (a) | 96. (b)  | 126. (a) |
| 7. (c)          | 37. (b) | 67. (a) | 97. (b)  | 127. (a) |
| 8. (c)          | 38. (c) | 68. (d) | 98. (b)  | 128. (c) |
| 9. (a)          | 39. (d) | 69. (a) | 99. (a)  | 129. (a) |
| 10. (a)         | 40. (d) | 70. (a) | 100. (a) | 130. (c) |
| 11. (b)         | 41. (a) | 71. (a) | 101. (b) | 131. (a) |
| 12. (c)         | 42. (a) | 72. (a) | 102. (a) | 132. (a) |
| 13. (a)         | 43. (d) | 73. (a) | 103. (a) | 133. (b) |
| 14. (c)         | 44. (d) | 74. (a) | 104. (a) | 134. (a) |
| 15. (b)         | 45. (c) | 75. (a) | 105. (c) | 135. (a) |
| 16. (a)         | 46. (a) | 76. (a) | 106. (c) | 136. (a) |
| 17. (a)         | 47. (b) | 77. (b) | 107. (a) | 137. (a) |
| 18. (a)         | 48. (c) | 78. (a) | 108. (a) | 138. (a) |
| 19. (a)         | 49. (a) | 79. (a) | 109. (a) | 139. (a) |
| 20. (c)         | 50. (b) | 80. (a) | 110. (b) | 140. (a) |
| 21. (b)         | 51. (a) | 81. (a) | 111. (b) | 141. (a) |
| 22. (a)         | 52. (d) | 82. (a) | 112. (c) | 142. (a) |
| 23. (a)         | 53. (b) | 83. (a) | 113. (a) | 143. (a) |
| 24. (a)         | 54. (b) | 84. (a) | 114. (a) | 144. (a) |
| 25. (c)         | 55. (a) | 85. (a) | 115. (a) | 145. (a) |
| 26. (a)         | 56. (a) | 86. (a) | 116. (b) | 146. (a) |
| 27. (c)         | 57. (d) | 87. (a) | 117. (a) | 147. (c) |
| 28. (c)         | 58. (a) | 88. (b) | 118. (a) | 148. (c) |
| 29. (a)         | 59. (a) | 89. (c) | 119. (a) | 149. (a) |
| 30. (c)         | 60. (a) | 90. (d) | 120. (a) | 150. (c) |
| <b>151. (a)</b> |         |         |          |          |