Report - Assignment No 3

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EE19B105

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1 Abstract

In this Assignment, we study the effect of different noise distributions (Gaussian functions with different σ) applied to the Bessel Function on the fitting process.We plot different graphs and analyse the relationship between error in estimations of coefficients(mainly A and B) with change in σ

2 Introduction

We generate a file fitting.dat with 10 columns with first column as time while the remaining columns are data with a different noise amounts

The functions used is a linear combination of Bessel Function and y = t

$$f(t) = A * J_2(t) + B * t + n(t)$$

where A and B are constants and have values of 1.05 and -0.105 respectively and n(t) is the noise function with a given σ:

The noise is given to be normally distributed, i.e., its probability distribution is given by

$$Pr(n(t)|\sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp(\frac{-n(t)^2}{2\sigma^2})$$

where σ = logspace(-1,-3,9)

3 Objectives

3.1 Generating and Loading Data

The data is generated using the code provided in the assignment.It computes all the values of the sample function with 9 different values of standard deviation.The data is stored in a .dat file.The first column represents time values and the rest 9 columns represent the function value with different amount of noise added.

3.2 Plotting data

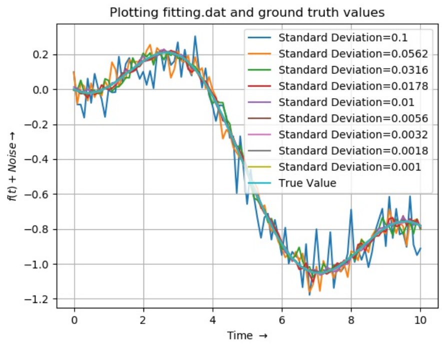

Figure 1: Various Functions

3.3 Error plots

We visualise the error in the measurement using the *errorbar()* function. The graph has been obtained by plotting the first column in the data file which corresponds to $sigma = 0.1$ The true value has also been plotted for reference.

$$\epsilon_{ij} = \frac{1}{101}\sum_{k=0}^{101}(f(t) - g(t_k, A_i, B_j))^2$$

Here I am plotting the error bar for $\sigma = 0.1$:

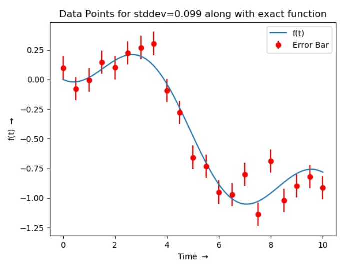

Figure 2: Error bars for $\sigma = 0.1$ along with exact function

3.4 Generating Matrix

We transform this problem into a matrix multiplication as it becomes faster for computing

$$g(t, A, B) = \begin{pmatrix} J_2(t_1) & t_1 \\ J_2(t_2) & t_1 \\ ... & ... \\ J_2(t_n) & t_n \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = M \cdot p$$

3.5 Contour plots of the error function

True value function is plotted by defining a function:

$$g(t, A, B) = AJ_2(t) + Bt$$

Next we assume that there exists some function which fits the noise data with general form $g(t, A, B) = AJ_2(t) + Bt$

Next we will find the (A,B) values by minimising the absolutes error between the predicted values from given (A,B) and data column.

Contour Plot of $MS\ error$ for w.r.t data column1 for range of (A,B): From

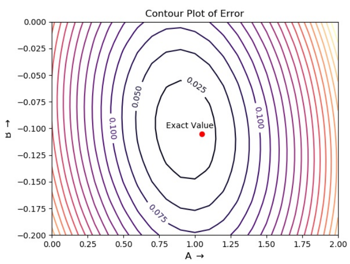

Figure 3: contour plot for ϵ_{ij}

the above plot we can clearly see that there exists a single minimum.

3.6 Error plots for different scales

3.6.1 Linear scale

We plot the graph of error with changing values of σ_n

The error in the estimates of A and B are non-linear with respect to σ_n

3.6.2 Log log scale

Now we plot the graph in log vs log scale

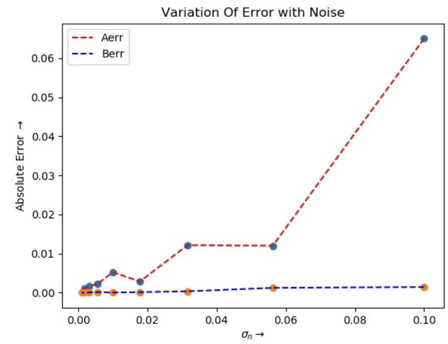

Figure 4: A and B error in linear scale

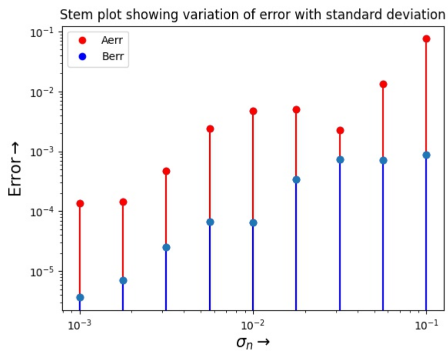

Figure 5: A and B error in log log scale

4 Results

If we calculate the absolute error for the predicted points w.r.t to the data points, in the log log scale we observe that there will be a linear variation. The previous plots show the error in A and B for different data columns: In the first plot B error is appearing to be constant but in reality it increases by a small amount.

5 Conclusions

For the given noisy data the best possible estimates for A and B were obtained by minimizing the absolute error.It is observed that error changes approximately linearly with respect to σ in the log log scale