EE19B105 Pranav Phatak

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Assignment 2

1) order by asymptotic growth rate.

i) $4n\log n + 2n \le 4n\log n + 2n(\log n) = \Theta(n\log n)$

ii) $2^{10} = c = \Theta(1)$

iii) $2^{\log n} \le 2^{\log_2 n} = n = \Theta(n)$

(if $\log n$ is $\log_a n$ where $a \geq 2$)

iv) $4n = \Theta(n)$

v) $3n + 100\log n \le 3n + 100n = 103n = \Theta(n)$

vi) $2^n = \Theta(2^n)$

vii) $n^2 + 10n \le n^2 + 10n^2 = 11n^2 = \Theta(n^2)$

viii) $n^3 = \Theta(n^3)$

ix) $n\log n = \Theta(n\log n)$

$\Rightarrow$ order is

$2^{10} < 2^{\log n} = 4n = 3n + 100\log n < 4n\log n + 2n = n\log n$
$< n^2 + 10n < n^3 < 2^n$

2) Indicate if $f = O(g)$ or $f = \Omega(g)$ or $f = \Theta(g)$

a) $f = n - 100$, $g = n - 200$.

$c = 2 \Rightarrow (n-100) \le 2n - 400$ for $n \geq 300$

$c = 1/2 \Rightarrow \frac{1}{2}(n-200) \le n - 100$ for $n \geq 0$

So $f = \Theta(g)$

b) $f = 100n + \log n$, $g = n + (\log n)^2$

if limit $n \to \infty$, $\frac{f}{g} = c$ then $f = \Theta(g)$ $(c \neq 0)$

$$\lim_{n\to\infty} \frac{100n + \log n}{n + (\log n)^2} = \lim_{n\to\infty} \frac{100 + 1/n}{1 + \frac{2\log n}{n}} = \lim_{n\to\infty} \frac{100n + 1}{n + 2\log n}$$

$$= \lim_{n\to\infty} \frac{100}{1 + 2/n} = \lim_{n\to\infty} \frac{100n}{n+2} = 100 \Rightarrow f = \Theta(g)$$

c) $f = \log 2n$, $g = \log 3n$

$\Rightarrow f = \log 2 + \log n$, $g = \log 3 + \log n$

$$\lim_{n\to\infty} \frac{f}{g} = \frac{\log 2 + \log n}{\log 3 + \log n} = \frac{0 + 1/n}{0 + 1/n} = 1$$

$$\Rightarrow \underline{f = \Theta(g)}$$

d) $f = n^{1.01}$, $g = n \log^2 n$

Claim : $f = \Omega(g)$

$\Leftrightarrow cg \leq f$ for some $n \geq n_0$

$\Leftrightarrow cn \log^2 n \leq n^{1.01}$

$\Leftrightarrow \log^2 n \leq \frac{n^{1/100}}{c} \Leftrightarrow \log n \leq \frac{n^{1/200}}{\sqrt{c}}$

Take $c = 1$ $\Leftrightarrow \log n \leq n^{1/200}$

$$\lim_{n\to\infty} \frac{\log n}{n^{1/200}} = \lim_{n\to\infty} \frac{1/n}{n^{-199/200}} \times 200 = \lim_{n\to\infty} \frac{200}{n^{1/200}}$$

$$= 0$$

$\Rightarrow$ By definition of limit ; $\exists\ m \in \mathbb{N} \ni \forall n \geq m$,

$\left| \frac{\log n}{n^{1/200}} \right| < \varepsilon$ (where ε can be arbitrarily small)

$\because \frac{\log n}{n^{1/200}} > 0$; choose $\varepsilon = 1$

$\Rightarrow \frac{\log n}{n^{1/200}} < 1 \quad \forall n > n_0$ by definition.

$\Rightarrow \underline{f = \Omega(g)}$.

If we try to show $f = O(g)$; again by using same definition of limit we will get contradiction.