Analytical evaluation of extrapolation error for *Homography* and *Cross ratio*

Pranav Kant Gaur

January 13, 2014

Abstract

This document describes *Normalized DLT* algorithm and derivation of error expression. Further, it also shows the *extrapolation* error for cross ratio approach.

1 Problem statement

Given *projective* mapping between points in two 2D coordinate systems(say S_w and S_i). Points have error associated with them that is $x^i_{we} = x^i_{wo} + \delta e^x_w$ (similarly, for all coordinates on S_w and S_i). Derive error expression for Homography mapping.

2 Normalized Direct linear transformation(DLT)

Direct linear transformation algorithm is used to define transformation between 2 projective planes. This transformation is represented in form of 3X3 matrix called *Homography* or *Collineation.* Input data is in the form of pairs of real world points,(x_w, y_w) and corresponding image space projections,(x_i, y_i). Scale of data in real world coordinate system can be different than that in image coordinate system. Therefore, to avoid possible *numerical instability*, input data is normalized.

2.1 Benefits of data normalization

1. Reduces condition number of involved matries, thereby increasing numerical stability. It consequentrly results in increased accuracy of solution.

2. Normalization enhances rate of convergence of iterative algorithms.

2.2 Algorithm

Normalized DLT algorithm only introduces data normalization, which is a kind of *preconditioning* of the input.

1. Normalize data set by translating and scaling the data(both real world and image space coordinates)
2. Compute homography, H_n using DLT algorithm
3. Denormalize H_n to get actual homography, H

2.3 DLT algorithm

1. Represent relation between object and image points(x_w, y_w) and (x_i, y_i) respectively in form:
$$Ah = 0 \tag{1}$$
where,
A is $2nX9$ matrix and h is $9X1$ matrix and n is the number of (object point,image point pairs).
2. Compute Singular value decomposition(SVD) of A, such that $A = U \sum V*$
3. The right singular vector,v_i corresponding to *smallest* singular value,σ represents desired h
4. Convert h(9X1 form) to H(3X3 form) to recover homography

3 Homography estimation algorithm: Error propogation study

Since, acquired image points have error(or *noise*) this error will propogate to final solution. In this section, we will derive formal expression for propogated error for a chosen homography computation algorithm. Here, we will focus on error analysis of SVD computation algorithm since it forms the major computation part of homography estimation in DLT algorithm.