Optics and Lasers in Engineering 51 (2013) 538–541

Contents lists available at SciVerse ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

An accurate calibration method for a camera with telecentric lenses

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ARTICLE INFO

Article history:
Received 12 September 2012
Received in revised form
3 December 2012
Accepted 12 December 2012
Available online 10 January 2013

Keywords:
Camera calibration
Telecentric lenses
Lens distortion

ABSTRACT

It is very important to achieve an accurate calibration to obtain high measurement accuracy for a telecentric imaging system. However, the conventional camera calibration methods typically based on a pinhole model are not suitable for telecentric lenses due to the unique behavior of orthographic projection of telecentric lenses. In this paper, we propose a new method to accurately calibrate the telecentric imaging system. An analytical camera model for telecentric lenses is presented with considering the major sources of lens distortions. Based on this model, a two-step calibration procedure is described for the telecentric imaging system. We use a camera with a telecentric lens to verify our model and study the impacts of different distortion models for properly characterizing the distortion of telecentric lens. The experimental results show the maximum distortion of the system reduces 26 times by the presented calibration technology.

1. Introduction

Telecentric lenses have the unique property of purely orthographic projections of scene points and maintaining a constant magnification over a specific range of object distances. Thus, they are being widely used in many machine vision applications especially for accurate dimensional measurements of three-dimensional (3D) parts and components of different heights [1,2]. Although high quality telecentric lenses normally show very low distortion degree, in the range of 0.1%, which seems to be very small, it would actually result into measurement errors approaching the size of one pixel of a camera. This must not happen in precise measurement. In fact, such as radial distortion and trapezoidal distortion are inevitably in telecentric lenses system. Therefore, the calibration of a camera with telecentric lenses is essential for metric measurement in high accuracy application.

Many techniques for camera calibration have been proposed. Hall and Faugeras-Toscani use a least-squares technique to obtain the parameters of the linear model. And later, nonlinear calibrating methods are developed for lens distortion. The widely used method proposed by Tsai is based on a two-step technique modeling only with radial lens distortion [3]. Weng presents a camera model with three different types of lens distortion and a two-step calibration procedure [4]. Zhang proposes a flexible camera calibration technique which uses images of a 2D template taken from different camera positions and orientations [5]. All these camera calibration methods use the pinhole model based on perspective projection, while telecentric lenses act as orthographic projection. Therefore, these calibration methods are not available for a camera with telecentric lenses. Zhu develops a system for deformation measurement with telecentric lens, and proposes a simple calibration method based on linear fitting [6]. However, this calibration does not take into account the lens distortion and could only get the intrinsic parameter magnification, in which the extrinsic parameters are not given. To the best of our knowledge, it has not an accurate calibration method for telecentric lens based camera.

According to the work principle of telecentric lenses, we proposed a calibration method for a camera with telecentric lenses in this paper. A camera model based on orthographic projection is presented, and the major sources of telecentric lens distortion including radial, decentering, and thin prism distortions are taken into account. We adopt a two-step approach to calibrate the telecentric lens system. In the first step, all the extrinsic parameters and an intrinsic parameter are estimated using a closed-form solution based on a distortion free camera mode. In the second step, a nonlinear optimization based on the camera model with distortions is carried out using the solution of the first step as an initial guess, and all the parameters are computed and refined. As different types of distortion are considered in the second step, the impacts of different distortions are studied.

The principle of telecentric imaging system is described and the mathematical expression of the camera model is proposed in Section 2. The calibration procedure is described in detail in

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http://dx.doi.org/10.1016/j.optlaseng.2012.12.008

Section 3. In Section 4, the performance of the calibration is shown on experimental measurements, then the results are analyzed to reveal the impacts of different distortions, which demonstrates the validity of the proposed method. Finally, conclusions are presented in Section 5.

2. Principle

2.1. Telecentric imaging

In a telecentric lens system, a small aperture stop is located at the focal point of the object lens [7]. Therefore, only the light rays that are approximately parallel to the optical axis of the lens pass through the aperture stop and form the image. As the image is formed by the parallel projection of the object onto the image plane, the image magnification does not depend on the object distance. Telecentricity in object and image space can be achieved by combining two single-sided telecentric lenses, as shown in Fig. 1. The two lenses are separated by the sum of their focal lengths f_1, f_2. The aperture stop is placed in the focal plane between the two lenses. A bilateral telecentric lens accurately reproduces dimensional relationships within its telecentric depth, and it is not susceptible to small differences in the distance between the lens and the camera's sensor. The magnification, $m=f_2/f_1$ from geometrical optics, is one of the most important parameters of a telecentric lens for imaging, which must be calibrated for high-precision measurements.

2.2. Camera model with telecentric lens

The imaging model of a camera with a telecentric lens can be illustrated by Fig. 2. (X_w, Y_w, Z_w) is the 3D coordinate of the object point P in the 3D world coordinate system. (X_c, Y_c, Z_c) is the 3D coordinate of the object point P in the 3D camera coordinate system. (x_u, y_u) is the image coordinate of $P(X_c, Y_c, Z_c)$ if a perfect orthographic projection model is used. (x_d, y_d) is the actual image coordinate which differs from (x_u, y_u) due to lens distortion. (u, v) is the image coordinate of the computer in pixels.

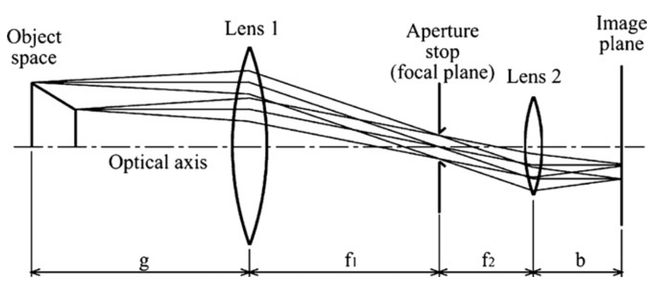

Fig. 1. The schematic of a bilateral telecentric lens.

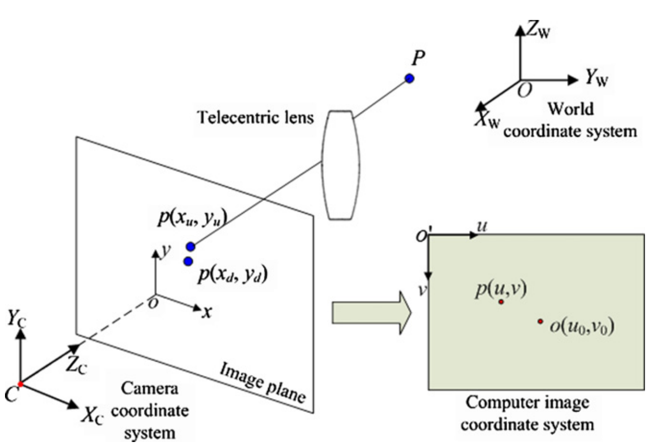

Fig. 2. The imaging model of a camera with a telecentric lens.

The most important premise for an accurate calibration is the correct mathematical expression of the camera model. The pinhole model of wide angle cameras was not applicable for the telecentric imaging system. Telecentric lenses perform scaled orthographic projection [8], thus the projection of an arbitrary point P to the ideal (undistorted) image plane in metrical units is expressed as

$$\begin{bmatrix} x_u \\ y_u \\ 1 \end{bmatrix} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} \tag{1}$$

where m is the effective magnification of telecentric lens, which needs to be calibrated. And the relationship between the world and camera coordinate systems is given by

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \tag{2}$$

where $R=[r_{ij}]$ is the rotation matrix and $T=[\ t_x\ t_y\ t_z\ ]^{\mathrm{T}}$ is the translation matrix.

Without any loss of generality, we set the image coordinate system oxy coincident with the computer the image coordinate system. Thus, the transformation from image coordinate (x, y) to computer image coordinate (u, v) in pixels is described by

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} 1/du & 0 & 0 \\ 0 & 1/dv & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{3}$$

where du and dv are the sizes of a pixel in the x and y directions respectively.

Combining Eqs. (1)–(3), the orthographic projection of telecentric lenses is formed and expressed by the equation:

$$\begin{bmatrix} x_u \\ y_u \\ 1 \end{bmatrix} = \begin{bmatrix} m/du & 0 & 0 \\ 0 & m/dv & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \tag{4}$$

It is obvious that there is not a principal point (u_0, v_0) for telecentric lenses from Eq. (4), as telecentric lenses perform parallel projection and there is not a projection center.

2.3. Lens distortion

There are mainly three types of distortion in telecentric lens, namely radial, decentering, and thin prism distortions. The first one is caused by imperfect lens shape and manifests itself by radial positional error only, whereas the second and the third types of distortion are generally caused by improper lens and camera assembly and generate both radial and tangential errors in point positions. For each kind of distortion, an infinite series is required. However, experiments show that the terms of order higher than 3 could be neglected. Therefore, the effective distortion can be expressed by

$$\begin{cases} \delta_x = k_1 x_u (x_u{}^2 + y_u{}^2) + h_1 (3x_u{}^2 + y_u{}^2) + 2h_2 x_u y_u + s_1 (x_u{}^2 + y_u{}^2) \\ \delta_y = k_1 y_u (x_u{}^2 + y_u{}^2) + 2h_1 x_u y_u + h_2 (x_u{}^2 + 3y_u{}^2) + s_2 (x_u{}^2 + y_u{}^2) \end{cases}$$
$$\begin{bmatrix} x_d \\ y_d \end{bmatrix} = \begin{bmatrix} x_u \\ y_u \end{bmatrix} + \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} \tag{5}$$

where (x_d, y_d) is the actual image coordinate on the image plane; k_1, h_1, h_2, s_1 and s_2 are the distortion coefficients.

The parameters of telecentric lense imaging model which were expressed by Eqs. (4) and (5) can be categorized into extrinsic parameters and intrinsic parameters. Different from traditional camera calibration, there are only five extrinsic parameters: the Euler angles yaw θ, pitch φ, and tilt ψ for rotation R, the two components t_1, t_2 for the translation vector T. And there are six intrinsic parameters: effective magnification m, lens distortion coefficients k_1, h_1, h_2, s_1 and s_2. We use d to denote the set of distortion parameters:

$$d=(k_1,h_1,h_2,s_1,s_2)^{\tau} \tag{6}$$

The problem of camera calibration is to determine the optimal values for the camera intrinsic and extrinsic parameters based on a number of points (x_{wi}, y_{wi}, z_{wi}) and their corresponding pixel locations (u_i, v_i). From Eq. (1), it is obvious that the camera frame z-axis position does not affect the projection of the point (X_w, Y_w, Z_w) to the image plane, thus providing the ability of the z-axis system movement without requiring further calibration. In this paper, a ceramic plate with a precise planar pattern is employed for calibration.

3. Calibration

Since the homography expressed in Eq. (1) is much different from pinhole model, the existed methods for camera calibration are not suitable [2–4]. We improved a two-step procedure for the calibration of telecentric imaging system.

Step 1: Let $d=0$, a closed-form solution is obtained based on the distortion-free camera mode. And then intrinsic parameter m and all the five extrinsic parameters will be estimated.

According to Eq. (1), we have

$$\frac{x}{y}=\frac{r_{11}X_w+r_{12}Y_w+r_{13}Z_w+t_x}{r_{21}X_w+r_{22}Y_w+r_{23}Z_w+t_y} \tag{7}$$

As the calibration points are on a common plane, the (x_{wi}, y_{wi}, z_{wi}) coordinate system can be chosen such that $z_{wi}=0$ without loss of generality. For each point i with $(x_{wi}, y_{wi}, 0)$ as the 3D object coordinate, the corresponding image coordinate(x_i, y_i) can be computed from computer image coordinate (u_i, v_i) by Eq. (3). Linear equations can be set up and expressed by Eq. (8), which is denoted as $ML=X$:

$$\begin{bmatrix} y_1X_{w1} & y_1Y_{w1} & y_1 & -x_1X_{w1} & -x_1Y_{w1} \\ y_2X_{w2} & y_2Y_{w2} & y_2 & -x_2X_{w2} & -x_2Y_{w2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ y_NX_{wN} & y_NY_{wN} & y_N & -x_NX_{wN} & -x_NY_{wN} \end{bmatrix} \begin{bmatrix} r_{11}/t_y \\ r_{12}/t_y \\ t_x/t_y \\ r_{21}/t_y \\ r_{22}/t_y \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \tag{8}$$

As the number of control points N is much larger than five, the over-determined system of linear equations can be solved using a least-squares technique for L. And the five unknowns r_1/t_2, r_2/t_2, t_1/t_2, r_4/t_2, r_5/t_2 are determined.

Since R is a rotation matrix with three degrees of freedom, r_{11}, r_{12}, r_{21}, and r_{22} depend on θ, φ, and ψ. Thus, there are five dependent unknown in L, and $r_{11}, r_{12}, r_{21}, r_{22}, t_x, t_y$ can be computed by L [2]. Then the rest components of R can be determined by the conditions of $RR^T=I$ and $\det(R)=1$.

The effective magnification m can be computed by Eq. (9), which is obtained from Eq.(1).

$$[m]\begin{bmatrix} r_{11}X_{wi}+r_{12}Y_{wi}+t_x \\ r_{21}X_{wi}+r_{22}Y_{wi}+t_y \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \tag{9}$$

Step 2: Considering the lens distortion, the calibration problem is equivalent to an optimization problem in which the calibration parameters are determined in order to minimize an objective function F:

$$F=\sum_{i=1}^{N} e_i^2=\sum_{i=1}^{N}\|p_i-\tilde{p}_i(R,t_x,t_y,m,k_1,h_1,h_2,s_1,s_2)\|^2 \tag{10}$$

where p_i is the computer image coordinate of P_i; $\tilde{p}_i(R, t_x, t_y, m, k_1, h_1, h_2, s_1, s_2)$ is the projection of point P_i according to Eqs. (4) and (5). A rotation R is parameterized by a vector of 3 parameters, which is realized by Rodrigues' formula. Minimizing function F is a nonlinear minimization problem, which is solved with the Levenberg–Marquardt algorithm [9]. The initial guess of R, t_1, t_2, m has been computed in Step 1, and the rest initial guess of distortion parameters k_1, h_1, h_2, s_1, s_2 can be simply set to 0. Furthermore, by minimizing Eq. (10), all parameters are refined and higher calibration accuracy can be achieved.

4. Experiment and discussions

To verify the proposed method, an experiment system is established to evaluate the accuracy of camera calibration. A telecentric lens GCO230105 with magnification 0.16 and distortion $\leq 0.4\%$ and a DH–HV 1351UC camera with resolution 1280×1024 are employed. The size of a pixel is 5.2 μm × 5.2 μm. We use a ceramics plate with a precise circle pattern for calibration, as shown in Fig. 3(a).

The target plate is placed at the working distance of the telecentric lens, and an image is taken by the camera. The centers of circles are extracted as the control points using ellipse fitting algorithm in sub-pixel accuracy, as shown in Fig. 3(b). The experiment results are shown in Table 1. The calibrated magnification m of the telecentric lens approaches to the nominal magnification 0.16.

We calculate the distance between the control point p_i and$\tilde{p}_i$in pixels and convert it to the object space of P_i in metric to estimate the measurement accuracy [10]. Distortion is calculated as the percent difference between p_i and $\tilde{p}_i$.The standard deviation and the maximum of residual in pixels and in metric are listed in Table 2 respectively, which indicate the accuracy of measurement.

The nominal distortion of this telecentric lens GCO230105 is below 0.4%, which agrees with the result of distortion free in the proposed method. From Table 2, it is clear that the proposed calibration method with distortion model significantly improves the measurement accuracy. The results show the distortion can be reduced about 14 times by considering radial distortion and about 26 times by considering radial and thin prism distortion. The standard deviation of residual is reduced about 9 times and 15 times respectively. As thin prism distortion arises from imperfection in lens design and manufacturing as well as assembly, it plays a more important role in telecentric lens than in traditional lens whereas decentering distortion has less impact, which is much different from traditional camera calibration. Therefore the distortion types we need to consider and correct in order of importance are from radial distortion, thin prism distortion to decentering distortion for the telecentric imaging system.

As mentioned above, there are few methods proposed for accurately calibrating telecentric lens. To our knowledge, only Opto Engineering® provides a telecentric lens calibration software, CVTClib [11]. According to the information of Opto Engineering®, CVTClib can lower distortion to 0.007% for a telecentric lens with distortion $\leq 0.1\%$, which means the maximum distortion can be reduced about 14 times. However, the measurement accuracy could be further improved about 1 times by our method.

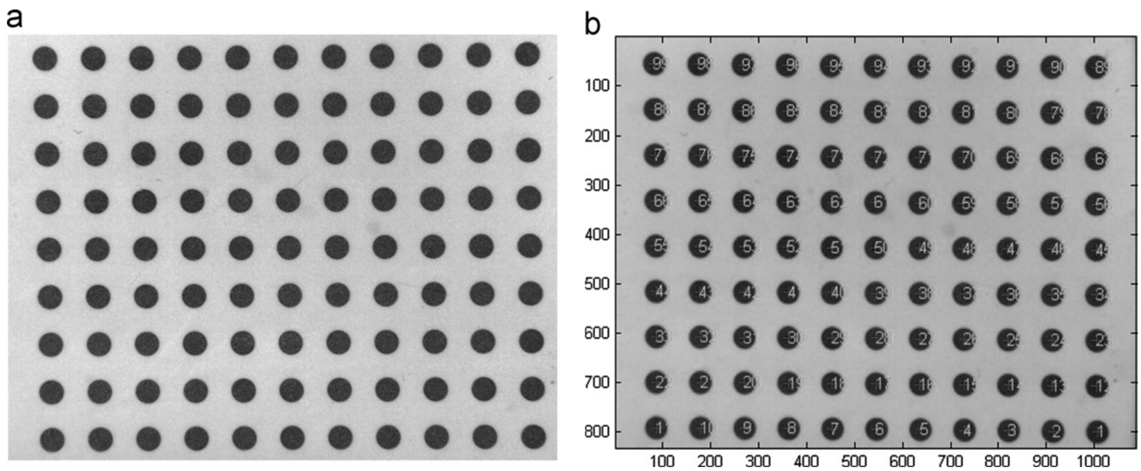

Fig. 3. Target plate with precise circle pattern for calibration. (a) Photo of the plate with 9×11 circles, with separation of 3 mm. (b) centers of circles extracted as the control points.

Table 1
Calibration results with different distortion camera models.

Item	Distortion free	Radial only	Radial and decentering	Radial and thin prism	Radial and decentering and thin prism
Magnification (m)	0.16028	0.15972	0.15976	0.15972	0.15976
Radial distortion [k_1]	0	0.00060	0.00055	0.00061	0.00055
Decentering distortion [p_1,p_2]	[0,0]	[0,0]	[0.00026, −0.00015]	[0,0]	[0.00027, −0.00017]
Thin prism distortion [s_1,s_2]	[0,0]	[0,0]	[0,0]	[−0.00011, 0.00018]	[−0.00011, 0.00018]

Table 2
Measurement accuracy for the telecentric lens with different distortion camera models.

Residual	Distortion free	Radial only	Radial and decentering	Radial and thin prism	Radial and decentering and thin prism
σ (pix., μm)	0.75913, 24.6	0.08763, 2.85	0.08611, 2.80	0.05466, 1.78	0.05184, 1.69
Max (pix., μm)	3.3931, 110	0.2344, 7.6	0.2140, 7.0	0.1352, 4.4	0.1317, 4.3
Distortion(%)	0.3	0.02	0.02	0.01	0.01

5. Conclusion

In conclusion, we propose a calibration method for the telecentric imaging system. Different from the traditional pinhole model, a telecentric lens camera model with the major sources of distortion is presented based on orthographic projection. We improve a two-step approach to accomplish the calibration, which consists of a closed-form solution, followed by a nonlinear refinement. Experimental results prove the effectiveness of the proposed method, which significantly improves the accuracy 15 times and reduces distortion 26 times by calibration. The distortion we need to consider by importance is from radial distortion, thin prism distortion to decentering distortion in telecentric lens calibration and thin prism distortion should not be ignored, which differs in traditional perspective camera.

Acknowledgments

The authors would like to appreciate the financial support from the Natural Science Foundation of China Grant 60805017 and the Natural Science Foundation of SZU (Grant no. 201110).

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