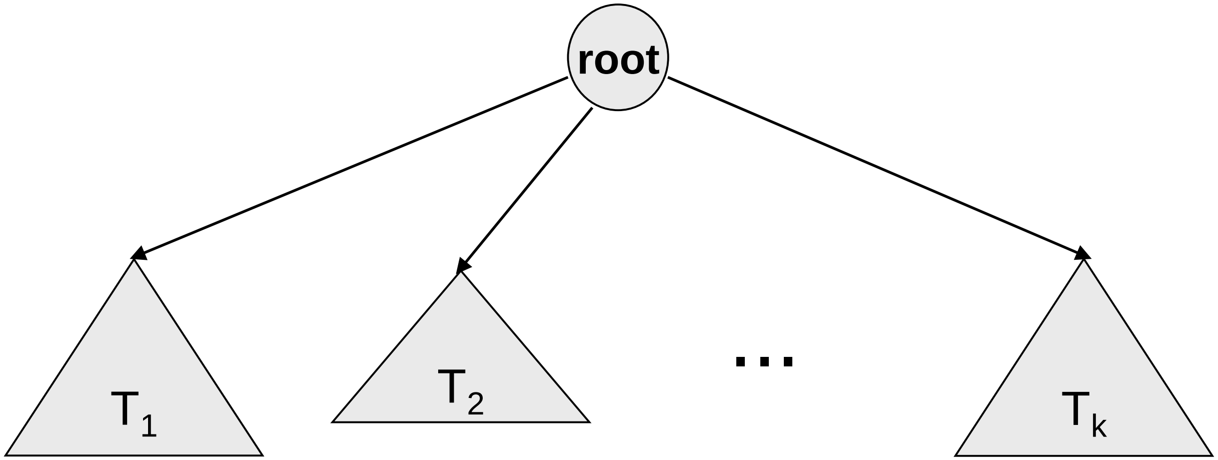
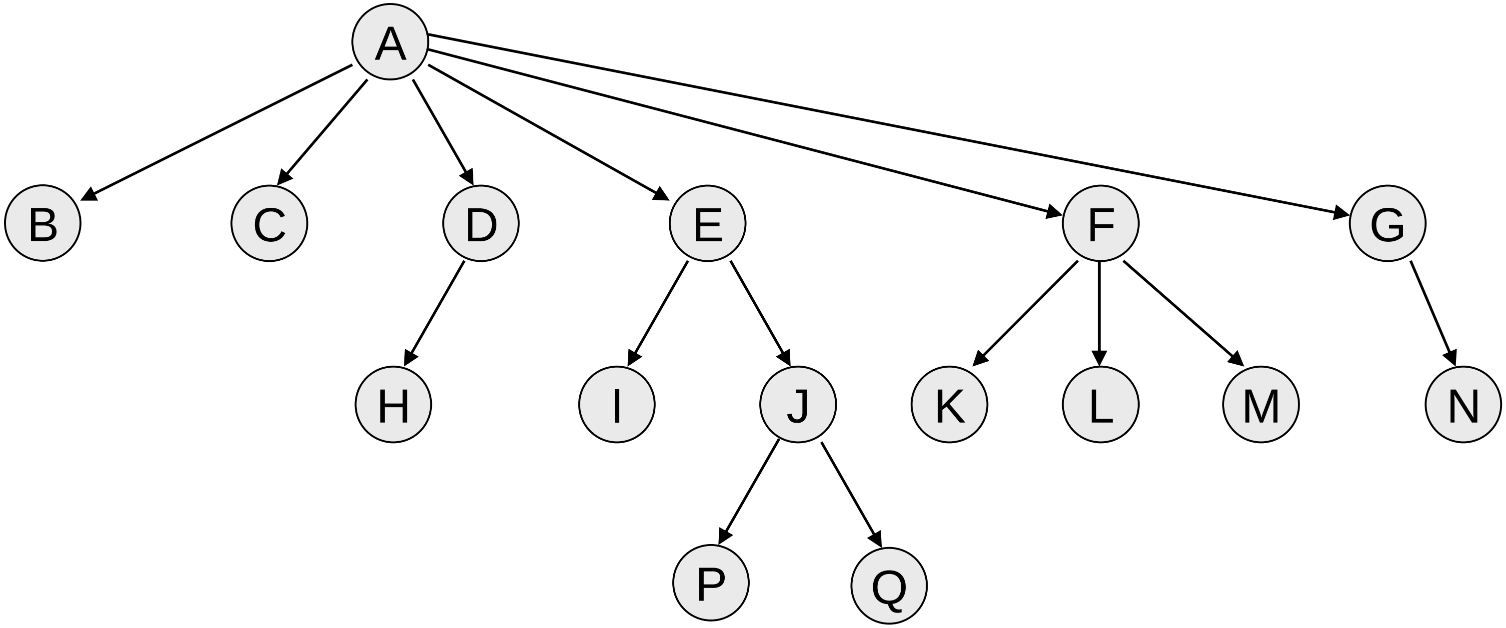
Trees

What is a Tree?

- A tree is a collection of nodes with the following properties:
  - The collection can be empty and the tree is called a null tree.
  - Otherwise, a tree consists of a distinguished node r, called *root*, and the remaining nodes are partitioned into k subsets representing subtrees T_1, T_2, … , T_k, each of whose roots are connected by a *directed edge* from r.
- The root of each sub-tree is said to be *child* of r, and r is the *parent* of each sub-tree root.
- If a tree is a collection of N nodes, then it has N-1 edges.



Preliminaries



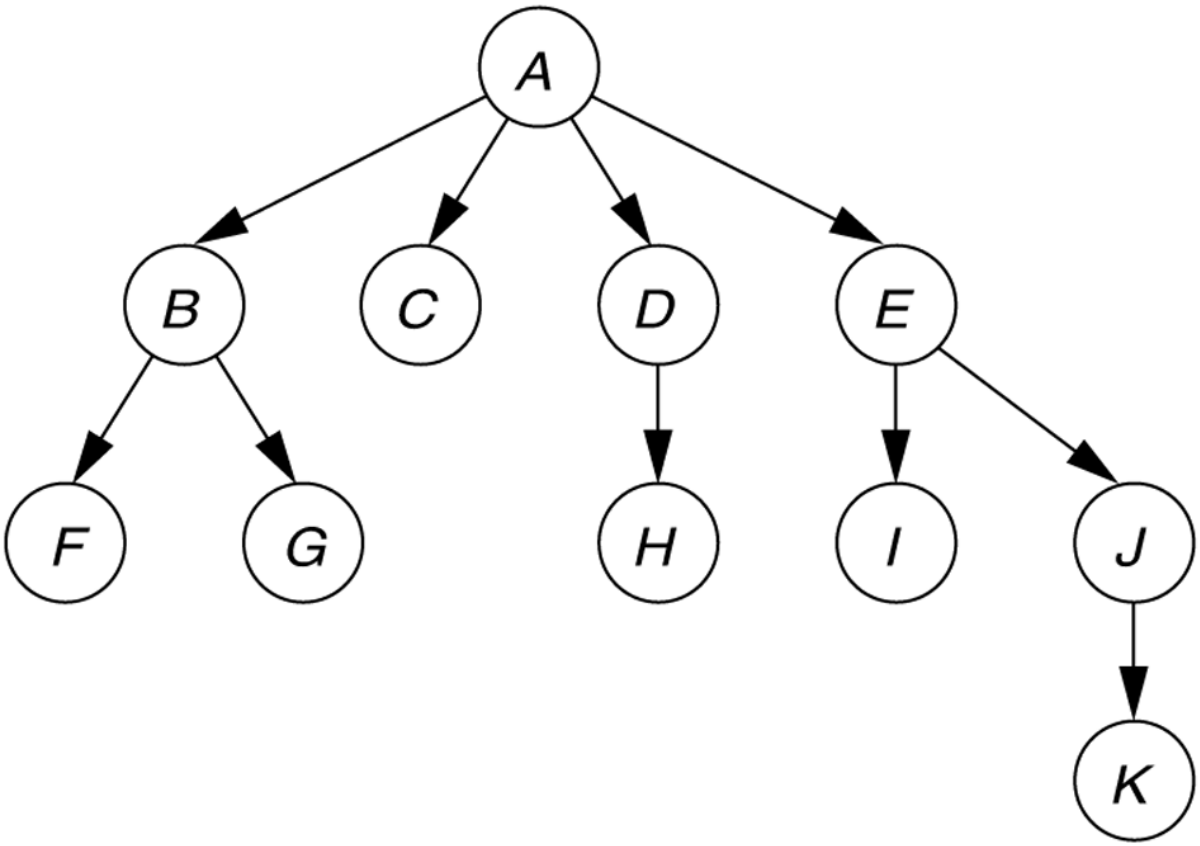
- Node *A* has 6 *children*: B, C, D, E, F, G.
- B, C, H, I, P, Q, K, L, M, N are *leaves* in the tree above.
- K, L, M are *siblings* since F is parent of all of them.

Preliminaries (continued)

- A ***path*** from node n_1 to n_k is defined as a sequence of nodes $n_1, n_2, \ldots, n_k$ such that n_i is parent of n_{i+1} $(1 \leq i < k)$
  - The ***length*** of a path is the number of edges on that path.
  - There is a path of length zero from every node to itself.
  - There is exactly one path from the root to each node.
- The ***depth*** of node n_i is the length of the path from *root* to node n_i
- The ***height*** of node n_i is the length of longest path from node n_i to a *leaf*.
- If there is a path from n_1 to n_2, then n_1 is ***ancestor*** of n_2, and n_2 is ***descendent*** of n_1.
  - If $n_1 \neq n_2$ then n_1 is *proper ancestor* of n_2, and n_2 is *proper descendent* of n_1.

Figure 1

A tree, with height and depth information



Node	Height	Depth
A	3	0
B	1	1
C	0	1
D	1	1
E	2	1
F	0	2
G	0	2
H	0	2
I	0	2
J	1	2
K	0	3