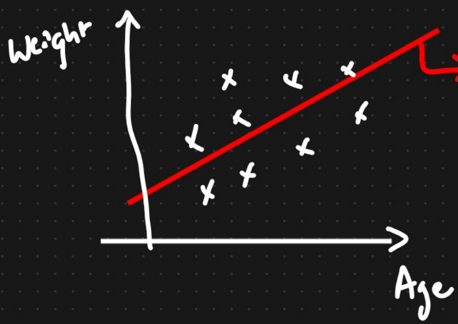
Tutorial 2 - Logistic Regression Maths Intuition

Linear Regression → Regression problem statement

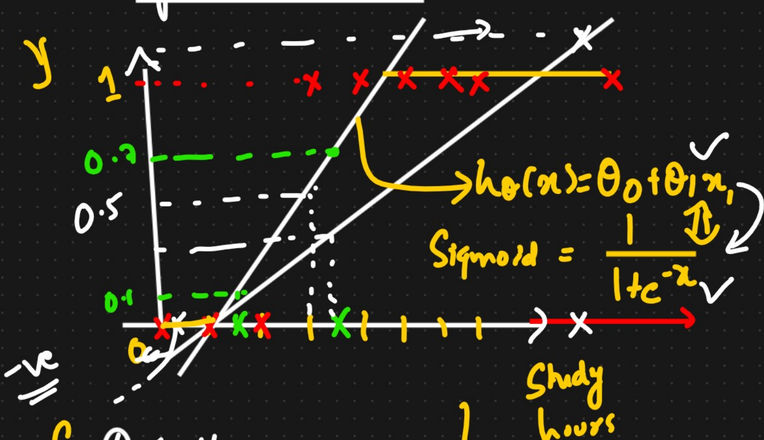


Cost function

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x)^{(i)} - y^{i}\right)^2 \downarrow\downarrow$$

You cannot solve

Logistic Regression → Classification



{ ① Outlier
② >1 or <0 }

Study hours	Pass/Fail
4	Pass
2	Fail
0	Fail
1	Fail
2.5	Pass

O/P → Binary Classification

O/P Classification

0 to 1

$$h_\theta(x) = g(\theta_0 + \theta_1 x) \qquad z = \theta_0 + \theta_1 x$$

$$g = \frac{1}{1+e^{-z}}$$

$h_\theta(x) = \theta_0 + \theta_1 x_1$

Sigmoid

$$h_\theta(x) = \frac{1}{1+e^{-z}} \qquad z = \theta_0 + \theta_1 x_1$$

Logistic Hypothesis function

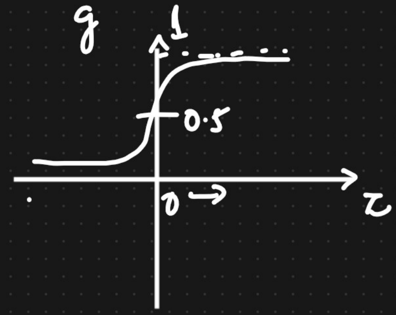
$$\boxed{h_\theta(x) = \frac{1}{1+e^{-(\theta_0 + \theta_1 x_1)}}}$$

$z < 0$

$g(z) < 0.5$

Sigmoid

$$g = \frac{1}{1+e^{-z}}$$



$z \geq 0$
$g(z) \geq 0.5$

Trainset $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \dots (x^{m}, y^{m})\}$

$$x \in \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad y \in \{0, 1\}$$

$$z = \theta_0 + \theta_1 x$$
$$= \theta^T x$$

$$h_\theta(x) = \frac{1}{1+e^{-(\theta_0+\theta_1 x)}}$$

$$h_\theta(x) = \frac{1}{1+e^{-(\theta^T x)}} \Rightarrow \text{Hypothesis Just}$$

Linear Regression

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x)^{(i)} - y^{(i)}\right)^2$$

$h_\theta(x) = \theta^T x$
$= \theta_0 + \theta_1 x$

Logistic Regression

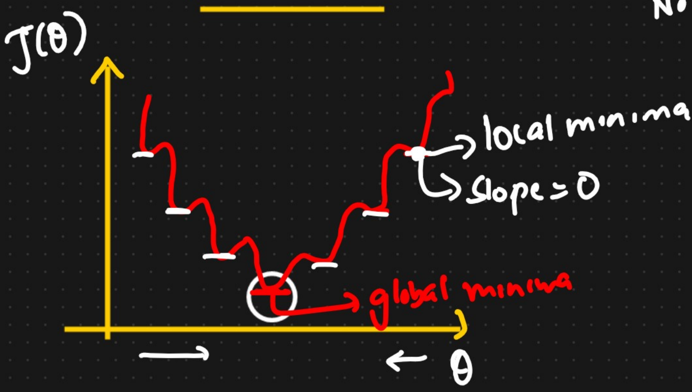
Cost $(h_\theta(x), y)$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \underbrace{\left(h_\theta(x)^{i} - y^{(i)}\right)^2}_{\text{Non convex function}}$$

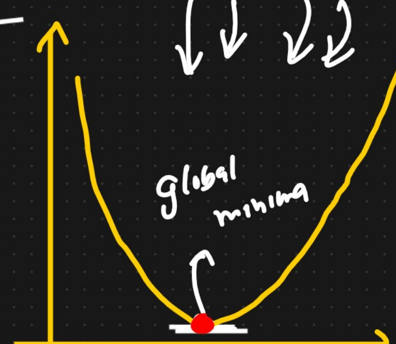
$$h_\theta(x) = \frac{1}{1+e^{-(\theta^T x)}}$$

Convex function

Non Convex



No local minima



$$J(\theta_0,\theta_1)=\frac{1}{2m}\sum_{i=1}^{m}\underbrace{\left(h_\theta(x)^{(i)}-y^{(i)}\right)^2}_{\Downarrow}$$

$$h_\theta(x)^{(i)}=\frac{1}{1+e^{-z}} \qquad z=\theta_0+\theta_1 x$$

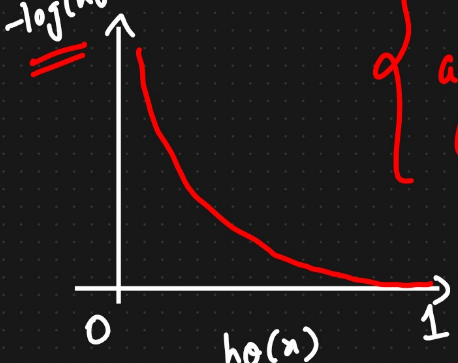
$$\mathrm{Cost}(h_\theta(x^{(i)}),y)$$

$$\mathrm{Cost}(h_\theta(x)^{(i)},y)=\begin{cases}-\log(h_\theta(x)) & \text{if } y=1\\ -\log(1-h_\theta(x)) & \text{if } y=0\end{cases} \Rightarrow \text{Cost function}$$

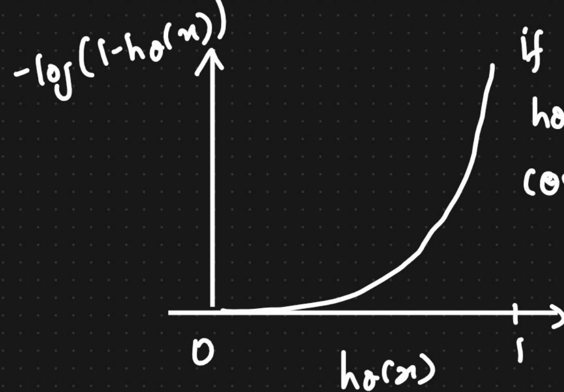
$\Rightarrow$

$\Rightarrow$

if $y=1$



$\left\{\begin{array}{l}\text{if } y=1\\ \text{and } h_\theta(x)=1\\ \text{Cost}=0\end{array}\right\}$



$\left\{\begin{array}{l}\text{if } y=0\\ h_\theta(x)=0\\ \text{cost}=0\end{array}\right\}$

$$\left\{\mathrm{Cost}(h_\theta(x),y)=-y\log(h_\theta(x))-(1-y)\log(1-h_\theta(x))\right\}$$

if $y=1$

$= -\log(h_\theta(x))$

if $y=0$

$= -\log(1-h_\theta(x))$

$$J(\theta_0,\theta_1)=\frac{-1}{2m}\sum_{i=1}^{m}\left(y^{(i)}\log h_\theta(x)^{(i)}+(1-y^{(i)})\log\left(1-h_\theta(x)^{(i)}\right)\right)$$

$\Downarrow$

$\downarrow$

Cost function

$\Downarrow$

Convergence Repeat

{

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

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