Modelling the Change in Acceleration of the Melting Greenland Ice Sheet

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Introduction and Rationale

The idea that we can use math to solve real-world problems fascinates me. Specifically, I marvel at the idea of data analysis as it represents a basis on which humans can attempt to predict the future. Data analytics have been an uprising field that is imperative to studies in a scientific or mathematical field. However, in order to conduct proper data analytics, the presentation of statistics is necessary. Data analytics is said to be the most accurate way for humans to tell the future. The idealistic goal is that society has impeccable data analytics, which allows us to account for unprecedented variables that could impact various trends.

Modelling patterns that occur on our planet is important in predicting how our society needs to plan in order to cope with the modelled representation of what our future could look like. The possibility of demonstrating what a more probable outcome of the future may look like helps us alter our current decisions if we believe that we need to change the current trajectory that we are on.

I selected global warming as an area of exploration as it is a topic that is a key focus for data analysts around the world. This is because global warming is directly correlated with the idea of portraying ongoing trends that could lead to a specific result, therefore attempting to predict the future. A tangible representation of global warming is the rapid melting of ice bodies which lead to a rise in ocean levels.

As a concerned member of society, I chose to utilize my interest in data analysis to model the changing rate of melting in the Greenland Ice Sheet. The Greenland Ice Sheet is the second largest ice body in the world and has enough ice mass to increase global water levels by 7.2

meters, therefore representing why its exponential increase in melting for the past decades, has raised concern.

The melting glaciers and ice sheets tend to include many variables that contribute to their decreasing surface mass. The primary reason being the increase in temperature of their surroundings. However, their surroundings are tougher to measure as one has to account for the air around them, the location, their current size, and the temperature of the water around them. That data is accessible, but I chose to replicate mass loss in an ice sheet versus time because it directly demonstrates how long it may take for the ice sheet to melt.

Therefore, due to my interest in applying math to solving real world problems as well as my interest in data analysis, I use a series of models to represent and analyze the data of the Greenland Ice Sheet's loss in ice mass over a time period of 18 years. In order to do so, I must analyze the curve of various time intervals in the data in order to get a more accurate understanding of the shift in behavior of the ice sheet's melting. My initial belief is that the acceleration of melting will increase as time increases due to the idea there is an increasing global population, therefore expediting the rate of global warming.

Aim and Approach

The aim of this paper is to model the changing behavior of the exponential rate of ice mass loss in the Greenland Ice Sheet. After sorting through the data available and reading various articles, it is my understanding that between the years 1997-2014, the ice sheet melted at an exponential rate. After taking a physics course in 10^{th} grade, I grasped the concept that an exponential increase in an object's distance vs. time curve represents a change in velocity, therefore acceleration. Like this situation, the ice is melting at an exponential rate in an ice mass vs. time graph, therefore representing a changing rate of change, demonstrating acceleration. However, since this is a real-life situation that is impacted by many exterior factors that may hinder or expedite the ice melting process, the acceleration during various time intervals is changing.

The aim is to demonstrate the change in acceleration of the melting of the ice sheet throughout its exponential ice mass loss in order to depict information that can be used to make inferences about the future longevity of the ice sheet's existence.

I split the 18 data points into three separate intervals in order to demonstrate the changing acceleration throughout the intervals: Years 0-5, years 6-11, and years 12-17. I could have separated the data into more intervals in order to increase the accuracy in my analysis of the changing acceleration, however, for the sake of this paper, I felt that utilizing three intervals was feasible as it demonstrates the behavioral change without being redundant.

I utilize quadratic regression in order to demonstrate a best fit curve for each interval. This will allow me to demonstrate the behavior of the melting ice for each interval through producing a trend line from which I can calculate its rate of change and acceleration. The trend line will be plotted on a graph with x-values, indicating years after 1997, and y-values indicating the cumulative ice mass loss of the Greenland Ice Sheet in gigatons. I utilize excel to depict the graphs of the regression as well as to compute the data points.

To demonstrate the accuracy of the regression curve that I calculated, I use the r^2 value of the curve. The r^2 value tells me the amount of variance from the original data points and the points produced by the regression curve through producing a number between 0 and 1. The closer the number is to 1, the higher the accuracy of the regression curve.

In order to calculate the rate of change for the data points, I calculate the derivative of the function for the interval, then plot the graph using Microsoft Excel. Since I am calculating the quadratic regression of the intervals, the derivative of the function will be in the form of $y=mx + b$. From there, I calculate the second derivative of the function through finding the slope of the derivative in order to find the acceleration of that interval. After doing the same process for all three intervals, I can now compare the differing accelerations and analyze how the accelerations have changed throughout the regression curves for each of the intervals.

Data Collection

I have collected data on the Greenland Ice Sheet's loss of ice mass from the European Environment Agency. They are constantly updating their data collection, and as I am writing this paper, the data they had available was up till 2014. They measure both the Greenland and Antarctic Ice sheet and provide a series of graphs and intricate data sets that represent how these ice bodies have melted over the decades. The Greenland Ice Sheet is the fastest melting ice sheet, in comparison to the Antarctic Ice sheet, therefore making it the largest contributor to the rise in ocean levels.

The data originally was provided in the form of multiple intervals within each year, but for the clarity of this paper, I chose to take annual data points, which represent the cumulative ice mass loss. The reason I chose 1997 as the starting point, is because that was the first full year where the ice mass gained was negative, indicating that this is when the loss in ice mass started. The amount shown for the y-value of 2014, is the cumulative amount of ice mass that was lost up until 2014. Therefore, the difference in between each y-value represents the annual ice mass loss.

The graphs I chose to represent the data have an x-axis that represents the years since 1997, and a y-axis that represents the cumulative amount, in gigatons, of ice mass that has been lost from the Greenland Ice Sheet.

Number of years since 1997	Amount of Ice Mass Loss from Greenland Ice Sheet (Gigatons)
0	41.76
1	92.47
2	162.57
3	248.32
4	337.41
5	439.6
6	587.68
7	785.42
8	998.58
9	1219.77
10	1447.37
11	1679.35
12	1924.41
13	2207.87
14	2515.69
15	2822.92
16	3125.18
17	3400.29

Table 1 – Cumulative Ice Mass Loss in Greenland Ice Sheet (gigatons) from 1997-2014.

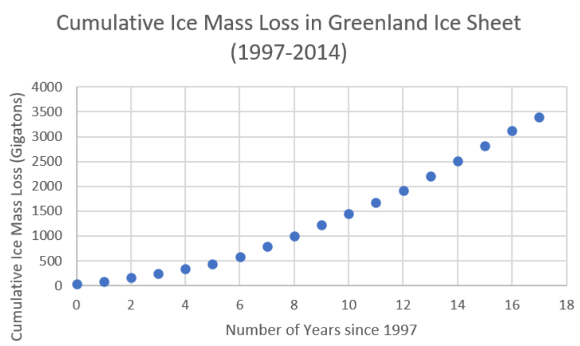

Figure 1 – Plot of data of cumulative ice mass loss of Greenland Ice Sheet from 1997-2014

Tools

Exponential Function

By looking at the curving shape of the line that connected the data points, I inferred that the trend line would be exponential. For a set of values to be exponential, the slope between different sets of points in the graph must vary. In order to see if it is exponential, I must utilize the formula to calculate the slope between the three intervals of points I have selected.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Year 0: (0, 41.76)

Year 5: (5, 439.6)

$$\frac{439.6 - 41.76}{5 - 0} = 79.57$$

Year 6: (6, 587.68)

Year 11: (11, 1,679.35)

$$\frac{1{,}679.35 - 587.68}{11 - 6} = 218.334$$

Year 12: (12, 1924.41)

Year 17: (17, 3400.29)

$$\frac{3400.29 - 1924.41}{17 - 12} = 295.18$$

$$79.57 \ < 218.334 \ < 295.18$$

Since the slopes of each interval are increasing, that inclined me to believe that the speed at which the ice is melting is accelerating through each interval. Although I can get a rough estimate of the acceleration of the melting ice through averaging out the difference in slopes, that measure would not indicate the changing behavior of the acceleration.

R^2 Measure

Pearson's correlation coefficient, r, is used to determine the strength of correlation between two variables. Although important, in this situation it is known that the ice is melting over time, demonstrating that there will already be a strong positive correlation. Therefore, my focus shifts to finding best fit lines that represent the rate of change of the rapidly melting ice sheet. R^2 is used to depict the variation between the best fit line and the original data points. As someone who finds data analysis interesting, I find this helpful as it demonstrates which trend lines are correct representations of past data, allowing one to predict the future in a more accurate manner. I must utilize this formula in order to check the amount of variance in my trend curves.

$$r^2 = \left(\frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}\right)^2$$

An r^2 value closer to 1 represents a more accurate fit of the data points as it demonstrates that the regression that was used encompasses most or all of the data points that were inputted.

Quadratic Regression

Since I have deduced that the trend is exponential, quadratic regression is suitable for my investigation as it attempts to create an accurate best fit for all data points available through implementing a quadratic function. This formula allows the creation of trend curves that can be used to predict data that is not included in the dataset by providing the best fit line for all the points.

$$f(x) = ax^2 + bx + c$$

The use of specifically quadratic regression attempts to create a polynomial function to the degree of 2. If the degree was increased, then the accuracy of the best fit line from the regression formula would be increased. If the r^2 values depict that the best fit lines have high variance from the data points, then I may be inclined to use regression to a higher degree in order to increase accuracy.

The importance of finding a best fit line in the context of modelling the melting Greenland Ice Sheet is important because I can calculate the rate of change in the best fit line in order to gain a sense of the behavior of the curve. Distinguishing the behavior of the curves will help me analyze whether the rate of change in the ice sheet is slowing down or speeding up, therefore demonstrating if global warming is accelerating or decelerating the melting process of the ice sheet.

In order to find the variables necessary for quadratic regression, I utilize this system of equations.

$$a\sum x^4 + b\sum x^3 + c\sum x^2 = \sum x^2 y$$
$$a\sum x^3 + b\sum x^2 + c\sum x = \sum xy$$
$$a\sum x^2 + b\sum x + cn = \sum y$$

(n = number of data points)

The Power Rule – Calculating Derivatives and Second Derivatives

I have demonstrated that this graph is exponential by showing the changing slope throughout different intervals. Since it is exponential, that means that the rate of change is accelerating or decelerating, therefore demonstrating a changing rate of change. Derivatives are defined as $\frac{dy}{dx}$ meaning the instantaneous rate of change. Since we know that the graph is exponential, the change is not constant and is accelerating at a certain amount. In order to calculate the acceleration of the ice melting, I must first calculate the rate of change through calculating the derivative of the whole graph, then calculate the derivative of the derivative to see how fast the change is changing. Once I calculate the second derivative for all three intervals, I can see how the behavior of the acceleration is changing throughout the graph. In order to calculate the derivative, I use the power rule.

$$f'(x) = \frac{dy}{dx}x^n = n * x^{n-1}$$

Once I find the second derivative for each interval, I can compare the accelerations of each of the intervals. This will demonstrate how the behavior of the melting in the Greenland Ice Sheet has changed throughout the years.

Process and Results

Years 0-5

Year Number	Cumulative Ice Mass Loss (Gigatons)
0	41.76
1	92.47
2	162.57
3	248.32
4	337.41
5	439.6

Table 2 – Years 0-5 of Table 1, representing f(x)

$\sum x$	$\sum y$	$\sum x^2$	$\sum x^4$	$\sum xy$	$\sum x^2y$	$\sum x^3$	$\sum y^2$	n
15	1322.13	55	979	4710.21	19366.19	225	405480.1	6

Table 3 – Coefficients used to calculate regression and r^2 for f(x)

$$979a + 225b + 55c = 19366.19$$

$$225a + 55b + 15c = 4710.21$$

$$55a + 15b + 6c = 1322.13$$

$$a = 5.95$$

$$b = 50.51$$

$$c = 39.5$$

$$f(x) = 5.95x^2 + 50.5x + 39.5$$

This represents the quadratic regression for the years 0-5.

$$r^2 = \left(\frac{6(4710.21)-(15)(1322.13)}{\sqrt{[6(55)-(15)^2][6(405480.1)-(1322.13)^2]}}\right)^2 = 1.00$$

The r^2 value of 1 represents assurance in the regression line that was calculated, therefore not representing a need for me to calculate further degrees of regression.

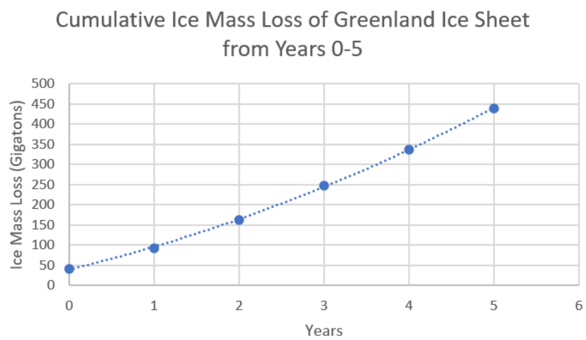

Figure 2 – Plot of f(x) with original data points

Now in order to find the rate of change in the interval, I must find the derivative of f(x).

$$f'(x)= 2 * 5.95x^{2-1} + 1 * 50.5x^{1-1} + 0$$

$$f'(x) = 11.9x + 50.5$$

This represents the rate of change for the interval of years 0-5. The positive linear equation that the derivative produces represents the increasing speed of the ice melting through the years 0-5.

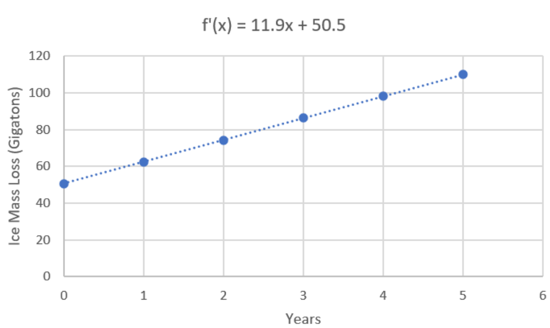

Figure 3 – Plot of f'(x) with original x-values

Now in order to find the change in the rate of change, also known as acceleration, I must find the second derivative of f(x).

$$f''(x) = 1 * 11.9x^{1-1} + 0$$

$$f''(x) = 11.9$$

This represents the acceleration of the amount of ice that melted per year in the Greenland Ice Sheet from years 0-5. The positive value of 11.9 represents that the ice sheet is accelerating positively.

Years 6-11

Year Number	Cumulative Ice Mass Loss (Gigatons)
6	587.68
7	785.42
8	998.58
9	1219.77
10	1447.37
11	1679.35

Table 4 - Years 6-11 of Table 1, representing g(x)

$\sum x$	$\sum y$	$\sum x^2$	$\sum x^4$	$\sum xy$	$\sum x^2y$	$\sum x^3$	$\sum y^2$	n
51	6718.17	451	38995	60937.14	570290.9	4131	8362350	6

Table 5 - Coefficients used to calculate regression and r^2 for g(x)

$$38995a + 4131b + 451c = 570290.9$$

$$4131a + 451b + 51c = 60937.14$$

$$451a + 51b + 6c = 6718.17$$

$$a = 4.09$$

$$b = 149.51$$

$$c = -458.43$$

$$g(x) = 4.09x^2 + 149.51x - 458.43$$

This represents the quadratic regression for the years 6-11.

$$r^2 = \left(\frac{6(60937.14)-(51)(6718.17)}{\sqrt{[6(451)-(51)^2][6(8362350)-(6718.17)^2]}}\right)^2 = 1.00$$

The r^2 value of 1, represents assurance in the accuracy of the regression line that was calculated, therefore not representing a need for me to calculate further degrees of regression.

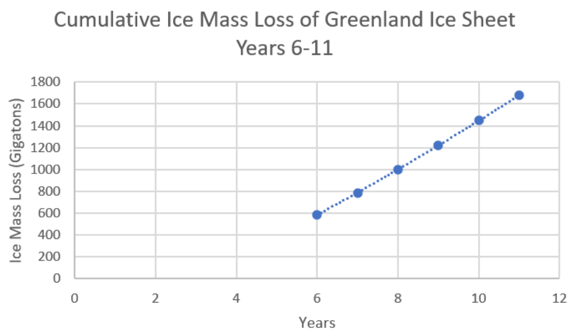

Figure 4 - Plot of g(x) with original data points

Now in order to find the rate of change in the interval, I must find the derivative of g(x).

$$g'(x)= 2 * 4.09x^{2-1} + 1 * 149.51x^{1-1} + 0$$

$$g'(x) = 8.18x + 149.51$$

This represents the rate of change for the interval of years 6-11. The positive linear equation that the derivative produces represents the increasing speed of the ice melting through the years 6-11.

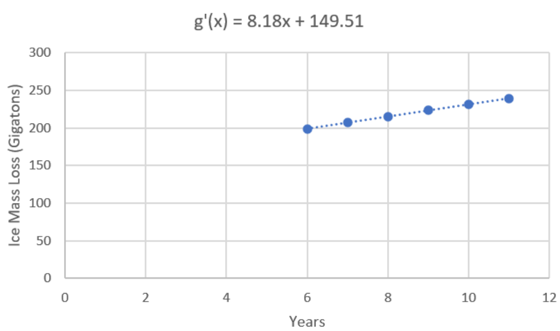

Figure 5 - Plot of g'(x) with original x-values

Now in order to find the change in the rate of change, also known as acceleration, I must find the second derivative of g(x).

$$g''(x) = 1 * 8.18x^{1-1} + 0$$

$$g''(x) = 8.18$$

This represents the acceleration of the amount of ice that melted per year in the Greenland Ice Sheet from years 6-11. The positive value of 8.18 represents that the ice sheet is accelerating positively. However, this is a decrease in acceleration from the previous interval's acceleration of 11.9. The deceleration between intervals disproves my initial belief that there was going to be an increasing acceleration between each interval.

Years 12-17

Year Number	Cumulative Ice Mass Loss (Gigatons)
12	1924.41
13	2207.87
14	2515.69
15	2822.92
16	3125.18
17	3400.29

Table 6 - Years 12-17 of Table 1, representing h(x)

$\sum x$	$\sum y$	$\sum x^2$	$\sum x^4$	$\sum xy$	$\sum x^2y$	$\sum x^3$	$\sum y^2$	n
87	15996.36	1279	287395	237166.5	3561207	19053	44204339.4	6

Table 7 - Coefficients used to calculate regression and r^2 for h(x)

$$287395a + 19053b + 1279c = 3561207$$

$$19053a + 1279b + 87c = 237166.5$$

$$1279a + 87b + 6c = 15996.36$$

$$a = -1.14$$

$$b = 331.54$$

$$c = 1896.5$$

$$h(x) = -1.14x^2 + 331.54x + 1896.5$$

This represents the quadratic regression for the years 12-17. The negative *a* value of -1.14 is a change from the other two positive values of *a* from the other intervals. This represents a quadratic function where the parabola is going to be faced down.

$$r^2 = \left(\frac{6(237166.5)-(87)(15996.36)}{\sqrt{[6(1279)-(87)^2][6(44204339.4)-(15596.36)^2]}}\right)^2 = 1.00$$

The r^2 value of 1 represents that there is a high amount of confidence in the regression line that was calculated, therefore not representing a need for me to calculate further degrees of regression.

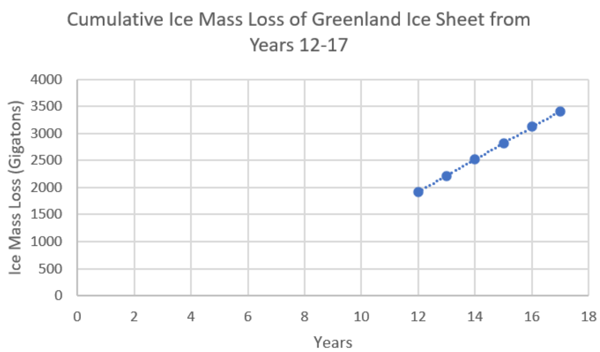

Figure 6 - Plot of h(x) with original data points

Now in order to find the rate of change in the interval, I must find the derivative of h(x).

$$h'(x)= 2 * -1.14x^{2-1} + 1 * 331.541x^{1-1} + 0$$

$$h'(x) = -2.28x + 331.54x$$

This represents the rate of change for the interval of years 12-17. The negative linear equation that the derivative produces represents the decreasing speed of the ice melting through the years 12-17. This indicates that the total rate of the ice melting is slowing down in this interval.

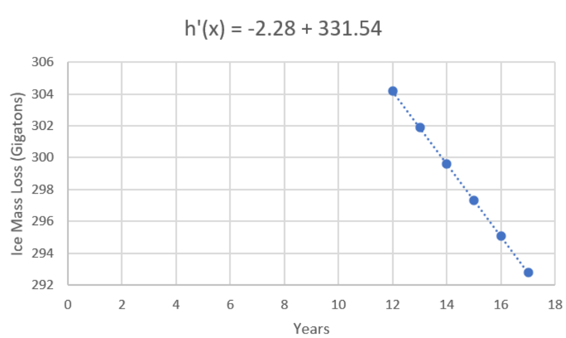

Figure 7 - Plot of h'(x) with original x-values

Now in order to find the change in the rate of change, also known as acceleration, I must find the second derivative of h(x).

$$h''(x) = 1 * -2.28x^{1-1} + 0$$

$$h''(x) = -2.28$$

This represents the acceleration of the amount of ice that melted per year in the Greenland Ice Sheet from years 12-17. The negative value of the second derivative of h(x) represents that the interval is part of a parabola that is decelerating. This value is also less than g(x)'s second derivative, representing a decline in acceleration between the two intervals. This means that according to the intervals that I have chosen, the following years will represent a deceleration in ice mass loss. This does not necessarily mean that the ice is going to gain mass, however, it most likely means that the rate of change is slowing down, disproving my initial belief that the ice mass loss is accelerating throughout the intervals.

Results

$$f''(x) = 11.9$$

$$g''(x) = 8.18$$

$$h''(x) = -2.28$$

$$f''(x) > g''(x) > h''(x)$$

The decrease in each of the interval's second derivative demonstrates that the rate of change is slowing down. Although the melting of the ice is experiencing an exponential increase, that increase is not constant as this concludes that the rate of change in the melting of the Greenland Ice Sheet is slowing down.

Reflection

The aim of my investigation was to depict the changing behavior of the Greenland Ice Sheet's melting process so that we can get a further understanding of the future longevity of the ice sheet's existence. The melting of the Greenland Ice Sheet is an extremely important event that is occurring in the world as its melting can contribute to a significant rise in ocean levels, contributing to the destruction of various pieces of land. Therefore, the melting of the Greenland Ice Sheet is a direct indicator of one of the largest problems the world faces today, global warming. As someone that has an interest in data analysis, this investigation was particularly interesting to me as it provided me with the experience of undergoing the manipulation of data in order to create trends that help the world become aware of large-scale problems.

Since the data points represent an exponential curve, I utilized quadratic regression for all three of the intervals that I had chosen in order to produce a trend curve that I could analyze. The accuracy of the curve was measured through the use of r^2 which computes a value that represents how variant the original data is from the predicted data. If the accuracy was low, I was planning on using a higher degree of regression as that is known to produce a higher level of accuracy. Luckily, all my quadratic regression curves approximately produced an r^2 value of 1, depicting no variance from the original data.

After hearing various panics about the global warming crisis, I was inclined to believe that the Greenland Ice Sheet's rate of melting was accelerating rapidly. However, after conducting this investigation, my inclination was proved wrong.

The second derivative of each of the interval's functions represent the change in the rate of change of their respective quadratic functions. That means that it represents the acceleration throughout the function. Based off my intuition, I had believed that the acceleration between intervals was going to increase, meaning that the rate of change was increasing. However, the reality of the interval's accelerations was that they decreased every interval. Years 1997-2002 represented a period that had the most acceleration in its ice melting throughout all three 6-year intervals. Years 2003-2008 followed as it had the next greatest acceleration. And finally, to my surprise, years 2009-2014 represented a deceleration as its second derivative was a negative value, meaning that the ice melting had reached a plateau, or was on the trend to gain back ice mass.

The decrease in acceleration between each 6-year interval is surprising to me as it is known that in recent years, the ice melting is accelerating due to the increase in carbon emissions. Although the deceleration of the ice melting rate is a good sign for the world, we cannot get our hopes up because there are multiple factors that could break the ice melting function out of its trend, indicating why it is significant to represent the accelerating or decelerating behavior of the Greenland Ice Sheet.

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Appendix

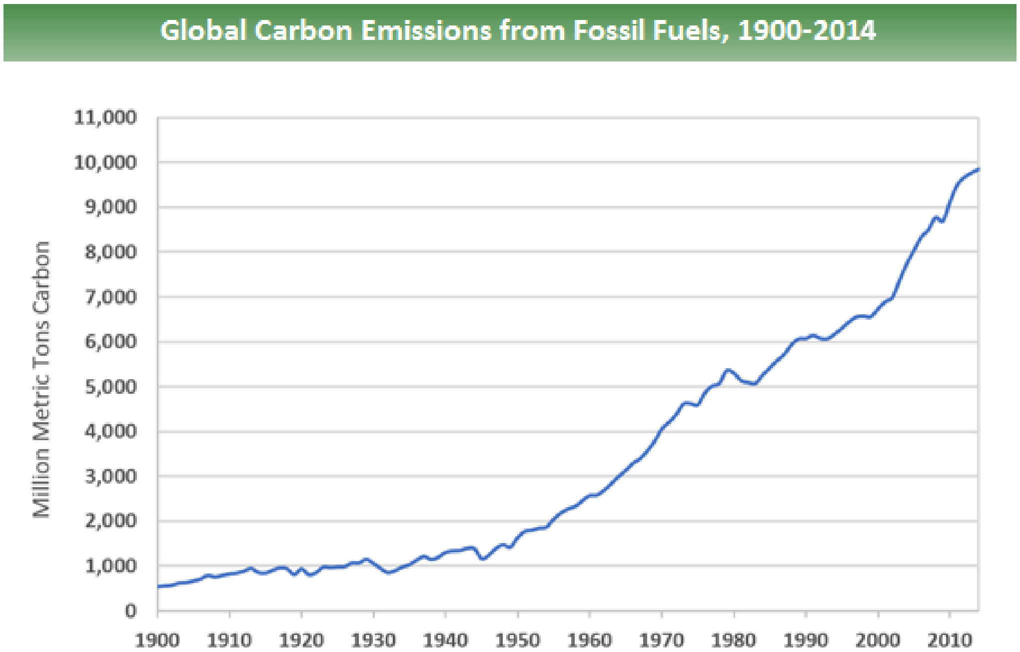

Figure 8 – Represents exponential increase in global carbon emissions from 1900-2014