key ideas:
- Current Stellar Composition
- reaction rates/cross-sections for key reaction pathways
- effective Simulation techniques

1.2 Nomenclature

→ Z: atomic/proton number
N: neutron number
A: nucleon/mass number

$$A = Z + N$$

→ same Z, same N, same nuclear property.

→ symbol: ${}^{A}_{Z}X_{N}$ (where X is the element symbol)

→ A distinct kind of atom, defined by specific atomic and neutron number, is called a nuclide.

nuclide with → same A, different Z and N → isobars

different A, same Z → isotopes

different A, same N → isotones

→ chart of the nuclides

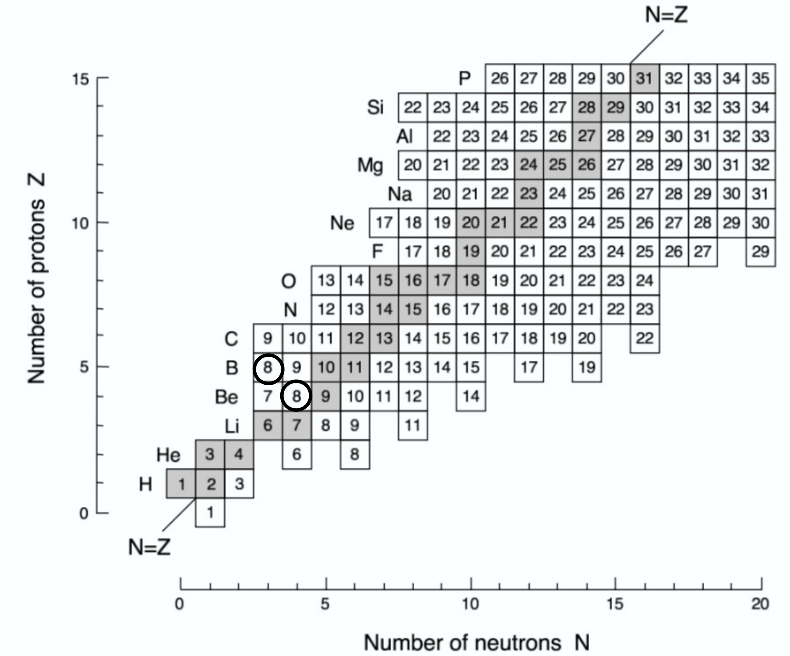


Fig. 1.1 Section of the chart of the nuclides, showing the lightest species with $Z \leq 15$ and $N \leq 20$. The shaded squares represent stable nuclides, while the open squares correspond to unstable nuclides with half-lives in excess of 1 ms. The only exceptions are the nuclides ^{8}Be and ^{9}B which have much shorter half-lives. Note that no stable nuclides exist with a mass number of $A = 5$ or 8.

→ shaded box are considered has two half lives.

1.3 **Solar system abundances**

→ Solar system formed from the collapse of a gaseous nebula that has uniform chemical and isotopic distribution.

→ **Method 1**: Solar photosphere → outest layer of a star, 5800K, most light can escape.

Method 2: Primitive meteorites (CI carbonaceous chondrites)

(These two methods give result with uncertainties less than 10% for the most elements)

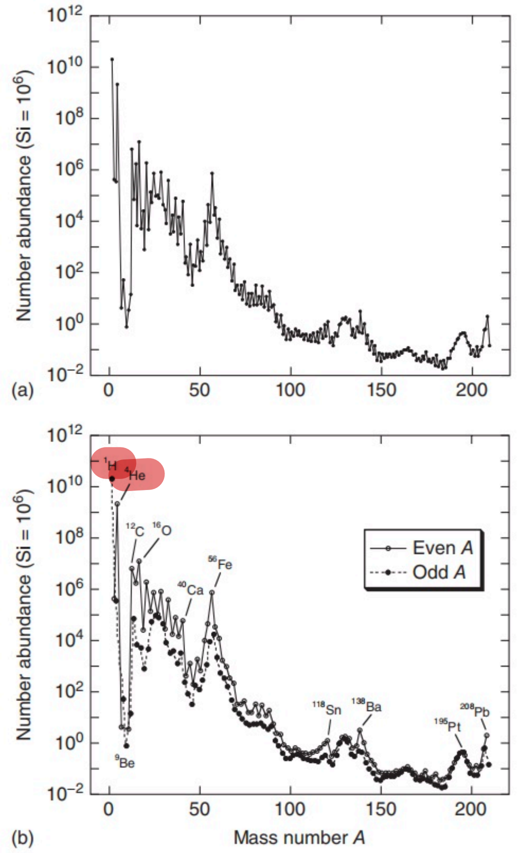


Figure 1.2 Abundances of the nuclides in the solar system at its birth. Number abundances are normalized to the number of silicon atoms (Si = 10^6). Data from Lodders (2003). (a) Sum of all nuclide abundances at a given value of A versus mass number. The maximum in the A = 50–65 region is referred to as the *iron peak*. (b) Separate abundance contributions from nuclides with an even or an odd value of A versus mass number. Even-A nuclides are in general more abundant than odd-A nuclides.

→ normalised (Si: 10^6)

→ ^{1}H (71.1%)
^{4}He (27.4%)
^{12}C + ^{16}O (1.5%)

→ first two minimum:
A = 5–11 : Li, Be, B
A = 41–49 : Sc

→ iron peak: A = 50–65.

→ before iron, fusion. 吸/放
after iron, fission. 裂

→ generally decreasing abundances as A increases.

→ significance: Since almost all nuclides are formed within the star, therefore analysing the solar system abundance offers powerful clues for stellar history and evolution.

1.5 **Nuclear mass and Binding energy**

$$\boxed{m_{nuc} = Zm_p + Nm_n - \Delta m}$$

'nuclear mass'

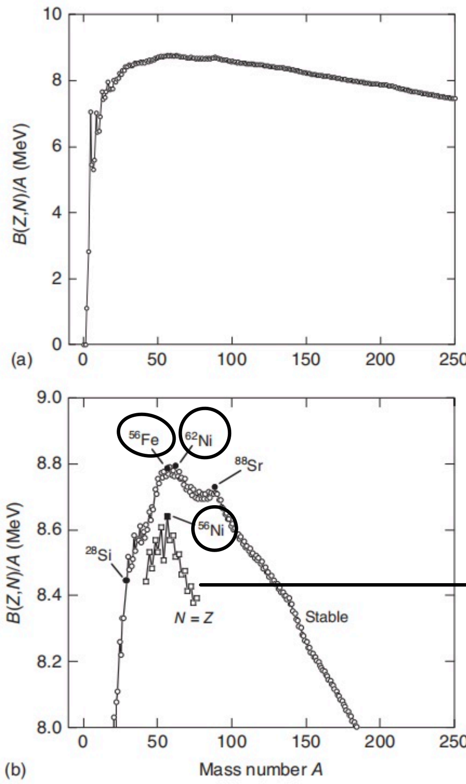
mass defects ($\Delta E = \Delta m c^2$)

'Binding energy'

(It's the energy released in assembling a given nucleus from its constituent nucleons)

'or say it in the reverse way.'

$$\boxed{\Delta E = B(Z,N) = (Zm_p + Nm_n - \Delta m)c^2}$$



squares are all radioactive.

Figure 1.9 Experimental binding energy per nucleon, $B(Z,N)/A$, versus mass number, A. (a) Value of $B(Z,N)/A$ for the most tightly bound nuclide at a given mass number A. (b) Expanded section; the round symbols have the same meaning as in part (a), that is, they correspond to the most tightly bound nuclide of a given mass number; the nuclide with the largest binding energy per nucleon is ^{62}Ni ($B/A = 8.795$ MeV); the square symbols show $B(Z,N)/A$ values for $N = Z$ nuclides above $A = 40$, which are all radioactive; the $N = Z$ species with the largest binding energy per nucleon is ^{56}Ni ($B/A = 8.643$ MeV). Data from Wang *et al.* (2012).

Energetics of Nuclear reactions

→ Commonly, we will encounter reactions:

$0 + 1 \longrightarrow 2 + 3$ (or noted as $0(1,2)3$)

If 0,1 are identical to 2,3, then it is elastic/inelastic scattering.

If light is involved in the reactants, then it is called photodisintegration reaction

light is involved in the reactants products, then it is called reactive capture reaction

2.1 Cross Sections

→ Def: It's a quantitative measure of the probability that an interaction will occur.

(effective interaction area of a incident and target particles)

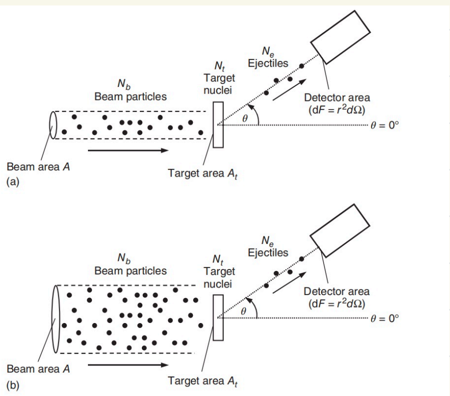


Figure 2.1 Typical nuclear physics counting experiment, showing a beam of $\mathcal{N}_b$ particles per unit time, $\mathcal{N}_t$ nonoverlapping target nuclei within the beam area A, $\mathcal{N}_e$ interaction products and a detector of area dF. The detector is located at an angle of θ with respect to the incident beam direction. The two situations are as follows: (a) the target area is larger than the beam area; and (b) the beam area is larger than the target area.

Assumptions here: $N_e = \underline{N_R}$ ⟶ reaction number.

⟶ say | if emitted product = incident product ⟶ elastic scattering

| if emitted product = incident product ⟶ reaction.

$[\sigma] = m^2$

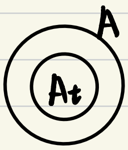
$$\sigma = \frac{\text{number of interaction per time}}{(\text{number of incident particle per area per time})(\text{number of target nuclei within the beam})}$$

$$= \frac{N_R/t}{N_b/(tA)\cdot N_t}$$

① If $A > A_t$

$$N_R/t = \frac{N_b N_t}{t A_t}\sigma$$

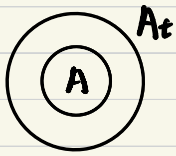
$$\sigma = \frac{N_R A_t}{N_b N_t}$$



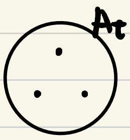
② If $A < A_t$

$$N_R/t = \frac{N_b N_t}{tA}\sigma$$

$$\sigma = \frac{N_R A}{N_b N_t}$$



Plus, for a homogeneous target.



→ "Distributed evenly" $\frac{N_t}{A_t}$ is easy to obtain.

Then, for the emitted particles (assumed we used $A<A_t$ case)

$$\frac{N_R}{t} = \frac{N_e}{t}$$

$$= \frac{N_b N_t}{tA}\sigma$$

Then, $$\frac{N_e(d\Omega)}{t} = \frac{N_b N_t}{tA} d\sigma$$

$$= \frac{N_b N_t}{tA}\left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

So, $$\frac{d\sigma}{d\Omega} = \frac{N_e(d\Omega) A}{N_b N_t d\Omega}$$

Also define $N_{et} = \frac{N_e}{N_t}$.

then we can obtain: $$\frac{d\sigma}{d\Omega} = \frac{N_{et}(d\Omega) A}{N_b d\Omega}$$

$$= \frac{N_{et}(d\Omega)/t}{(N_b/t)(1/A)}\frac{1}{d\Omega}$$

Plus we know current density.

$$j_b = \frac{N_b}{tA}$$

$$j_{et} = \frac{N_{et}(d\Omega)}{t\,d\Omega} = \frac{N_{et}(d\Omega)}{dF} \quad \left(dF = d\Omega\, r^2,\ d\Omega = \frac{dF}{r^2}\right)$$

One can finally gets: $\sigma = \frac{N_{et}/t}{j_b}$ $\quad \frac{d\sigma}{d\Omega} = \frac{j_{et} r^2}{j_b}$ $\quad \boxed{\sigma = \int\left(\frac{d\sigma}{d\Omega}\right)d\Omega}$

3.1 Cross section and reaction rate

$\longrightarrow$ reaction rate:

$$\text{reaction rate} = \frac{N_R}{t\,V}$$

$$= \frac{\sigma N_b N_t}{A\,t\,V}$$

$$= \sigma \frac{N_t}{V} \frac{N_b}{A \cdot t}$$

$$\text{reaction rate} = \sigma v \left(\frac{N_t}{V}\right)\left(\frac{N_b}{V}\right)$$

where v is the relative velocity of interacting particles.

$\longrightarrow$ particle-induced reactions.

Consider the reaction: $0 + 1 \longrightarrow 2 + 3$

where neither 4 particles is photon. (all particles have rest mass)

$$r_{01} = \sigma(v)\, v\, N_0 N_1$$

where $N_0 \equiv \frac{N_t}{V}$, $N_1 \equiv \frac{N_b}{V}$, named as number densities of interacting particles.

For thermodynamical equillibrium, v is not constant.

But we have $P(v)dv$ in the small range.

with $\int_0^\infty P(v)dv = 1$ to normalise.

$\longrightarrow$ non-identical

So, $\langle r_{01} \rangle = N_0 N_1 \int_0^\infty v P(v) \sigma(v) dv = N_0 N_1 \langle \sigma v \rangle_{01}$

(where $N_0 N_1$ are non-identical number density, and $N_0 N_1$ are total number of pairs)

$\longrightarrow$ identical

$$\langle r_{01} \rangle = \frac{N_0(N_0-1)}{2} \xrightarrow[\text{large}]{N_0 \text{ is}} \frac{N_0^2}{2}$$

In sum up: we obtain

$$\langle r_{01} \rangle = \frac{N_0 N_1 \langle \sigma v \rangle_{01}}{1 + \delta_{01}}$$

Now we made two assumptions:
1. the velocity distribution follows the Maxwell-Boltzmann distribution.
2. we used the center-of-mass frame.

So we have: $P(v)dv = \left(\frac{m_{01}}{2\pi kT}\right)^{\frac{3}{2}} e^{-m_{01} v^2/2kT}\, 4\pi v^2 dv$

(where $m_{01} = \frac{m_0 m_1}{(m_0 + m_1)}$)

Then we wanna express it in terms of energy.
we have: $E = \frac{m_{01} v^2}{2}$

$$\frac{dE}{dv} = m_{01} v$$ (we want to energy distribution)

$$P(v)dv = P(E)dE$$
$$= \left(\frac{m_{01}}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{E}{kT}} 4\pi \frac{2E}{m_{01}} \frac{dE}{m_{01}} \sqrt{\frac{m_{01}}{2E}}$$

$$P(E)dE = \frac{2}{\sqrt{\pi}} \frac{1}{(kT)^{3/2}} \sqrt{E}\, e^{-\frac{E}{kT}} dE$$

Then $\langle \sigma v \rangle_{01} = \int_0^\infty v P(E) dE$
$$= \left(\frac{8}{\pi m_{01}}\right)^{\frac{1}{2}} \frac{1}{(kT)^{3/2}} \int E\, \sigma(E)\, e^{-\frac{E}{kT}} dE$$

Substitute this back to r_{01} will result in the rate of reaction.

$\longrightarrow$ other types of reactions, see details back in textbook...