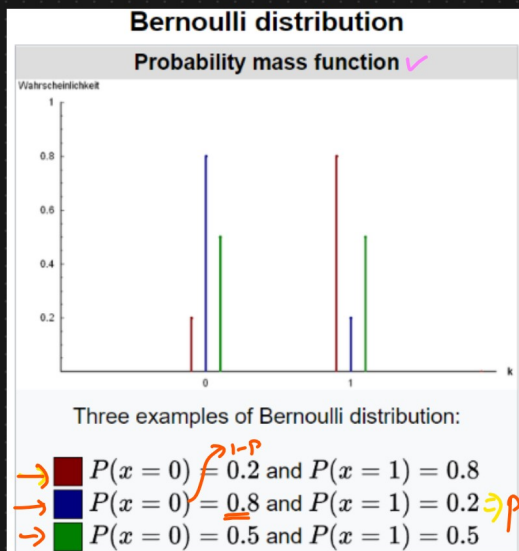


# Bernoulli Distribution

**Definition:** The Bernoulli distribution is the simplest discrete probability distribution. It represents the probability distribution of a random variable that has exactly two possible outcomes: success (**with probability  $p$** ) and failure (**with probability  $1-p$** ). It is used to model binary outcomes, such as a coin flip or a yes/no question.



① Discrete Random Variable (pmf)

② Outcomes are Binary

Eg: ① Tossing a coin  $\{H, T\}$

$$\Pr(X=H) = 0.5 = p$$

$$\Pr(X=T) = 1 - 0.5 = 0.5 = q$$

$p, q$

$$q = (1-p)$$

② Whether the person will Pass/Fail

$$\Pr(X=Pass) = 0.4$$

$$\Pr(X=Fail) = 1 - 0.4 = 0.6$$

## Parameters

$$0 \leq p \leq 1$$

$$q = 1 - p$$

$$K = \{0, 1\} \Rightarrow 2 \text{ outcomes.}$$

$$\Pr(\text{Success}) \Rightarrow K=1$$

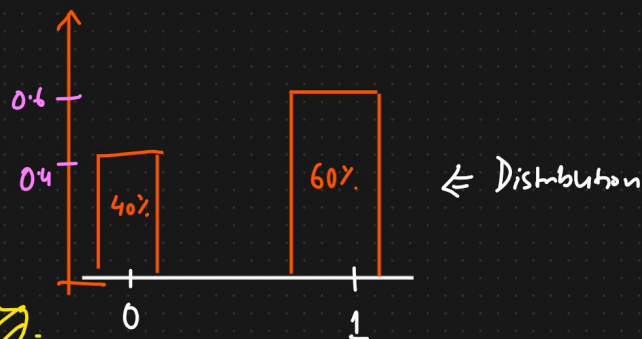
$$\Pr(\text{Fail}) \Rightarrow K=0$$

PMF : Company has launched a new Smartphone 'A'

K

$$(1) \text{ Use} = 60\% \Rightarrow p$$

$$(2) \text{ Not use} = 40\% \Rightarrow q = 1 - p$$



$$PMF = p^K \cdot (1-p)^{1-K}$$

if  $K=1$

$$Pr(K=1) = p^1 (1-p)^{1-1} \Rightarrow p //$$

$$Pr(K=0) = p^0 (1-p)^1 \Rightarrow (1-p) = q //$$

Simplified

$$pmf \begin{cases} q = 1-p & \text{if } K=0 \\ p & \text{if } K=1 \end{cases}$$

④ Mean of Bernoulli Distribution (using below formula .)

$$E(x) = \sum_{k=0}^1 k \cdot p(k)$$

$$K = \{0, 1\}$$

$$= 0 \times 0.40 + 1 \times 0.60$$

$$p = 0.6$$

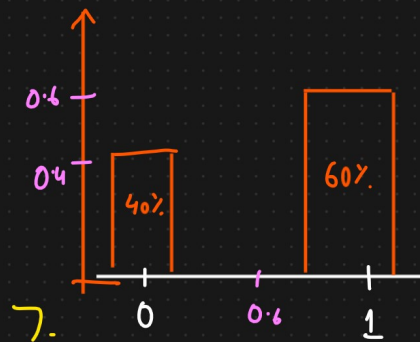
$$= 0 + 0.60$$

$$q = 0.4$$

$$= 0.60 \Rightarrow p //$$

$$Pr(1) = 0.6$$

$$Pr(0) = 0.4$$



④ Median of Bernoulli Distribution

$$\text{Median} \begin{cases} 0 & \text{if } p < 1/2 \\ \rightarrow [0, 1] & \text{if } p = 1/2 \\ 1 & \text{if } p > 1/2 \end{cases}$$

$$\left\{ \begin{array}{l} \text{median} = 0 \text{ if } q > p \\ \text{median} = 0.5 \text{ if } q = p \\ \text{median} = 1 \text{ if } q < p \end{array} \right.$$

④ Mode :

$p > q \Rightarrow p$  will be the mode  
else  $q$  will be the mode.

⑧ Variance :

$K=0$  and  $1$

$\Pr(K=0) = 0.4 \Rightarrow q$   $\Pr(K=1) = 0.6 \Rightarrow p$

$$\sigma^2 = 0.40 * (0 - 0.6)^2 + 0.6 (1 - 0.6)^2$$

$$= 0.40 + 0.36 + 0.6 (0.16)$$

$$\sigma^2 = 0.24 \Rightarrow \Pr(K=0) \neq \Pr(K=1)$$

$q \neq p$

$$\sigma^2 = pq$$
$$\sigma = \sqrt{pq}$$