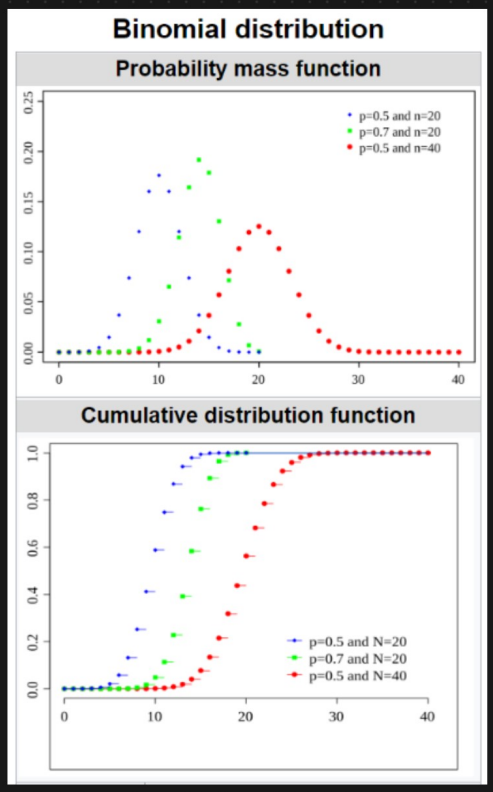
Binomial Distribution (it is bernoulli distribution with value n=1)

In probability theory and statistics, the binomial distribution with parameters n and p is the discrete probability distribution of the number of successes in a sequence of n independent experiments, each asking a yes–no question, and each with its own Boolean-valued outcome: success (with probability p) or failure (with probability q = 1-p). A single success/failure experiment is also called a Bernoulli trial or Bernoulli experiment, and a sequence of outcomes is called a Bernoulli process; for a single trial, i.e., n = 1, the binomial distribution is a Bernoulli distribution. The binomial distribution is the basis for the popular binomial test of statistical significance.



works for

① Discrete Random Variable

② Every outcome of the experiment is binary

③ These experiments are performed for n trials

Eg: Tossing a coin 10 times → n=10.

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Notation : $B(n, p)$

Parameters : $n \in \{0, 1, 2, \cdots\}$ ⇒ no. of trails or experiment

$p \in [0, 1]$ → Success probability for each trial

$q = 1 - p$

Support : $k \in \{0, 1, 2, 3 \cdots n\}$ ⇒ Number of successes

PMF :

$$Pr(k, n, p) = {}^{n}C_{k}\, p^{k} (1-p)^{n-k}$$

for $k = 0, 1, 2, \ldots n$ where

$$^{n}C_{k} = \frac{n!}{k!(n-k)!}$$

⇒ Binomial Coefficient.

Mean : $n \cdot p$

Variance : npq.

σ : $\sqrt{npq}$

Eg : Coin flip

No. of trial $(n) = 5$

Probability of success $(p) = 0.5$

No. of Success (k) = Varies from 0 to 5

such ques. can be asked in interviews

(a) What is the probability of getting exactly 3 heads in 5 flips?

$n = 5 \quad k = 3$

$$P_Y(X = 3) = {}^{5}C_{3} (0.5)^{3} (1 - 0.5)^{5-3} = 0.3125.$$

(b)

Example : Quality Control

Scenario : Inspecting 10 items in a factory where each item has a 10% chance of being defective

(a) No. of Trials $(n) = 10$

(b) Probability of Success $(p) = 0.1$ (defective item)

(c) No. of Successes (k) = Varies from 0 to 10

Question: What is the probability of finding exactly 2 defective items in a sample of 10?

$$Pr(X=2) = {}^{10}C_2 (0.1)^2 (1-0.1)^{10-2} \approx 0.1937 //.$$