EC60007: COMPUTATIONAL NEUROSCIENCE
PROJECT-1

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(1) Reduction of the Van-der-Pol (VDP) equation into two first order differential equations with variables y and u^{-1}(dy/dt):

VDP equation: $\frac{d^2y}{dt^2} - \mu(1 - y^2)\frac{dy}{dt} + y = 0$ for $\mu > 0$

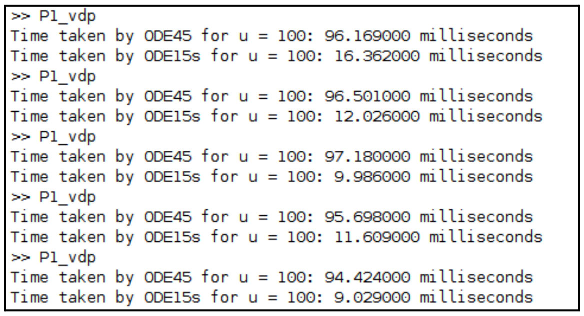
Let $y_1 = y$ and $y_2 = \mu^{-1}(\frac{dy}{dt}) = \mu^{-1}(\frac{dy_1}{dt})$

Then , first order 1 diff equation: $\frac{dy_1}{dt} = \mu y_2$;

second order 1 diff equation: $\frac{dy_2}{dt} = \mu(1 - y_1^2)y_2 - \frac{y_1}{\mu}$ (by substituting $\frac{dy_1}{dt}$ as μy_2 in the original VDP equation)

(2) Comparison of simulation time for VDP with $\mu = 100$ for the ODE45 and ODE15s solvers:

The VDP equation is stiff for μ = 100 (basically as μ increases, time to simulate the equation increases). We observe that the simulation time using ODE15s method is considerably lower than than simulation time using ODE45 method, as illustrated below in the output of my code for multiple runs :



```
>> P1_vdp
Time taken by ODE45 for u = 100: 96.169000 milliseconds
Time taken by ODE15s for u = 100: 16.362000 milliseconds
>> P1_vdp
Time taken by ODE45 for u = 100: 96.501000 milliseconds
Time taken by ODE15s for u = 100: 12.026000 milliseconds
>> P1_vdp
Time taken by ODE45 for u = 100: 97.180000 milliseconds
Time taken by ODE15s for u = 100: 9.986000 milliseconds
>> P1_vdp
Time taken by ODE45 for u = 100: 95.698000 milliseconds
Time taken by ODE15s for u = 100: 11.609000 milliseconds
>> P1_vdp
Time taken by ODE45 for u = 100: 94.424000 milliseconds
Time taken by ODE15s for u = 100: 9.029000 milliseconds
```

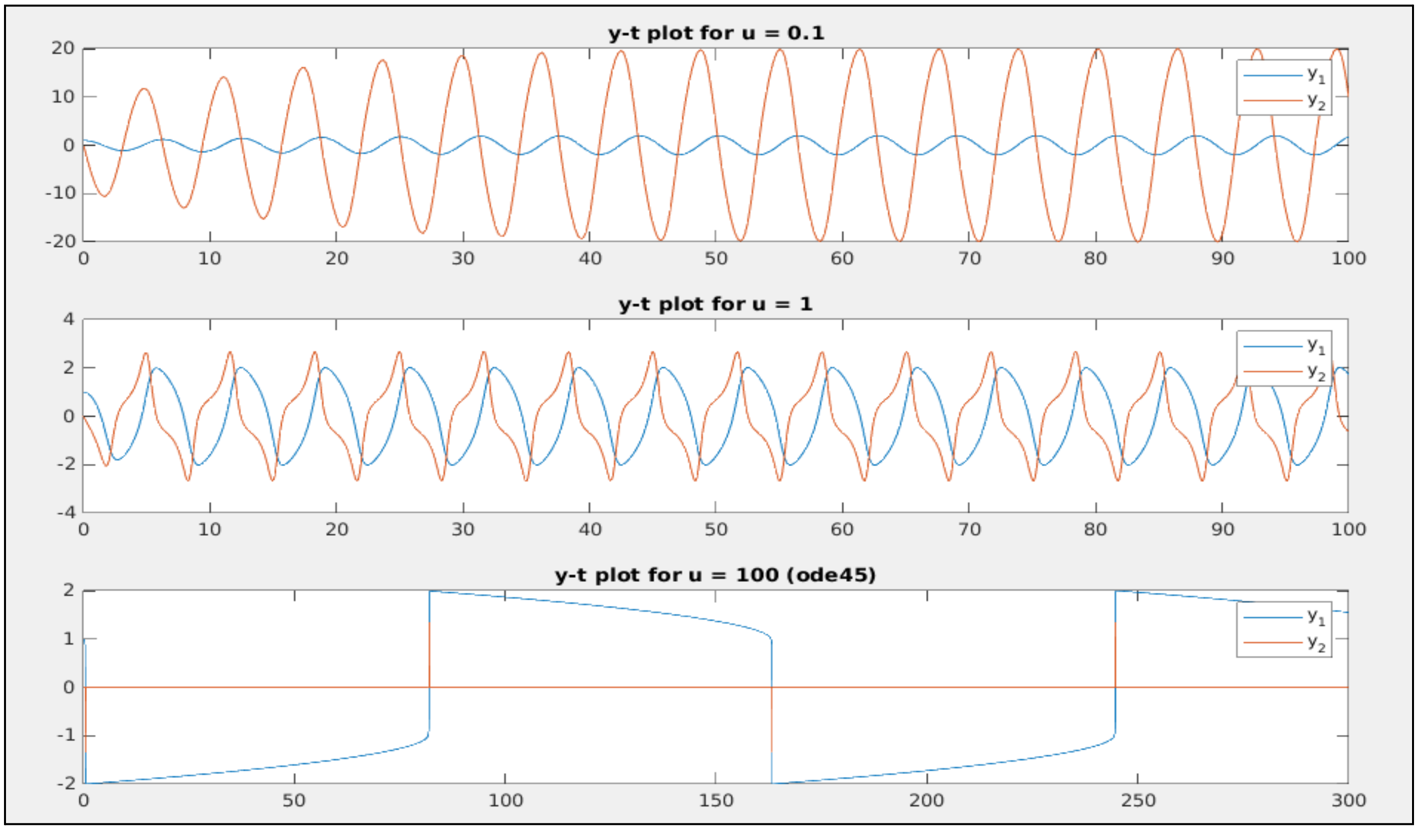
(3) For $u = 0.1$, the value of y is a bit damped initially, but later oscillates between two fixed values, while as μ increases, there is negligible damping throughout the period of simulation. The phase plane diagrams reveal that for $u = 0.1$, the system oscillates many times before reaching a stable oscillatory path, while for $\mu = 1$, the system attains a stable oscillatory path in very few oscillations, which even decrease for $\mu = 100$.

MATLAB script:

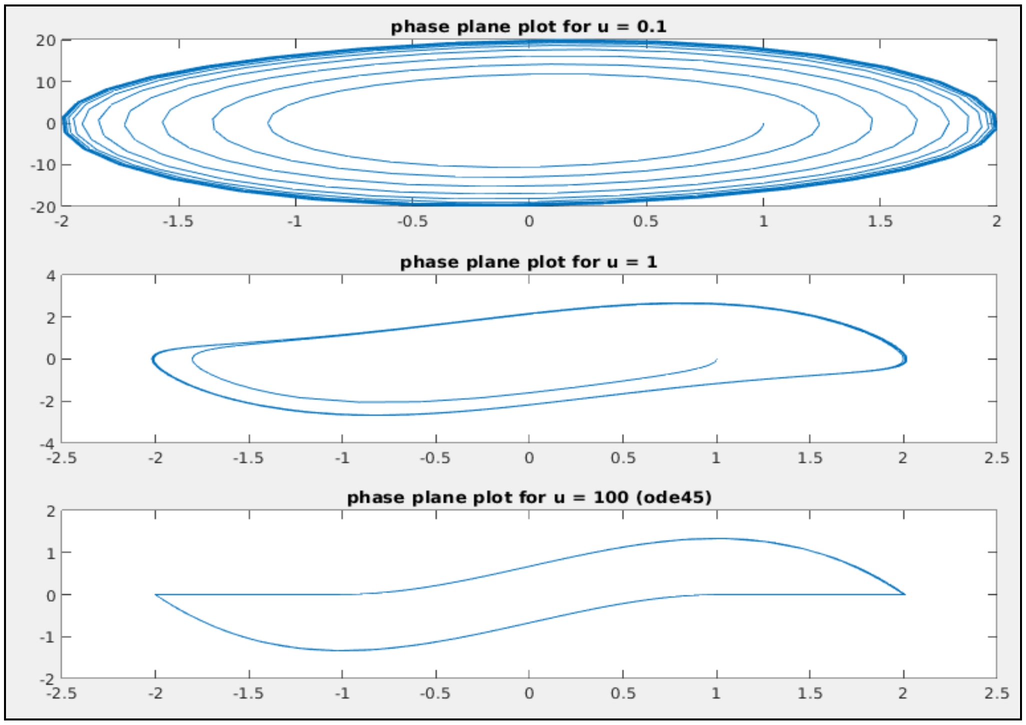
```matlab
function P1_vdp()
    T_span_1 = [0 100];
    T_span_2 = [0 300];
    y1_0 = 1;
    y2_0 = 0;
    Y_0 = [y1_0; y2_0];

    % ODE simulation using ode45 for u = 1
    [T1, Y1] = ode45(@vdp_1, T_span_1, Y_0);

    % ODE simulation using ode45 for u = 0.1
    [T2, Y2] = ode45(@vdp_2, T_span_1, Y_0);

    % ODE simulation using ode45 for u = 100
    tStart_ode45 = tic;
    [T3, Y3] = ode45(@vdp_3, T_span_2, Y_0);
    tEnd_ode45 = toc(tStart_ode45);

    % ODE simulation using ode15 for u = 100 (stiff case)
    tStart_ode15 = tic;
    [T4, Y4] = ode15s(@vdp_3, T_span_2, Y_0);
    tEnd_ode15 = toc(tStart_ode15);

    % Comparing time taken by ODE45 and ODE15 for u = 100 case
    fprintf("Time taken by ODE45 for u = 100: %f milliseconds\n", tEnd_ode45*(10^3));
    fprintf("Time taken by ODE15s for u = 100: %f milliseconds\n", tEnd_ode15*(10^3));

    % y-t plots for different u
    figure()
    subplot(3, 1, 1); plot(T2, Y2(:, 1), T2, Y2(:, 2));
    legend('y_1','y_2');title("y-t plot for u = 0.1");
    subplot(3, 1, 2); plot(T1, Y1(:, 1), T1, Y1(:, 2));
    legend('y_1','y_2');title("y-t plot for u = 1");
    subplot(3, 1, 3); plot(T3, Y3(:, 1), T3, Y3(:, 2));
    legend('y_1','y_2');title("y-t plot for u = 100 (ode45)");

    % phase-plane analysis plots for the 4 cases same as above
    figure()
    subplot(3, 1, 1); plot(Y2(:, 1), Y2(:, 2)); title("phase plane plot for u = 0.1");
    subplot(3, 1, 2); plot(Y1(:, 1), Y1(:, 2)); title("phase plane plot for u = 1");
    subplot(3, 1, 3); plot(Y3(:, 1), Y3(:, 2)); title("phase plane plot for u = 100 (ode45)");
end

% VDP for u = 1
function der_vec = vdp_1(t, y)
    der_vec = [y(2); (1- (y(1)^2))*y(2) - y(1)];
end

% VDP for u = 0.1
function der_vec = vdp_2(t, y)
    der_vec = [0.1*y(2); 0.1*(1- (y(1)^2))*y(2) - (y(1)/(0.1))];
end

% VDP for u = 100
function der_vec = vdp_3(t, y)
    der_vec = [100*y(2); 100*(1- (y(1)^2))*y(2) - (y(1)/(100))];
end
```

Figures:

(1) Y vs t plots:



(2) Phase plane plots


phase plane plot for u = 0.1
phase plane plot for u = 1
phase plane plot for u = 100 (ode45)