Q1) $\mathcal{L}(x,\lambda) = \frac{1}{2}x^T P x + q^T x + r + \lambda_1(x_1 - 1) + \lambda_2(-x_1 - 1)$
$+ \lambda_3(x_2 - 1) + \lambda_4(-x_2 - 1)$
$+ \lambda_5(x_3 - 1) + \lambda_6(-x_3 - 1)$

KKT conditions:

a) Primal feasibility: $-1 \le x_i^* \le 1$ (satisfied at the given x^*)

b) From complementary slackness: $\lambda_3^* = \lambda_4^* = \lambda_2^* = \lambda_5^* = 0$

c) Dual feasibility: $\lambda^* \geq 0$

d) $\nabla_x \mathcal{L}(x,\lambda) = 0 \Rightarrow Px^* + q + \begin{bmatrix} \lambda_1^* - \lambda_2^* \\ \lambda_3^* - \lambda_4^* \\ \lambda_5^* - \lambda_6^* \end{bmatrix} = 0$

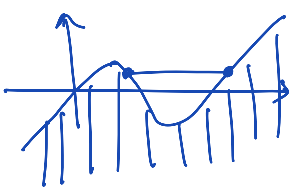
$\Rightarrow \lambda_1^* = 1, \ \lambda_6^* = 12$

Thus, we can find λ^* s.t. (x^*, λ^*) satisfy KKT conditions, which is sufficient for optimality.

Q2) cost function is convex because each $|x_i + c|$ is a convex function, (affine function + 1-norm)
and sum of two convex functions is convex.
Feasibility set is a convex set as it is defined via lineare inequality constraints.

Q3) a) convex set as it is epigraph of $e^{x_1 - 1}$ which is a convex function.

b) not a convex set.
one can easily find two points whose connecting line is outside the set.



c) convex set as it is intersection of two convex sets
$\{x_1^2 + x_2^2 \le 4\}$ and $\{x_1 \geq 0\}$

(Q4) since the function is convex,

$$f(x) \geq f(y) + \nabla f(y)^T (x-y)$$
$$f(y) \geq f(x) + \nabla f(x)^T (y-x)$$

adding both yields the result.

(Q5) converting to standard form

$$\min_{x \in \mathbb{R}^3, s} \quad x_1 + x_3$$
$$\text{s.t.} \quad x_1 + 2x_2 + s = 5$$
$$x_1 + 2x_3 = 6$$
$$x \geq 0, \; s \geq 0$$

$c = [1 \; 0 \; 1 \; 0]$

$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

dual of $\min_{\bar{x}} c^T \bar{x}$ s.t. $A\bar{x} = b$, $\bar{x} \geq 0$ is $\max_{y} b^T y$ s.t. $A^T y \leq C$ $\equiv$ $\max_{y_1, y_2} 5y_1 + 6y_2$ s.t. $y_1 + y_2 \leq 1$, $2y_1 \leq 0$, $2y_2 \leq 1$, $y_1 \leq 0$

which is equivalent to

$$\max_{y \in \mathbb{R}^2} \quad 5y_1 + 6y_2$$
$$\text{s.t.} \quad y_1 + y_2 \leq 1$$
$$y_1 \leq 0$$
$$y_2 \leq 1/2$$

optimal dual sol^n: $[0, \frac{1}{2}]$, optimal value $= 3$

(Q6) $\mathcal{L}(x, \lambda) = x^2(1+\lambda) - 6\lambda x + 1 + 8\lambda$

$\nabla_x \mathcal{L}(x, \lambda) = 0 \Rightarrow x^* = \frac{3\lambda}{1+\lambda}$ minimizes $\mathcal{L}(x, \lambda)$

$d(\lambda) = \mathcal{L}(x^*, \lambda) = \frac{-9\lambda^2}{1+\lambda} + 1 + 8\lambda$

optimal dual sol^n: $\max_{\lambda \geq 0} d(\lambda)$ is attained at $\lambda^* = 2$

dual optimal value $= 5$

$x^* = 2$ & optimal value of primal is 5.

Strong duality holds.