1. What is a random variable in probability theory?
A **random variable** is a numerical outcome of a random process or experiment. It assigns a real number to each outcome in a sample space.

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2. What are the types of random variables?
There are two main types:

- **Discrete Random Variables**: Take on countable values (e.g., 0, 1, 2…).
- **Continuous Random Variables**: Take on infinitely many values within a given range (e.g., height, weight, temperature).

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3. What is the difference between discrete and continuous distributions?

- **Discrete distributions** deal with countable outcomes. Example: Number of heads in 5 coin tosses.
- **Continuous distributions** involve uncountable outcomes in a range. Example: Measuring the time it takes to run a mile.

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4. What are probability distribution functions (PDF)?
A **PDF** (Probability Density Function) describes the likelihood of a continuous random variable taking on a specific value. It represents the density of probability over a range, not a precise value.

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5. How do cumulative distribution functions (CDF) differ from probability distribution functions (PDF)?

- **PDF** gives the probability density at a specific point.
- **CDF** gives the probability that a variable is less than or equal to a certain value. CDF(x)=P(X≤x)CDF(x) = P(X \leq x)CDF(x)=P(X≤x)

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6. What is a discrete uniform distribution?
A **discrete uniform distribution** assigns equal probability to each outcome in a finite set. Example: Rolling a fair die — each face has a 1/6 chance.

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7. What are the key properties of a Bernoulli distribution?

- Only two possible outcomes: success (1) or failure (0)
- One trial
- Mean = ppp, Variance = p(1−p)p(1 - p)p(1−p), where ppp is the probability of success

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8. What is the binomial distribution, and how is it used in probability?
A **binomial distribution** models the number of successes in a fixed number of independent Bernoulli trials. It’s used for situations with two outcomes per trial (success/failure).

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9. What is the Poisson distribution and where is it applied?
A **Poisson distribution** models the number of events occurring in a fixed interval of time or space, assuming the events occur independently and at a constant rate. Common in queuing theory, call arrivals, etc.

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10. What is a continuous uniform distribution?
It assigns equal probability to all values within a specified range [a,b][a, b][a,b]. The PDF is constant:
f(x)=1b−a for a≤x≤bf(x) = \frac{1}{b - a} \text{ for } a \le x \le bf(x)=b−a1 for a≤x≤b

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11. What are the characteristics of a normal distribution?

- Bell-shaped and symmetric
- Mean = median = mode
- Defined by mean (μ) and standard deviation (σ)
- Empirical Rule: ~68% within 1σ, ~95% within 2σ, ~99.7% within 3σ

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12. What is the standard normal distribution, and why is it important?
It is a normal distribution with mean 0 and standard deviation 1. It allows for standardization of values (Z-scores) and simplifies calculations in hypothesis testing.

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13. What is the Central Limit Theorem (CLT), and why is it critical in statistics?
The **CLT** states that the sampling distribution of the sample mean approaches a normal distribution as the sample size increases, regardless of the original distribution. It enables inference using normal distribution.

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14. How does the Central Limit Theorem relate to the normal distribution?
It justifies using the normal distribution for means and sums of large samples, even when the population distribution is not normal.

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15. What is the application of Z statistics in hypothesis testing?
Z-statistics help determine whether to reject the null hypothesis by comparing the Z-score to critical values based on the significance level.

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16. How do you calculate a Z-score, and what does it represent?
Z=X−μσZ = \frac{X - \mu}{\sigma}Z=σX−μ
It represents how many standard deviations a value XXX is from the mean μ\muμ.

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17. What are point estimates and interval estimates in statistics?

- **Point estimate**: A single value used to estimate a population parameter (e.g., sample mean).
- **Interval estimate**: A range (e.g., confidence interval) within which the parameter likely lies.

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18. What is the significance of confidence intervals in statistical analysis?
Confidence intervals express the range of plausible values for a population parameter. A 95% CI means we are 95% confident that the true parameter lies within the interval.

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19. What is the relationship between a Z-score and a confidence interval?
Z-scores are used to calculate confidence intervals for known standard deviation:
CI=x¯±Z⋅σnCI = \bar{x} \pm Z \cdot \frac{\sigma}{\sqrt{n}}CI=x¯±Z⋅nσ

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20. How are Z-scores used to compare different distributions?
By converting raw values to Z-scores, we can compare values from different distributions on a standardized scale.

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21. What are the assumptions for applying the Central Limit Theorem?

- Random sampling
- Independent observations
- Sample size is sufficiently large ($n \geq 30$ generally)
- Finite mean and variance

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22. What is the concept of expected value in a probability distribution?
The **expected value** is the long-term average outcome of a random variable.
For discrete:
E(X)=∑xi⋅P(xi)E(X) = \sum x_i \cdot P(x_i)E(X)=∑xi⋅P(xi)
For continuous:
E(X)=∫x⋅f(x) dxE(X) = \int x \cdot f(x) \, dxE(X)=∫x⋅f(x)dx

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23. How does a probability distribution relate to the expected outcome of a random variable?
The probability distribution defines the likelihood of each outcome, and the expected value is a weighted average of all possible outcomes, weighted by their probabilities.