7CCSMSUF: Software Engineering and Underlying Technologies for Financial Systems

Kevin Lano

kevin.lano@kcl.ac.uk

Week 4: Financial products and analyses

- Wide range of financial products are traded by financial institutions: bonds, shares, derivative securities and structured products such as collateralised debt obligations (CDOs).
- Financial engineering involves: (i) model a product in terms of its cash flows, risks, value, volatility, etc., (ii) evaluate significant properties of a product, such as risk of default in a CDO.

Financial products and analyses

- E.g., an institution may be interested in creating new/variant product.

  Mathematical models are constructed and different scenarios executed on the models.

- Efficient algorithms are required to compute predictions of models for specific products.

- May be a trade-off between accuracy and timeliness, as e.g., in high-frequency trading. Heuristics + approximations may be used if exact computational procedures not available.

Specific products

- *Bonds*: loan of capital funds from investor to bond issuer, in return for coupon payments over fixed term, + return of loan principal at end of term.
- *Shares*: purchase of part-ownership stake in a publicly-traded company, a share in equity of the company.
- *Derivative securities*: financial product whose value is based upon value of an underlying asset, e.g., a physical commodity such as gold, or another financial product.
- *Collateralised Debt Obligations (CDOs)*: aggregated portfolios of debts.

Bonds

- Bonds enable a company or government to raise funds from investors. Repayments may be based on a fixed or floating interest rate. The *principal* is amount borrowed. *term* is period over which investment is paid back.
- For businesses, bonds have advantage that investors do not acquire stake in the company.
- Value of bond as investment depends upon interest rates: increase in interest rates will reduce value of fixed interest rate bonds, since higher returns will be available elsewhere.

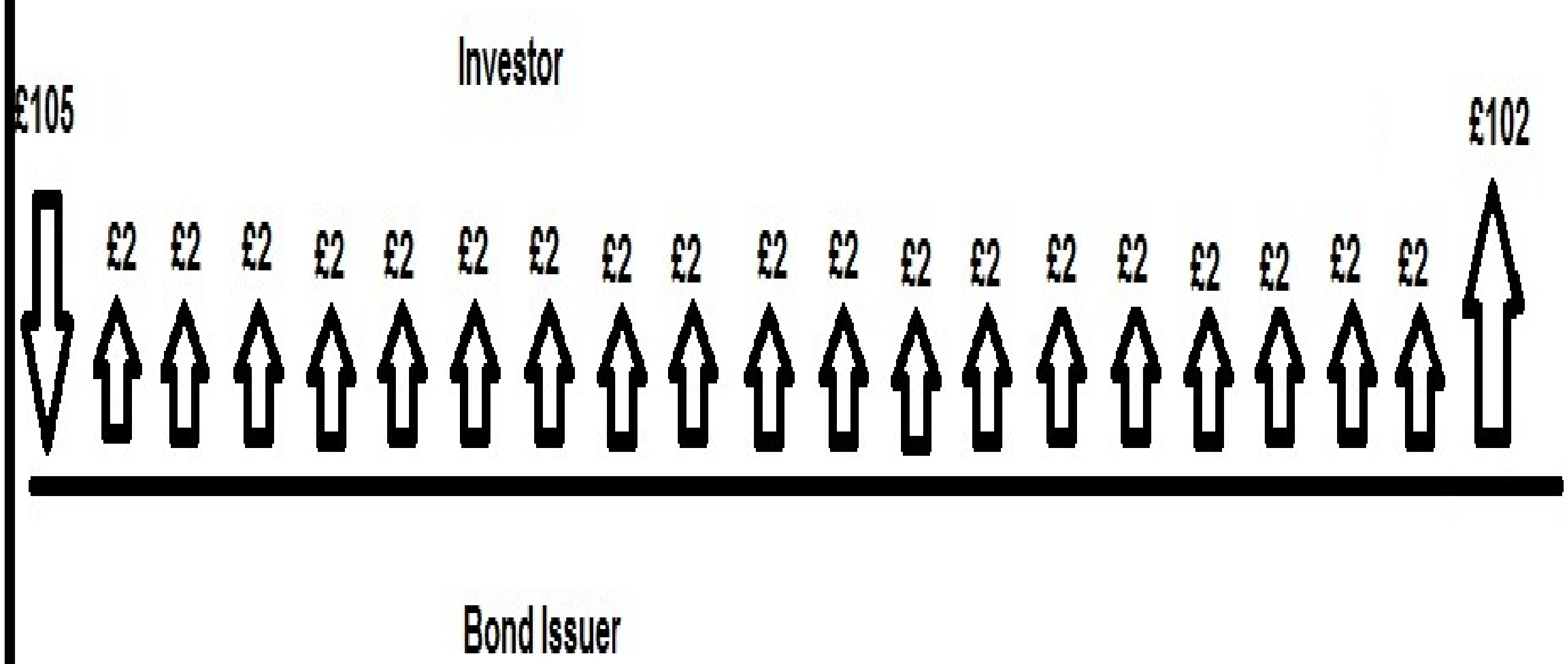
Bonds

- A bond is purchased for a given price including an initial investment amount, the *principal* of bond.
- E.g., UK government bonds at dmo.gov.uk.
- To calculate bond values, usually consider nominal principal amount of 100. Bond *term* expressed as, e.g., 5 years, 10 years, etc.
- Common form is a *bullet bond*, where coupon payments are paid to investor throughout bond term, the last payment being final coupon plus return of principal.
- In fixed-coupon bond the coupon is defined as annual percentage c of investment amount, split into payments at regular intervals ($0.5 * c$ twice per year, or c once per year, etc) during term.

Fixed-coupon Bonds

- In terms of cash flows, initial payment of *price* is flow from investor (lender) to bond issuer (borrower).
- Coupon payments are regular cash flows of $coupon/f$ from borrower to investor. f is frequency of coupon payments per year. There are $f * term$ payments of the $coupon/f$ amount in total. At term, also cash flow of 100 from borrower to investor.

E.g., investment of £100 in 20-year 2% annual coupon bond, with price of £105. Investor receives coupon £2 at end of each of first 19 years, then £102 at term.



Cash flows for a fixed-coupon bond

Bond analysis

- Analysis of bonds computes their value under different market situations & risk-free interest rates r.
- Basic quantity is bond *payout*: total amount which an investor receives back.
- For fixed coupon bonds with a whole-integer term we have

$$payout = term * coupon + 100$$

  E.g., £140 in the example.

Bond analysis: discounting

- Value of money received at future time (e.g., N years in future) is reduced or discounted compared to same amount received today – because we could invest the amount if we had it now. Amount X received N years in future, assuming annual risk-free interest rate r, is worth $X/(1+r)^N$ today.
- We say X is *discounted* to $X/(1+r)^N$.
- If amount Y available today, investor could invest it at rate r, and amount would have grown to $Y*(1+r)^N$ after N years. Thus amount X in N years time is equivalent to $X/(1+r)^N$ today, given risk-free interest rate r.

Bond analysis: discounting

- For fixed-coupon annual coupon bond, present value, $value(r)$, is sum of discounted payments over term, assuming interest rate r:

$$value(r) = (\Sigma_{t=1}^{term} coupon/(1+r)^t) + 100/(1+r)^{term}$$

- As r increases, $value(r)$ decreases.
- E.g., $value(0.01)$ at 1% interest rate of our 20-year bond is £118. $value(0.02)$ is £100.

Bond analysis

- Price of bond based on assumption by seller what interest rate r will be over term: price is set close to $value(r)$.
- Rate r at which market price of bond equals $value(r)$ is *internal rate of return (IRR)* of bond, or its *yield*:

$$value(irr) = price$$

- Investment is profitable for investor if actual risk-free rate is lower than IRR: i.e., bond returns more value than an alternative risk-free return.

The IRR of our example is 0.017, ie., 1.7%.

Bond analysis – traded bonds

- Some bonds may be traded during their term.
- Value of bond at point tm during term is based on remaining future cash flows due from it (if tm falls on a coupon payment date):

$$value(r, tm) = (\Sigma_{t=1}^{term-tm} coupon/(1+r)^t) + 100/(1+r)^{term-tm}$$

for annual fixed coupon bond, tm an integer, $tm \geq 1$, $tm < term$.

E.g., at 10 years point with $r = 0.01$, our bond is worth approx £109.

Discrete versus continuous compounding

- We used *discrete compounding* of interest rates: amount X grows to $X * (1 + r)^N$ if invested for N years at rate r. $r * amount$ added to $amount$ once at end of each year.
- Alternative is to continually apply r: X becomes

$$X * e^{r*N}$$

- With continuous compounding, discounting becomes $X * e^{-r*N}$ instead of $X/(1 + r)^N$.

Index-linked bonds

- Index-linked bonds also have varying cash flows. Such bonds link coupon rates to some price index/inflation index in order to cancel out effects of inflation on value of coupon payments.
- E.g., a UK index-linked gilt has coupon payments and principal payments adjusted in line with retail price index (RPI).
- At date t, indexation factor $ifact_t$ is computed as ratio of index rate at time t compared to rate at issue date s of bond: $ifact_t = index_t/index_s$.
- Principal is then adjusted to $100 * ifact_t$ and coupons recalculated based on adjusted principal:

$$coupon_t = c * 100 * ifact_t$$

- Thus valuation formula at t becomes:

$$value(r) = (\Sigma_{i=1}^{term} coupon_t/(1+r)^i) + 100 * ifact_t/(1+r)^{term}$$

for annual repayment index-linked bond with $term \geq 1$ years remaining until bond matures. This value is $ifact_t * value_0(r)$ where $value_0(r)$ is usual valuation formula

$$value_0(r) = (\Sigma_{i=1}^{term} c * 100/(1+r)^i) + 100/(1+r)^{term}$$

- This is approximation, since actual indexation factor applied to future coupon payments is not known at time t.

Yield curves

- Professional valuation of bonds uses predicted interest rates over term of bond.
- Variation of yield with length of an investment is the *term structure of interest rates* or a *yield curve*.
- To estimate a yield curve, identify set of market data points from financial assets, with similar risk levels, within one country.
- E.g., government bonds with same issue date but varying term.
- Compute effective duration and IRR of bonds, to obtain set of $(time, rate)$ points.
- Computational optimisation procedures can be applied to fit a curve to these data points.

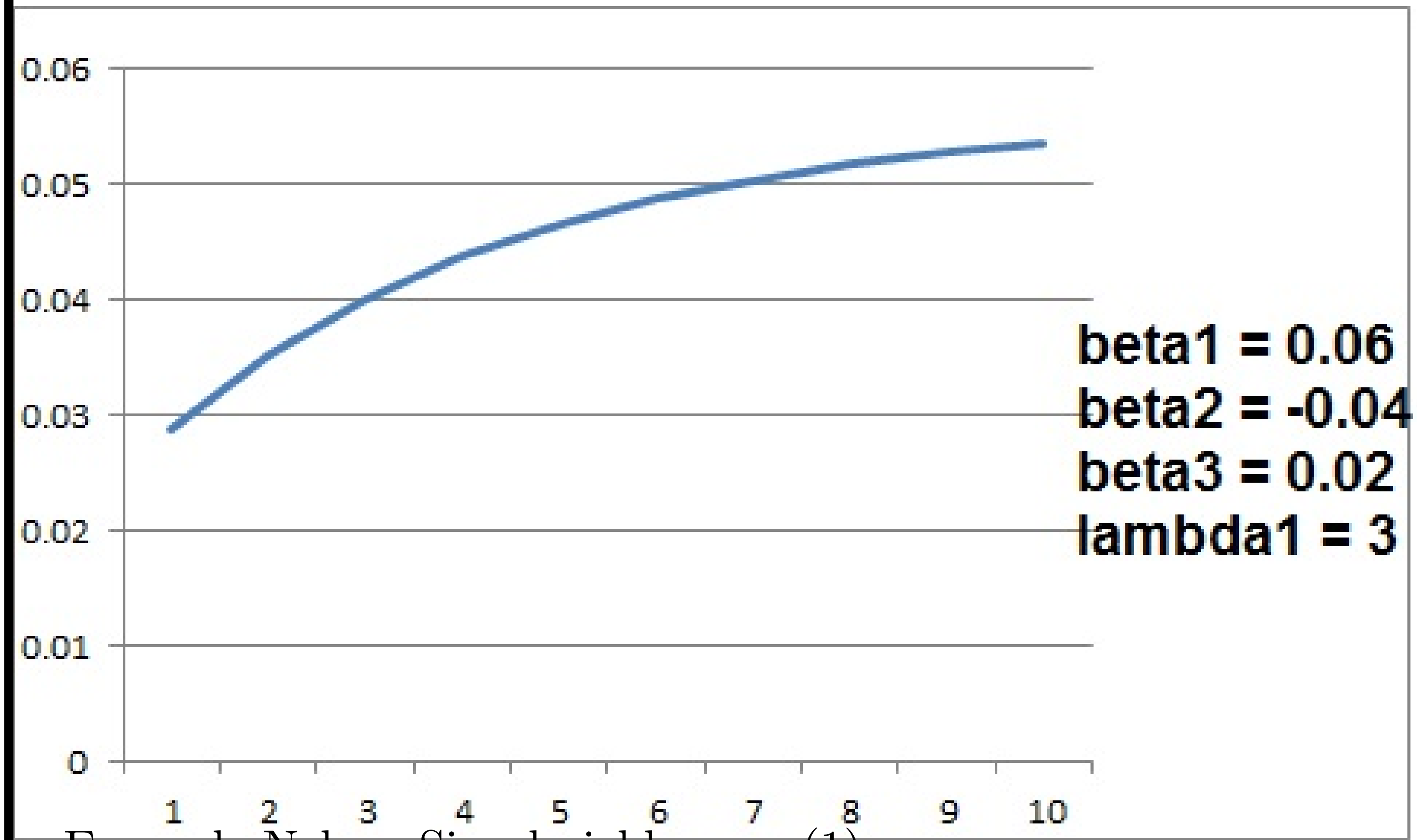
Yield curve models

- Different models exist (e.g., Nelson-Siegel formula) for shape of yield curve.
- Nelson-Siegal (NS) model for yield curves is:

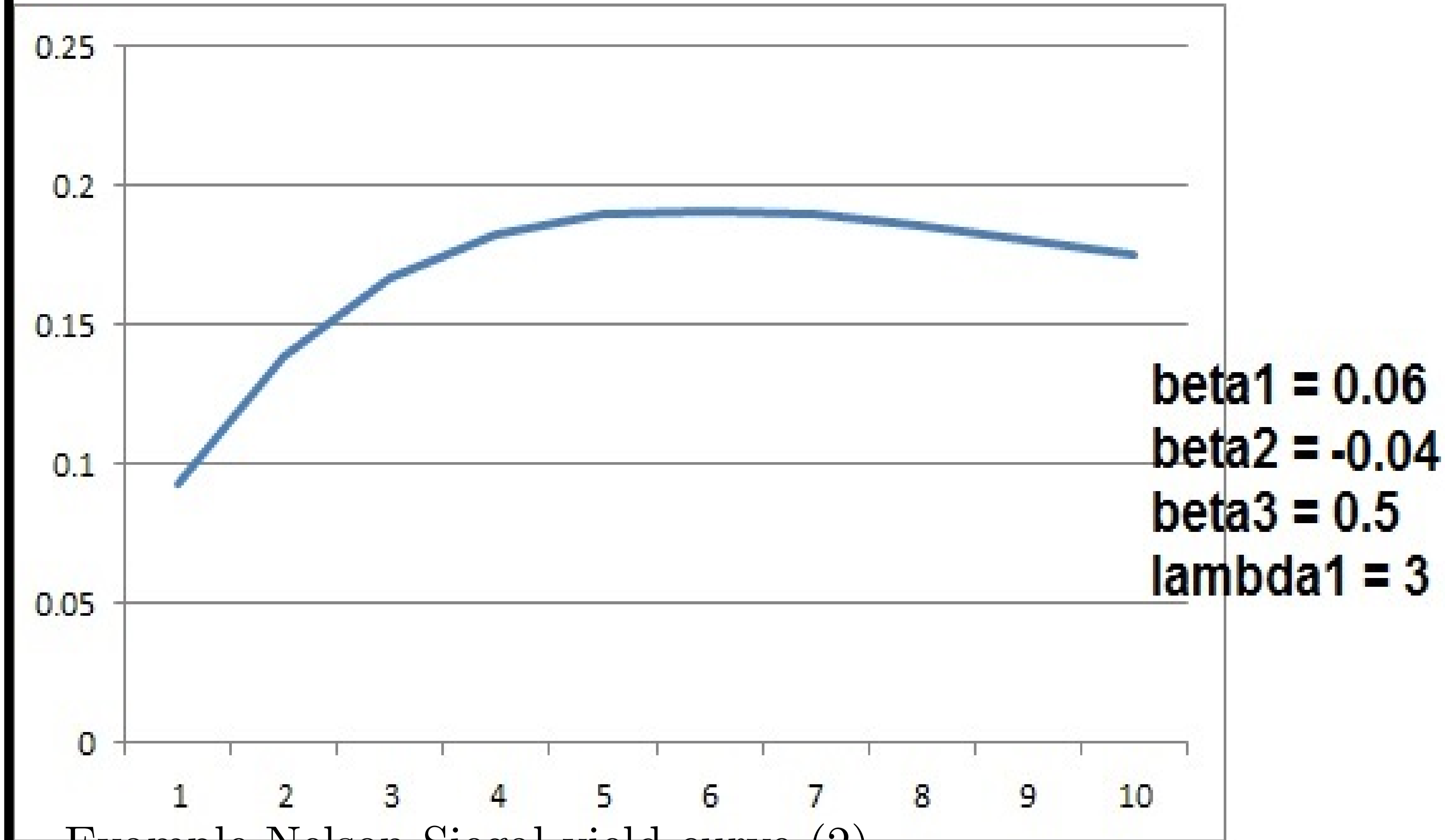
$$y(t) = \beta_1 + \beta_2 * (1 - exp(-t/\lambda_1))/(t/\lambda_1) + \beta_3 * ((1 - exp(-t/\lambda_1))/(t/\lambda_1) - exp(-t/\lambda_1))$$

  For yield $y(t)$ of duration t. β_1, β_2, β_3 and λ_1 are real-valued constants.

- $\beta_1 > 0$ is long-term yield.
- $\beta_1 + \beta_2 > 0$ is short-term.
- 'Hump' position determined by $\lambda_1 > 0$. β_3 controls curvature.



Example Nelson-Siegal yield curve (1)



Example Nelson-Siegal yield curve (2)

Yield curve models

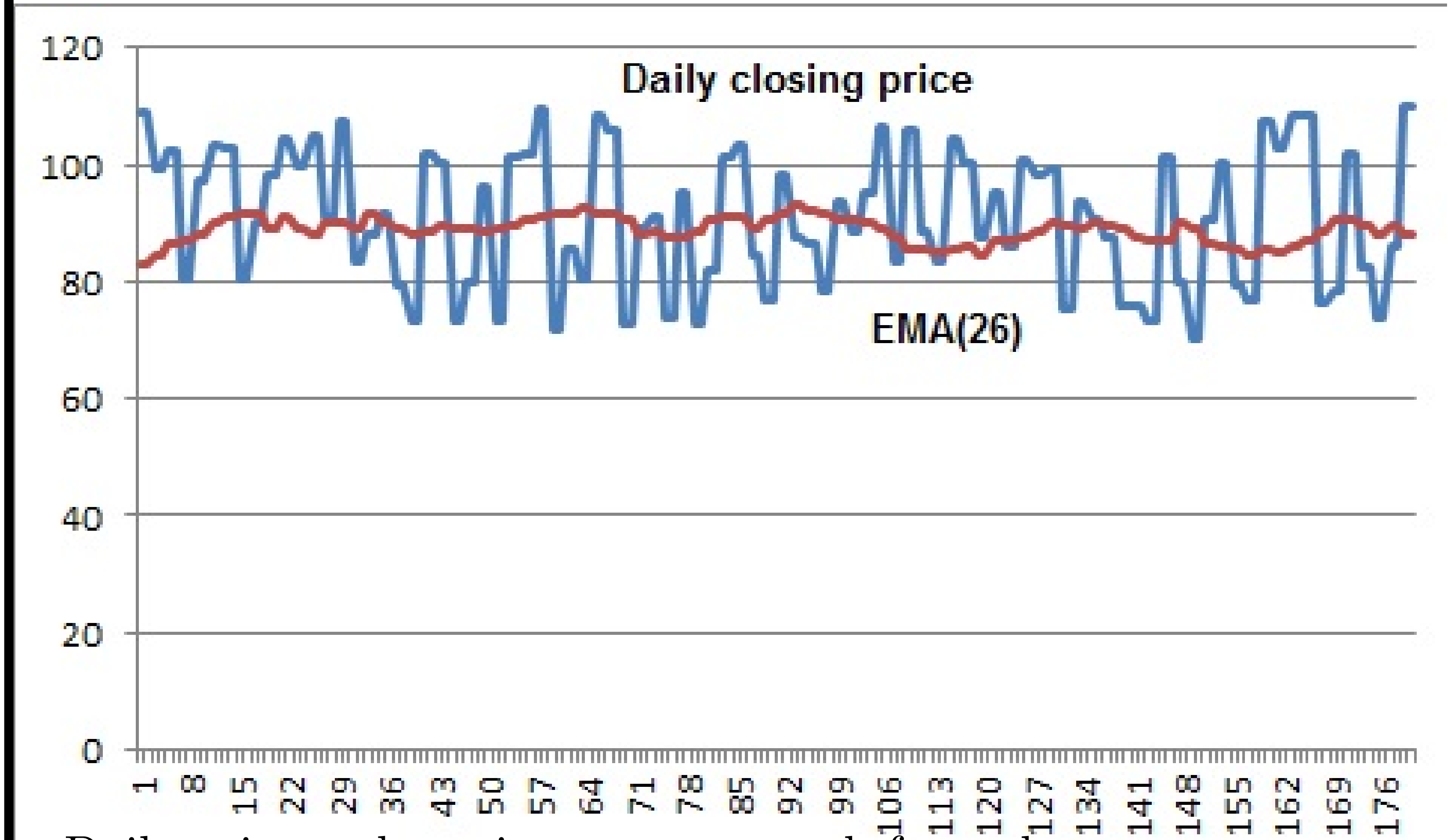
- The problem is to estimate parameters β_i and λ_1 which make curve best fit a set of given market data points: termed 'fitting the curve' to the data.
- The Nelson-Siegal-Svensson (NSS) model adds another term with parameters β_4 and λ_2.
- From yield curve, price of bond of duration d can be calculated by finding $y(d)$ and computing $value(y(d))$.

Yield curve models

- Yield curves tend to be upwards sloping because investors prefer more liquid (shorter term) investments where possible, hence require higher yields on longer-term investments.
- Longer-term investments also carry higher risk of default and hence higher yield is required to compensate for this risk.
- Inverted yield curves (with short-term yields higher than long-term) can indicate lack of confidence in economic future – historically have preceded recessions.

Shares and stocks

- Shares represent part share ownership in equity of a company: the surplus of its assets over its liabilities.
- Selling shares is another means for a business to raise funds.
- Share price analysis is concerned with loss or gain in a portfolio of shares over different periods of time.
- *Technical analysis* seeks to use past performance data to predict future performance
- *Fundamental analysis* seeks to evaluate key properties of a business as a guide to its share performance.
- Apart from ROI, investors may also be concerned with environmental & social value (ESG) aspects of investment.



Daily price and moving average graph for a share

Shares and stocks

- There is dispute over merits of technical analysis – usual mathematical assumption is that share prices follow a Markov process, i.e., future price depends only on current price and not on previous history.
- Nonetheless, different heuristic or algorithmic approaches have been used for technical analysis share price prediction: neural nets; genetic algorithms; K nearest-neighbour, etc.
- Historical share information also used to estimate volatility σ of a share (how much its price is likely to change).

Derivative securities

- Derivative securities include wide variety of financial instruments whose value depends upon an underlying asset.
- *Forward contracts* are agreement to buy or sell an asset at fixed price at fixed future time. Generally traded over the counter (OTC).
- *Futures contracts* permit variation of maturity date within some limits, are traded on an exchange.
- *Options* give holder the right (but not obligation) to buy (call option) or sell (put option) an asset on or before a date for a fixed price.
- *American* options can be exercised at any time up to expiration date.
- *European* options can only be exercised on expiration date.

Put options example

American put option prices of Google, May 8, 2013, when share price was $871.37:

Strike price	June 2013	Sept 2013	Dec 2013
820	5.50	24.90	37.50
840	8.90	31.80	45.10
860	14.80	40.10	53.90
880	24.40	49.80	63.70
900	36.20	60.90	75.00

Prices for options to sell one Google share at strike price, on or before these future dates.

Derivative securities

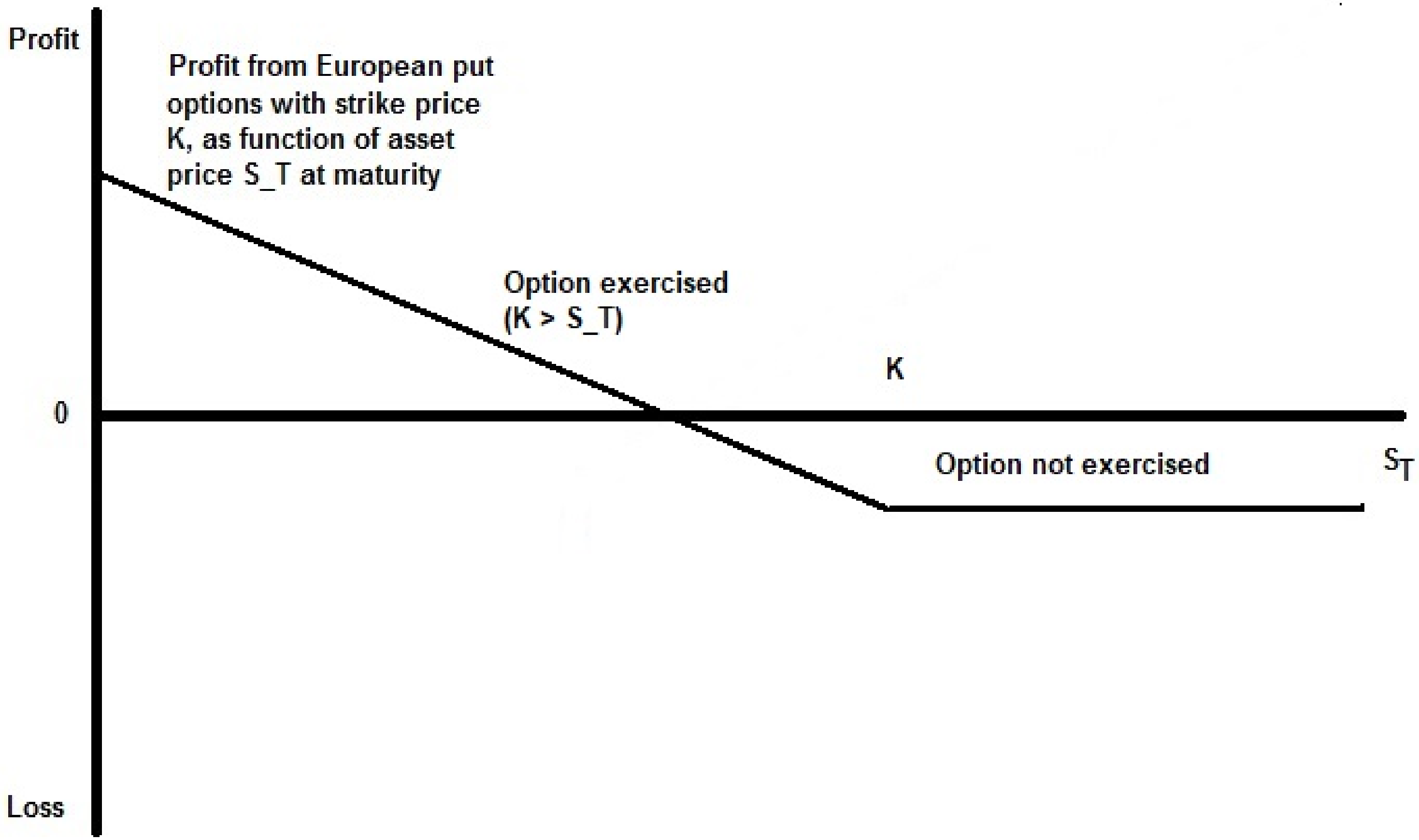
- Derivative securities can be based on almost any underlying variable: (i) traded securities, e.g., shares, bonds, gold; (ii) financial variables such as interest rates, exchange rates; (iii) other variables such as climatic variables, election results, etc.

  "The range of derivatives contracts is limited only by the imagination of man (or sometimes, so it seems, madmen)." (Warren Buffett)

- Derivative securities can be used for *hedging*: limiting losses from a contract or financial position, or for *speculation*: using derivative to reduce amount of funds needed to speculate on an asset, i.e., to increase leverage of investor's funds.

Hedging example using put options

- If I own 100 Google shares at $871 in May 2013, can buy put options to reduce losses in case of decline in share price:
- American put options, strike price $880, maturity December 2013. Cost $64 per share.
- If price declines to $800 by November 2013, would lose $71 per share (without the hedge)
- With hedge, can sell at $880 – gain of $9, loss of $64, so overall loss of $55 per share.



Profit from European put options

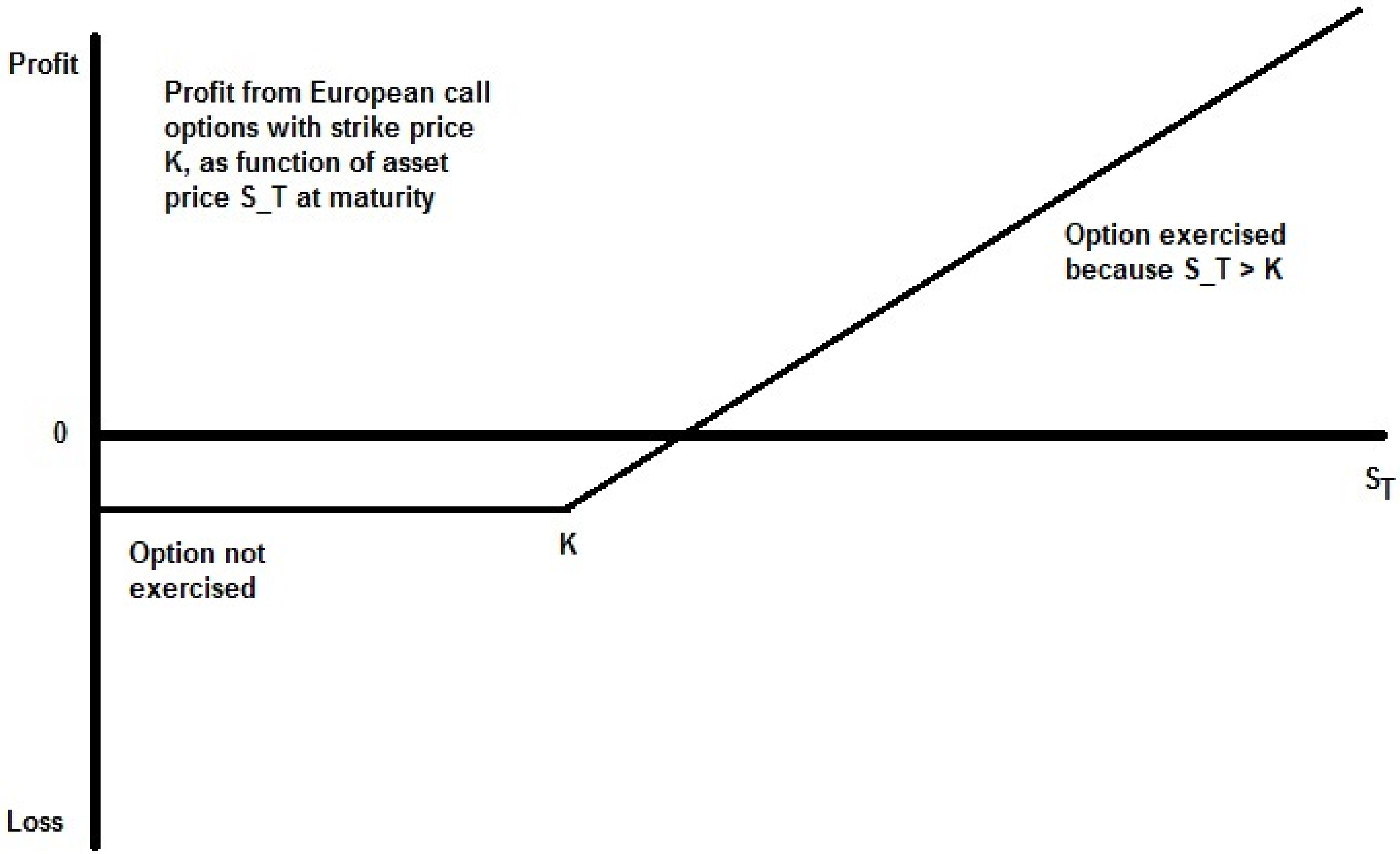
Derivative securities

- Options have become essential feature of financial markets since stock options first traded on an exchange in 1973.
- Consequently, valuation of options + other derivative securities is of major importance.
- Valuation of derivative securities takes into account value of underlying asset, underlying interest rates, and other factors.
- In some cases a precise formula for the value has not been found, and approximations are used.

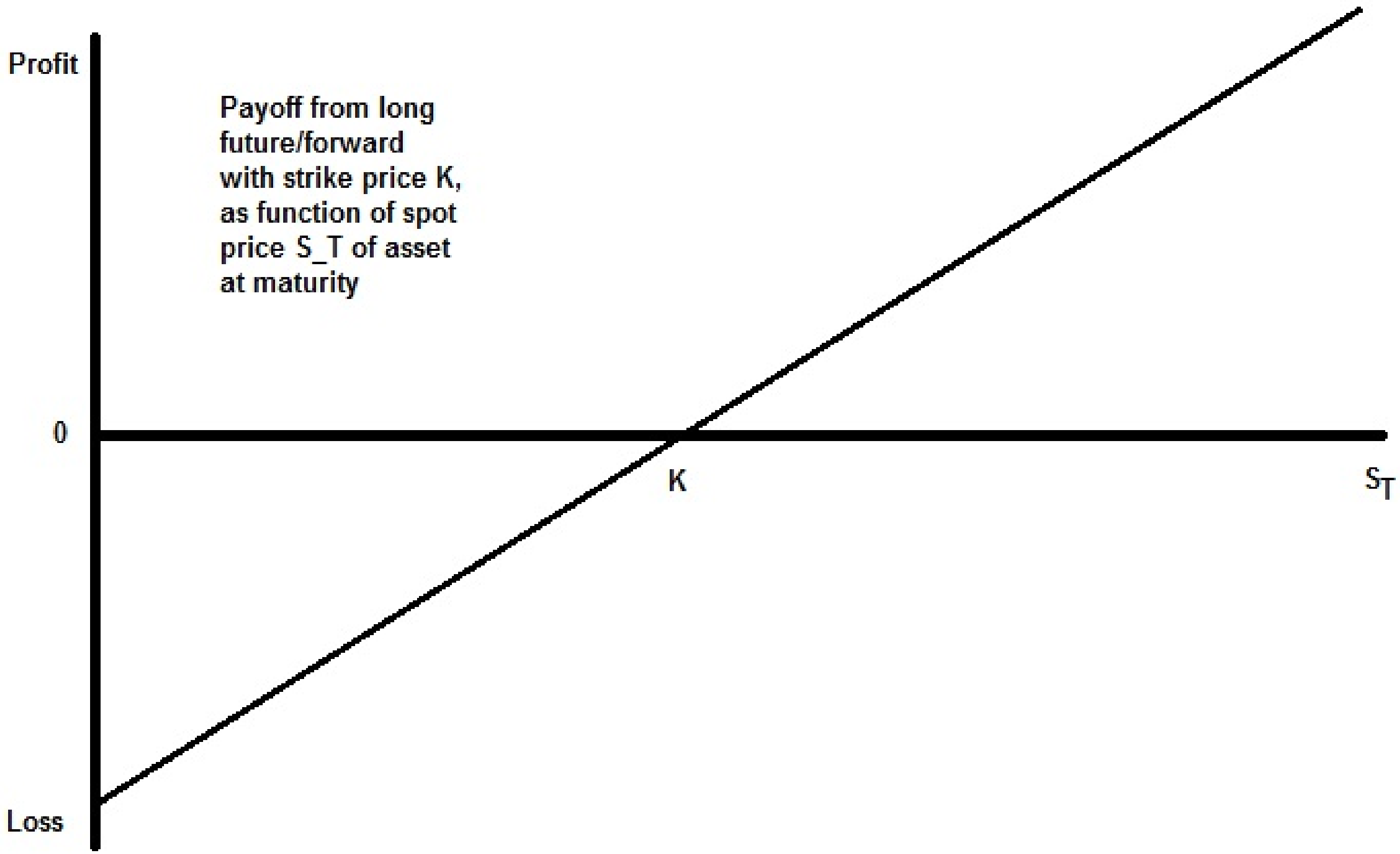
Speculation example using call options

- Instead of buying 100 Google shares at \$871 in May 2013, can buy call options to speculate on increase in share price:
- American call options, strike price \$860, maturity December 2013. Cost \$66 per share.
- If price increases to \$1000 by November 2013, would gain (140 - 66)\$ per share (= \$74)
- Higher leverage – gain of \$74, for investment of \$66 per share, instead of gain of \$129 for outlay of \$871 per share.

Option speculation can also produce greater gains (and losses) than direct investment.



Change in call option profit with underlying price at term



Change in long future profit with underlying price at term

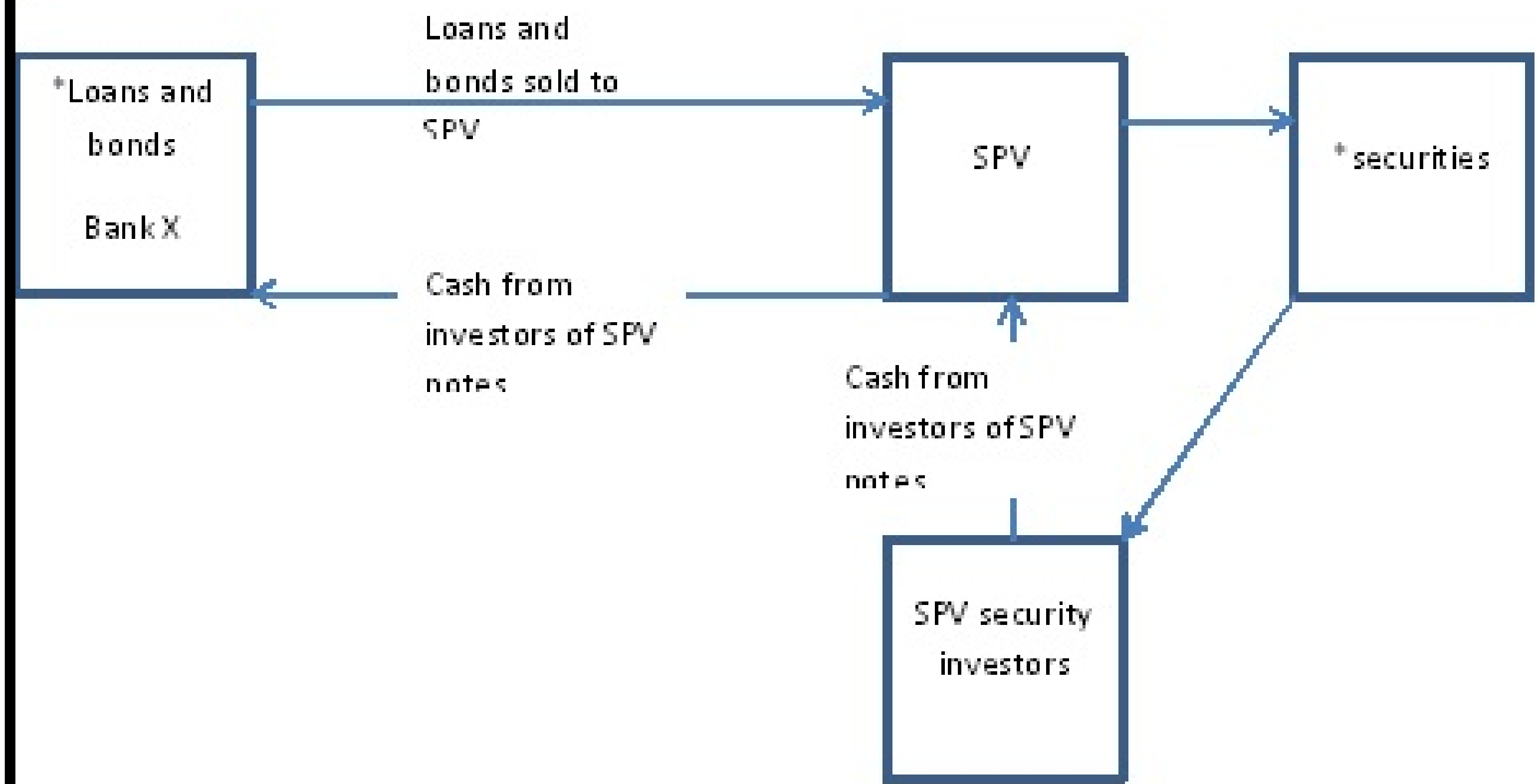
Exotic options

Non-standard derivatives created for specialised purposes.

- *Bermudan option*: American option where exercise restricted to specific dates
- *Forward start options*: term starts at a future time
- *Barrier options*: options terminated/initiated by asset price reaching a certain price H
- *Lookback option*: payoff depends on maximum/minimum asset price during life of option
- *Asian option*: payoff depends on average asset price during life of option.

Collateralised debt obligations

- A collateralised debt obligation (CDO) is an aggregated set of debts (e.g., mortgage loans or business loans made by a bank), income from debts then used as basis for further financial products.
- Termed *securitisation* of the loans.
- Aggregation of debts with different (and uncorrelated) default characteristics has benefit of reducing impact of defaults.
- Analysis of risk associated with CDOs is highly critical for institutions that trade in them: global financial crisis of 2008 was consequence of incorrect estimations of risks for CDOs based on sub-prime mortgage loans.


*Loans and
bonds
Bank X
Loans and
bonds sold to
SPV
SPV
*securities
Cash from
investors of SPV
notes
Cash from
investors of SPV
notes
SPV security
investors

CDO example

Collateralised debt obligations

- To set up a CDO from a collection of debts, a bank X would establish special purpose vehicle (SPV), probably incorporated in tax advantage jurisdiction like the Cayman Islands, considered bankruptcy remote to bank X.
- SPV issues securities backed from cash flows from underlying loan/bond pool (e.g., mortgage payments, loan interest payments).
- These securities are like bonds, but with less assurance of recovering principal.
- Typical loan pool might consist of several hundred up to thousands of loans used for backing a single security issued by SPV.

Collateralised debt obligations

- Issued securities are collateralised by cash flows from more primitive loan pool.
- Cash obtained from investors are used to facilitate bank X undertaking more loans and bonds and changing structure of its balance sheet, risk-weighted assets and capital associated with assets.
- In particular, economic risks of assets are removed from balance sheet of X, meaning that it gains ability to re-lend asset amount without violating Basel regulations requiring a certain percentage of assets to be retained as equity (e.g., 8%). This increases effective leverage ratio of bank (ratio of assets to equity).

Collateralised debt obligations

- Analysis of CDOs involves computing risk of losses from single and multiple defaults within aggregated pools of loans, some defaults may be (partly) correlated and others assumed to be uncorrelated.
- If there are n loans in pool of loans owned by bank X, need to find probability that m from n number of defaults occur, $m \leq n$.
- Information useful for bank to understand risks it faces from borrower defaults.
- Information may be used by rating agencies to determine credit rating of securities issued by SPV.

Collateralised debt obligations

- One means to organise CDO is to group loans into sectors, e.g., all loans to other financial institutions in one sector, loans to oil and gas businesses in another sector, etc.
- Assumption is made that defaults within one sector cannot cause defaults in another, but only possibly cause defaults within same sector (termed 'default infection').
- Risk contributions allow a bank to assess relative riskiness of one sector with respect to another.
- If there is a downturn in economy which affects one sector more than another then a bank might be more inclined to lend to companies in least affected sector more than in the most affected sectors.

Default infection model (Davis and Lo theorem)

Probability of k defaults out of n loans:

$$\begin{aligned} P[N = k] &= C_k^n \alpha_{nk}^{pq} \\ &\textit{where} \\ \alpha_{nk}^{pq} &= p^k (1-p)^{n-k} (1-q)^{k(n-k)} \\ &\quad + \Sigma_{i=1}^{k-1} C_i^k p^i (1-p)^{n-i} (1-(1-q)^i)^{k-i} (1-q)^{i(n-k)} \end{aligned}$$

p is individual default probability, q default infection probability.

C_i^n is combinatorial constant: number of ways of choosing i elements from n.

$$C_i^n = \frac{n!}{i! * (n-i)!}$$

Summary

- We have given overview of some important financial products and their analysis.
- We will use examples of these financial analyses in subsequent parts to show how different software engineering techniques and underlying technologies can be applied to financial software problems.