Assignment 2 ICSE class 12 2019

Pranav B (AI21BTECH11023)

Question 7

A 13 m long ladder is leaning against a wall, touching the wall at a certain height from the ground level.The bottom of the ladder is pulled away from the wall,along the ground at the rate of 2 m/s. How fast is the height of the ladder decreasing when the foot of the ladder is 5 m away from the wall?

1 Solution:

The various parameters involved in this question are listed in Table

TABLE 0
VARIABLES USED

Parameter	Variable	Value
length of the rod	s	13
height from ground level to tip of the rod at time t	h(t)	???
distance from wall to foot of the rod at time t	x(t)	???

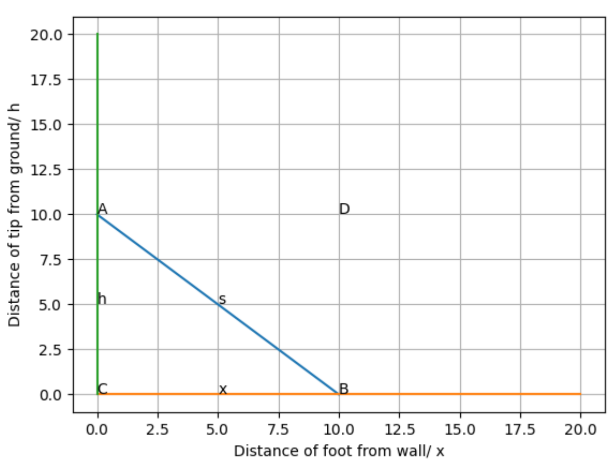

Fig. 0. Figure depicting the ladder:AB=s,BC=$x(t)$,AC=$h(t)$

Consider the position vectors,

$$\mathbf{A} = \begin{pmatrix} 0 \\ h(t) \end{pmatrix} \tag{1.1}$$

$$\mathbf{B} = \begin{pmatrix} x(t) \\ 0 \end{pmatrix} \tag{1.2}$$

Also at any time t,

$$\|\mathbf{A} - \mathbf{B}\| = s \tag{1.3}$$

$$\implies \|\mathbf{A} - \mathbf{B}\|^2 = s^2 \tag{1.4}$$

$$\implies x(t)^2 + h(t)^2 = s^2 \tag{1.5}$$

$\because$ length of rod is always constant

Consider another position vector, $\mathbf{D} = \begin{pmatrix} x(t) \\ h(t) \end{pmatrix}$

$$\|\mathbf{D}\| = s \tag{1.6}$$

$$\implies \|\mathbf{D}\|^2 = s^2 \tag{1.7}$$

Diffrentiating the equation on both sides with respect to time,

$$\frac{d(\|\mathbf{D}\|^2)}{dt} = \frac{d(s^2)}{dt} \tag{1.8}$$

$$\implies 2\mathbf{D}^\top \frac{d(\mathbf{D})}{dt} = 0 \tag{1.9}$$

$$\frac{d(\mathbf{D})}{dt} = \begin{pmatrix} \frac{dx}{dt} \\ \frac{dh}{dt} \end{pmatrix} \tag{1.10}$$

$$\implies \begin{pmatrix} x(t) & h(t) \end{pmatrix} \begin{pmatrix} \frac{dx}{dt} \\ \frac{dh}{dt} \end{pmatrix} = 0 \tag{1.11}$$

$$\implies x(t)\frac{d(x(t))}{dt} + h(t)\frac{d(h(t))}{dt} = 0 \tag{1.12}$$

Given in question at some $t = t_0$, $x(t)_{t=t_0} = 5m$
From equation (1.5)

$$5^2 + h(t)^2 = 13^2 \tag{1.13}$$

$$\implies h(t) = 12 \tag{1.14}$$

$$\implies h(t)_{t=t_0} = 12m \tag{1.15}$$

Also $\frac{d(x(t))}{dt}_{t=t_0} = 2m/s$

$\therefore$ From equation (1.12) at $t = t_0$

$$\frac{d(h(t))}{dt}_{t=t_0} = -\frac{5}{6}m/s \qquad (1.16)$$