

# AI1110: Assignment 2

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by the length of the diagonal of the rectangle.

## QUESTION 7

A 13 m long ladder is leaning against a wall, touching the wall at a certain height from the ground level. The bottom of the ladder is pulled away from the wall, along the ground at the rate of 2 m/s. How fast is the height of the ladder decreasing when the foot of the ladder is 5 m away from the wall?

## SOLUTION:

The various parameters involved in this question are listed in Table

TABLE 0  
VARIABLES USED

Parameter	Variable	Value
length of the rod	s	13
height from ground level to tip of the rod at time t	h(t)	???
distance from wall to foot of the rod at time t	x(t)	???

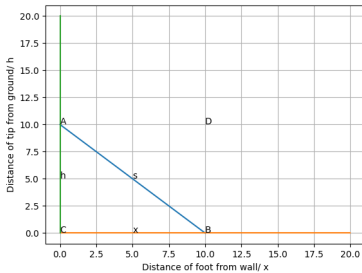


Fig. 0. Figure depicting the ladder:  $AB=s, BC=x(t), AC=h(t)$

**Baudhāyana Sulba Sūtra:** States that, if given a rectangle of side lengths  $a, b$  and diagonal length  $c$  the sum of the area of squares made by the side lengths of rectangle equals the area of square made

$$\Rightarrow a^2 + b^2 = c^2 \quad (0.0.1)$$

Consider the position vectors,

$$\mathbf{A} = \begin{pmatrix} 0 \\ h(t) \end{pmatrix} \quad (0.0.2)$$

$$\mathbf{B} = \begin{pmatrix} x(t) \\ 0 \end{pmatrix} \quad (0.0.3)$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.0.4)$$

Also at any time t,

$$\|\mathbf{A} - \mathbf{B}\| = s \quad (0.0.5)$$

$\therefore$  length of rod is always constant

Consider another position vector,  $\mathbf{D} = \begin{pmatrix} x(t) \\ h(t) \end{pmatrix}$ , clearly ACBD is a rectangle

$$\therefore \|\mathbf{A} - \mathbf{C}\| = \|\mathbf{B} - \mathbf{D}\| \quad (0.0.6)$$

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{B} - \mathbf{C}\| \quad (0.0.7)$$

$$(\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) = 0 \quad (0.0.8)$$

$\therefore$  From Baudhāyana Sulba Sūtra,

$$\|\mathbf{B} - \mathbf{C}\|^2 + \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{B}\|^2 \quad (0.0.9)$$

Differentiating with respect to time on both sides:

$$\frac{d(\|\mathbf{B} - \mathbf{C}\|^2 + \|\mathbf{A} - \mathbf{C}\|^2)}{dt} = \frac{d(\|\mathbf{A} - \mathbf{B}\|^2)}{dt} \quad (0.0.10)$$

$$\Rightarrow (\mathbf{B} - \mathbf{C})^\top \frac{d(\mathbf{B} - \mathbf{C})}{dt} + (\mathbf{A} - \mathbf{C})^\top \frac{d(\mathbf{A} - \mathbf{C})}{dt} = 0 \quad (0.0.11)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} x(t) \\ 0 \end{pmatrix} \quad (0.0.12)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 0 \\ h(t) \end{pmatrix} \quad (0.0.13)$$

$$\frac{d(\mathbf{B} - \mathbf{C})}{dt} = \begin{pmatrix} \frac{d(x(t))}{dt} \\ 0 \end{pmatrix} \quad (0.0.14)$$

$$\frac{d(\mathbf{A} - \mathbf{C})}{dt} = \begin{pmatrix} 0 \\ \frac{d(h(t))}{dt} \end{pmatrix} \quad (0.0.15)$$

$$\implies x(t) \frac{d(x(t))}{dt} + h(t) \frac{d(h(t))}{dt} = 0 \quad (0.0.16)$$

Given in question at some  $t = t_0$ ,  $x(t)_{t=t_0} = 5m$

From equation (0.0.9),

$$5^2 + \|\mathbf{A} - \mathbf{C}\|^2 = 13^2 \quad (0.0.17)$$

$$\implies \|\mathbf{A} - \mathbf{C}\| = 12m \quad (0.0.18)$$

$$\implies h(t)_{t=t_0} = 12m \quad (0.0.19)$$

Also  $\frac{d(x(t))}{dt}_{t=t_0} = 2m/s$

$\therefore$  From equation (0.0.16) at  $t = t_0$

$$\frac{d(h(t))}{dt}_{t=t_0} = -\frac{5}{6}m/s \quad (0.0.20)$$