Assignment 2 ICSE class 12 2019

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Question 7

A 13 m long ladder is leaning against a wall, touching the wall at a certain height from the ground level.The bottom of the ladder is pulled away from the wall,along the ground at the rate of 2 m/s. How fast is the height of the ladder decreasing when the foot of the ladder is 5 m away from the wall?

1 Solution:

The various parameters involved in this question are listed in Table

TABLE 0
Variables used

Parameter	Variable	Value
length of the rod	s	13
height from ground level to tip of the rod at time t	h(t)	???
distance from wall to foot of the rod at time t	x(t)	???

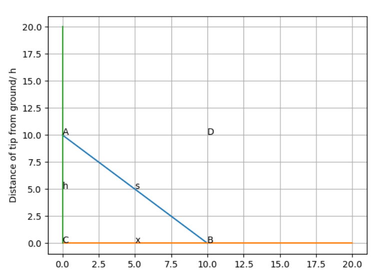

Fig. 0. Figure depicting the ladder:AB=s,BC=$x(t)$,AC=$h(t)$

Baudhāyana Sulba Sūtra: States that, if given a rectangle of side lengths a,b and diagonal length c the sum of the area of squares made by the side lengths of rectangle equals the area of square made by the length of the diagonal of the rectangle.

$$\implies a^2 + b^2 = c^2 \tag{1.1}$$

Consider the position vectors,

$$\mathbf{A} = \begin{pmatrix} 0 \\ h(t) \end{pmatrix} \tag{1.2}$$

$$\mathbf{B} = \begin{pmatrix} x(t) \\ 0 \end{pmatrix} \tag{1.3}$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1.4}$$

Also at any time t,

$$\|\mathbf{A} - \mathbf{B}\| = s \tag{1.5}$$

$\because$ length of rod is always constant

Consider another position vector, $\mathbf{D} = \begin{pmatrix} x(t) \\ h(t) \end{pmatrix}$,clearly ACBD is a rectangle

$$\because \|\mathbf{A} - \mathbf{C}\| = \|\mathbf{B} - \mathbf{D}\| \tag{1.6}$$

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{B} - \mathbf{C}\| \tag{1.7}$$

$$(\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) = 0 \tag{1.8}$$

$\therefore$ From Baudhāyana Sulba Sūtra,

$$\|\mathbf{B} - \mathbf{C}\|^2 + \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{B}\|^2 \tag{1.9}$$

Diffrentiating with respect to time on both sides:

$$\frac{d(\|\mathbf{B} - \mathbf{C}\|^2 + \|\mathbf{A} - \mathbf{C}\|^2)}{dt} = \frac{d(\|\mathbf{A} - \mathbf{B}\|^2)}{dt} \tag{1.10}$$

$$\implies (\mathbf{B} - \mathbf{C})^\top \frac{d(\mathbf{B} - \mathbf{C})}{dt} + (\mathbf{A} - \mathbf{C})^\top \frac{d(\mathbf{A} - \mathbf{C})}{dt} = 0 \tag{1.11}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} x(t) \\ 0 \end{pmatrix} \tag{1.12}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 0 \\ h(t) \end{pmatrix} \tag{1.13}$$

$$\frac{d(\mathbf{B} - \mathbf{C})}{dt} = \begin{pmatrix} \frac{d(x(t))}{dt} \\ 0 \end{pmatrix} \tag{1.14}$$

$$\frac{d(\mathbf{A} - \mathbf{C})}{dt} = \begin{pmatrix} 0 \\ \frac{d(h(t))}{dt} \end{pmatrix} \tag{1.15}$$

$$\implies x(t)\frac{d(x(t))}{dt} + h(t)\frac{d(h(t))}{dt} = 0 \tag{1.16}$$

Given in question at some $t = t_0$, $x(t)_{t=t_0} = 5m$
From equation (1.9),

$$5^2 + \left\|\mathbf{A} - \mathbf{C}\right\|^2 = 13^2 \tag{1.17}$$

$$\implies \left\|\mathbf{A} - \mathbf{C}\right\| = 12m \tag{1.18}$$

$$\implies h(t)_{t=t_0} = 12m \tag{1.19}$$

Also $\frac{d(x(t))}{dt}_{t=t_0} = 2m/s$

$\therefore$ From equation (1.16) at $t = t_0$

$$\frac{d(h(t))}{dt}_{t=t_0} = -\frac{5}{6}m/s \tag{1.20}$$