AI1110: Assignment 2

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ICSE class 12 paper 2019

Question 7

A 13 m long ladder is leaning against a wall, touching the wall at a certain height from the ground level.The bottom of the ladder is pulled away from the wall,along the ground at the rate of 2 m/s. How fast is the height of the ladder decreasing when the foot of the ladder is 5 m away from the wall?

Solution:

The various parameters involved in this question are listed in Table

TABLE 0
VARIABLES USED

Parameter	Variable	Value
length of the rod	s	13
height from ground level to tip of the rod at time t	h(t)	???
distance from wall to foot of the rod at time t	x(t)	???

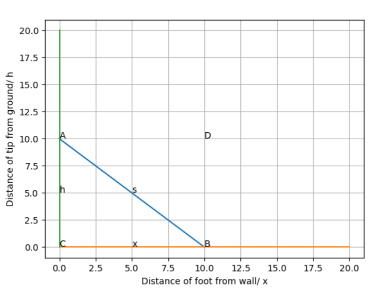

Fig. 0. Figure depicting the ladder:AB=s,BC=$x(t)$,AC=$h(t)$

Baudhāyana Sulba Sūtra: States that, if given a rectangle of side lengths a,b and diagonal length c the sum of the area of squares made by the side lengths of rectangle equals the area of square made by the length of the diagonal of the rectangle.

$$\implies a^2 + b^2 = c^2 \tag{0.0.1}$$

Consider the position vectors,

$$\mathbf{A} = \begin{pmatrix} 0 \\ h(t) \end{pmatrix} \tag{0.0.2}$$

$$\mathbf{B} = \begin{pmatrix} x(t) \\ 0 \end{pmatrix} \tag{0.0.3}$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.4}$$

Also at any time t,

$$\|\mathbf{A} - \mathbf{B}\| = s \tag{0.0.5}$$

$\because$ length of rod is always constant

Consider another position vector, $\mathbf{D} = \begin{pmatrix} x(t) \\ h(t) \end{pmatrix}$,clearly ACBD is a rectangle

$$\because \|\mathbf{A} - \mathbf{C}\| = \|\mathbf{B} - \mathbf{D}\| \tag{0.0.6}$$

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{B} - \mathbf{C}\| \tag{0.0.7}$$

$$(\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) = 0 \tag{0.0.8}$$

$\therefore$ From Baudhāyana Sulba Sūtra,

$$\|\mathbf{B} - \mathbf{C}\|^2 + \|\mathbf{A} - \mathbf{C}\|^2 = \|\mathbf{A} - \mathbf{B}\|^2 \tag{0.0.9}$$

Diffrentiating with respect to time on both sides:

$$\frac{d(\|\mathbf{B} - \mathbf{C}\|^2 + \|\mathbf{A} - \mathbf{C}\|^2)}{dt} = \frac{d(\|\mathbf{A} - \mathbf{B}\|^2)}{dt} \tag{0.0.10}$$

$$\implies (\mathbf{B} - \mathbf{C})^\top \frac{d(\mathbf{B} - \mathbf{C})}{dt} + (\mathbf{A} - \mathbf{C})^\top \frac{d(\mathbf{A} - \mathbf{C})}{dt} = 0 \tag{0.0.11}$$

$$\mathbf{B}-\mathbf{C}=\begin{pmatrix}x(t)\\0\end{pmatrix} \tag{0.0.12}$$

$$\mathbf{A}-\mathbf{C}=\begin{pmatrix}0\\h(t)\end{pmatrix} \tag{0.0.13}$$

$$\frac{d(\mathbf{B}-\mathbf{C})}{dt}=\begin{pmatrix}\frac{d(x(t))}{dt}\\0\end{pmatrix} \tag{0.0.14}$$

$$\frac{d(\mathbf{A}-\mathbf{C})}{dt}=\begin{pmatrix}0\\\frac{d(h(t))}{dt}\end{pmatrix} \tag{0.0.15}$$

$$\implies x(t)\frac{d(x(t))}{dt}+h(t)\frac{d(h(t))}{dt}=0 \tag{0.0.16}$$

Given in question at some t=t_0, $x(t)_{t=t_0}=5m$
From equation (0.0.9),

$$5^2+\left\|\mathbf{A}-\mathbf{C}\right\|^2=13^2 \tag{0.0.17}$$

$$\implies \left\|\mathbf{A}-\mathbf{C}\right\|=12m \tag{0.0.18}$$

$$\implies h(t)_{t=t_0}=12m \tag{0.0.19}$$

Also $\frac{d(x(t))}{dt}_{t=t_0}=2m/s$

$\therefore$ From equation (0.0.16) at $t=t_0$

$$\frac{d(h(t))}{dt}_{t=t_0}=-\frac{5}{6}m/s \tag{0.0.20}$$