AI1110
Assignment 9

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Outline

1. Question
2. Solution

Papoulis Chapter 9 problem 9.9

Show that the process $x(t) = c\omega(t)$ is WSS iff $E(c) = 0$ and $\omega(t) = e^{j(\omega t+\theta)}$

Solution

A stochastic process $x(t)$ is called wide-sense stationary if its mean is constant and $R_x(t_1, t_2) = R_x(T)$, where $T = t_1 - t_2$.

$$\eta_c = 0 \tag{1}$$
$$\implies \eta_{x(t)} = \eta_c e^{j(\omega t+\theta)} = 0 \tag{2}$$
$$R_{xx}(t+\tau, t) = {\sigma_c}^2 e^{j\omega\tau} \tag{3}$$

Hence $x(t)$ is WSS, Now lets prove the converse:

$\eta_{x(t)} = \eta_c w(t)$ is independent of t;hence$\eta_c = 0$.The function $R_x(t_1, t_2) = {\sigma_c}^2 w(t_1) w^*(t_2)$ depends only on $\tau = t_1 - t_2$;Hence $\omega(t+\tau)\omega^*(t) = g(\tau)$, with $\tau = 0$ this yields

$$\left|\omega(t)\right|^2 = g(0) = \text{constant} \tag{4}$$

$$\omega(t+\tau)\omega^*(t) = a^2 e^{j[\phi(t+\tau) - \phi(t)]} \tag{5}$$

Hence the difference $\phi(t+\tau) - \phi(t)$ depends only on τ
From this it follows that,if $\phi(t)$ is continuous then,$\phi(t)$ is a linear function of t. Assuming $\phi(t)$ to be differentiable we get

$$\phi'(t+\tau) = \phi'(t) \tag{6}$$

$$\text{putting t=0} \tag{7}$$

$$\phi'(\tau) = \phi'(0) = \text{constant} \tag{8}$$