AI1110
Assignment 10

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Outline

1. **Question**

2. **Solution**

Papoulis Chapter 9 problem 9.35

Show that if $y(t) = x(t + a) - x(t - a)$ then,

$R_y(\tau) = 2R_x(\tau) - R_x(\tau + 2a) - R_x(\tau - 2a)$

$S_y(\omega) = 4S_x(\omega)\sin^2(a\omega)$

Solution

The process $y(t) = x(t + a) - x(t - a)$ is the output of a system with input $x(t)$ and system function
$H(\omega) = e^{ja\omega} + e^{-ja\omega} = 2j\sin(a\omega)$
From corollary of convolution theorem,

$$S_{yy}(\omega) = S_{xx}(\omega)H(\omega)H^*(\omega) \tag{1}$$

$$\implies S_{yy}(\omega) = S_{xx}(\omega)|H(\omega)|^2 \tag{2}$$

$$\therefore S_{yy}(\omega) = S_{xx}(\omega)(2\sin^2(a\omega)) \tag{3}$$

$$\implies S_y(\omega) = 4S_x(\omega)\sin^2(a\omega) \tag{4}$$

$$\implies S_y(\omega) = (2 - e^{2ja\omega} - e^{-2ja\omega})S_x(\omega) \tag{5}$$

$$\therefore R_y(\tau) = 2R_x(\tau) - R_x(\tau + 2a) - R_x(\tau - 2a) \tag{6}$$