AI1110
Assignment 5

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Outline

Papoulis Chapter 2 example 2.17

If we toss a coin twice, we generate the four outcomes hh, ht, th, and tt

Solution

(a) To construct an experiment with these outcomes, it suffices to assign probabilities to its elementary events. With $a$and b two positive numbers such that $a + b = 1$, we assume that

$$P(hh) = a^2 \quad P(ht) = P(th) = ab \quad P(tt) = b^2$$

These probabilities are consistent with the axioms because

$$a^2 + ab + ab + b^2 = (a + b)^2 = 1 \tag{1}$$

In the experiment so constructed, the events
H_1 = heads at first toss = (hh, ht)
H_2 = heads at second toss = (hh, th)
The intersection H_1 H_2 of these two events consists of the single outcome (hh) Hence

$$P(H_1H_2) = P(hh) = a^2 = P(H_1)P(H_2) \tag{2}$$

This shows that the events H_1 and H_2 are independent.

(b) The experiment in part (a) of this example can be specified in terms of the probabilities $P(H_1) = P(H_2) = a$ of the events H_1 and H_2 and the information that these events are independent.
Indeed as we have shown the events H_1' and H_2 and the events H_1' and H_2' are also independent. Furthermore,

$$H_1H_2 = (hh) \quad H_1H_2 = (ht) \quad H_1H_2 = (th) \quad H_1H_2 = (tt)$$

and $P(H_1') = 1 - P(H_1) = 1 - a, P(H_2') = 1 - P(H_2) = 1 - a$. Hence

$$P(hh) = a^2 \quad P(ht) = a(I - a) \quad P(th) = (I - a)a \quad P(tt) = (1 - a)^2$$