

Assignment:Random Variables

Pranav B (AI21BTECH11023)

1 SOLUTION:

(1.3) Since U is an uniform random variable distribution, $P_U(x_i) = P_U(x_j) = k, \forall i, j$
CDF of $P_U(x) = F_U(x)$

$$= \int P_U(x) dx \quad (1.1)$$

$$= \int k dx \quad (1.2)$$

$$\text{wkt } \int_0^1 k dx = 1 \quad (1.3)$$

$$\therefore k = 1 \quad (1.4)$$

$$\therefore F_U(x) = x \quad (1.5)$$

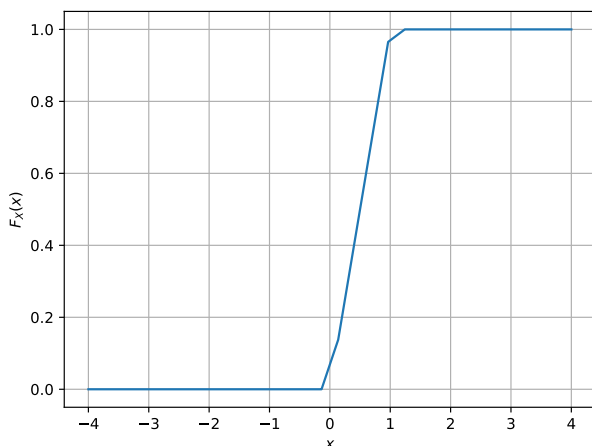


Fig. 0. CDF for (1)

$$(1.5) dF_U(x) = dx$$

$$\therefore E[U^k] = \int_{-\infty}^{\infty} x^k dx \quad (1.6)$$

$$E[U] = \int_0^1 x dx = \frac{1}{2} \quad (1.7)$$

$$E[U^2] = \int_0^1 x^2 dx = \frac{1}{3} \quad (1.8)$$

$$\therefore P_X(x) = 0, \forall x \in (1, \infty) \cap (-\infty, 0) \quad (1.9)$$

$$Var(X) = E[U^2] - (E[U])^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \quad (1.10)$$

- PDF is symmetric about $x = 0$
- graph is bell shaped
- mean of graph is situated at the apex point of the bell

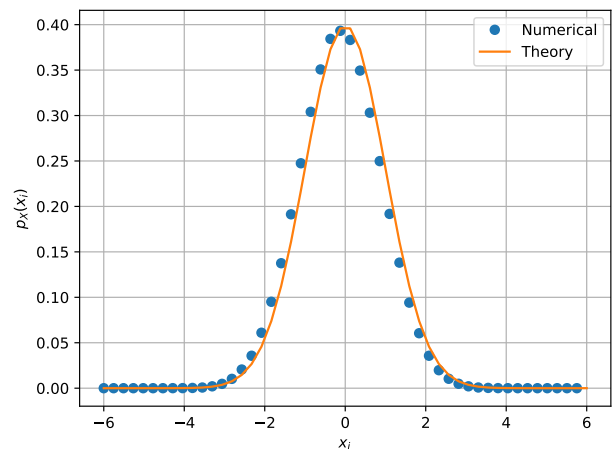


Fig. 0. PDF for (2)

$$(2.5) \text{ Given } p_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$F_X(x) = \int_{-\infty}^x p_X(x) dx$$

$$= \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \quad (1.11)$$

$$= \frac{1}{2} \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) \quad (1.12)$$

$$E[x] = \int_{-\infty}^{\infty} x p_X(x) dx \quad (1.13)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x e^{-\frac{x^2}{2}} dx \quad (1.14)$$

$$\because x e^{-\frac{x^2}{2}} \text{ is a odd function,} \quad (1.15)$$

$$E[x] = 0$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 p_X(x) dx \quad (1.16)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x (x e^{-\frac{x^2}{2}}) dx \quad (1.17) \quad (3.2)$$

Using integration by parts:

$$= x \int x e^{-\frac{x^2}{2}} dx - \int \frac{d(x)}{dx} \int x e^{-\frac{x^2}{2}} dx \quad (1.18)$$

$$I = \int x e^{-\frac{x^2}{2}} dx \quad (1.19)$$

$$\text{Let } \frac{x^2}{2} = t \quad (1.20)$$

$$\implies x dx = dt \quad (1.21)$$

$$\implies \int e^{-t} dt = -e^{-t} + c \quad (1.22)$$

$$\therefore \int x e^{-\frac{x^2}{2}} = -e^{-\frac{x^2}{2}} + c \quad (1.23)$$

Using (1.23) in (1.18)

$$= -x e^{-\frac{x^2}{2}} + \int e^{-\frac{x^2}{2}} dx \quad (1.24)$$

$$\text{Also, } \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi} \quad (1.25)$$

$$\therefore \text{ substituting limits we get, } E[x^2] = 1 \quad (1.26)$$

$$\operatorname{Var}(X) = E[x^2] - (E[x])^2 = 1 - 0 \quad (1.27)$$

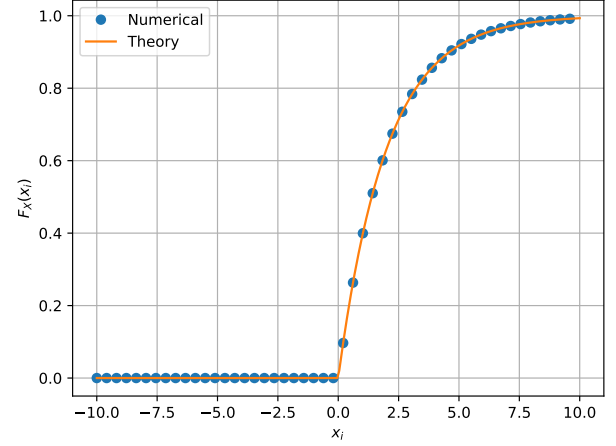


Fig. 0. CDF for (3)

$$F_V(x) = P(V \leq x) \quad (1.28)$$

$$= P(-2\ln(1 - U) \leq x) \quad (1.29)$$

$$= P(1 - e^{-\frac{x}{2}} \geq U) \quad (1.30)$$

$$P(U < x) = \int_0^x dx = x \quad (1.31)$$

$$\therefore P(1 - e^{-\frac{x}{2}} \geq U) = 1 - e^{-\frac{x}{2}}, \forall x \geq 0 \quad (1.32)$$