Assignment:Random Variables

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1 Solution:

(1.3)Since U is an uniform random variable distribution, $P_U(x_i) = P_U(x_j) = k$,$\forall i, j$
CDF of $P_U(x)$=$F_U(x)$

$$= \int P_U(x)dx \tag{1.1}$$

$$= \int kdx \tag{1.2}$$

$$\text{wkt} \int_0^1 kdx = 1 \tag{1.3}$$

$$\therefore k = 1 \tag{1.4}$$

$$\therefore F_U(x) = x \tag{1.5}$$

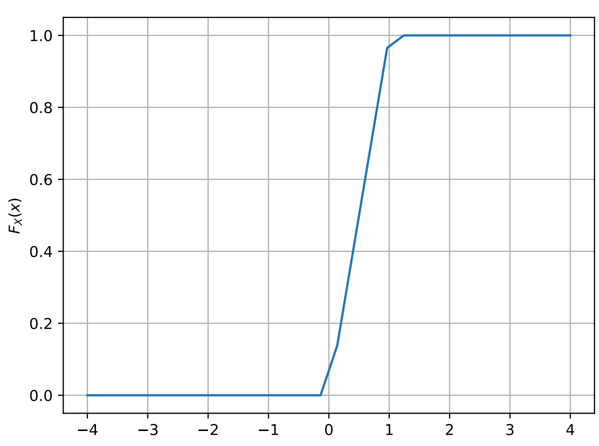

Fig. 0. CDF for (1)

(1.5) $dF_U(x) = dx$

$$\therefore E[U^k] = \int_{-\infty}^{\infty} x^k dx \tag{1.6}$$

$$E[U] = \int_0^1 xdx = \frac{1}{2} \tag{1.7}$$

$$E[U^2] = \int_0^1 x^2dx = \frac{1}{3} \tag{1.8}$$

$$\because P_X(x) = 0, \forall x \in (1, \infty) \cap (-\infty, 0) \tag{1.9}$$

$$Var(X) = E[U^2] - (E[U])^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12} \tag{1.10}$$

(2.3)

- PDF is symmetric about $x = 0$
- graph is bell shaped
- mean of graph is situated at the apex point of the bell

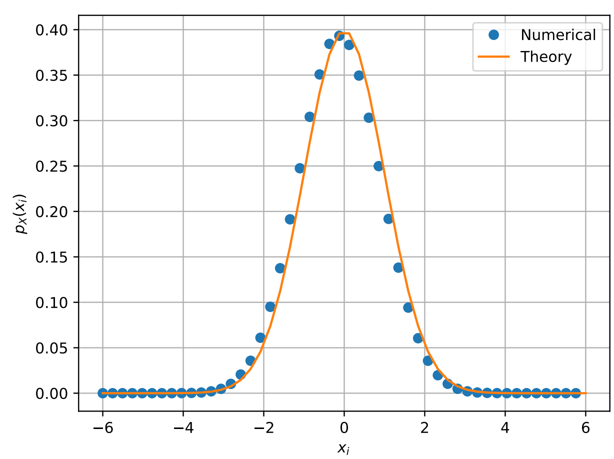

Fig. 0. PDF for (2)

(2.5) Given ,$p_X(x) = \frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}$
$F_X(x) = \int_{-\infty}^{x} p_X(x)dx$

$$= \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx \quad (1.11)$$

$$= \frac{1}{2} erf\left(\frac{x}{\sqrt{2}}\right) \quad (1.12)$$

$$E[x] = \int_{-\infty}^{\infty} x p_X(x) dx \quad (1.13)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x e^{-\frac{-x^2}{2}} \quad (1.14)$$

$$\because x e^{-\frac{-x^2}{2}} \text{is a odd function,} \quad (1.15)$$

$$E[x] = 0$$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 p_X(x) dx \quad (1.16)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x(x e^{-\frac{-x^2}{2}}) dx \quad (1.17)$$

Using integration by parts:

$$= x \int x e^{-\frac{-x^2}{2}} dx - \int \frac{d(x)}{dx} \int x e^{-\frac{-x^2}{2}} dx \quad (1.18)$$

$$I = \int x e^{-\frac{-x^2}{2}} \quad (1.19)$$

$$\text{Let} \frac{x^2}{2} = t \quad (1.20)$$

$$\implies x dx = dt \quad (1.21)$$

$$\implies = \int e^{-t} dt = -e^{-t} + c \quad (1.22)$$

$$\therefore \int x e^{-\frac{-x^2}{2}} = -e^{-\frac{-x^2}{2}} + c \quad (1.23)$$

Using (1.23) in (1.18)

$$= -xe - \frac{-x^2}{2} + \int e^{-\frac{-x^2}{2}} dx \quad (1.24)$$

$$\text{Also,} \int_{-\infty}^{\infty} e^{-\frac{-x^2}{2}} dx = \sqrt{2\pi} \quad (1.25)$$

$$\therefore \text{substituting limits we get,} E[x^2] = 1 \quad (1.26)$$

$$Var(X) = E[x^2] - (E[x])^2 = 1 - 0 \quad (1.27)$$

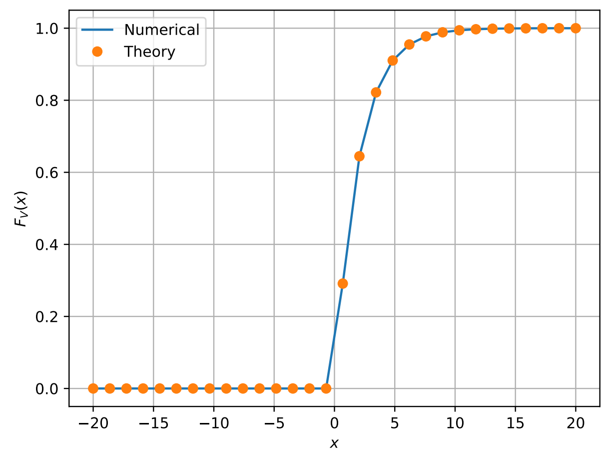

Fig. 0. CDF for (3)

(3.2)

$$F_V(x) = P(V \le x) \quad (1.28)$$

$$= P(-2ln(1-U) \le x) \quad (1.29)$$

$$= P(1 - e^{\frac{-x}{2}} \ge U) \quad (1.30)$$

$$P(U < x) = \int_0^x dx = x \quad (1.31)$$

$$\therefore P(1 - e^{\frac{-x}{2}} \ge U) = 1 - e^{\frac{-x}{2}}, \forall x \ge 0 \quad (1.32)$$