Circuits and Transforms

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Contents

***Abstract*—This manual provides a simple introduction to Transforms**

1 Definitions

1. The unit step function is

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases} \tag{1.1}$$

2. The Laplace transform of $g(t)$ is defined as

$$G(s) = \int_{-\infty}^{\infty} g(t)e^{-st}\,dt \tag{1.2}$$

2 Laplace Transform

1. In the circuit, the switch S is connected to position P for a long time so that the charge on the capacitor becomes $q_1\,\mu C$. Then S is switched to position Q. After a long time, the charge on the capacitor is $q_2\,\mu C$.

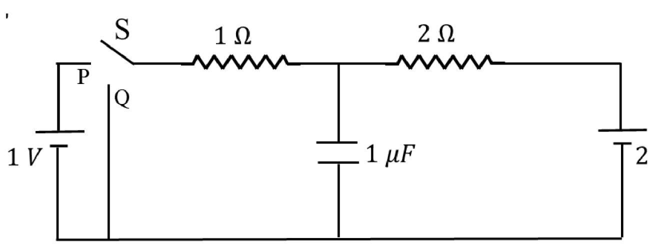

Fig. 2.1

2. Find q_1.
Solution: : At steady state capacitor behaves like an open circuit.Assume current i flows in the circuit in anti-clockwise direction,using KVL

$$-2 + 2i + i + 1 = 0 \tag{2.1}$$
$$\therefore i = \frac{1}{3}A \tag{2.2}$$

Assume V_C be voltage across the capacitor at that time,

$$\implies -2 + 2i = -V_C \tag{2.3}$$
$$\therefore V_C = \frac{4}{3}V \tag{2.4}$$
$$V_C = \frac{q_1}{C} \tag{2.5}$$
$$\implies q_1 = \frac{4}{3}\mu C \tag{2.6}$$

3. Show that the Laplace transform of $u(t)$ is $\frac{1}{s}$ and find the ROC.
Solution: :

$$\mathcal{L}[u(t)] = \int_{-\infty}^{\infty} u(t)e^{-st}dt \tag{2.7}$$
$$= \int_{0}^{\infty} e^{-st}dt \tag{2.8}$$
$$= \frac{1}{s} \tag{2.9}$$

ROC: Re(s)> 0

4. Show that

$$e^{-at}u(t) \overset{\mathcal{H}}{\longleftrightarrow} L\frac{1}{s+a}, \quad a > 0 \tag{2.10}$$

and find the ROC.
Solution: :

$$\mathcal{L}[e^{-at}u(t)] = \int_{-\infty}^{\infty} u(t)e^{-(s+a)t}dt \tag{2.11}$$
$$= \int_{0}^{\infty} e^{-(s+a)t}dt \tag{2.12}$$
$$= \frac{1}{s+a} \tag{2.13}$$

ROC: Re(s)> $-a$

5. Now consider the following resistive circuit transformed from Fig. 2.1 where

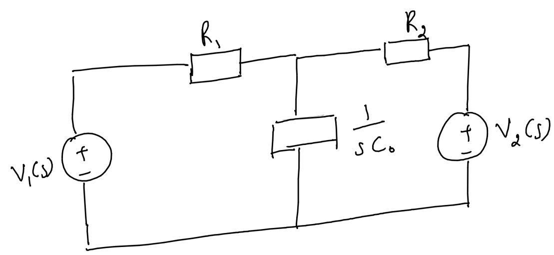

Fig. 2.2

$$u(t) \overset{\mathcal{H}}{\longleftrightarrow} LV_1(s) \tag{2.14}$$

$$2u(t) \overset{\mathcal{H}}{\longleftrightarrow} LV_2(s) \tag{2.15}$$

Find the voltage across the capacitor $V_{C_0}(s)$.
Solution: :

$$\mathcal{L}[V_1] = V_1(s) = \frac{1}{s} \tag{2.16}$$

$$\mathcal{L}[V_2] = V_2(s) = \frac{2}{s} \tag{2.17}$$

$$\mathcal{L}[C] = \frac{1}{Cs} \tag{2.18}$$

Using KVL,

$$i = i_1 + i_2 \tag{2.19}$$

$$\frac{-2}{s} + 2i_2 - \frac{i_1}{sC} = 0 \tag{2.20}$$

$$\frac{i_1}{sC} + i + \frac{1}{s} = 0 \tag{2.21}$$

$$\implies i_1(s) = -\frac{2}{s + \frac{3}{2C}} \tag{2.22}$$

$$\implies V_{C_0}(s) = \frac{i_1(s)}{sC} \tag{2.23}$$

$$= \frac{2}{C}\frac{1}{s(s + \frac{3}{2C})} \tag{2.24}$$

6. Find $v_{C_0}(t)$. Plot using python.
Solution: :Taking laplace inverse on both sides of above equation

$$v_{C_0}(t) = \mathcal{L}^{-1}\left[\frac{2}{C}\frac{1}{s(s + \frac{3}{2C})}\right] \tag{2.25}$$

$$= \frac{4}{3}\mathcal{L}^{-1}\left[\frac{1}{s} - \frac{1}{s + \frac{3}{2C}}\right] \tag{2.26}$$

$$= \frac{4}{3}[1 - e^{-\frac{3}{2C}}]u(t) \tag{2.27}$$

Run the following code

```
wget https://github.com/Pranavb060504/
    SIgnalProcessing/blob/main/
    circuits_and_transforms/vt.py
```

Use the following command in the terminal to run the code

```
python3 vt.py
```

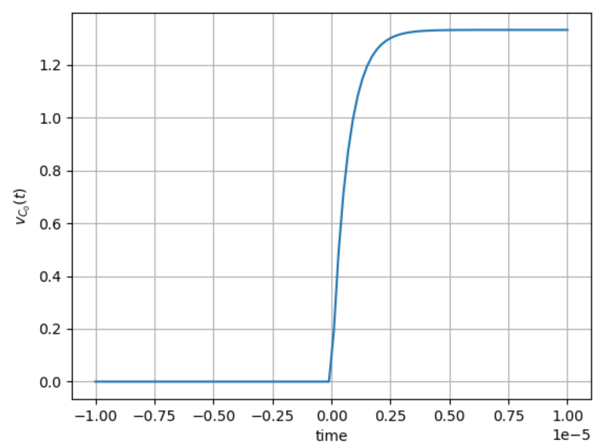

Fig. 2.3

7. Verify your result using ngspice.
Solution: : Run the following code

```
wget https://github.com/Pranavb060504/
    SIgnalProcessing/blob/main/
    circuits_and_transforms/C1.cir
```

Use the following command in the terminal to run the code

```
ngspice C1.cir
tran 0.0001ms 0.1ms
plot (V(Cin)−V(Cout))
```

3 Initial Conditions

1. Find q_2 in Fig. 2.1.
Solution: : at t=∞,capacitor is fully charged,

$$3i = 2 \tag{3.1}$$
$$i = \frac{2}{3}A \tag{3.2}$$
$$V_{C_0} = 2 - \frac{4}{3} = \frac{2}{3}V \tag{3.3}$$
$$q_2 = CV = \frac{2}{3}\mu C \tag{3.4}$$

2. Draw the equivalent s-domain resistive circuit when S is switched to position Q. Use variables R_1, R_2, C_0 for the passive elements.
3. $V_{C_0}(s) = ?$
Solution: :by junction law

$$i_1 + i_2 + i_3 = 0 \tag{3.5}$$
$$\frac{V_c}{R_1} + \frac{V_c - \frac{2}{s}}{R_2} + \frac{V_c - \frac{4}{3s}}{\frac{1}{sC_0}} = 0 \tag{3.6}$$
$$V_C(s) = \frac{\frac{1}{C} + \frac{4s}{3}}{s(s + \frac{3}{2C})} \tag{3.7}$$

4. $v_{C_0}(t) = ?$ Plot using python.
Solution: : Taking laplace inverse on both sides

$$v_{C_0}(t) = \mathcal{L}^{-1}\left[\frac{A}{s} + \frac{B}{s + \frac{3}{2C}}\right] \tag{3.8}$$
$$A = \frac{2}{3}, B = \frac{2}{3} \tag{3.9}$$
$$\therefore v_{C_0}(t) = \frac{2}{3}(1 + e^{\frac{-3}{2C}t})u(t) \tag{3.10}$$

Run the following code

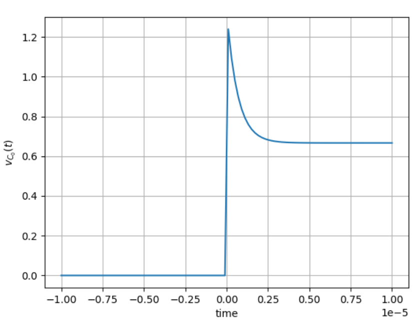

Fig. 3.1

```
wget https://github.com/Pranavb060504/
    SIgnalProcessing/blob/main/
    circuits_and_transforms/vt1.py
```

Use the following command in the terminal to run the code

```
python3 vt1.py
```

5. Verify your result using ngspice.
Solution: : Run the following code

```
wget https://github.com/Pranavb060504/
    SIgnalProcessing/blob/main/
    circuits_and_transforms/C2.cir
```

Use the following command in the terminal to run the code

```
ngspice C2.cir
tran 0.0001ms 0.1ms
plot (V(Cin)−V(Cout))
```

6. Find $v_{C_0}(0-), v_{C_0}(0+)$ and $v_{C_0}(\infty)$.
Solution: : for t=0^- switch is still at P , so

$$V_C(0^-) = \frac{4}{3}V \tag{3.11}$$

from formula

$$V_C(0^+) = \frac{2}{3}(1+1)(1) = \frac{4}{3} \tag{3.12}$$
$$V_C(\infty) = \frac{2}{3} \tag{3.13}$$