# EE 305 Project Report

## Introduction:

The parallel-plate waveguide is a classic configuration in electromagnetic eld theory which consists of two metal plates separated by a distance H. The material between the plates can be electric, magnetic, and/or lossy assuming a dielectric constant $\varepsilon_r$, relative permeability $\mu_r$, electrical conductivity $\sigma$, and a magnetic conductivity ɣ.

Between the plates, there exists an electromagnetic field with two possible polarizations. Under TE polarization, the electric field $\bar{E}$ is parallel to the plates and has only a z-oriented component. Under TM polarization, the magnetic field is z-oriented. In either case, the electric field is governed by the vector Helmholtz equation in two dimensions, given by

$$(\Delta)^2 E + k^2 E = 0 \text{ where k is intrinsic wavenumber}$$

Here, we are considering a parallel-plate waveguide made of two perfectly conducting plates separated by a lossless dielectric medium with constitutive parameters $\varepsilon_r$, $\mu_r$. The plates are assumed to be infinite along x- and z-directions, and separated by a distance of *b* in y-direction. Let's consider a y-polarized $TM_n$ mode wave propagating in +z-direction. Now, we need to determine the cut off frequency of the propagating wave for different modes:

We know, the governing equation for TM analysis i.e. $H_z = 0$ is

$$\frac{d^2E_{0,z}}{dx^2} + \frac{d^2E_{0,z}}{dy^2} - k_c^2 E_{0,z} = 0 \text{ where } k_c^2 = k^2 - \beta^2 \qquad \text{....}\textit{equation-1}$$

After solving the above equation, we can calculate the remaining components as:

$$H_{0,x} = \frac{j\omega\varepsilon}{k_c^2}\frac{dE_{0,z}}{dy},\ H_{0,y} = \frac{j\omega\varepsilon}{k_c^2}\frac{dE_{0,z}}{dx},\ E_{0,x} = \frac{-j\beta}{k_c^2}\frac{dE_{0,z}}{dx},\ E_{0,y} = \frac{-j\beta}{k_c^2}\frac{dE_{0,z}}{dy} \text{ and } H_z = 0$$

Since the waveguide is in direction of x-axis, we can write $\frac{d}{dx} = 0 \ and\ \frac{d^2}{dx^2} = 0$

∴ we can write equation-1 as: $\frac{d^2E_{0,z}}{dy^2} - k_c^2 E_{0,z} = 0$

The general solution of the above equation: $E_{0,z} = Asin(k_c y) + Bcos(k_c y)$

Since the Electric Field component $E_{0,z}$ is tangential to the interfaces ∴ we can apply boundary conditions directly. The first boundary condition:

$E_{0,z}(x, 0) = Asin(0) + Bcos(0) = B = 0$ which give $B = 0$

The second boundary condition: $E_{0,z}(x, b) = Asin(k_c b) = 0$

We can't choose A = 0 since it would give a trivial solution. Hence, the term $sin(k_c b)$ must be 0 at y = b.

$sin(k_c b) = 0$ which gives $k_c b = n\pi$ where $n = 0, 1, 2, 3, ........$

Hence, the cut off wave number is $k_c = \frac{n\pi}{b}\ where\ n = 0, 1, 2, 3, ....$

Here, n = 0 is allowed since it doesn't force the field to be entirely zero but does however force the field to be perfectly uniform. Thus, $TM_0$ is the TEM mode.

### Cut-off Frequency:

We have earlier defined $k_c^2 = k^2 - \beta^2$ in equation-1. Now, we got $k_c = \frac{n\pi}{b}$.

Hence, we can write: $k^2 - \beta^2 = (\frac{n\pi}{b})^2$

Now, we can write phase constant $\beta$ as $\beta_n = \sqrt{k^2 - (\frac{n\pi}{b})^2}$ $where\, n = 0, 1, 2, 3,...$

For this to exist, $\beta$ must be real. i.e. $\beta > 0$ or $k > k_c$. Hence, cut-off condition will be:

$$\omega\sqrt{\mu\varepsilon} = k_c = \frac{n\pi}{b} => 2\pi f_c\sqrt{\mu\varepsilon} = \frac{n\pi}{b} => f_c = \frac{n}{2b\sqrt{\mu\varepsilon}} = \frac{k_c}{2\pi\sqrt{\mu\varepsilon}}$$

Hence, the cut off frequency $f_c = \frac{n}{2b\sqrt{\mu\varepsilon}}$ where $n = 0, 1, 2, 3,..$

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