4th order Runge-Kutta method

Code for 4th order Runge-Kutta method:

Consider transient 2D conduction problem governed by equation

$$\frac{\partial \theta}{\partial t} = \alpha \left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right]$$

Take a rectangular domain of width L and height W

The boundary conditions are:

At x = 0: $\frac{\partial \theta}{\partial x} = 0$

At x = L: $\theta = 0$

At y = 0: $\frac{\partial \theta}{\partial y} = 0$

At y = W: $\theta = 0$

Initial condition:

At t = 0: $\theta_i(x, y) = 1$

Write a CFD code for finding the temperature variation. This problem has an analytical solution which is given by a series solution expressed as

$$\frac{\theta(x,y)}{\theta_i} = 4\left[\sum_{n=0}^{\infty} \frac{(-1)^n}{\lambda_n L} e^{-\alpha {\lambda_n}^2 t} \cos \lambda_n x\right]\left[\sum_{m=0}^{\infty} \frac{(-1)^m}{\lambda_m L} e^{-\alpha {\lambda_m}^2 t} \cos \lambda_m y\right] \quad (1)$$

where λ_n and λ_m are given by

$$\lambda_n = (2n+1)\frac{\pi}{2L}, \qquad \text{n=0, 1, 2, 3, . . .}$$

$$\lambda_m = (2m+1)\frac{\pi}{2W}, \qquad \text{m=0, 1, 2, 3, . . .}$$

Take L = 2, W = 1, θ_i= 1 and α = 1. In your code you also write a subroutine to calculate temperature variation using the above analytical expression given by Eq. (1).

Your results should contain at least the following:

(a) Compare the temperature contours obtained by the code and with those obtained by using the above analytical expression at t = 0.1(show the temperature contours figures side by side).

(b) Compare the temperature contours obtained by the code and with those obtained by using the above analytical expression at t = 0.2(show the temperature contours figures side by side).

(c) Compare the temperature contours obtained by the code and with those obtained by using the above analytical expression at t = 1.0(show the temperature contours figures side by side).

(d) Compare the temperature variation with y along mid-vertical plane x = 0.5 from the code and the above analytical expression for t=0.1. Plot both temperature profiles in the same figure.

(e) Compare the temperature variation with y along mid-horizontal plane y = 0.5 from the code and the above analytical expression for t=0.5. Plot both temperature profiles in the same figure.

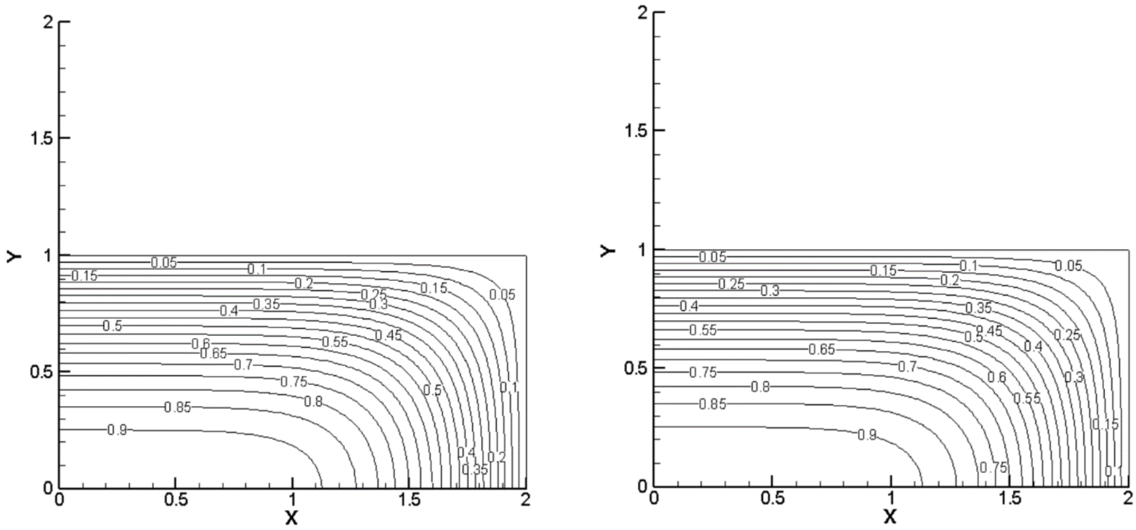
(f) Temperature variation at point (x, y) = (L/4, W/4) with respect to time till steady state is reached.

$$\phi^*_{n+\frac{1}{2}} = \phi^n + \frac{\Delta t}{2} f(t_n, \phi^n)$$

$$\phi^{**}_{n+\frac{1}{2}} = \phi^n + \frac{\Delta t}{2} f\left(t_n, \phi^*_{n+\frac{1}{2}}\right)$$

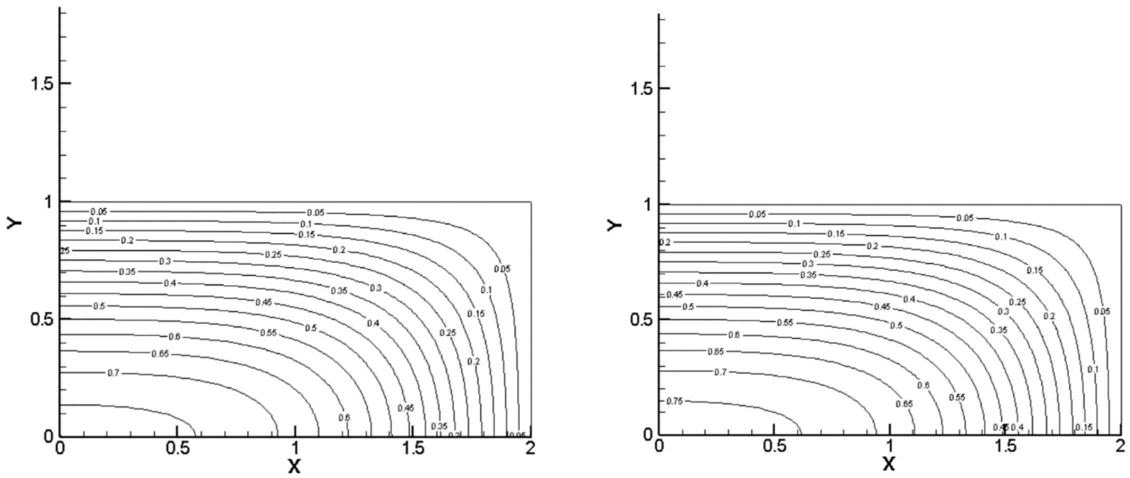
$$\phi^*_{n+1} = \phi^n + \Delta t f\left(t_n, \phi^{**}_{n+\frac{1}{2}}\right)$$

$$\phi^{n+1} = \phi^n + \frac{\Delta t}{6}\left[f(t_n, \phi^n) + 2f\left(t_n, \phi^*_{n+\frac{1}{2}}\right) + 2f\left(t_n, \phi^{**}_{n+\frac{1}{2}}\right) + f(t_{n+1}, \phi^*_{n+1})\right]$$



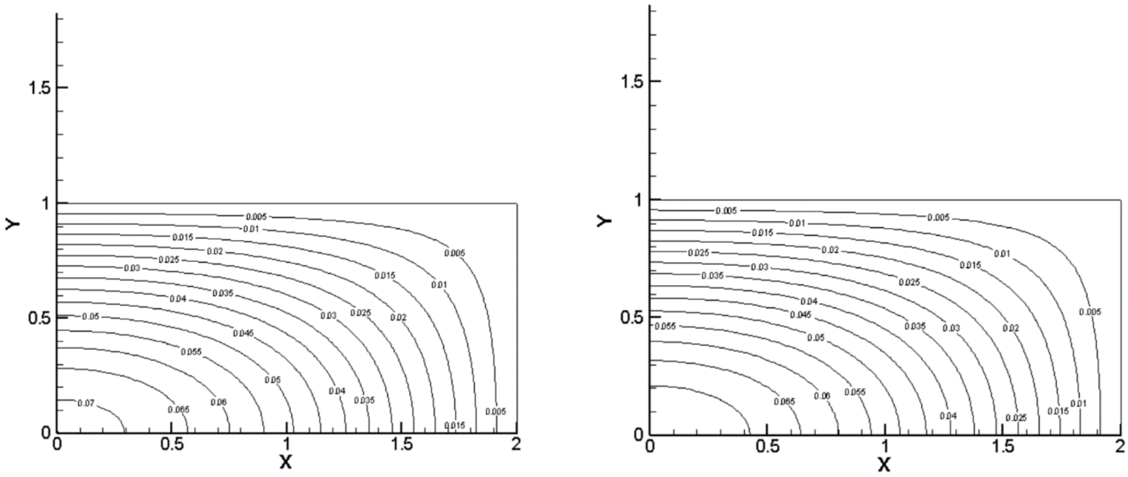
a. Time integral result b. Analytical result

Figure 1: Temperature contours at t=0.1



a. Time integral result b. Analytical result

Figure 2: Temperature contours at t=0.2



a. Time integral result b. Analytical result

Figure 3: Temperature contours at t=1

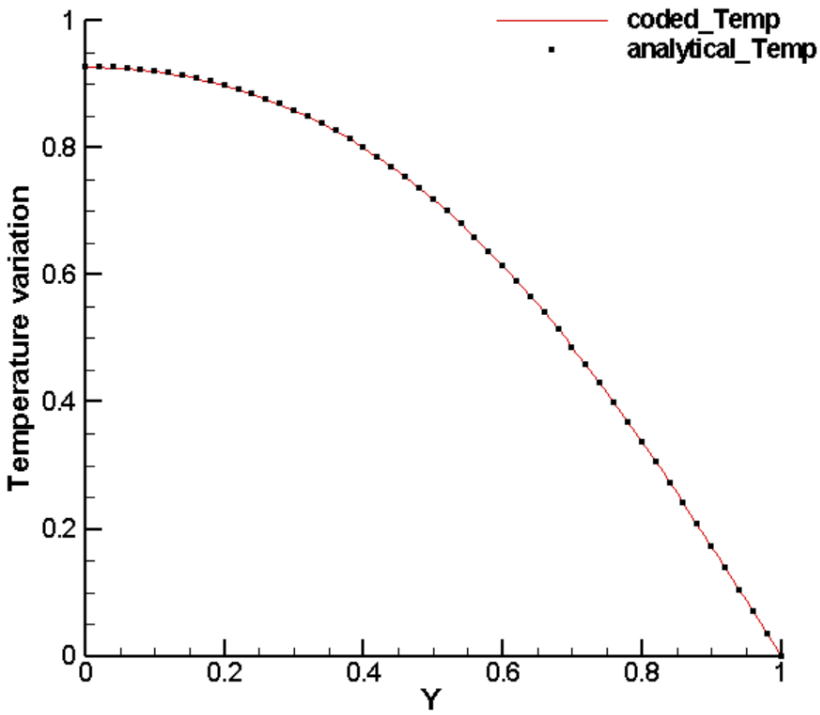


Figure 4: The temperature variation with y along mid-vertical plane x = 0.5L

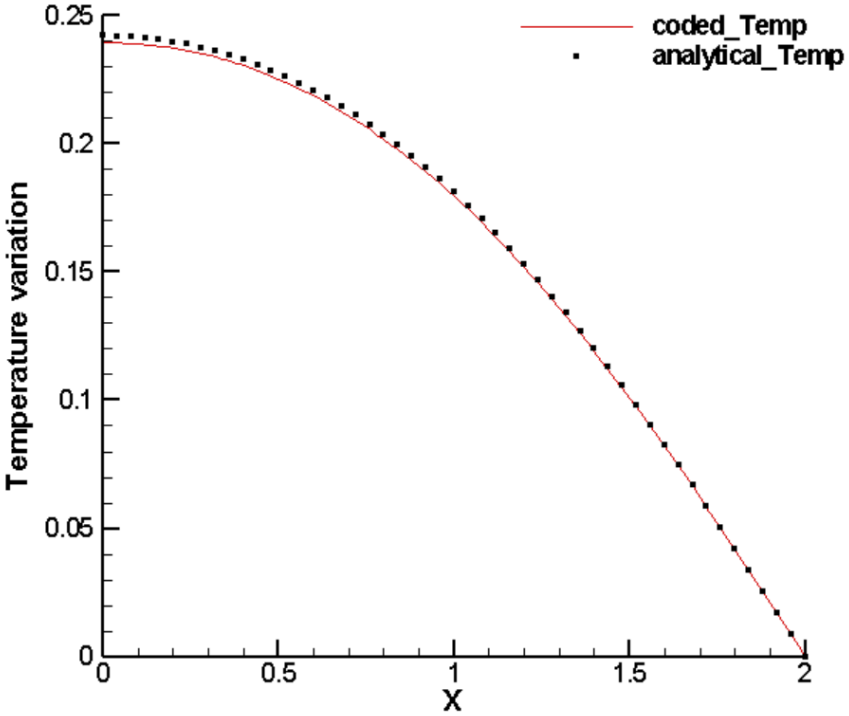


Figure 5: The temperature variation with x along mid-horizontal plane y = 0.5W

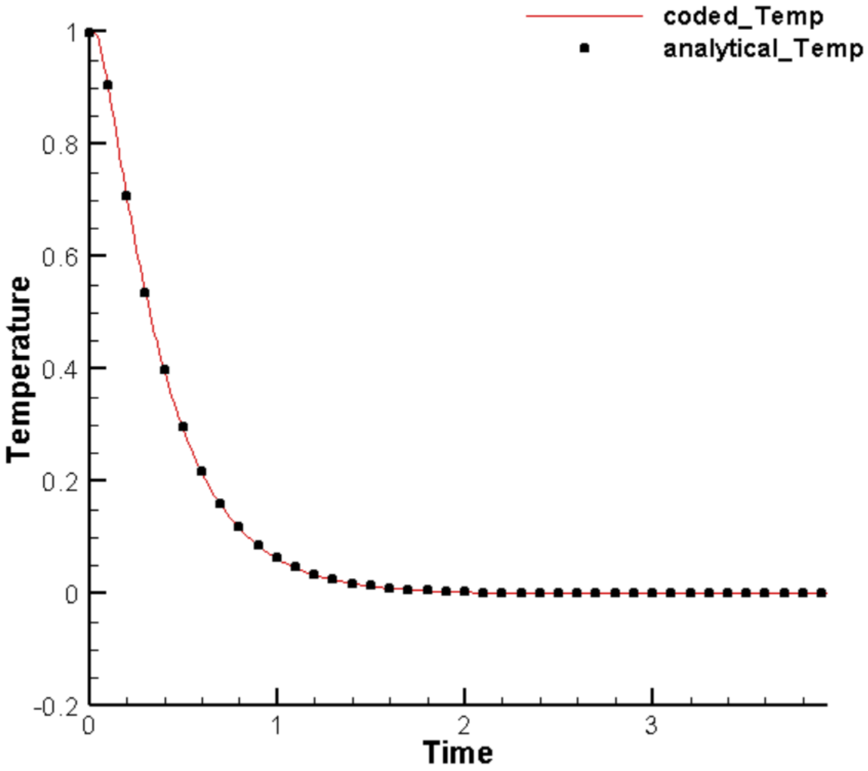


Figure 6: Temperature variation at point (x, y) = (L/4, W/4) with respect to time