

Chapter 1

Incomplete LU decomposition

Question-1:

Code for Incomplete LU decomposition (ILU):

Consider 2D conduction problem governed by equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Take a square domain of width and height of 1 unit. The boundary conditions are:

At $x = 0$: $T = 0$

At $x = 1$: $T = 0$

At $y = 0$: $T = 0$

At $y = 1$: $T = 1$

Discretise the governing equation and solve the obtained system of linear equations by writing code for Incomplete LU decomposition (ILU). This problem has an analytical solution which is given by series solution given by

$$T(x, y) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{-1^{n+1} + 1}{n} \sin n\pi x \frac{\sinh n\pi y}{\sinh n\pi} \quad \text{Eq. (1)}$$

In your code you also write a subroutine to calculate temperature variation using the above analytical expression given by Eq. (1)

Your results should contain at least the following:

- Compare the temperature contours obtained by the code and with those obtained by using the above analytical expression (show the temperature contours figures side by side).
- Compare the temperature variation with y along mid-vertical plane $x = 0.5$ from the code and the above analytical expression. Plot both temperature profiles in the same figure.
- Compare the temperature variation with y along mid-horizontal plane $y = 0.5$ from the code and the above analytical expression. Plot both temperature profiles in the same figure.

1.1 Physical System

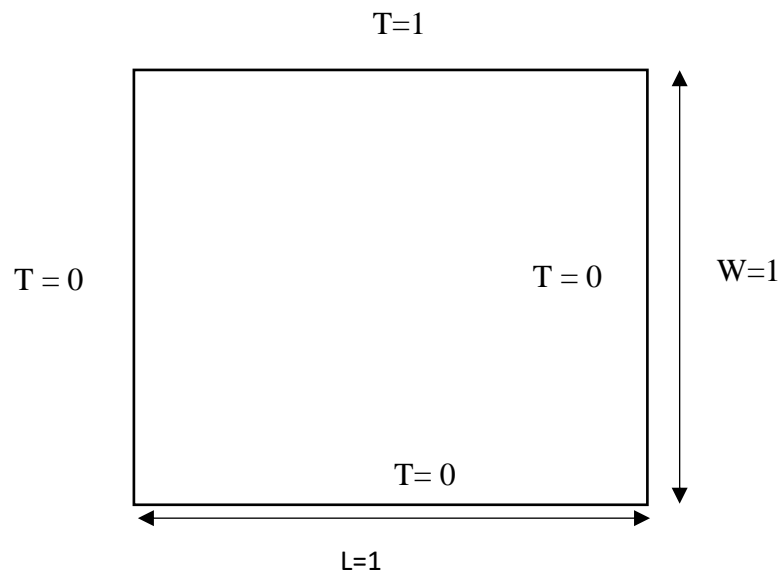


Figure 1.1: The physical system

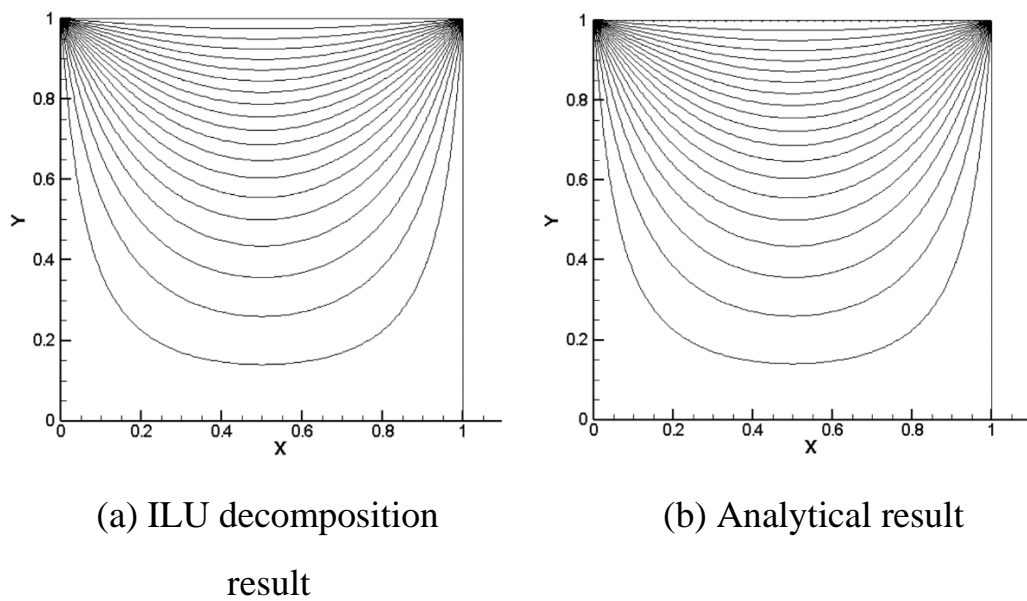


Figure 1.2: Temperature contours

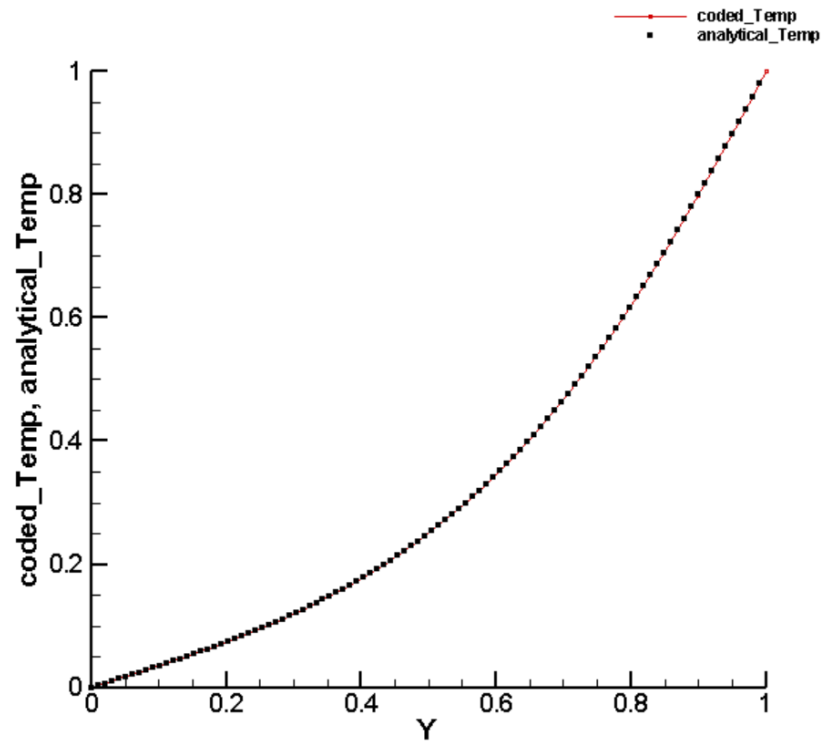


Figure 1.3: Temperature variation with y along mid-vertical plane $x = 0.5$

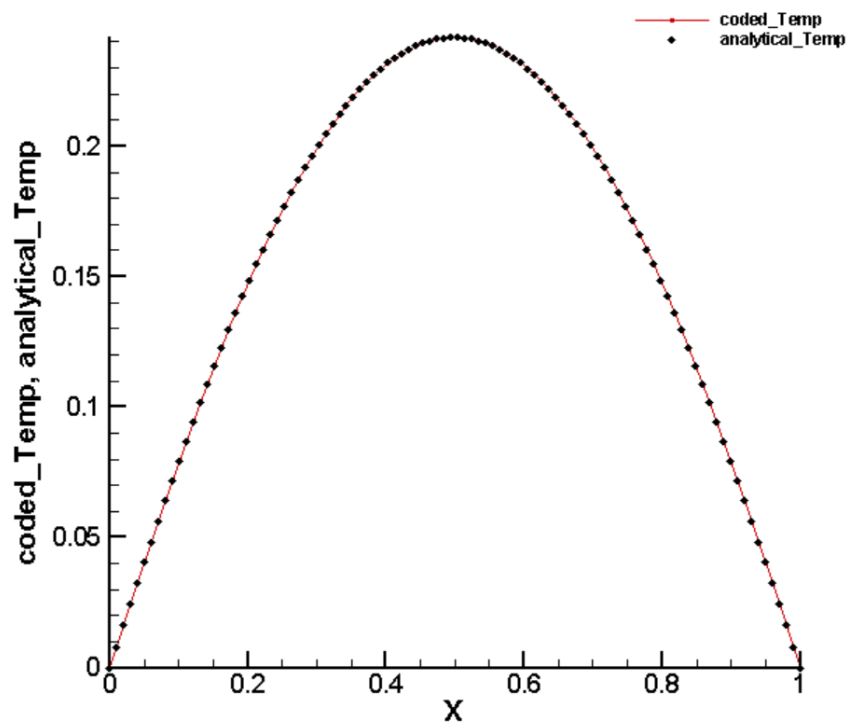


Figure 1.4: Temperature variation with x along mid-horizontal plane $y = 0.5$

Chapter 2

Strongly Implicit Procedure

Question-2:

Code for Strongly Implicit procedure (SIP):

Consider 2D conduction governed by equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

Take a domain of width L and height W. The boundary conditions are:

At x = 0: T = 0

At x = L: T = 0

At y = 0: T = 0

At y = W: T = $T_m \sin \frac{\pi x}{L}$ Take L = 2 and W = 2.

Take $T_m = 2$. Discretise the governing and solve the obtained system of linear equations by writing code for Strongly Implicit procedure (SIP). This problem has an analytical solution which is given by expression

$$\theta(x, y) = T_m \frac{\sinh \frac{\pi y}{L}}{\sinh \frac{\pi y}{W}} \sin \frac{\pi x}{L} \quad \text{Eq. (2)}$$

In your code you also write a subroutine to calculate temperature variation using the above analytical expression given by Eq. (2). Your results should contain at least the following:

(a) Compare the temperature contours obtained by the code and with those obtained by using the above analytical expression (show the temperature contours figures side by side).

(b) Compare the temperature variation with y along mid-vertical plane $x = L/2$ from the code.

and the above analytical expression. Plot both temperature profiles in the same figure.

(c) Compare the temperature variation with x along mid-horizontal plane $y = W/2$ from the code and the above analytical expression. Plot both temperature profiles in the same figure.

2.1 Physical System

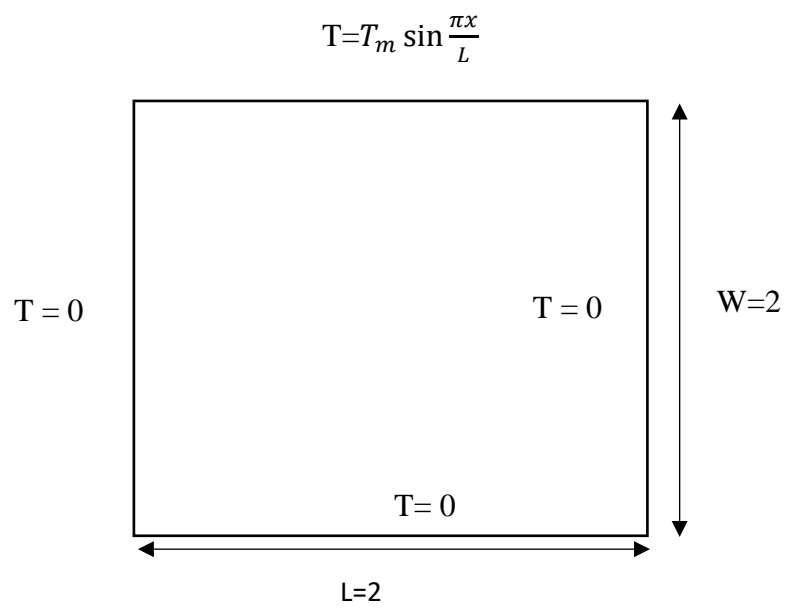


Figure 2.1: The physical system

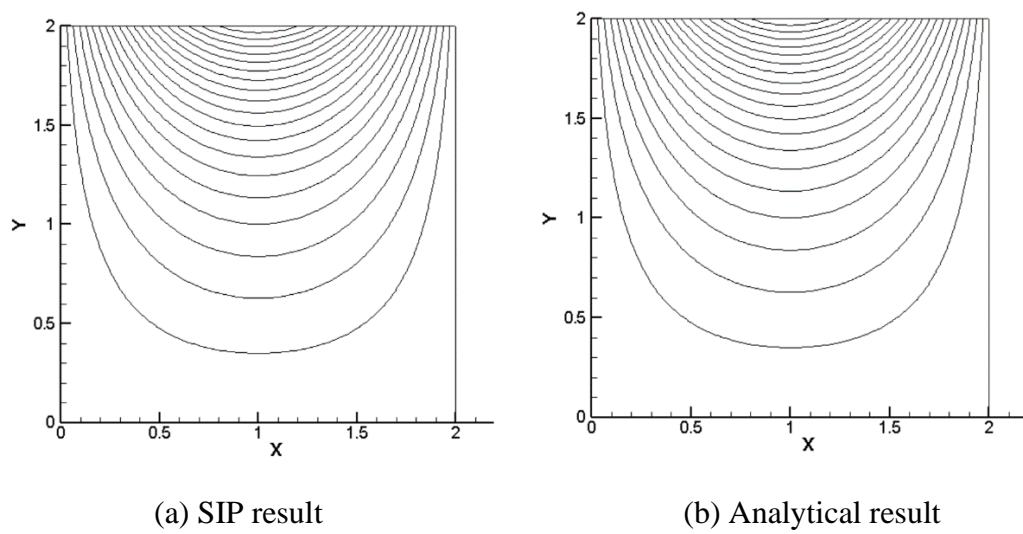


Figure 2.2: Temperature contours

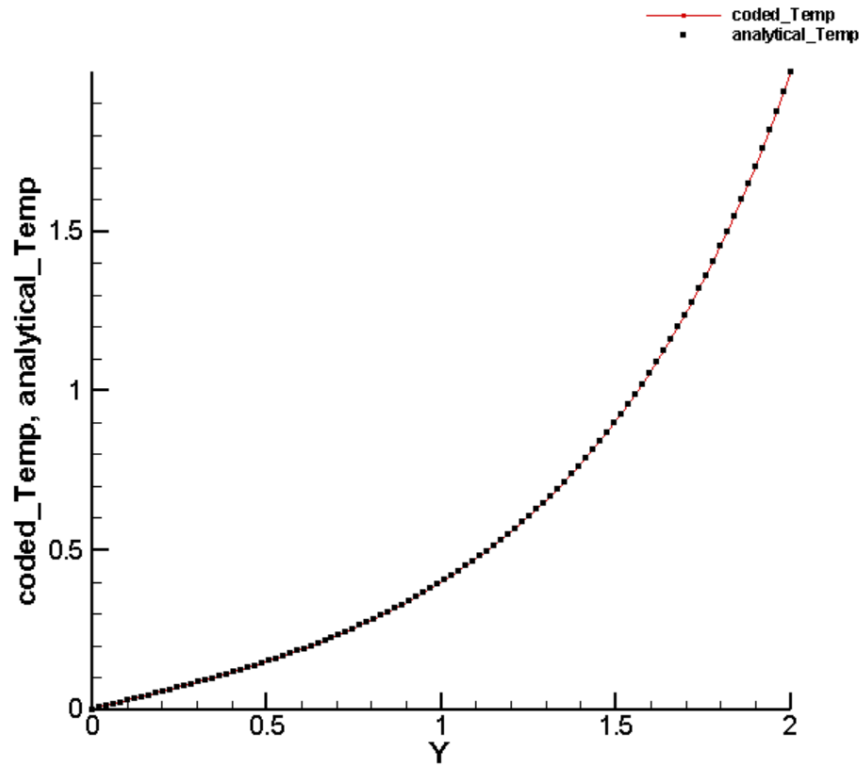


Figure 2.3: Temperature variation with y along mid-horizontal plane $x = 0.5$

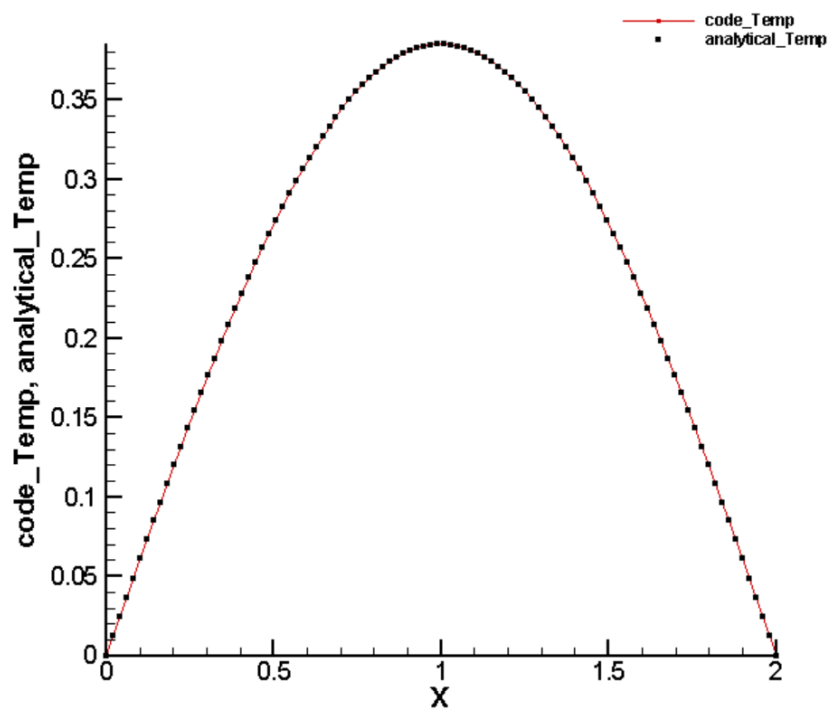


Figure 2.4: Temperature variation with x along mid-horizontal plane $y = 0.5$

Chapter 3

Conjugate Gradient Method

Question-3:

Code for conjugate gradient method:

Consider 2D conduction problem governed by equation:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = -8\pi^2 [\sin 2\pi x \sin 2\pi y]$$

Take a square domain of width and height of 1 unit. The boundary conditions for all the boundaries are $T = 0$. Discretise the equation. Then solve the obtained system of linear equations by writing code for conjugate gradient method. This problem has an analytical solution which is given by:

$$T(x, y) = \sin 2\pi x \sin 2\pi y \quad \text{Eq. (3)}$$

In your code you also write a subroutine to calculate temperature variation using the above analytical expression given by Eq. (3). Your results should contain at least the following:

- Compare the temperature contours obtained by the code and with those obtained by using the above analytical expression (show the temperature contours figures side by side).
- Compare the temperature variation with y along mid-vertical plane $x = 0.5$ from the code and the above analytical expression. Plot both temperature profiles in the same figure.
- Compare the temperature variation with y along mid-horizontal plane $y = 0.5$ from the code and the above analytical expression. Plot both temperature profiles in the same figure.

3.1 Physical System

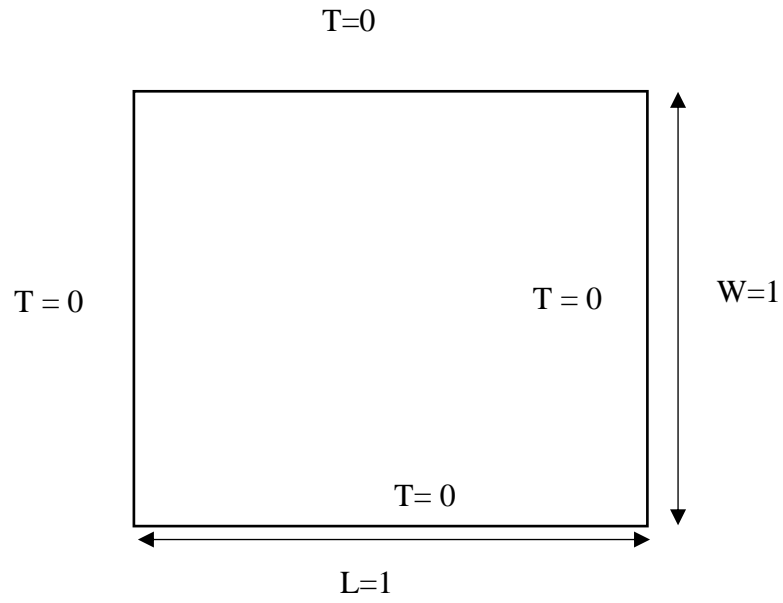


Figure 3.1: The physical system

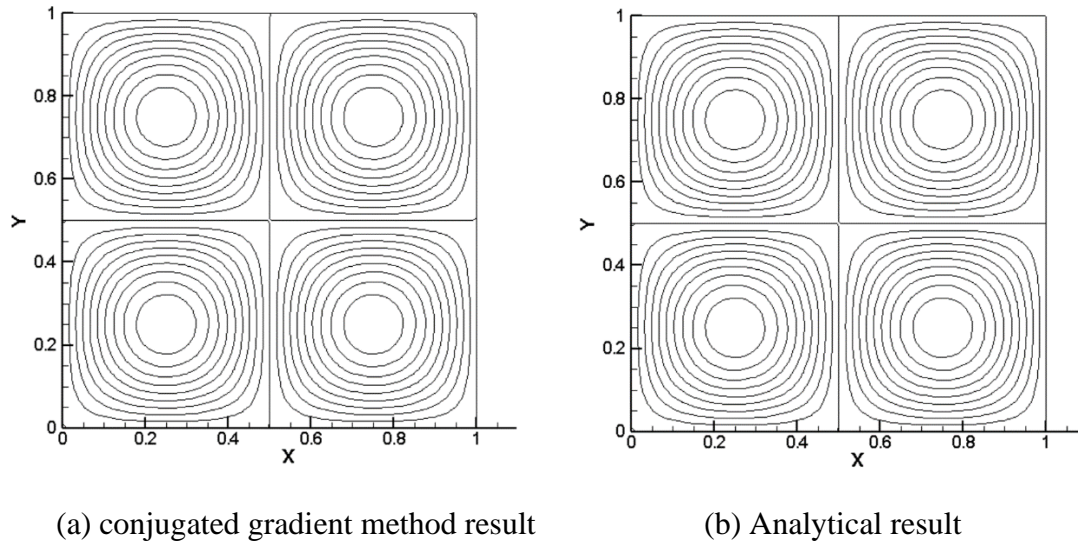


Figure 3.2: Temperature contours

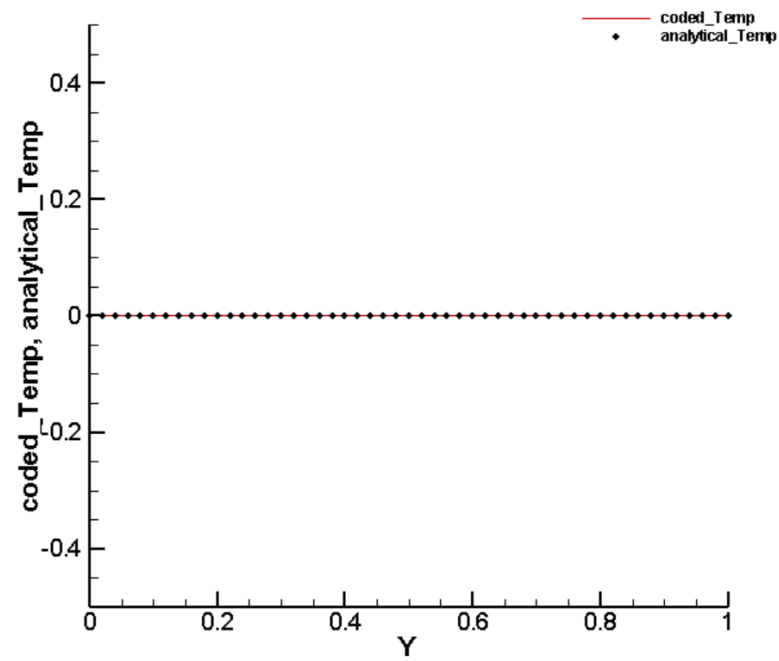


Figure 3.3: Temperature variation with y along mid-vertical plane $x = 0.5$

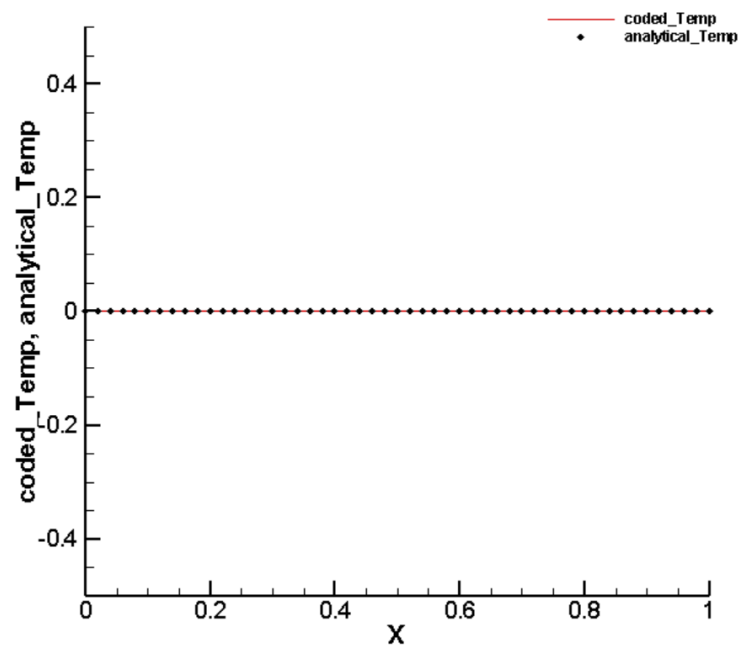


Figure 3.4: Temperature variation with x along mid-horizontal plane $y = 0.5$