

# Encryption steps

- step1: generate a pair of keys
- step2: keep the private key / secret key (SK) and distribute the public key (PK) – place PK in a public register or other accessible file
- step3: Bob encrypts the message with Alice's PK
- step4: upon receiving the ciphertext (CT), Alice decrypt CT with SK

# Public-Key Encryption: Definition

- Three parts:

- $\text{KeyGen}() \rightarrow PK, SK$ : Generate a public/private keypair, where  $PK$  is the public key, and  $SK$  is the private (secret) key
- $\text{Enc}(PK, M) \rightarrow C$ : Encrypt a plaintext  $M$  using public key  $PK$  to produce ciphertext  $C$
- $\text{Dec}(SK, C) \rightarrow M$ : Decrypt a ciphertext  $C$  using secret key  $SK$

$y = x^2$   
 $x = \sqrt{y}$

- Properties

- Correctness: Decrypting a ciphertext should result in the message that was originally encrypted
  - $\text{Dec}(SK, \text{Enc}(PK, M)) = M$  for all  $PK, SK \leftarrow \text{KeyGen}()$  and  $M$   
 $\rightarrow$  plaintext
- Efficiency: Encryption/decryption should be fast
- Security: 1. Alice (the challenger) just gives Eve (the adversary) the public key, and Eve doesn't request encryptions. Eve cannot guess out anything; 2. computationally infeasible to recover  $M$  with  $PK$  and ciphertext ( $PK, \text{ciphertext}$ )

$M \xleftarrow{C}$  one to one mapping

$PK \xrightarrow{*} \text{decrypt}$   
 $SK \xrightarrow{*} \text{encrypt}$

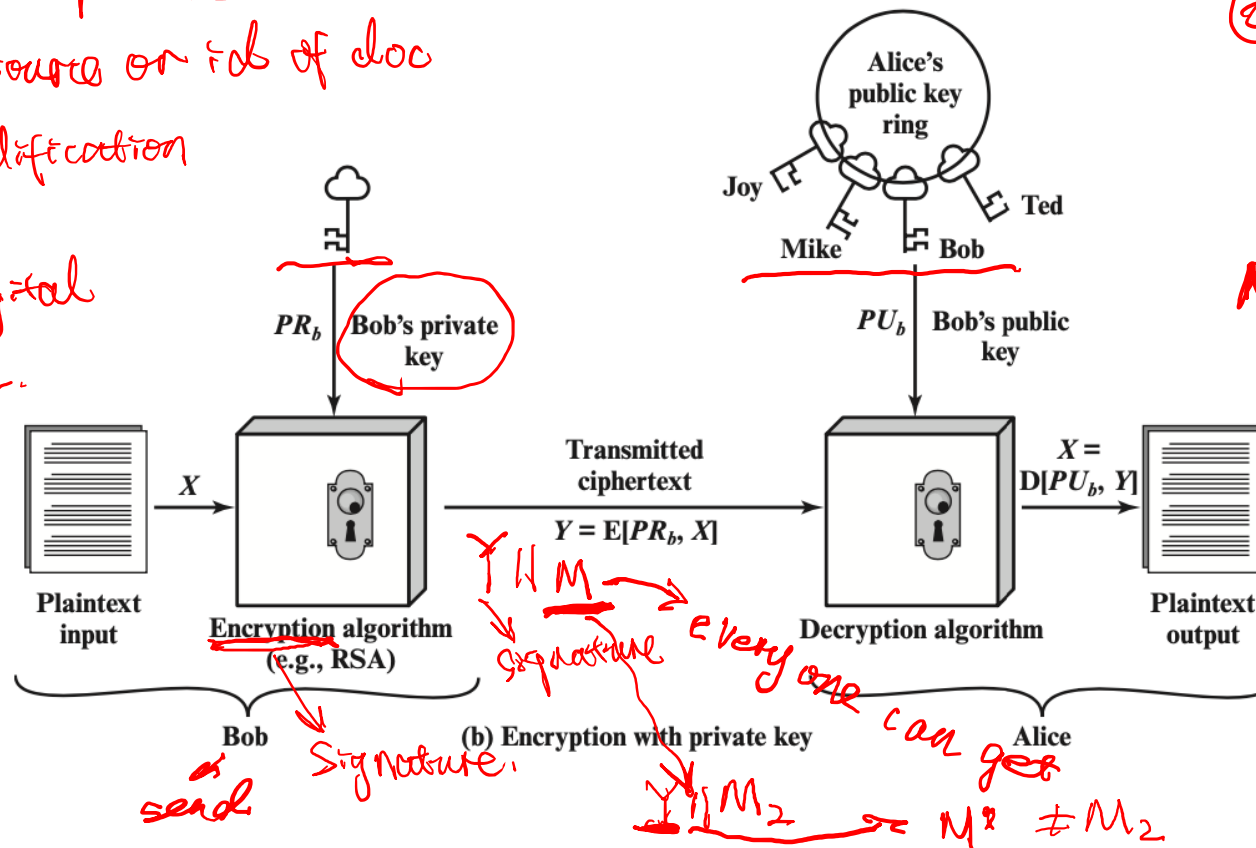
# Public-Key Cryptography - Signature

Definition of Signature

{ prove the source or id of doc  
 detect modification

integrity

i.e. pdf digital signature.



① key

② Algorithm

→ RSA

$$M' = \text{Sig}[PK, M]$$

$$M \neq M'$$

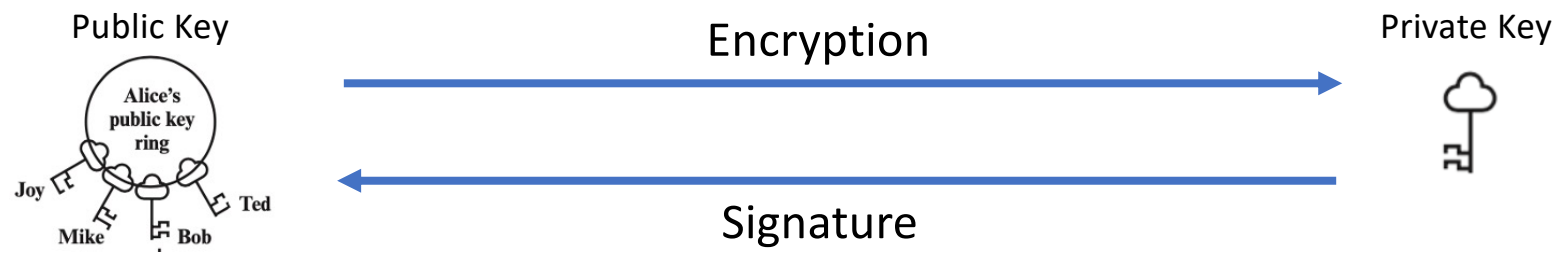
if =

① no modification

② signed by Bob

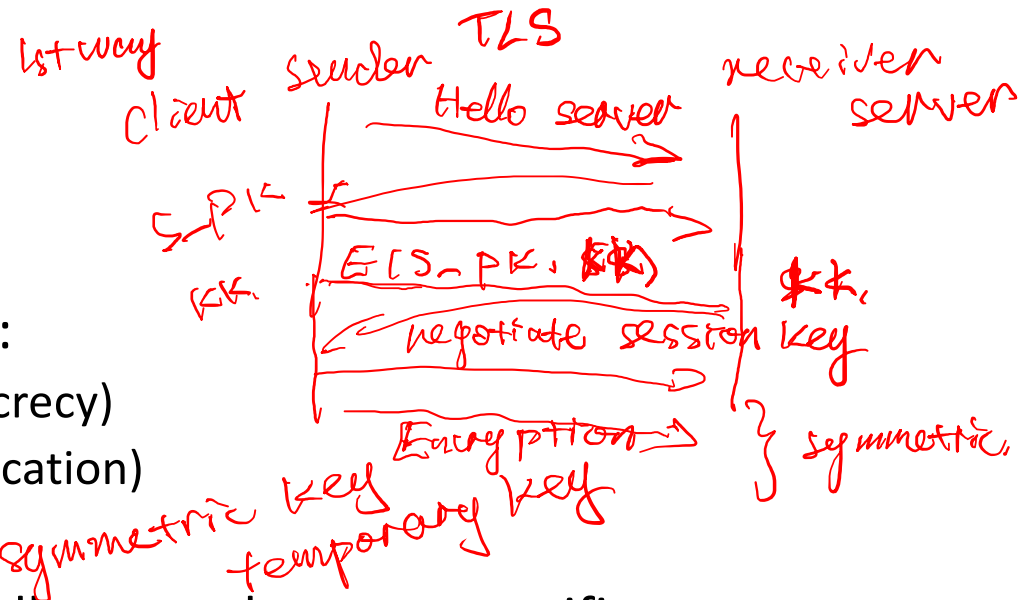
$$Y_2 || M_2 \rightarrow M' \neq M_2$$

# Review



# Public-Key application

- can classify uses into 3 categories:
  - encryption/decryption (provide secrecy)
  - digital signatures (provide authentication)
  - key exchange (of session keys)
- some algorithms are suitable for all uses; others are specific to one
- Either of the two related keys can be used for encryption, with the other used for decryption



| Algorithm      | Encryption/Decryption | Digital Signature | Key Exchange |
|----------------|-----------------------|-------------------|--------------|
| RSA            | Yes                   | Yes               | Yes          |
| Diffie-Hellman | No                    | No                | Yes          |
| DSS            | No                    | <del>Yes</del>    | No           |
| Elliptic curve | Yes                   | Yes               | Yes          |

replace  $\rightarrow$  key length shorter

② 2nd way  
Diffie-Hellman

# Security of Public Key Schemes

- Keys used are **very large** (>512bits)
  - like private key schemes brute force **exhaustive search** attack is always theoretically possible
- Security relies on a large enough difference in **difficulty** between easy (en/decrypt) and hard (cryptanalyze) problems
  - more generally the hard problem is known, it's just made too hard to do in practice
- Requires the use of **very large numbers**, hence is **slow** compared to private/symmetric key schemes

2048 bits  
 $\frac{2048}{8} = 256 \text{ Kbytes}$   
 $\approx 6096$

1024 bits

$$n = p \cdot q$$

$p, q$  prime

Encrypt  
Decrypt (without key)  
attacker