Paper / Subject Code: 50922 / Discrete Structures & Graph Theory

QP CODE : 30226

**(3 Hours)**   **Total Marks : 80**

N.B. : (1) Question Number 1 is compulsory
   (2) Solve any three questions from the remaining questions
   (3) Make suitable assumptions if needed
   (4) Assume appropriate data whenever required. State all assumptions clearly.

1. a. Define the following with suitable example   5
      a) Ring b) Cyclic Group c) Monoid d) Normal Subgroup e) Planar Graph

   b. Check whether $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$ is a tautology   5

   c. Determine the number of positive integers n where $1 \le n \le 100$ and n is not divisible by 2,3 or 5.   5

   d. Prove by mathematical induction that   5
      $2+5+8+\ldots\ldots+(3n-1) = n(3n+1)/2$

2 a Define Equivalence Relation. Let A be a set of integers, Let R be a Relation on AXA   8
      defined by (a,b) R (c,d) if and only if $ad = bc$. Prove that R is an Equivalence Relation

   b. Let A={a, b, c, d, e}   8

$$MR = \begin{matrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

Find the transitive closure of it using Warshall's algorithm.

   c Let G be a group. Prove that the identity element 'e' is unique.   4

3 a Prove that set G = {1,2,3,4,5,6} is a finite abelian group of order 6 with respect to   8
      multiplication module 7

   b Give the exponential generating function for the sequences   8
      i) {1,1,1…}
      ii) {0,1,0,-1,0,1,0,-1…}.

   c Determine whether the following graphs are isomorphic. Justify your answer.   4
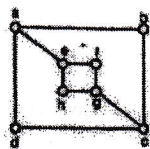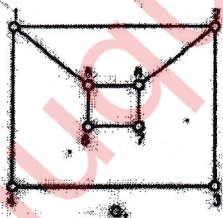
4. a A Function $f: R - \left\{\dfrac{7}{3}\right\} \rightarrow R - \left\{\dfrac{4}{3}\right\}$ is defined as $f(x) = (4x - 5)/(3x - 7)$ 8

      Prove that f is Bijective and find the rule   for f^{-1}

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b   Show that (2,5) encoding function $e:B^2 \rightarrow B^5$ defined by   8
e(00)=00000
e(01)=01110
e(10)=10101
e(11)=11011
is a group code.

c   Check whether Euler cycle and Euler Path exist in the Graph given below. If yes Mention them   4
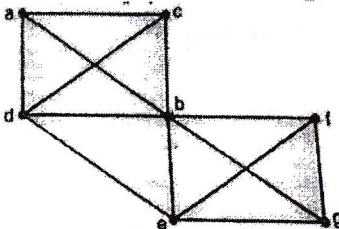
5.   a   Consider the Set A={1,2,3,4,5,6} under multiplication Modulo 7.   8
   1) Prove that it is a Cyclic group.

   2) Find the orders and the Subgroups generated by {2,3}and {3,4}

b   State and explain the extended Pigeonhole principle. How many friends must you have to   8
guarantee that at least five of them will have birthdays in the same month.

c   Functions f,g,h are defined on a set X={a,b,c} as   4
f={(a,b),(b,c),(c,a)}
g={(a,b),(b,a),(b.b)}
h={(a,a),(b,b),(c,a)}
   i)   Find fog, gof . Are they equal?
   ii)   Find fogoh and fohog?

6.   a   Draw the Hasse Diagram of D_{72} and D_{105} and check whether they are Lattice.   8

b   Define Bounded Lattice and Distributive Lattice. Check if the following diagram is a Distributive   8
Lattice or not
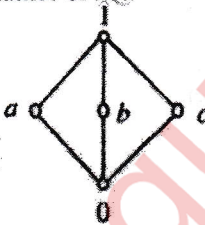
c   Define the following with suitable example.   4
   a)Hamiltonian path   b) Euler Circuit c) Sub Lattice d)Group e) Surjective Function

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