Topic: Find all six trig functions given the quadrant of the angle

Question: Find the value of the trig function.

If θ is an angle in the second quadrant and $\csc\theta = \sqrt{7}$, what is the value of $\sec\theta$?

Answer choices:

A $\sqrt{-\frac{6}{7}}$

B $-\frac{6}{7}$

C $-\frac{7}{6}$

D $-\sqrt{\frac{7}{6}}$

Solution: D

Since we've been given the value of $\csc\theta$, the natural first step is to find the value of $\sin\theta$ by using the reciprocal identity

$$\sin\theta = \frac{1}{\csc\theta}$$

Substituting $\sqrt{7}$ for $\csc\theta$, we obtain

$$\sin\theta = \frac{1}{\sqrt{7}}$$

The next step is to determine the value of $\cos\theta$:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

Substituting $1/\sqrt{7}$ for $\sin\theta$:

$$\cos^2\theta = 1 - \left(\frac{1}{\sqrt{7}}\right)^2 = 1 - \frac{1}{7} = \frac{6}{7}$$

The cosine of any angle in the second quadrant is negative, so

$$\cos\theta = -\sqrt{\frac{6}{7}}$$

Next, we'll find the value of $\sec\theta$ by using the reciprocal identity

$$\sec\theta = \frac{1}{\cos\theta}$$

Substituting the value we found for $\cos\theta$,

$$\sec\theta = \frac{1}{-\sqrt{\frac{6}{7}}} = -\sqrt{\frac{7}{6}}$$

Topic: Find all six trig functions given the quadrant of the angle

Question: Which of the following most closely approximates $\cot 11.0^\circ$, if $\cos 11.0^\circ = 0.98$?

Answer choices:

A 4.64

B 4.76

C 4.92

D 4.98

Solution: C

We need to find an expression for cosine that's in terms of only cotangent. We know that

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

We'll set that aside for a moment, and instead work with the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$. We'll manipulate it.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sqrt{\sin^2\theta} = \pm\sqrt{1 - \cos^2\theta}$$

$$\sin\theta = \pm\sqrt{1 - \cos^2\theta}$$

Now we'll plug this value for $\sin\theta$ into the equation above for $\cot\theta$.

$$\cot\theta = \frac{\cos\theta}{\pm\sqrt{1 - \cos^2\theta}}$$

Square both sides.

$$\cot^2\theta = \frac{\cos^2\theta}{1 - \cos^2\theta}$$

Because we've been told that $\cos 11.0^\circ = 0.98$, we'll plug this into the equation we've created.

$$\cot^2\theta = \frac{(0.98)^2}{1 - (0.98)^2} = \frac{0.9604}{1 - 0.9604} = \frac{0.9604}{0.0396} \approx 24.25^\circ$$

$$\cot\theta \approx \pm\sqrt{24.25^\circ}$$

$$\cot\theta \approx \pm 4.92^\circ$$

Note that an angle of 11.0° is in the first quadrant. Since the cotangent of any angle in the first quadrant is positive, we have

$$\cot\theta \approx 4.92^\circ$$

Topic: Find all six trig functions given the quadrant of the angle

Question: What are the possible values of the trig function?

$\tan\theta$ if $\cos\theta = -0.218$

Answer choices:

A 3.27 and -3.27

B 4.48 and -4.48

C 2.14 and -2.14

D 5.46 and -5.46

Solution: B

We'll ultimately use the definition of $\tan\theta$:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

In order to do that, we need to compute the possible values of $\sin\theta$:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

Substituting -0.218 for $\cos\theta$ gives

$$\sin^2\theta = 1 - (-0.218)^2 \approx 1 - 0.0475 = 0.953$$

Therefore, either

$$\sin\theta = \sqrt{0.953} \approx 0.976$$

or

$$\sin\theta = -\sqrt{0.953} \approx -0.976$$

Since $\cos\theta$ is negative, θ is in either the second quadrant or the third quadrant. If θ is in the second quadrant, $\sin\theta$ is positive (hence $\sin\theta \approx 0.976$). If θ is in the third quadrant, $\sin\theta$ is negative (hence $\sin\theta \approx -0.976$).

By definition,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Substituting the values of $\sin\theta$ and $\cos\theta$, we find that if θ is in the second quadrant, then

$$\tan\theta \approx \frac{0.976}{-0.218} \approx -4.48$$

If θ is in the third quadrant, then

$$\tan\theta \approx \frac{-0.976}{-0.218} \approx 4.48$$

Thus the possible values for $\tan\theta$ are 4.48 and -4.48.