Topic: Domain and range of the six circular functions

Question: At which of the following angles is the tangent function undefined?

Answer choices:

A $\quad \theta = 3\pi$

B $\quad \theta = \dfrac{5}{3}\pi$

C $\quad \theta = \dfrac{7}{2}\pi$

D $\quad \theta = \dfrac{9}{4}\pi$

Solution: C

The tangent function is defined as

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Therefore, the only angles θ at which the tangent function is undefined are those at which $\cos \theta = 0$. This occurs at

$$\theta = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi, \frac{9}{2}\pi, \ldots$$

$$\theta = \frac{1}{2}\pi(1, 3, 5, 7, 9, \ldots)$$

$$\theta = \frac{1}{2}\pi k$$

for some odd positive integer k. The only answer choice in which θ can be expressed in this way is answer choice C,

$$\theta = \frac{7}{2}\pi$$

that is, for $k = 7$.

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Question: Which of the following circular functions cannot take the value?

$$-0.756$$

Answer choices:

A   The sine function

B   The tangent function

C   The cotangent function

D   The cosecant function

Solution: D

The cosecant function is defined as

$$\csc\theta = \frac{1}{\sin\theta}$$

In interval notation, the range of the cosecant function is $(-\infty, -1] \cup [1,\infty)$, which excludes all numbers in the interval $(-1,1)$. Since the number -0.756 is in the interval $(-1,1)$, the cosecant function cannot take the value -0.756.

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Question: Choose the best approximation.

If θ is an angle in the second quadrant such that $\cos\theta = -0.412$, which of the following most closely approximates the value of $\tan\theta$?

Answer choices:

A -2.21

B 2.01

C -0.452

D 0.496

Solution: A

We're given the value of $\cos\theta$, so we'll first find the value of $\sin\theta$:

$$\sin^2\theta + \cos^2\theta = 1$$

Substituting the value of $\cos\theta$, we have

$$\sin^2\theta + (-0.412)^2 = 1$$

$$\sin^2\theta = 1 - (-0.412)^2$$

Now $(-0.412)^2 \approx 0.170$, so

$$\sin^2\theta \approx 1 - 0.170$$

$$\sin^2\theta \approx 0.830$$

Well,

$$\sqrt{0.830} \approx 0.911,$$

so either

$$\sin\theta \approx 0.911$$

or

$$\sin\theta \approx -0.911$$

Since θ is in the second quadrant, $\sin\theta$ is positive, so $\sin\theta \approx 0.911$. Therefore,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \approx \frac{0.911}{-0.412} \approx -2.21$$