Topic: Law of cosines

Question: Find the other measures.

If the lengths of two of the sides of a triangle are $a = 28$ and $b = 37$, and the measure of the included angle is 110°, what is the length c of the third side of the triangle, and what are the measures of the other two interior angles?

Answer choices:

A $c \approx 12.3$, and the measures of the other two interior angles are approximately 11.9° and 58.1°

B $c \approx 39.8$, and the measures of the other two interior angles are approximately 23.1° and 46.9°

C $c \approx 53.5$, and the measures of the other two interior angles are approximately 29.4° and 40.6°

D $c \approx 66.2$, and the measures of the other two interior angles are approximately 51.3° and 18.7°

Solution: C

The included angle is angle C. By the law of cosines,

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Substituting the data:

$$c^2 = 28^2 + 37^2 - \left[2(28)(37)\cos 110^\circ\right]$$

$$c^2 = 784 + 1{,}369 - \left[56(37)\cos 110^\circ\right]$$

$$c^2 = 2{,}153 - (2{,}072\cos 110^\circ)$$

Using a calculator, we find that

$$\cos 110^\circ \approx -0.342$$

Therefore,

$$c^2 \approx 2{,}153 - (2{,}072)(-0.342) = 2{,}153 + (2{,}072)(0.342) \approx 2{,}153 + 709 = 2{,}862$$

Since c is a length (and thus has to be positive), we get

$$c \approx \sqrt{2{,}862} \approx 53.5$$

To find angle A (the interior angle opposite the side of length a), we'll use the following form of the law of cosines,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

and solve for $\cos A$:

$$\cos A = \frac{a^2 - (b^2 + c^2)}{-2bc}$$

Substituting the data:

$$\cos A \approx \frac{28^2 - (37^2 + 53.5^2)}{-2(37)(53.5)}$$

$$\cos A \approx \frac{784 - (1,369 + 2,863)}{-74(53.5)}$$

$$\cos A \approx \frac{784 - 4,232}{-3,959}$$

$$\cos A \approx \frac{-3,448}{-3,959}$$

$$\cos A \approx \frac{3,448}{3,959}$$

$$\cos A \approx 0.871$$

With a calculator, we find that the measure of angle A is approximately 29.4°.

Thus the measure of the third interior angle (angle B) is (approximately)

$$180^\circ - (110^\circ + 29.4^\circ) = 180^\circ - 139.4^\circ = 40.6^\circ$$

Topic: Law of cosines

Question: What are the measures of the interior angles?

If the lengths of the sides of a triangle are $a = 17$, $b = 24$, and $c = 31$, what are the measures of the three interior angles (A, B, and C) of the triangle which are opposite the sides of length a, b, and c, respectively?

Answer choices:

A $A \approx 33.0^\circ$, $B \approx 50.3^\circ$, and $C \approx 96.7^\circ$

B $A \approx 46.3^\circ$, $B \approx 55.6^\circ$, and $C \approx 78.1^\circ$

C $A \approx 37.4^\circ$, $B \approx 63.5^\circ$, and $C \approx 79.1^\circ$

D $A \approx 33.9^\circ$, $B \approx 61.9^\circ$, and $C \approx 84.2^\circ$

Solution: A

To get the measure of angle A, we'll use the following form of the law of cosines,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

and solve for $\cos A$:

$$\cos A = \frac{a^2 - (b^2 + c^2)}{-2bc}$$

Substituting the data:

$$\cos A \approx \frac{17^2 - (24^2 + 31^2)}{-2(24)(31)}$$

$$\cos A \approx \frac{289 - (576 + 961)}{-48(31)}$$

$$\cos A \approx \frac{289 - 1{,}537}{-1{,}488}$$

$$\cos A \approx \frac{-1{,}248}{-1{,}488}$$

$$\cos A \approx \frac{1{,}248}{1{,}488}$$

$$\cos A \approx 0.839$$

With a calculator, we find that the measure of angle A is approximately 33.0°.

To get the measure of angle B, we'll use the following form of the law of cosines,

$$b^2 = a^2 + c^2 - 2ac\cos B$$

and solve for $\cos B$:

$$\cos B = \frac{b^2 - (a^2 + c^2)}{-2ac}$$

Substituting the data:

$$\cos B \approx \frac{24^2 - (17^2 + 31^2)}{-2(17)(31)}$$

$$\cos B \approx \frac{576 - (289 + 961)}{-34(31)}$$

$$\cos B \approx \frac{576 - 1{,}250}{-1{,}054}$$

$$\cos B \approx \frac{-674}{-1{,}054}$$

$$\cos B \approx \frac{674}{1{,}054}$$

$$\cos B \approx 0.639$$

With a calculator, we find that the measure of angle B is approximately 50.3°.

Thus the measure of angle C is (approximately)

$$180^\circ - (33.0^\circ + 50.3^\circ) = 180^\circ - 83.3^\circ = 96.7^\circ$$

Topic: Law of cosines

Question: Only one of the following triples of numbers consists of lengths of the sides of some triangle. Which triple is it?

Answer choices:

A $(24,4,19)$

B $(10,25,32)$

C $(34,16,16)$

D $(47,35,11)$

Solution: B

First, we'll show that there is indeed a triangle with sides of length 10, 25, and 32.

Let $a = 10$, $b = 25$, and $c = 32$.

$$|a - b| = |10 - 25| = 15 < 32 = c = 32 < 35 = 10 + 25 = a + b$$

$$|a - c| = |10 - 32| = 22 < 25 = b = 25 < 42 = 10 + 32 = a + c$$

$$|b - c| = |25 - 32| = 7 < 10 = a = 10 < 57 = 25 + 32 = b + c$$

Now we'll show that none of the other answer choices works.

For answer choice A, let $a = 24$, $b = 4$, and $c = 19$. Then

$$|a - b| = |24 - 4| = 20 \nless 19 = c$$

For answer choice C, let $a = 34$, $b = 16$, and $c = 16$. Then

$$a = 34 \nless 32 = 16 + 16 = b + c$$

For answer choice D, let $a = 47$, $b = 35$, and $c = 11$. Then

$$|a - c| = |47 - 11| = 36 \nless 35 = b$$