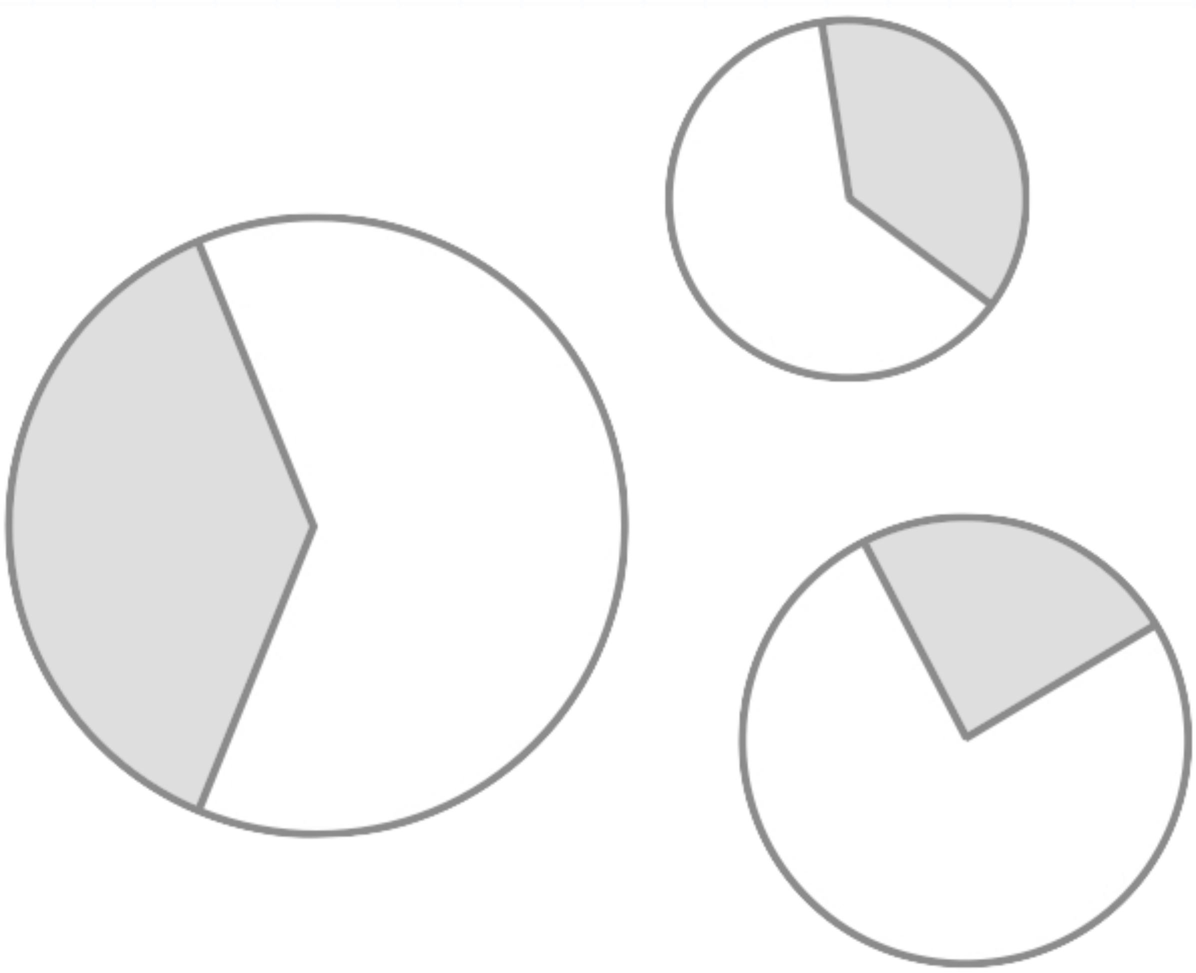
Area of a circular sector

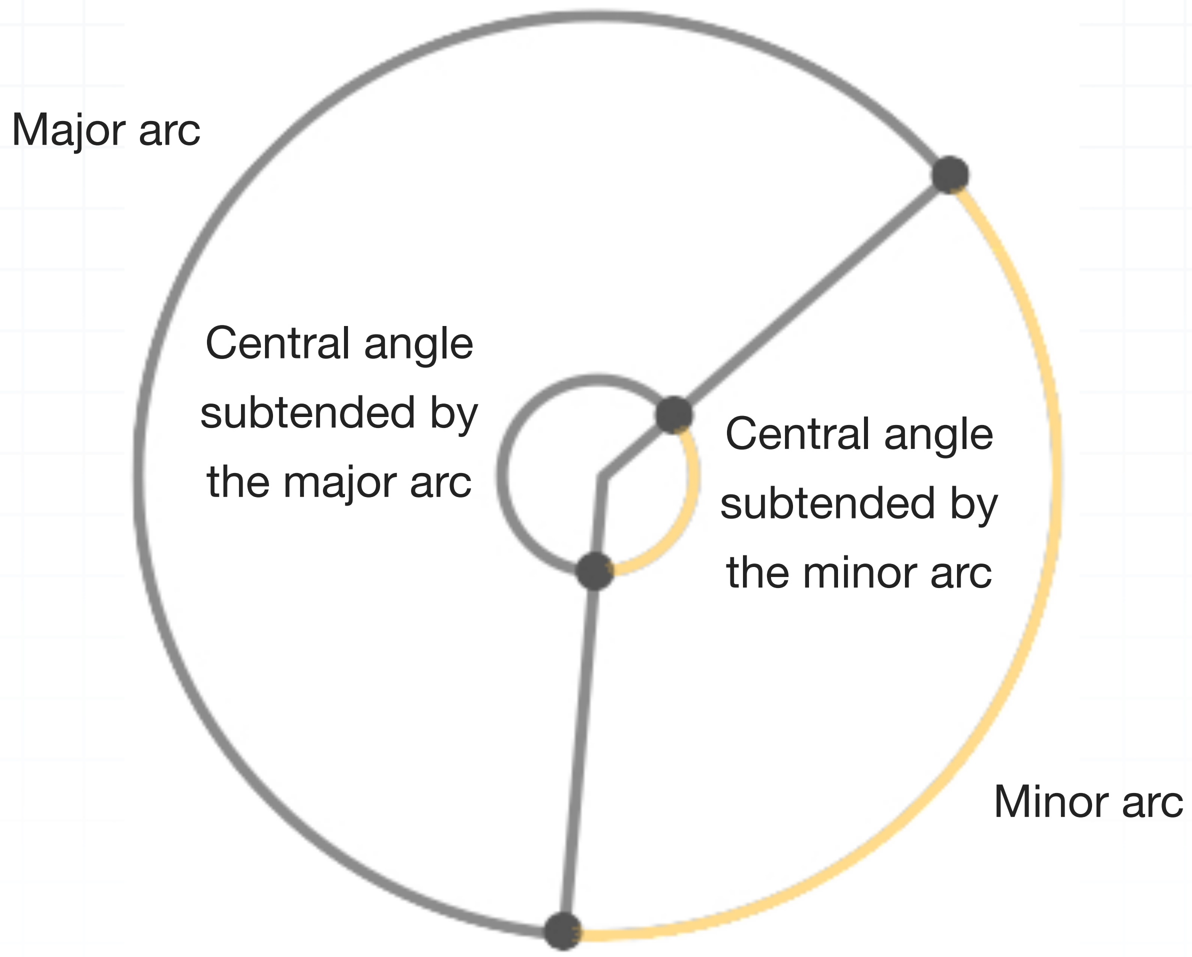
Think of a circular sector as a wedge in a circle, like a piece in a pie. Whenever you have one sector in a circle, there's always another sector: the rest of the circle.

Each sector is bounded by an arc along the perimeter of the circle. When the arcs have the same length, it means both arcs are semicircles, and that the sectors have the same area of half of the circle. Otherwise, if the arcs aren't the same length, the shorter arc is the minor arc, and the longer one is the major arc.

The gray regions below are circular sectors, but so are the white regions.



The angle inside a circular sector is called the central angle. The arc across the circular sector “subtends” the central angle; the central angle is subtended by the arc.



This also means that the area of a circular sector is proportional to the size of the central angle θ. If θ is in radians, then

$$\text{Area of sector} = r^2\left(\frac{\theta}{2}\right)$$

If θ is in degrees, then

$$\text{Area of sector} = \pi r^2\left(\frac{\theta}{360}\right)$$

Example

Find the area A in square inches of the circular sector with a central angle of $\pi/4$ radians, if the circle has a radius of 9 inches.

Since θ is in radians, the area of this circular sector is

$$A = r^2\left(\frac{\theta}{2}\right)$$

$$A = (9^2)\left(\frac{\frac{\pi}{4}}{2}\right)$$

$$A = 81\left(\frac{\pi}{8}\right)$$

$$A = \frac{81}{8}\pi$$

Next, let's work out an example where the central angle θ is given in degrees.

Example

Think of a circle of radius 10 centimeters, and calculate the area A in square centimeters of a circular sector with a central angle of 80°.

Since θ is in degrees,

$$A = \pi r^2 \left(\frac{\theta}{360} \right)$$

$$A = \pi \left(10^2 \right) \left(\frac{80}{360} \right)$$

$$A = \pi(100) \left(\frac{2}{9} \right)$$

$$A = \pi \left(\frac{200}{9} \right)$$

You might also run into circular sector problems where you're asked to solve for the area of the circle. Usually, you'll be given the coordinates for the center of the circle and the coordinates of one point on the circle.

In terms of Cartesian coordinates (x, y), the standard form for the equation of a circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is the center of the circle, r is its radius, and (x, y) represents the coordinates of any point on the circle.

Example

Think of a circle that passes through the point $(9, -7)$ and has its center at $(4,5)$. Find the area A of a sector of this circle that has a central angle of $\pi/15$ radians.

Since the center of the circle is at $(4,5)$, notice that every point on this circle satisfies the equation

$$(x-4)^2+(y-5)^2=r^2$$

where r is the radius. Moreover, this circle passes through the point $(9,-7)$, so by letting $(x,y)=(9,-7)$, you can find the radius.

$$(9-4)^2+(-7-5)^2=r^2$$

$$5^2+(-12)^2=r^2$$

$$25+144=r^2$$

$$169=r^2$$

What this tells us is that $|r|=\sqrt{169}=13$, so either $r=13$ or $r=-13$. The radius of a circle is a length, so it can't be negative, and you know $r=13$.

We know the central angle of the circular sector is $\pi/15$ radians, so we'll plug everything into the formula and solve for the area A of the circle.

$$A=r^2\left(\frac{\theta}{2}\right)$$

$$A=\left(13^2\right)\left(\frac{\frac{\pi}{15}}{2}\right)$$

$$A=169\left(\frac{\pi}{30}\right)$$

$$A = \frac{169}{30}\pi$$