Domain and range of the six circular functions

There are six trig functions, sometimes called the “circular functions”. The first two are the sine and cosine functions, and in this lesson we’ll define the other four. We’ll also look at the domain and range for each of the six.

The sine function

Because the sine function just represents the y-value of the coordinate point that’s sitting on the unit circle at whatever angle we’re interested in, the domain of the sine function is just the set of all angles, and we can say the domain of sine is $(-\infty, \infty)$.

The range of the sine function is $[-1,1]$, since the unit circle spans $y = [-1,1]$.

The cosine function

The cosine function has the same domain and range as the sine function, because $\cos\theta$ is defined as the x-value of the point where the angle θ intersects the unit circle.

And the range of the cosine function is $[-1,1]$, since the unit circle spans $x = [-1,1]$.

The other four functions

The **secant** function is the reciprocal of cosine and is defined in terms of cosine as

$$\sec\theta = \frac{1}{\cos\theta}$$

The **cosecant** function is the reciprocal of sine and is defined in terms of sine as

$$\csc\theta = \frac{1}{\sin\theta}$$

The **tangent** function is the reciprocal of cotangent and is defined in terms of sine and cosine as

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

The **cotangent** function is the reciprocal of tangent and is defined in terms of sine and cosine as

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

The domain and range of all six circular functions are summarized in the table below, where they are given in both set notation and interval notation.

Function	Definition	Domain	Range
sine	$\sin(\theta)$ = y-coordinate of point where terminal side of θ intersects unit circle	set of all angles θ $(-\infty, \infty)$	$\{r\colon \lvert r\rvert \le 1\}$ $[-1, 1]$
cosine	$\cos\theta$ = x-coordinate of point where terminal side of θ intersects unit circle	set of all angles θ $(-\infty, \infty)$	$\{r\colon \lvert r\rvert \le 1\}$ $[-1, 1]$
secant	$\sec\theta = \frac{1}{\cos\theta}$	$\left\{\theta\colon \text{for all integers } k,\ \theta \ne \frac{(2k+1)\pi}{2}\right\}$ $\bigcup_{k=-\infty}^{\infty} \left(\frac{(2k+1)\pi}{2}, \frac{(2k+3)\pi}{2}\right)$	$\{r\colon \lvert r\rvert \ge 1\}$ $(-\infty, -1] \cup [1, \infty)$
cosecant	$\csc\theta = \frac{1}{\sin\theta}$	$\{\theta\colon \text{for all integers } k,\ \theta \ne k\pi\}$ $\bigcup_{k=-\infty}^{\infty} (k\pi, (k+1)\pi)$	$\{r\colon \lvert r\rvert \ge 1\}$ $(-\infty, -1] \cup [1, \infty)$
tangent	$\tan\theta = \frac{\sin\theta}{\cos\theta}$	$\left\{\theta\colon \text{for all integers } k,\ \theta \ne \frac{(2k+1)\pi}{2}\right\}$ $\bigcup_{k=-\infty}^{\infty} \left(\frac{(2k+1)\pi}{2}, \frac{(2k+3)\pi}{2}\right)$	set of all real numbers r $(-\infty, \infty)$
cotangent	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	$\{\theta\colon \text{for all integers } k,\ \theta \ne k\pi\}$ $\bigcup_{k=-\infty}^{\infty} (k\pi, (k+1)\pi)$	set of all real numbers r $(-\infty, \infty)$