Topic: Find the value or say where it's undefined

Question: Where is secant undefined?

For what angle is $\sec \theta$ undefined in the interval?

$$\left(-\frac{53\pi}{6}, -\frac{25\pi}{3}\right)$$

Answer choices:

A $\theta = -9\pi$

B $\theta = -\frac{17\pi}{2}$

C $\theta = -\frac{15\pi}{2}$

D $\theta = -8\pi$

Solution: B

By one of the reciprocal identities,

$$\sec\theta = \frac{1}{\cos\theta}$$

Thus the angles θ at which $\sec\theta$ is undefined are those at which $\cos\theta = 0$. This occurs if and only if θ is of the form $k\pi/2$ where k is an odd integer.

Note that

$$-\frac{19\pi}{2} = -\frac{57\pi}{6} < -\frac{53\pi}{6} < -\frac{51\pi}{6}\left(= -\frac{17\pi}{2}\right) < -\frac{50\pi}{6}\left(= -\frac{25\pi}{3}\right) < -\frac{45\pi}{6} = -\frac{15\pi}{2}$$

The key inequalities above are

$$-\frac{19\pi}{2} < -\frac{53\pi}{6} < -\frac{17\pi}{2} < -\frac{25\pi}{3} < -\frac{15\pi}{2}$$

Thus $-17\pi/2$ is in the interval $(-53\pi/6, -25\pi/3)$.

Since -17 is an odd integer,

$$\cos\left(-\frac{17\pi}{2}\right) = 0$$

Therefore, $\sec(-17\pi/2)$ is undefined.

The odd integers closest to -17 are -19 and -15, hence the angles closest to $-17\pi/2$ at which the secant function is undefined are $-19\pi/2$ and $-15\pi/2$. We have shown that both of those angles lie outside the interval $(-53\pi/6, -25\pi/3)$.

Therefore, $-17\pi/2$ is the only angle θ in the interval $(-53\pi/6, -25\pi/3)$ at which $\sec\theta$ is undefined.

Topic: Find the value or say where it's undefined

Question: Say whether the function is defined, then find its value.

Determine whether $\tan(29\pi/6)$ is defined. If it is, what is its value?

Answer choices:

A $\tan\frac{29\pi}{6}$ is defined and equal to $\frac{2}{\sqrt{3}}$

B $\tan\frac{29\pi}{6}$ is defined and equal to $-\frac{1}{\sqrt{3}}$

C $\tan\frac{29\pi}{6}$ is defined and equal to $-\frac{\sqrt{3}}{2}$

D $\tan\frac{29\pi}{6}$ is undefined

Solution: B

First, note that

$$\frac{9\pi}{2} = \frac{27\pi}{6} < \frac{29\pi}{6} < \frac{30\pi}{6} = 5\pi$$

An angle of $9\pi/2$ radians is on the positive y-axis, and an angle of 5π radians is on the negative x-axis. Thus an angle of $29\pi/6$ radians is in the second quadrant (hence it isn't on any of the coordinate axes), so $\tan(29\pi/6)$ is defined. Furthermore,

$$\frac{29\pi}{6} = \frac{(24+5)\pi}{6} = \frac{24\pi}{6} + \frac{5\pi}{6} = 4\pi + \frac{5\pi}{6}$$

Since $29\pi/6$ differs from $5\pi/6$ by 4π, which is an integer multiple of 2π, we know that

$$\tan\frac{29\pi}{6} = \tan\frac{5\pi}{6}$$

Also,

$$\pi - \frac{5\pi}{6} = \frac{\pi}{6}$$

so $\pi/6$ is the reference angle for both $5\pi/6$ and $\pi/6$. Applying the reference angle theorem (and using the fact that $5\pi/6$ is in the second quadrant and $\pi/6$ is in the first quadrant, hence that $\sin(5\pi/6) = \sin(\pi/6)$ and $\cos(5\pi/6) = -\cos(\pi/6)$), we obtain

$$\tan\frac{29\pi}{6}=\tan\frac{5\pi}{6}=\frac{\left(\sin\frac{5\pi}{6}\right)}{\left(\cos\frac{5\pi}{6}\right)}=\frac{\left(\sin\frac{\pi}{6}\right)}{\left(-\cos\frac{\pi}{6}\right)}=\frac{\left(\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)}=-\frac{1}{\sqrt{3}}$$

Topic: Find the value or say where it's undefined

Question: What is the value of the function?

$$\csc \frac{19\pi}{4}$$

Answer choices:

A $-\sqrt{2}$

B $-\frac{1}{\sqrt{2}}$

C $\sqrt{2}$

D $\frac{1}{\sqrt{2}}$

Solution: C

Note that

$$\frac{19\pi}{4}=\frac{(16+3)\pi}{4}=\frac{16\pi}{4}+\frac{3\pi}{4}=4\pi+\frac{3\pi}{4}$$

Therefore, an angle of $19\pi/4$ radians differs from an angle of $3\pi/4$ radians by 4π, which is an integer multiple of 2π, so

$$\csc\frac{19\pi}{4}=\csc\frac{3\pi}{4}$$

Also,

$$\pi-\frac{3\pi}{4}=\frac{\pi}{4}$$

so the reference angle for both $19\pi/4$ and $3\pi/4$ is $\pi/4$. Since $3\pi/4$ is in the second quadrant and $\pi/4$ is in the first quadrant (hence $\sin(3\pi/4)=\sin(\pi/4)$), the reference angle theorem gives

$$\csc\frac{19\pi}{4}=\csc\frac{3\pi}{4}=\frac{1}{\sin\frac{3\pi}{4}}=\frac{1}{\sin\frac{\pi}{4}}=\frac{1}{\left(\frac{1}{\sqrt{2}}\right)}=\sqrt{2}$$