Topic: Area of a triangle

Question: What is the area of a triangle?

The triangle with side lengths 44, 28, and 36

Answer choices:

A The area is approximately 495.

B The area is approximately 503.

C The area is approximately 618.

D The area is approximately 527.

© Krista King Math

Solution: B

By Heron's formula, the area of a triangle is

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b, and c are the lengths of the sides of the triangle and

$$s = \frac{1}{2}(a+b+c)$$

Let $a = 44$, $b = 28$, and $c = 36$. Then

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(44+28+36) = \frac{1}{2}(72+36) = \frac{1}{2}(108) = 54$$

Thus

$$s - a = 54 - 44 = 10$$

$$s - b = 54 - 28 = 26$$

$$s - c = 54 - 36 = 18$$

Substituting the data, we find that

$$\text{area} = \sqrt{54(10)(26)(18)} = \sqrt{540(26)(18)} = \sqrt{(14{,}040)(18)} = \sqrt{252{,}720} \approx 503$$

© Krista King Math

Topic: Area of a triangle

Question: What is the area of a triangle?

The lengths of two of the sides are 17 and 53 and the measure of the included angle is 81°

Answer choices:

A The area is approximately 560.

B The area is approximately 468.

C The area is approximately 371.

D The area is approximately 445.

© Krista King Math

Solution: D

To compute the area of this triangle, we'll use the law of sines for the area of a triangle:

$$\text{area} = \frac{1}{2}ab\ \sin C$$

where a and b are the lengths of two sides of the triangle and angle C is the included angle.

Let $a = 17$ and $b = 53$. Then the included angle (angle C) is the angle of measure 81°.

Substituting the data:

$$\text{area} = \frac{1}{2}(17)(53)\ \sin 81^\circ = \frac{(17)(53)}{2}\ \sin 81^\circ = \frac{901}{2}\ \sin 81^\circ$$

Using a calculator, we find that $\sin 81^\circ \approx 0.988$, so

$$\text{area} \approx \frac{901}{2}(0.988) \approx 445$$

© Krista King Math

Topic: Area of a triangle

Question: Find the area of a triangle.

The measures of two of the interior angles are 77° and 56° and the length of the side that's common to those two angles is 39.

Answer choices:

A The area is approximately 841.

B The area is approximately 492.

C The area is approximately 571.

D The area is approximately 708.

© Krista King Math

Solution: A

Since we know the length of only one of the sides of this triangle, we can't find its area by immediately applying the law of sines for the area of a triangle (which requires, among other things, knowing the lengths of two of the sides), much less can we find the area by immediately applying Heron's formula (which requires knowing the lengths of all three sides). However, we can easily determine the measure of the third interior angle of this triangle, and we can apply the (usual) law of sines to find the length of either of the two unknown sides. Once we've done both of those things, we'll be able to apply the law of sines for the area of a triangle.

Let angle A be the angle of measure 77°, and let angle B be the angle of measure 56°. Then the measure of the third interior angle (angle C) is

$$180^\circ - (77^\circ + 56^\circ) = 180^\circ - 133^\circ = 47^\circ$$

Now let a, b, and c be the lengths of the sides opposite angles A, B, and C, respectively. Then $c = 39$, and (at this point) the values of both a and b are unknown.

By the (usual) law of sines,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Substituting the data:

$$\frac{a}{\sin 77^\circ} = \frac{b}{\sin 56^\circ} = \frac{39}{\sin 47^\circ}$$

Multiplying both sides of the equation

© Krista King Math

$$\frac{a}{\sin 77^\circ} = \frac{39}{\sin 47^\circ}$$

by $\sin 77^\circ$, we get

$$a = (\sin 77^\circ)\left(\frac{39}{\sin 47^\circ}\right)$$

Using a calculator, we find that $\sin 77^\circ \approx 0.974$ and $\sin 47^\circ \approx 0.731$, so

$$a \approx (0.974)\left(\frac{39}{0.731}\right) \approx 52.0$$

Now we're ready to apply the law of sines for the area of a triangle:

$$\text{area} = \frac{1}{2}ac \sin B$$

Substituting the data:

$$\text{area} \approx \frac{1}{2}(52.0)(39)\sin 56^\circ \approx \frac{(52.0)(39)}{2} \sin 56^\circ \approx \frac{2{,}028}{2} \sin 56^\circ$$

Again with the use of a calculator, we find that $\sin 56^\circ \approx 0.829$, so

$$\text{area} \approx \frac{2{,}028}{2}(0.829) \approx 841$$

© Krista King Math