Topic: Oriented arc for a real number

Question: Which of the following most closely approximates the number of turns in a rotation about the center of the unit circle as the oriented arc (of the unit circle) is traced out?

$$t = -85.12$$

Answer choices:

A 13 full turns and 94% of an additional turn

B 14 full turns and 36% of an additional turn

C 13 full turns and 55% of an additional turn

D 12 full turns and 67% of an additional turn

Solution: C

A rotation of one full turn about the center of the unit circle corresponds to an arc length of absolute value 2π, so the number of turns in a rotation about the center of the unit circle as an oriented arc (of the unit circle) of length -85.12 is traced out is

$$\frac{|-85.12|}{2\pi} = \frac{85.12}{2\pi} \approx -13.55$$

That is, a rotation of 13 full turns, and about 55% of an additional turn, would be made about the center of the unit circle as an oriented arc of length -85.12 is traced out.

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Question: Which of the following most closely approximates the measure (in degrees) of the angle in standard position that corresponds to the oriented arc of the unit circle?

$$t = 71.4$$

Answer choices:

A $4{,}091^\circ$

B $3{,}826^\circ$

C $2{,}046^\circ$

D $4{,}331^\circ$

Solution: A

Let θ be the angle in standard position that corresponds to an oriented arc of the unit circle which is of length 71.4. Recall that the length of an oriented arc of the unit circle is numerically equal to the measure of θ (in radians), so $\theta = 71.4$ (radians). What remains to be done is to convert the measure of this angle in radians to its measure in degrees. To do this, we multiply 71.4 by the conversion factor $(180^\circ)/\pi$:

$$\theta = 71.4$$

$$\theta = 71.4(1)$$

$$\theta = 71.4\left(\frac{180^\circ}{\pi}\right)$$

$$\theta = \left[\frac{71.4(180)}{\pi}\right]^\circ$$

$$\theta = \left(\frac{12{,}852}{\pi}\right)^\circ$$

$$\theta \approx 4{,}091^\circ$$

Topic: Oriented arc for a real number

Question: Which of the following most closely approximates the length of an oriented arc (of the unit circle) that corresponds to an angle in standard position whose measure is in DMS (degrees, minutes, and seconds)?

$$-502^\circ 15' 0''$$

Answer choices:

A -4.523

B -5.029

C -9.318

D -8.766

Solution: D

Let $\theta = -502^\circ 15' 0''$, and recall that what this means is that

$$\theta = (-502^\circ) + (-15') + (0'')$$

Recall that the length of an oriented arc (of the unit circle) is numerically equal to the measure of θ (in radians). Thus what we need to do is convert the measure of θ in DMS to its measure in (decimal) degrees, and then convert that to its measure in radians.

To get the measure of θ in (decimal) degrees, and given that the seconds part of this angle is equal to 0, it suffices to convert its minutes part, $-15'$, to degrees, and then add the result to -502°. For this, we'll use the conversion factor $(1^\circ)/(60')$:

$$(-15') = (-15')(1)$$

$$(-15') = (-15')\left(\frac{1^\circ}{60'}\right)$$

$$(-15') = -\left(\frac{15}{60}\right)^\circ$$

$$(-15') = -\left(\frac{1}{4}\right)^\circ$$

Substituting this result into the expression for θ:

$$\theta = (-502^\circ) + (-15') + (0'')$$

$$\theta = (-502^\circ) + \left[-\left(\frac{1}{4}\right)^\circ\right] + 0^\circ$$

$$\theta = -\left[502 + \left(\frac{1}{4}\right)\right]^\circ$$

$$\theta = -\left[\frac{4(502) + 1}{4}\right]^\circ$$

$$\theta = -\left(\frac{2{,}008 + 1}{4}\right)^\circ$$

$$\theta = -\left(\frac{2{,}009}{4}\right)^\circ$$

To get the measure of θ in radians, we use the conversion factor $\pi/(180^\circ)$:

$$\theta = -\left(\frac{2{,}009}{4}\right)^\circ (1)$$

$$\theta = -\left(\frac{2{,}009}{4}\right)^\circ \left(\frac{\pi}{180^\circ}\right)$$

$$\theta = -\left[\frac{2{,}009}{4(180)}\right]\pi$$

$$\theta = -\frac{2{,}009}{720}\pi$$

$$\theta \approx -8.766$$

From this it follows that the length of an oriented arc (of the unit circle) that corresponds to an angle in standard position whose measure in DMS is $-502^\circ 15' 0''$ is -8.766.