# Find all six trig functions given the quadrant of the angle

If you know the value of one of the six trig functions, and you also know the quadrant where the angle is located, you're always able to find the values of the other five trig functions.

Because $\sin$ corresponds to the $y$-value and $\cos$ corresponds to the $x$-value, we'll know the sign of $\sin$ and $\cos$ just from the quadrant.

| **Quadrant** | $\sin$ | $\cos$ |
|---|---|---|
| I | + | + |
| II | + | - |
| III | - | - |
| IV | - | + |

And based on these signs, we'll know the signs of the other trig functions as well.

1. Because $\tan$ is $\sin/\cos$, $\tan$ will be positive in quadrants I and III, but negative in quadrants II and IV

2. Because $\csc$ is the reciprocal of $\sin$, they'll always have the same sign.

3. Because $\sec$ is the reciprocal of $\cos$, they'll always have the same sign.

4. Because $\cot$ is the reciprocal of $\tan$, they'll always have the same sign.

We'll go through a few examples, but let's talk about the steps we'd use to find the value of the other five trig functions if we only knew the value of $\sec\theta$, and the quadrant of the angle.

We start by finding $\cos\theta$ using the reciprocal identity

$$\cos\theta = \frac{1}{\sec\theta}$$

Then we'd find $\sin\theta$ using $\sin^2\theta + \cos^2\theta = 1$ and what we know about the sign of $\sin\theta$. Once we know the value of $\sin\theta$, we'd find $\csc\theta$ by using the reciprocal identity

$$\csc\theta = \frac{1}{\sin\theta}$$

We'd compute $\tan\theta$ using $\sin\theta$ and $\cos\theta$ and the quotient identity

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

We'd find $\cot\theta$ using the reciprocal identity

$$\cot\theta = \frac{1}{\tan\theta}$$

**Example**

For an angle $\theta$ in the third quadrant with $\sec\theta = -2.53$, find the values of the other five trig functions at $\theta$.

With the reciprocal identity

$$\cos\theta = \frac{1}{\sec\theta}$$

we can plug in the value of $\sec\theta$ to find $\cos\theta$.

$$\cos\theta = \frac{1}{-2.53} \approx -0.395$$

Now we'll rewrite the Pythagorean identity.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

We'll plug in the value of $\cos\theta$ to find $\sin\theta$.

$$\sin^2\theta \approx 1 - (-0.395)^2$$

$$\sin^2\theta \approx 1 - 0.156$$

$$\sin^2\theta \approx 0.844$$

$$\sin\theta \approx \pm\sqrt{0.844}$$

Since the sine of any angle in the third quadrant is negative,

$$\sin\theta \approx -\sqrt{0.844} \approx -0.919$$

With the reciprocal identity

$$\csc\theta = \frac{1}{\sin\theta}$$

we can plug in the value of $\sin\theta$ to find $\csc\theta$.

$$\csc\theta \approx \frac{1}{-0.919} \approx -1.09$$

We'll plug the values of $\sin\theta$ and $\cos\theta$ into the quotient identity to find $\tan\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \approx \frac{-0.919}{-0.395} \approx 2.33$$

With the reciprocal identity

$$\cot\theta = \frac{1}{\tan\theta}$$

we can plug in the value of $\tan\theta$ to find $\cot\theta$.

$$\cot\theta \approx \frac{1}{2.33} \approx 0.430$$

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Let's do an example where we're starting with the tangent function instead of the secant function.

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**Example**

For an angle $\theta$ in the fourth quadrant whose tangent is equal to $-6.79$, find the values of the other five trig functions.

When we know the value of $\tan\theta$ and the quadrant of the angle, we have to start with the quotient identity.

$$\frac{\sin\theta}{\cos\theta}=\tan\theta$$

We also need to rewrite the Pythagorean identity.

$$\sin^2\theta+\cos^2\theta=1$$

$$\cos^2\theta=1-\sin^2\theta$$

$$\cos\theta=\pm\sqrt{1-\sin^2\theta}$$

If we plug $\cos\theta=\pm\sqrt{1-\sin^2\theta}$ into the quotient identity, we get

$$\frac{\sin\theta}{\pm\sqrt{1-\sin^2\theta}}=\tan\theta$$

Squaring both sides, we get

$$\frac{\sin^2\theta}{1-\sin^2\theta}=\tan^2\theta$$

Multiplying both sides by $1-\sin^2\theta$ and then distributing the right side gives

$$\sin^2\theta=(\tan^2\theta)(1-\sin^2\theta)$$

$$\sin^2\theta=\tan^2\theta-(\tan^2\theta)(\sin^2\theta)$$

We want to solve this equation for $\sin\theta$.

$$\sin^2\theta+(\tan^2\theta)(\sin^2\theta)=\tan^2\theta$$

$$\sin^2\theta(1+\tan^2\theta)=\tan^2\theta$$

$$\sin^2\theta=\frac{\tan^2\theta}{1+\tan^2\theta}$$

Now we can simply take the value of $\tan\theta$ we started with and use it in this equation to find the value of $\sin\theta$. We'll use the quadrant of the angle to determine the sign of $\sin\theta$.

In this problem, we know that $\tan\theta$ is equal to $-6.79$, so we'll get

$$\sin^2\theta\approx\frac{(-6.79)^2}{1+(-6.79)^2}$$

$$\sin^2\theta\approx\frac{46.1}{1+46.1}$$

$$\sin^2\theta\approx 0.979$$

$$\sin\theta\approx\pm\sqrt{0.979}$$

The sine of any angle in the fourth quadrant is negative, so

$$\sin\theta\approx-\sqrt{0.979}\approx-0.989$$

We'll use the value of $\sin\theta$ in the Pythagorean identity to find $\cos\theta$.

$$\sin^2\theta+\cos^2\theta=1$$

$$\cos^2\theta=1-\sin^2\theta$$

$$\cos^2\theta\approx1-(-0.989)^2$$

$$\cos^2\theta\approx 0.0220$$

$$\cos\theta \approx \pm\sqrt{0.0220}$$

The cosine of any angle in the fourth quadrant is positive, so

$$\cos \approx \sqrt{0.0220} \approx 0.148$$

Using reciprocal identities, we can find the values of the last three trig functions.

$$\sec\theta = \frac{1}{\cos\theta} \approx \frac{1}{0.148} \approx 6.76$$

$$\csc\theta = \frac{1}{\sin\theta} \approx \frac{1}{-0.989} \approx -1.01$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-6.79} \approx -0.147$$