**Topic**: Linear and angular velocity

**Question**: Find the angular speed.

What is the angular speed $\omega$ of a wheel that rotates at a constant rate and sweeps out an angle of $543\pi$ radians in $14.6$ minutes?

**Answer choices**:

A $\omega = 37.2$ radians per second

B $\omega = 0.620$ radians per second

C $\omega = 0.620\pi$ radians per second

D $\omega = 1.83$ radians per second

**Solution**: C

Since the elapsed time $t$ ($14.6$ minutes) is given in units of minutes and all the answers choices are given in units of radians per second (not radians per minute), we need to convert the elapsed time ($14.6$) minutes to seconds. Since there are $60$ seconds in a minute, we'll use the conversion factor $(60 \text{ sec})/(1 \text{ min})$.

$$t = 14.6 \text{ min}$$

$$t = (14.6 \text{ min})(1)$$

$$t = (14.6 \text{ min})\left(\frac{60 \text{ sec}}{1 \text{ min}}\right)$$

$$t = \left[14.6(60)\right] \text{ sec}$$

$$t = 876 \text{ sec}$$

Now we're ready to compute the angular speed in radians per second:

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{(543\pi) \text{ rad}}{876 \text{ sec}}$$

$$\omega = \left(\frac{543}{876}\pi\right) \text{ radians per second}$$

$$\omega \approx 0.620\pi \text{ radians per second}$$

**Topic**: Linear and angular velocity

**Question**: Find the angular speed.

If a disc is rotating at the constant rate of $94.9$ revolutions per minute, what is its angular speed $\omega$ in units of radians per second?

**Answer choices**:

A $\omega = 8.62$ radians per second

B $\omega = 18.7\pi$ radians per second

C $\omega = 3.16\pi$ radians per second

D $\omega = 15.1$ radians per second

**Solution**: C

To convert from revolutions per minute to radians per second, we need to use the following facts: (a) There are $2\pi$ radians in a (full) revolution, and (b) there are $60$ seconds in a minute.

$$\omega = 94.9 \frac{\text{rev}}{\text{min}}$$

$$\omega = \left(94.9 \frac{\text{rev}}{\text{min}}\right)(1)(1)$$

$$\omega = \left(94.9 \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)$$

$$\omega = \left[\frac{94.9(2)}{60}\pi\right] \text{ radians per second}$$

$$\omega \approx 3.16\pi \text{ radians per second}$$

**Topic**: Linear and angular velocity

**Question**: Find the linear speed.

If a wheel of diameter $21.0$ inches is rotating at a rate of $0.543\pi$ radians per second, what is the linear speed $v$ of a point on the outside edge of the wheel?

**Answer choices**:

A $v = 1.49$ feet per second

B $v = 2.98$ feet per second

C $v = 17.9$ feet per second

D $v = 3.66$ feet per second

**Solution**: A

To compute $v$, we'll use the standard formula: $v = r\omega$. However, we have to take into account two other aspects of this problem. One of these is that we've been given the diameter (not the radius) of the wheel, so we have to compute the radius. The other is that all the answer choices are given in units of feet per second, but the diameter of the wheel is given in units of inches (not feet), so we have to convert inches to feet.

To get the radius of the wheel (which is equal to the distance of any point on the outside edge of the wheel from the center of the wheel), we simply divide the diameter by $2$ (recall that the radius of any circle is one-half the diameter), so

$$r = \frac{21.0}{2} = 10.5 \text{ inches}$$

In order to take into account the different units of length (inches vs. feet), we'll use the fact that there are $12$ inches in a foot. (In our calculation, we'll use “in” and “ft” as abbreviations for inches and feet, respectively.) Also, recall that we don't write “radians” or “rad” when we substitute the value of $\omega$ into the formula $v = r\omega$, since $\omega$ is understood to be in units of radians per unit time.

$$v = r\omega$$

$$v = (10.5 \text{ in})\left(\frac{0.543\pi}{\text{sec}}\right)$$

$$v = (10.5 \text{ in})(1)\left(\frac{0.543\pi}{\text{sec}}\right)$$

$$v = (10.5 \text{ in})\left(\frac{1 \text{ ft}}{12 \text{ in}}\right)\left(\frac{0.543\pi}{\text{sec}}\right)$$

$$v = \left[\frac{10.5(0.543)}{12}\pi\right] \text{ feet per second}$$

$$v \approx 0.475\pi \text{ feet per second}$$

Substituting the numerical value of $\pi$ $(3.1415\ldots)$, we obtain

$$v \approx 1.49 \text{ feet per second}$$