Trigonometry Workbook Solutions

Angles in circles

krista king MATH

ORIENTED ARC FOR A REAL NUMBER

1. Find the approximate length of an oriented arc of the unit circle that corresponds to the DMS angle.

$$\theta = 290^\circ 44' 24''$$

Solution:

Separate the components of the angle and convert them to degrees.

$$\theta = 290^\circ + 44' + 24''$$

$$\theta = 290^\circ + 44' \left(\frac{1^\circ}{60'}\right) + 24'' \left(\frac{1^\circ}{60'}\right) \left(\frac{1'}{60''}\right)$$

$$\theta = 290^\circ + 44 \left(\frac{1^\circ}{60}\right) + 24 \left(\frac{1^\circ}{60}\right) \left(\frac{1}{60}\right)$$

$$\theta = 290^\circ + \frac{44}{60}^\circ + \frac{24}{3{,}600}^\circ$$

$$\theta = 290^\circ + 0.73^\circ + 0.0067^\circ$$

$$\theta = 290.7367^\circ$$

$$\theta = 290.74^\circ$$

Then convert the angle from degrees to radians.

$$\theta = 290.74^\circ \left(\frac{\pi \text{ radians}}{180^\circ} \right)$$

$$\theta = 290.74 \left(\frac{\pi \text{ radians}}{180} \right)$$

$$\theta \approx 5.08 \text{ radians}$$

■ 2. Find the number of rotations (to the nearest integer) associated with an oriented arc with approximate length 63.615.

Solution:

To find the number of rotations, we'll call the angle θ, and then find

$$\frac{\theta}{2\pi}$$

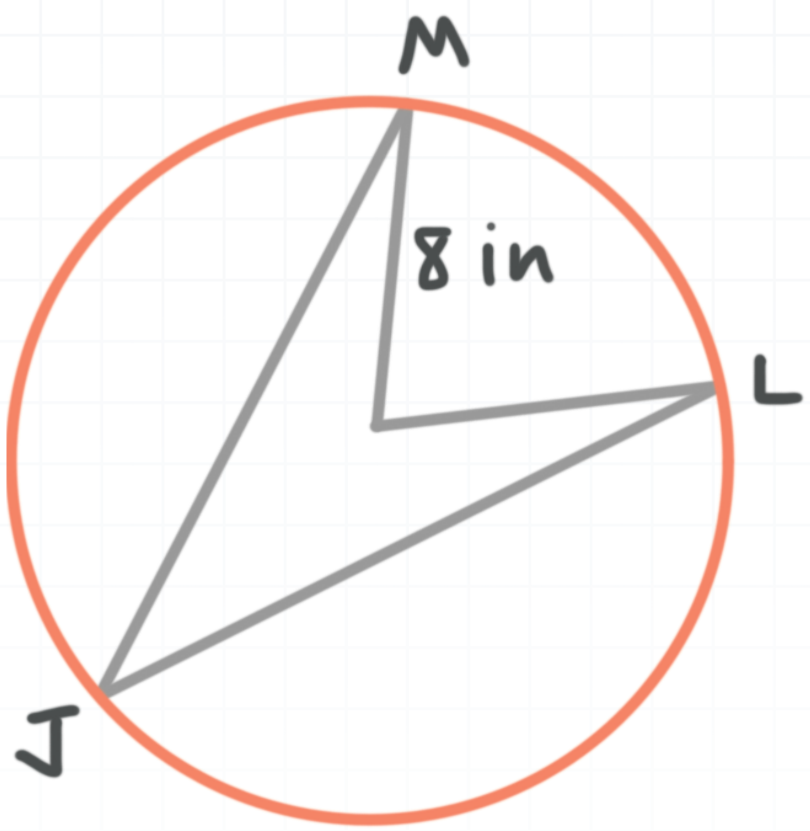
$$\frac{63.615}{2\pi}$$

$$10.12$$

So the arc length 63.615 represents approximately 10 rotations.

■ 3. Find the length of arc ML (to the nearest hundredth of an inch) on the circle, given that the radius is $r = 8$ inches.

$$m\angle MJL = \frac{\pi}{6}$$



Solution:

Find the interior angle associated with arc ML.

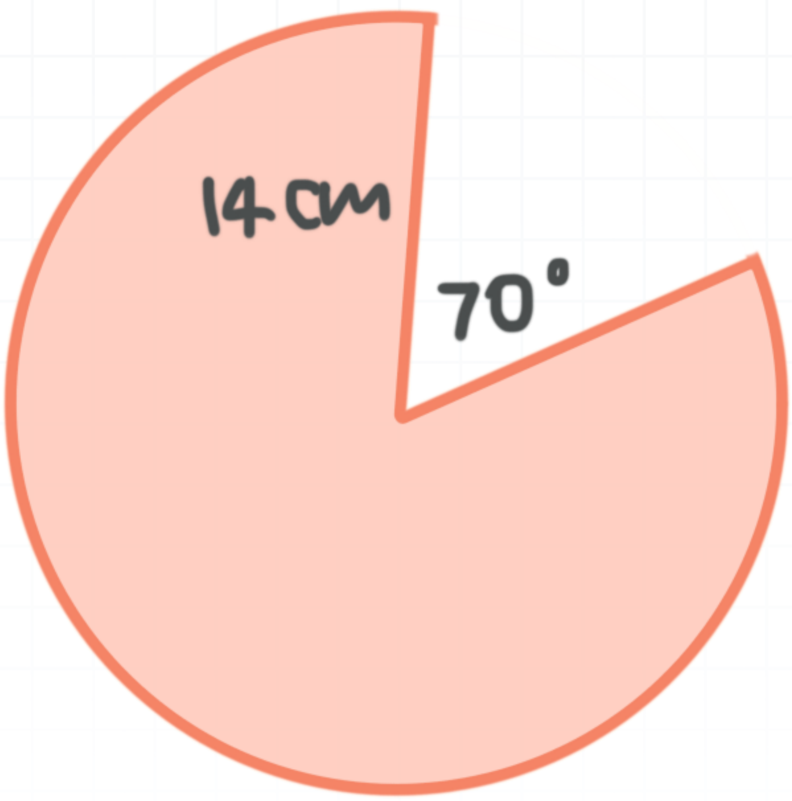
$$2(m\angle MJL) = 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$$

Then the length of arc ML is

$$s = r\theta = (8)\left(\frac{\pi}{3}\right) \approx 8.38 \text{ inches}$$

AREA OF A CIRCULAR SECTOR

1. Find the area of the shaded region.



Solution:

The angle of the circular sector is

$$\theta = 360^\circ - 70^\circ$$

$$\theta = 290^\circ$$

Plugging this angle and the radius into the formula for the area of a circular sector gives

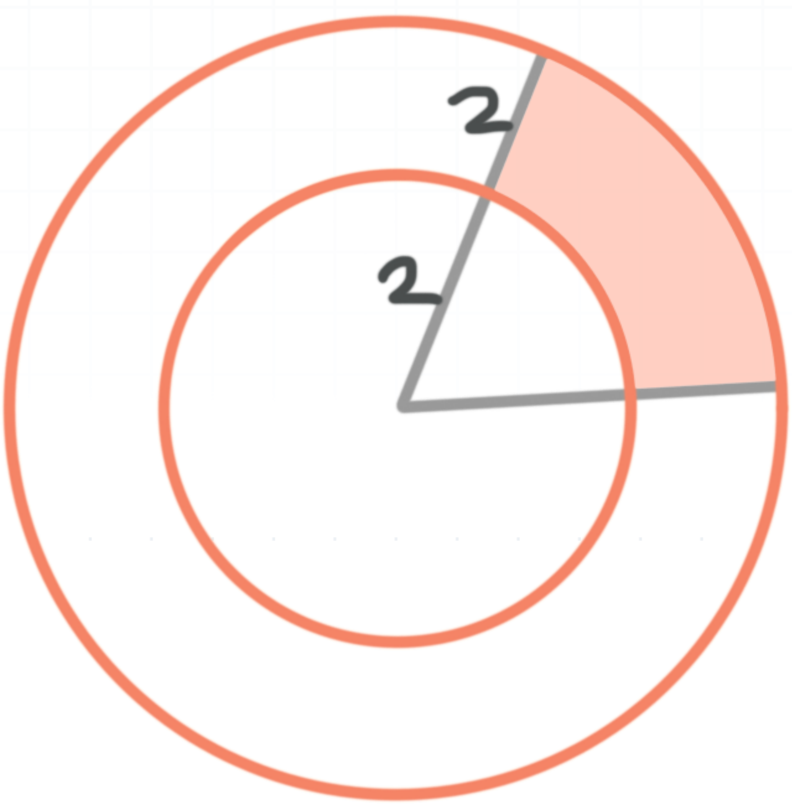
$$A = \pi r^2 \left(\frac{\theta}{360^\circ}\right)$$

$$A = \pi(14^2)\left(\frac{290^\circ}{360^\circ}\right)$$

$$A = 196\pi\left(\frac{29}{36}\right)$$

$A \approx 496.01$ square units

■ 2. Find the area of the shaded region between the concentric circles, if the angle that subtends the arc is 80°.



Solution:

If we find the area of the circular sector for the larger circle, given that its interior angle measure is 80° and its radius is 4, we get

$$A = \pi r^2 \left(\frac{\theta}{360^\circ} \right)$$

$$A = \pi (4^2) \left(\frac{80^\circ}{360^\circ} \right)$$

$$A = 16\pi \left(\frac{2}{9} \right)$$

$A \approx 11.16$ square units

The area of the circular sector for the smaller circle, given that its interior angle measure is 80° and its radius is 2.

$$A = \pi r^2 \left(\frac{\theta}{360^\circ} \right)$$

$$A = \pi (2^2) \left(\frac{80^\circ}{360^\circ} \right)$$

$$A = 4\pi \left(\frac{2}{9} \right)$$

$A \approx 2.79$ square units

So the area of the shaded region is

$$A = 11.16 - 2.79$$

$A = 8.37$ square units

3. A circle passes through $(-5,12)$ and has its center at the origin. Find the area A of a sector of the circle that has a central angle of $2\pi/5$ radians.

Solution:

Find the length of the radius, which will be the distance from $(0,0)$ to $(-5,12)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(-5 - 0)^2 + (12 - 0)^2}$$

$$d = \sqrt{(-5)^2 + (12)^2}$$

$$d = \sqrt{169}$$

$$d = 13$$

Then the area of the circular sector with radius $r = 13$ and central angle $2\pi/5$ is

$$A = \pi r^2 \left(\frac{\theta}{2\pi}\right)$$

$$A = \pi(13)^2 \left(\frac{\frac{2\pi}{5}}{2\pi}\right)$$

$$A = 169\pi\left(\frac{1}{5}\right)$$

$A \approx 106.18$ square units

LINEAR AND ANGULAR VELOCITY

■ 1. What is the angular speed, in radians per second, of a wheel that rotates at a constant rate and sweeps out an angle of $33\pi/4$ radians in 0.6 seconds?

Solution:

The angular speed is

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{\frac{33\pi}{4}}{0.6}$$

$\omega = 13.75\pi$ radians per second

■ 2. A saw has a circular blade with diameter 10 inches and it rotates at $5{,}000$ revolutions per minute. Find the approximate linear speed of the saw teeth (in ft/sec) as they contact the wood being cut.

Solution:

Find angular velocity.

$$\omega = 5000 \frac{\text{rev}}{\text{min}}$$

$$\omega = 5{,}000 \frac{\text{rev}}{\text{min}} \times 2\pi \frac{\text{rad}}{\text{rev}} \times \frac{1 \text{ min}}{60 \text{ sec}}$$

$$\omega = 5{,}000 \times 2\pi \times \frac{1}{60} \times \frac{\text{rad}}{\text{sec}}$$

$$\omega = 5{,}000 \times 2\pi \times \frac{1}{60} \times \frac{\text{rad}}{\text{sec}}$$

$$\omega = 166.67\pi \text{ radians/second}$$

Because the diameter of the blade is 10 inches, its radius is 5 inches, so linear speed is

$$v = \omega r$$

$$v = \left(166.67\pi \ \frac{\text{radians}}{\text{second}}\right)\left(5 \text{ inches } \frac{1 \text{ foot}}{12 \text{ inches}}\right)$$

$$v = \left(166.67\pi \ \frac{\text{radians}}{\text{second}}\right)\left(5 \ \frac{1 \text{ foot}}{12}\right)$$

$$v \approx 218.2 \text{ feet/second}$$

■ 3. The B75 wind turbine has a circular blade with diameter 154 meters that rotates at 18 rotations per minute. Find the angular speed of the blade in degrees per second.

Solution:

Find angular velocity in radians per second.

$$\omega = 18\ \frac{\text{rev}}{\text{min}}$$

$$\omega = 18\ \frac{\text{rev}}{\text{min}} \times 2\pi\ \frac{\text{rad}}{\text{rev}} \times \frac{1\ \text{min}}{60\ \text{sec}}$$

$$\omega = \frac{36\pi}{60}\ \frac{\text{rad}}{\text{sec}}$$

$$\omega = \frac{3\pi}{5}\ \text{radians/second}$$

Convert from radians to degrees.

$$\frac{3\pi}{5}\ \frac{\text{rad}}{\text{sec}} \times \frac{180^\circ}{\pi\ \text{rad}}$$

108 degrees/second

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