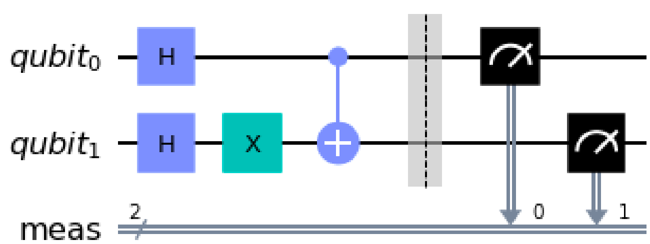
The Depth of a Quantum Circuit

Let's visit some history of Quantum Mechanics. Computers based on Quantum Mechanics laws were thought unworkable in the 1970s. The quantity of inaccuracy in each Qubit was the primary reason behind this. But thanks to the researchers working world wide for more than four decades we can now execute algorithms on Quantum Computers by converting them into Quantum Circuits. However far we may have come, the problem of error in Qubits still persists.

The amount of error scales linearly with single qubit gates and exponentially when we use CNOT gates for entangling multiple qubits. As a result, the bigger the number of gates in a Quantum Circuit, the lower the accuracy of Quantum States. So we want to have a circuit which can execute the algorithm we want which has the least number of gates. But how can you compare the number of gates in two Quantum Circuits with numerous qubits? Depth metric for the win!!

The depth of a Quantum Circuit is the length of the longest path from the preparation to the measurement gate, moving forward in time along qubit wires. Each gate in this longest path is considered as one unit. Generally theoreticians only consider Unitary operations to be a part of the depth. So the final measurement is often not included. But quantum programming packages such as QISKIT also include measurement while calculating the depth of the circuit. From my perspective I think it should be calculated as measurement also leads to error.

The reason we are actually looking for the longest path is because we want to see how much time it takes for a circuit to execute. Often one qubit has to wait due to entanglement with another. To give you an example, both the qubits in the below image start with the hadamard gate, then the qubit 0 waits until the X gate can be applied on qubit 1. So the total amount of time taken is proportional to the --H---X--CNOT-- path because the qubit 0 has to wait until qubit 1 is properly operated upon.

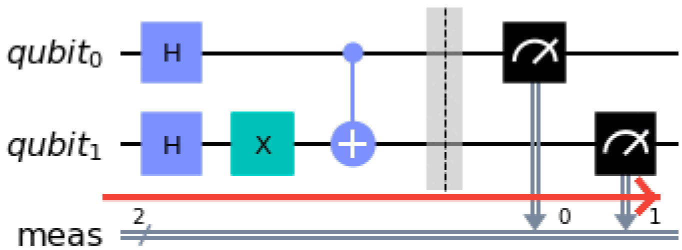


Calculating the depth of Quantum Circuits

All you have to do is find the longest path in the whole circuit, taking into consideration all the gates present, especially the CNOT gates. Remember we do not consider the *barrier* in the depth because it is *technically* not an operation on the quantum state rather, just an

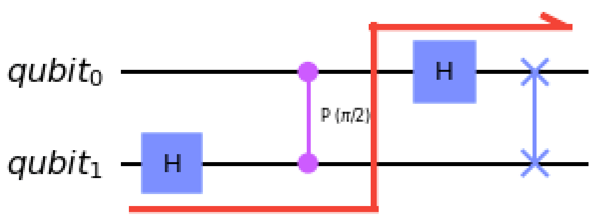
instruction to the transpiler.
Let’s take a look at a few examples,

1. For the above circuit I have marked the longest path that has to be taken and we can clearly see that the depth of this circuit is 4.



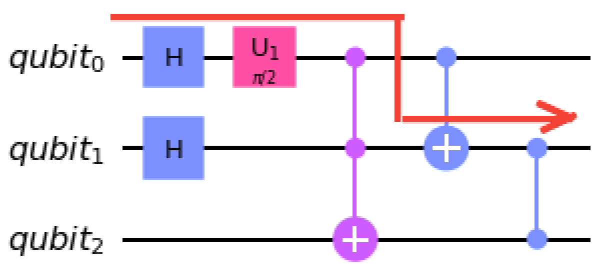
The red line indicates the longest path and the circuit depth

2. If we apply the famous Quantum Fourier transform algorithm for two qubits we obtain something like this.



QFT for two qubits

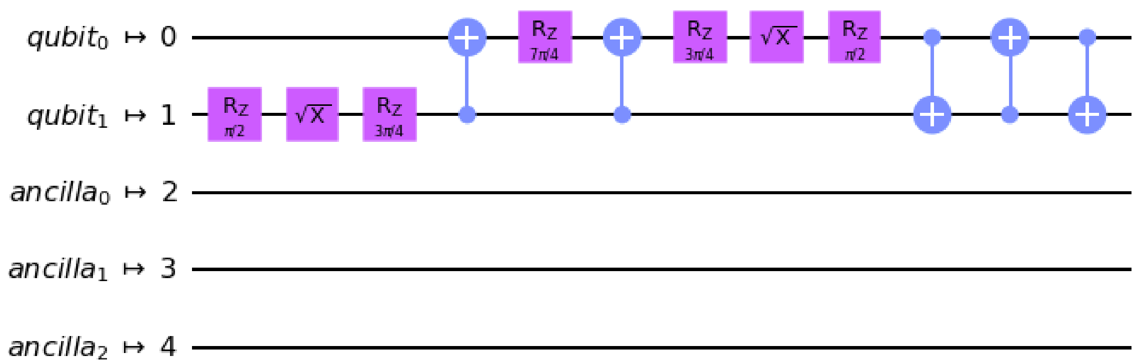
   Again I have used the red marker to show which is the longest path and we see that the depth of the circuit is 4.
3. This circuit is a random circuit and when you follow the longest path you will find that there are 5 gates and the depth here is 5.



Random circuit with a depth 5

On a Side Note:

One thing to keep in mind while calculating the depth of a Quantum Circuit is that when the circuit goes through the process of transpilation, the circuit's depth will probably change. For the 2nd example of QFT that we looked at above, when we map it to *ibmq_bargota* the circuit that we get is the following -



QFT circuit when transpiled onto ibmq_bargota

It is clearly visible that the depth increased from 4 to 12. This happens because the transpiler convertes the circuits according to the backends configuration.

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Hope you learned something new about Quantum Computing today.

Resources

1. https://www.nap.edu/read/25333/chapter/6
2. https://quantumcomputing.stackexchange.com/questions/5769/how-to-calculate-circuit-depth-properly
3. https://quantumcomputing.stackexchange.com/questions/14431/whats-meant-by-the-depth-of-a-quantum-circuit
4. https://medium.com/arnaldo-gunzi-quantum/how-to-calculate-the-depth-of-a-quantum-circuit-in-qiskit-868505abc104