UCLA Anderson

Quantitative Asset Management

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Spring 2025

My background

- Research Interests
  - Asset pricing and production networks
  - Financial Economics, Economic Theory, Macroeconomics.
  - Stock returns and idiosyncratic volatility
- Teaching
  - Investment Management (MBA/FEMBA) since Spring 2016
  - Quantitative Asset Management (MFE) since Spring 2017
  - Foundations of Finance (MBA) since Fall 2021
- More information about me: www.bernardherskovic.com

Rules of the game

- Read the syllabus!
- Grades in this class will be assigned on the basis of
  - class participation, attendance and in-class quizzes (20%)
  - four problem sets (20% each)–the lowest scored problem set will be dropped
  - final project (20%)
- Class attendance is required.
  - I will randomly cold students
- Study the required readings
  - a student will be randomly selected to summarize required readings
- Class participation is extremely encouraged!!

Problem Sets and Final Project

- Problem sets
  - Individual answers
  - Can discuss with your classmates
  - Indicate whom you collaborated with!
  - Submit your own code
- Final Project Presentations on Week 10

Useful information

- Class materials
  - Slides, readings, and assignments: BruinLearn
  - No textbook
  - Mostly academic papers published at the best finance or econ journals:

    Journal of Finance
    Journal of Financial Economics
    Review of Financial Studies
- My contact information
  - Email: bernard.herskovic@anderson.ucla.edu
- TA: Brian Park
  - Email: joongwoo.park.phd@anderson.ucla.edu

Quantitative Asset Management: course outline

Lecture 1	Mean-Variance Investing: Black-Litterman, $1/N$
Lecture 2	Risk Parity, Asset Growth, Profitability and 5 FF factors Due Sunday April 13: *Problem Set 1*
Lecture 3	Momentum
Lecture 4	Time Series Momentum and Volatility Due Sunday April 27: *Problem Set 2*
Lecture 5	Commodity, Short-selling, and Comomentum

Quantitative Asset Management: course outline

Lecture 6	Return Predictability Due Sunday May 11: : *Problem Set 3*
Lecture 7	BAB, QMJ, Currency
Lecture 8	Mutual Fund and Hedge Fund Performance Due Sunday May 25: *Problem Set 4*
Lecture 9	Factor Zoo and Data Mining
Lecture 10	Final Project Presentations Due Sunday June 1: *Final project report*

Lecture 1: Outline

Introduction: Contextualizing QAM

Problem Sets Overview

Annualizing return

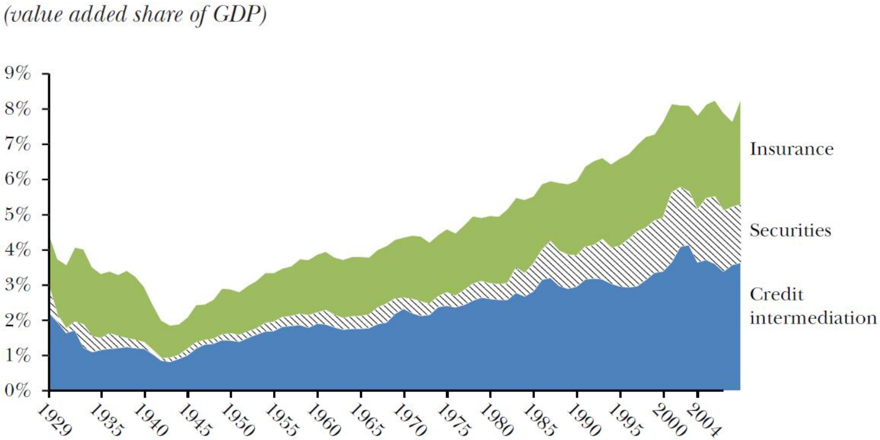
Market portfolio

Lecture

1. Review of Mean-Variance Analysis
   - Markowitz Model: No Risk-free Asset
   - Markowitz Model: With Risk-free Asset
   - Example: International Diversification Case Study
   - Pitfalls of Mean-Variance Analysis
2. Black Litterman Approach
3. $1/N$

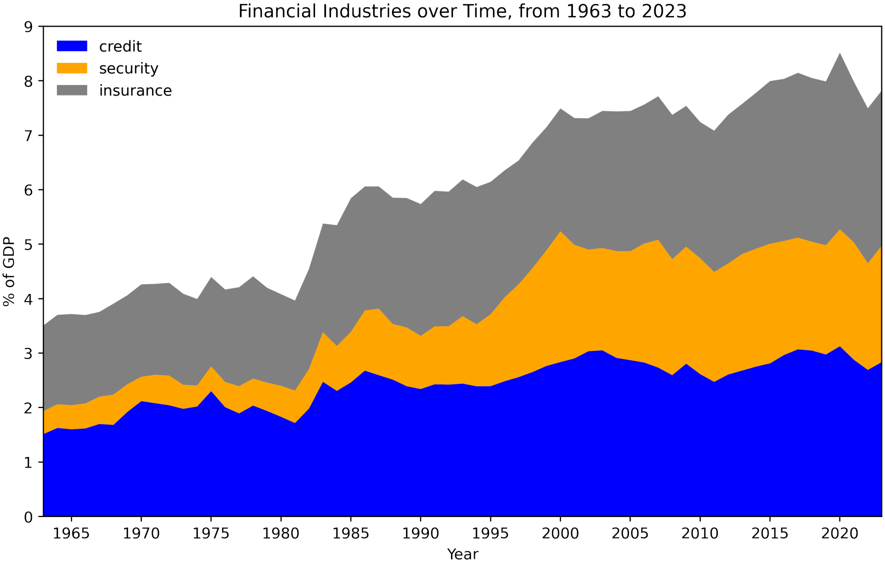
Introduction

The Growth of Financial Services



Greenwood and Sharfstein (2013, Journal of Economic Perspectives)

The growth of the Securities Industry, recent data from BEA



Investment Management Industry

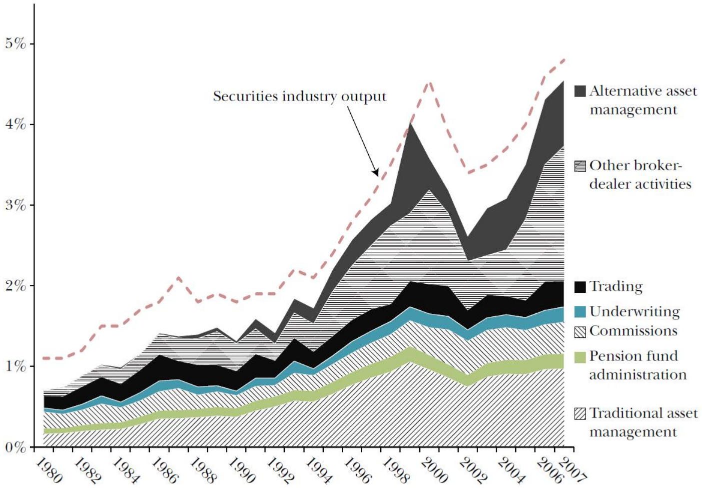
- Financial services experienced enormous growth

  From 5.3% of GDP in 1980s to 7.8% of GDP in 2023
- Much of the growth of finance is associated with two activities:
  1. Asset management
  2. Provision of household credit.
- Large fraction of growth comes from fees earned in asset management

  As asset values increase, fee increased as a fraction of GDP

The growth of the Securities Industry, 1980–2007

(revenues from different activities as a percent of GDP)



Greenwood and Sharfstein (2013, Journal of Economic Perspectives)

Sustainable Increase?

Active Management and Fees

- Neoclassical finance view: there is no alpha.

  Is there any evidence that all actively managed funds in the aggregate outperform? Not really: buy a passive index stock fund and save yourself 67 bps per annum (French 2008, JF Presidential Address)

- Post-neoclassical view: there is alpha but not after fees

  in a world in which some managers have skill, but it's hard to tell, it makes sense to chase performance. In 'equilibrium', the fees will eat up all of the alpha (skill is not alpha). (Berk and Green 2004, JPE)

What about the data?

There is alpha relative to the Market

- There is alpha relative to the market portfolio
  - Value (stocks, currencies)
  - Momentum (stocks, currencies, bonds)
  - Carry trades (currencies, bonds)
- Is this really alpha or are we capturing new types of beta?
  - Probably mostly new types of beta
  - Risk is multidimensional
- Creates role for active management:
  - How much do you want to pay for this service?

What about the data?

Asset Returns are Predictable

- Old notion of market efficiency: publicly available information does not predict returns
  - Clearly at odds with data
- Returns are predictable in
  - Stock markets: dividend yield, interest rates, price/earnings ratios all predict returns over longer holding periods (more than one year)
  - Bond markets: slope of the yield curve, forward spreads predict returns over the next year or so
  - Currency markets: interest rate spreads predict returns
- Creates role for active management

Portfolio Management

The CAPM does not 'work' anymore

- There are multiple priced factors (not just the market)
- Even if you only care about mean and variance, you should not hold the market in your basket of risky assets
- Portfolio management becomes more much interesting and exciting
- Maybe we need a large portfolio management industry: lots of sources of risk; different investors have different tolerances for different types of risk

Lecture 1: Outline

1. Review of Mean-Variance Analysis
   - Markowitz Model
   - Example: International Diversification Case Study
   - Pitfalls of Mean-Variance Analysis
2. Black Litterman Approach
3. $1/N$

Mean-Variance Analysis

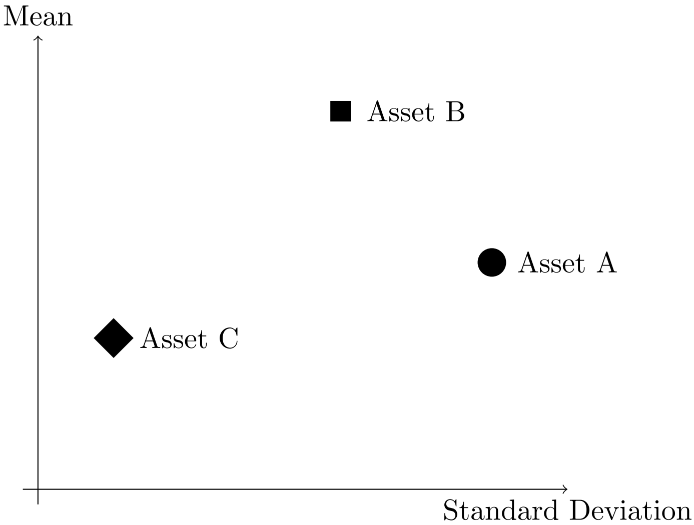
Notation

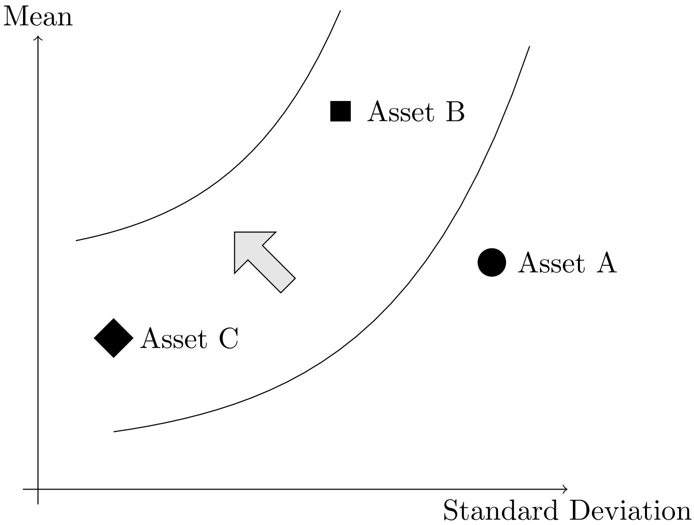
- Define
  - Number of assets: N
  - Column Vector of portfolio weights: w
  - Portfolio: $\omega' R$
  - Matrix variance covariance: $\mathrm{Var}(R) = \Sigma$
  - Expected returns: $\mathbb{E}[R] = \mu$
- The variance of the portfolio return:

$$w' \Sigma w$$

- The expected value of the portfolio return:

$$w' \mu$$


Mean
Asset B
Asset A
Asset C
Standard Deviation


Mean
Asset B
Asset A
Asset C
Standard Deviation

What do we need to know for mean-variance analysis?

Model inputs:

- Expected return
- Variance
- Co-variances (or correlations)

Mean-variance optimization

Optimization

$$\min_w \frac{1}{2} w' \Sigma w$$
$$\text{s.t.}$$
$$w'\mu \geq \bar{\mu}$$
$$w'\mathbf{1} = 1$$

where $\mathbf{1}$ is a column vector of ones

KKT conditions:

$$\Sigma w = \lambda_1 \mathbf{1} + \lambda_2 \mu$$
$$w'\mu \geq \bar{\mu}$$
$$w'\mathbf{1} = 1$$
$$\lambda_2 \geq 0$$
$$\lambda_2 (w'\mu - \bar{\mu}) = 0$$

See detailed derivations and proofs here

Mean-variance optimization (with risk aversion)

Optimization

$$\max_{w} w'\mu - \frac{\delta}{2} w'\Sigma w$$
$$\text{s.t.}$$
$$w'\mathbf{1} = 1$$

where $\mathbf{1}$ is a column vector of ones, δ is a risk aversion parameter

KKT conditions:

$$\delta\Sigma w = \mu + \lambda_1 \mathbf{1}$$
$$w'\mathbf{1} = 1$$

Using excess returns, weights on risky assets are

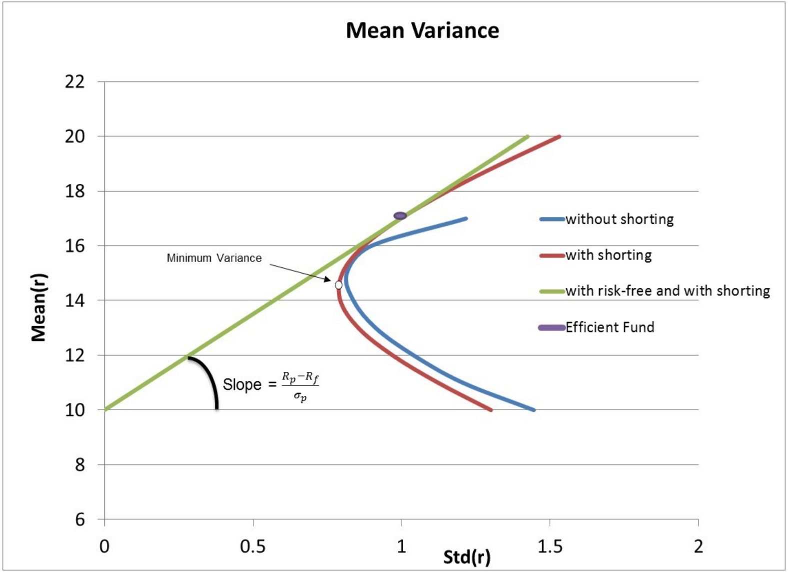
$$w^*_{ex} = \frac{1}{\delta}\Sigma^{-1}_{risky}\mu$$

One Fund Theorem

There is a single fund F of risky assets such that any efficient portfolio can be constructed as a combination of F and the risk-free asset

The slope of the line that connects the risk-free and F is the maximum Sharpe ratio:

$$slope = \frac{R_p - R_f}{\sigma_p}$$


Mean Variance
22
20
18
16
14
12
10
8
6
0
0.5
1
1.5
2
Std(r)
Mean(r)
Minimum Variance
Slope = $\frac{R_p - R_f}{\sigma_p}$
without shorting
with shorting
with risk-free and with shorting
Efficient Fund

Maximum Sharpe Ratio

To derive the maximum Sharpe ratio, we could simply maximize the slope..

$$slope = \frac{R_p - R_f}{\sigma_p} = \frac{\sum_{i=1}^{n} w_i R_i - R_f}{\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}}}$$

Maximize this slope by choosing the portfolio weights $w_i = 1, \ldots, n$ subject to the condition that the portfolio weights sum to one

Predictions from Mean-Variance Analysis

Portfolio Advice Summary

- All investors invest in the same fund of risky assets, regardless of risk aversion.
- Depending on risk aversion, they choose an appropriate mix of money market and risky fund investments
- What about the investment horizon?
  - Same portfolio advice applies for investors with longer horizons if we assume that investors are drawn from the same distribution each period
  - Your investment horizon (age) does not matter if returns are i.i.d. (independently and identically distributed over time)
- What about your labor income risk?
  - Does not matter
  - All you care about is mean and variance

International Diversification Case Study

International Diversification

- Suppose we consider investing in
  1. U.S. Equities (S&P 500)
  2. U.S. Bonds (10-year treasury)
  3. International Stocks
  4. Commodities (Goldman Sachs Commodity Index)
- Huge measurement challenge
- We need to come up with estimates for expected returns, volatilities and correlations for all asset classes
  - backward looking estimates : Historical data
  - forward looking estimates: Other variables like price/earnings, price/dividend ratios, cyclically adjusted price/earnings

Estimates of Risk and Returns

	US Equity	Foreign Equity	Bonds	Commodity
Avg. Excess Return	7.13	6.11	2.28	0.71
Volatility	15.37	14.95	8.27	20.27
Sharpe Ratio	0.46	0.41	0.28	0.04

Sample: 1970-01-31 to 2025-02-28. Excess Returns.

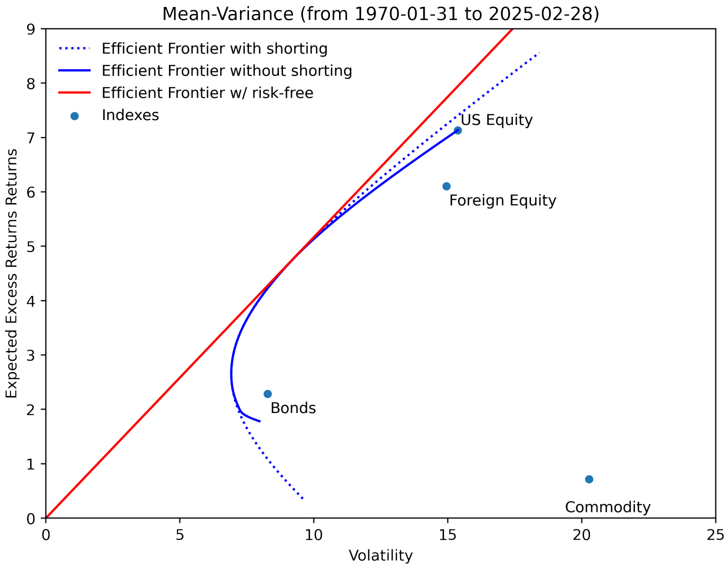
Source: Globalfinancialdata.com

Correlation Matrix of Excess Returns

	US Equity	Foreign Equity	Bonds	Commodity
US Equity	1.00	0.90	0.11	0.17
Foreign Equity	0.90	1.00	0.09	0.22
Bonds	0.11	0.09	1.00	-0.14
Commodity	0.17	0.22	-0.14	1.00

Sample: 1970-01-31 to 2025-02-28. Excess Returns.
Source: Globalfinancialdata.com

Mean-Variance Analysis



Efficient Fund

- The maximum Sharpe Ratio is 51.68%!
- The Efficient Fund consists of:
    1. 55.87% in domestic equity
    2. -5.00% in foreign equity
    3. 49.42% in bonds
    4. -0.29% in commodities

Pitfalls of Mean-Variance Analysis

Investment Management

- Investment managers rarely apply the Markowitz framework
- Pension funds typically use a 60/40 equity/bond allocation
- Why not used more often?
  1. Requires specifying expected return assumption for entire universe of assets (investment managers focus on small subset)
  2. Investment managers tend to find the weights counterintuitive

Mean-Variance Analysis

- Has obvious appeal
- Gives clear answer to tough questions
- So... why not just use as your benchmark portfolio?
- Issues and solutions:
  - Model Uncertainty:
    - Expected returns are not precisely estimated
    - Mean-variance optimization is sensitive to inputs
  - Black-Litterman Approach to Mean-Variance Analysis

Uncertainty about the Expected Returns

- We estimated an expected excess return of 0.59% monthly (7.13% annualized) on U.S. equities (sample mean)
- How precise is that estimate? Need to compute the standard error of the sample mean

Garbage in, Garbage out: Take care when using historical data

- When using historical data, don't use short samples (rolling windows) to compute average returns

  $\Longrightarrow$ noisy estimates of the true underlying means

  $\Longrightarrow$ cyclical bias in your estimate of returns

  - When past stock returns are high, future returns are more likely to be lower (see next lectures), at least over longer investment horizons (> 1 year)
  - Hence, you'll be making systematic mistakes by extrapolating from recent experience!

Means are hard to estimate

- Assume that the returns are independently and identically distributed over time
- The standard deviation of U.S equity returns is 4.44% monthly (15.37% annualized)
- The number of observations T is 662 months (∼55 years)
- Given these assumptions, the standard error on the sample mean is given by

$$\frac{\sigma}{\sqrt{T}} = \frac{4.44\%}{\sqrt{662}} = 0.17\% \text{ (monthly) or } 2.07\% \text{ (annualized)}$$

- 95% confidence interval: from 3.1% and 11.2%
- That's quite a range...

Ill-behaved Optimization Problem

- Mean-Variance Analysis: Badly behaved optimization problem
- Small changes in expected returns generate large changes in optimized portfolio weights that come out of the mean-variance analysis

Sensitivity of Efficient Fund

- Increase the expected return on U.S. equities by 2 percentage points
- The maximum Sharpe Ratio is 69.63%!
- The Efficient Fund consists of:
  1. 136.09% in domestic equity
  2. -81.24% in foreign equity
  3. 43.58% in bonds
  4. 1.57% in commodities

Sensitivity of Efficient Fund

- Decrease the expected return on U.S. equities by 2 percentage points
- The maximum Sharpe Ratio is 48.61%!
- The Efficient Fund consists of:
  1. -29.25% in domestic equity
  2. 76.10% in foreign equity
  3. 55.51% in bonds
  4. -2.36% in commodities

Black Litterman Approach (use a model!)

Market Weights

- We can compare to the implied portfolio weights to the market cap shares of different asset classes
- The market cap shares of different asset classes represent the equilibrium choices by the stand-in U.S. or world investor
- Good idea to compare the optimized weights to these equilibrium weights

Black Litterman

- N asset normally distributed

$$r \sim N(\mu, \Sigma)$$

- Equilibrium: w_{eq}
- Equilibrium-implied Risk premium:

$$\Pi_{eq} = \delta \Sigma w_{eq}$$

- Bayesian prior

$$\Pi | \mu \sim N(\mu, \tau \Sigma)$$

- Views (beliefs)

$$P\mu \sim N(Q, \Omega)$$

Black Litterman

- Applying Bayes' rule

  See Satchell and Scowcroft (2000)

$$\mu | \Pi \sim N\left(\bar{\mu}, \bar{M}^{-1}\right)$$

where

$$\begin{aligned}
\bar{\mu} &= \left[(\tau\Sigma)^{-1} + P'\Sigma^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Sigma^{-1}Q\right] \\
\bar{M} &= \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]
\end{aligned}$$

Black Litterman

- Thus, posterior distribution of returns is given by

$$r \sim N\left(\bar{\mu}, \bar{\Sigma}\right)$$

  where $\bar{\Sigma} = \Sigma + \bar{M}^{-1}$

- Given risk aversion δ, mean-variance optimization yields

$$w^* = \frac{1}{\delta}\bar{\Sigma}^{-1}\bar{\mu}$$

- which implies

$$w^* = \frac{1}{1+\tau}\left(w_{eq} + P'\Lambda\right)$$

  where

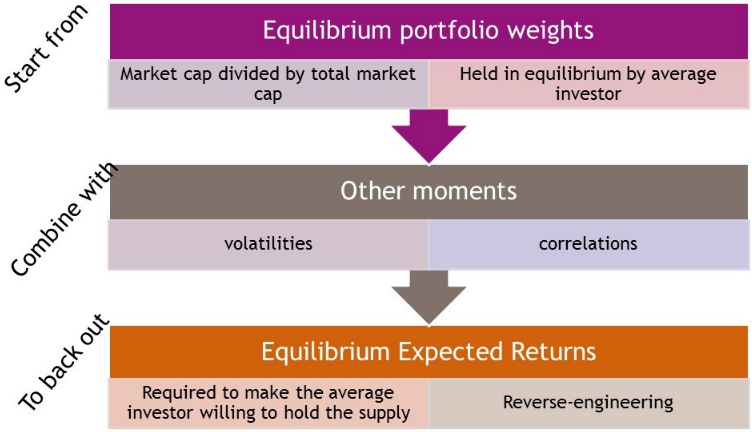
$$\Lambda = \tau\Omega^{-1}Q/\delta - A^{-1}P\frac{\Sigma}{1+\tau}w_{eq} - A^{-1}P\frac{\Sigma}{1+\tau}P'\tau\Omega^{-1}Q/\delta$$

Black-Litterman Approach

> Use market weights to infer expected returns,
> then consider small deviations and optimize

- Does not start from specifying assumptions about expected returns and volatilities
- Starts instead by backing out the expected returns that are consistent with the market cap weights (equilibrium expected returns)$\rightarrow$ that's essentially equivalent to imposing the CAPM
- Then consider small deviations from these equilibrium expected returns to derive optimized weights

Imposing the CAPM



Correlations

	Australia	Canada	France	Germany	Japan	UK
Canada	0.488					
France	0.478	0.664				
Germany	0.515	0.655	0.861			
Japan	0.439	0.310	0.355	0.354		
UK	0.512	0.608	0.783	0.777	0.405	
USA	0.491	0.779	0.668	0.653	0.306	0.652

	Equity Vol	Equilibrium Weight	Equilibrium Expected Return
Aus	16	1.6	3.9
Can	20.3	2.2	6.9
Fra	24.8	5.2	8.4
Germ	27.1	5.5	9.0
Jap	21.0	11.6	4.3
UK	20	12.4	6.8
US	18.7	61.5	7.6

Equilibrium expected returns

Start from returns that are consistent with the market weights (reflects the views of the average investor)

This is a neutral starting point

Introduce beliefs

Now, consider small deviations from these equilibrium returns (e.g., I'm 5% more bullish on France than the average investor)

Compute optimum portfolio

Re-compute the optimum portfolio weights

Global Equities: Black-Litterman approach

- Suppose we think German equities will outperform by 5%
  - We set the return on a long position in German equities and a short position in all other equities (weighted by market cap) equal to 5%
- We attach some confidence level to this view
- The new set of expected returns are a weighted average of the view and the equilibrium expected returns

	p	$\bar{\mu}$	w^*	$w^* - \frac{w_{eq}}{1+\tau}$
Australia	0.0	4.3	1.5	0.0
Canada	0.0	7.6	2.1	0.0
France	-29.5	9.3	-4.0	-8.9
Germany	100.0	11.0	35.4	30.2
Japan	0.0	4.5	11.0	0.0
UK	-70.5	7.0	-9.5	-21.3
USA	0.0	8.1	58.6	0.0

DeMiguel, Garlappi, Uppal (RFS, 2007)

Optimal Versus Naive Diversification: How Inefficient is the $1/N$ Portfolio Strategy?

What's the $1/N$ strategy?

- Equally-weighted portfolio
- This is a mean-variance solution if

$$\Sigma =$$

$$\mu =$$

- How would you test this strategy?

How to test the $1/N$ strategy?

- Compare naive strategy against 14 other strategies
- How are they going to compare?
  - Out-of-sample Sharpe ratio
  - Certainty-equivalent
  - Turnover
- Use different datasets (see Table 2)

- If mean variance is designed to deliver the highest Sharpe ratio, then how can the $1/N$ strategy outperform?

How to test the $1/N$ strategy?

Strategies (Table 1)

- Naive Portfolio: $1/N$
- Sample-based mean-variance
- Bayesian approach to estimation error
  - diffuse-prior
  - Bayes-Stein shrinkage
  - belief in asset-pricing model (data and model)
- Moment restrictions:
  - Minimum variance
  - value-weighted portfolio
  - asset-pricing model with unobservable factors (mp)
- Shortsale-constrained portfolios
  - Bayes-Stein constrained
  - Mean-variance constrained
  - Minimum-variance constrained
  - Generalized minimum-variance constrained
- Optimal combinations of portfolios (3)

Methodology

- Rolling windows: $M = 60$ or $M = 120$ months
- Measures of performance:
  1. Sharpe ratio
  $$SR_k = \frac{\hat{\mu}_k}{\hat{\sigma}_k}$$
  2. Certainty-equivalent (CEQ)
  $$\hat{\mu}_k - \frac{\gamma}{2}\hat{\sigma}_k^2$$
  3. Turnover
  $$\text{Turnover} = \frac{1}{T-M}\sum_{t=1}^{T-M}\sum_{j=1}^{N}|\hat{w}_{k,j,t+1} - \hat{w}_{k,j,t+}|$$

  Return-loss
  $$\text{return-loss}_k = \frac{\mu_{ew}}{\sigma_{ew}} \times \sigma_k - \mu_k$$

Results: Sharpe Ratios

Strategy	S&P sectors $N = 11$	Industry portfolios $N = 11$	Inter'l portfolios $N = 9$	Mkt/ SMB/HML $N = 3$	FF 1-factor $N = 21$	FF 4-factor $N = 24$
$1/N$	0.1876	0.1353	0.1277	0.2240	0.1623	0.1753
mv (in sample)	0.3848	0.2124	0.2090	0.2851	0.5098	0.5364
mv	0.0794	0.0679	−0.0332	0.2186	0.0128	0.1841
	(0.12)	(0.17)	(0.03)	(0.46)	(0.02)	(0.45)
bs	0.0811	0.0719	−0.0297	0.2536	0.0138	0.1791
	(0.09)	(0.19)	(0.03)	(0.25)	(0.02)	(0.48)
dm ($\sigma_\alpha = 1.0\%$)	0.1410	0.0581	0.0707	0.0016	0.0004	0.2355
	(0.08)	(0.14)	(0.08)	(0.00)	(0.01)	(0.17)
min	0.0820	0.1554	0.1490	0.2493	0.2778	−0.0183
	(0.05)	(0.30)	(0.21)	(0.23)	(0.01)	(0.01)
vw	0.1444	0.1138	0.1239	0.1138	0.1138	0.1138
	(0.09)	(0.01)	(0.43)	(0.00)	(0.01)	(0.00)
mp	0.1863	0.0533	0.0984	−0.0002	0.1238	0.1230
	(0.44)	(0.04)	(0.15)	(0.00)	(0.08)	(0.03)
mv-c	0.0892	0.0678	0.0848	0.1084	0.1977	0.2024
	(0.09)	(0.03)	(0.17)	(0.02)	(0.02)	(0.27)
bs-c	0.1075	0.0819	0.0848	0.1514	0.1955	0.2062
	(0.14)	(0.06)	(0.15)	(0.09)	(0.03)	(0.25)
min-c	0.0834	0.1425	0.1501	0.2493	0.1546	0.3580
	(0.01)	(0.41)	(0.16)	(0.23)	(0.35)	(0.00)
g-min-c	0.1371	0.1451	0.1429	0.2467	0.1615	0.3028
	(0.08)	(0.31)	(0.19)	(0.25)	(0.47)	(0.00)
mv-min	0.0683	0.0772	−0.0353	0.2546	−0.0079	0.1757
	(0.05)	(0.21)	(0.01)	(0.22)	(0.01)	(0.50)
ew-min	0.1208	0.1576	0.1407	0.2503	0.2608	−0.0161
	(0.07)	(0.21)	(0.18)	(0.17)	(0.00)	(0.01)

Results: Turnover

Strategy	S&P sectors $N = 11$	Industry portfolios $N = 11$	Inter'l portfolios $N = 9$	Mkt/ SMB/HML $N = 3$	FF- 1-factor $N = 21$	FF- 4-factor $N = 24$
$1/N$	0.0305	0.0216	0.0293	0.0237	0.0162	0.0198
			Panel A: Relative turnover of each strategy			
mv (in sample)	–	–	–	–	–	–
mv	38.99	606594.36	4475.81	2.83	10466.10	3553.03
bs	22.41	10621.23	1777.22	1.85	11796.47	3417.81
dm ($\sigma_\alpha = 1.0\%$)	1.72	21744.35	60.97	76.30	918.40	32.46
min	6.54	21.65	7.30	1.11	45.47	6.83
vw	0	0	0	0	0	0
mp	1.10	11.98	6.29	59.41	2.39	2.07
mv-c	4.53	7.17	7.23	4.12	17.53	13.82
bs-c	3.64	7.22	6.10	3.65	17.32	13.07
min-c	2.47	2.58	2.27	1.11	3.93	1.76
g-min-c	1.30	1.52	1.47	1.09	1.78	1.70
mv-min	19.82	9927.09	760.57	2.61	4292.16	4857.19
ew-min	4.82	15.66	4.24	1.11	34.10	6.80

Take-away

Mean-variance analysis is useful but has significant drawbacks when it comes to implementation out-of-sample

Mean-variance analysis ignores tail risk (or assumes investors do not care about tail risk)

Alternatives have been proposed.