Quantitative Asset Management

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Outline

1. Persistence in Mutual Fund Performance
   Carhart (1997, JF)
2. Time-varying betas
   Ang and Kristensen (2012, JFE)
3. Time-varying prices of risk
   Adrian, Crump, and Moench (2015, JFE)

Mutual Fund Performance

- Is mutual fund performance persistent?
- How to measure persistence?

Mutual Fund Performance

- What can explain persistence in mutual fund performance?

Mutual Fund Performance Persistence

- Carhart investigated persistence mutual fund performance
- Persistence is the key!
- Does previous performance predict alpha next year?

  If it does not, then there is no point in chasing alpha!
- Sort mutual funds by performance in the past 12 months into 10 bins
  - Form equal-weighted portfolios of mutual funds
  - What should we calculate for each portfolio? Why?
  - What are the main findings?

Portfolios of Mutual Funds

Equal-weighted Portfolios sorted on lagged one-year returns from 1963-1993. Table III.

Portfolio	Returns	*CAPM* α	market	Carhart α	market	SMB	HML	MOM
1A	0.75	0.27	1.08	-0.11	0.91	0.72	-0.07	0.33
1B	0.67	0.22	1.00	-0.10	0.86	0.59	-0.05	0.27
1C	0.63	0.17	1.02	-0.15	0.89	0.56	-0.05	0.27
1	0.68	0.22	1.03	-0.12	0.88	0.62	-0.05	0.29
10	0.01	-0.45	1.02	-0.40	0.93	0.32	-0.08	-0.09
10A	0.25	-0.19	1.00	-0.19	0.91	0.33	-0.11	-0.02
10B	0.02	-0.42	1.00	-0.37	0.91	0.32	-0.09	-0.09
10C	-0.25	-0.74	1.05	-0.64	0.98	0.32	-0.04	-0.17

Persistence in Performance

- There is still unexplained persistence: nearly 30bps
- Most from worst performing funds
- What else can explain the remaining persistence?
- Managers' claim:

  > 'expense and turnover do not reduce performance, since investor are paying for the quality of the manager's information, and because managers trade only to increase expected returns net of transaction costs"

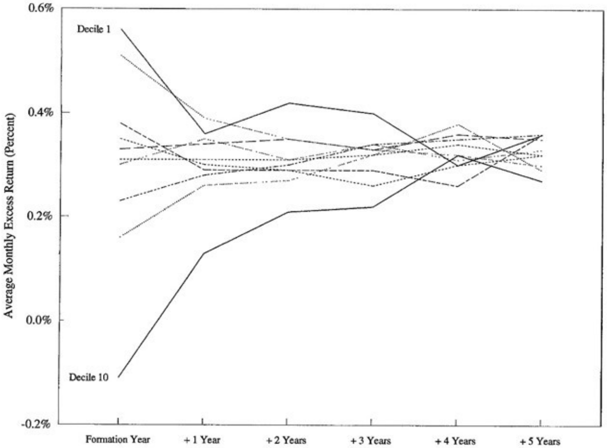
- Is this consist with the data according to Carhart (1997)?

Persistence in Performance

- Evidence for persistence in (out-)performance by mutual funds is not very compelling
- Once you remove the momentum effects, most of it gone
- There is strong evidence of persistent underperformance by some mutual funds:
  - True negative alpha is a also puzzle because you should not be able to consistently underperform the market, since another could short your strategy! (see Cochrane, 2013).

Persistence?

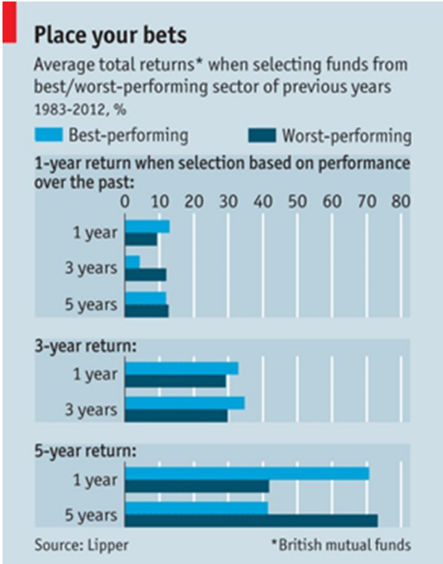
Figure 5



Post-formation portfolio returns. Source: Carhart (1997)

Momentum and mean-reversion in mutual funds

- Sort mutual funds by worst/best performing sectors
  - Past year
  - 3 years
  - 5 years
- Outperformance over the next year of the best funds
  - Momentum effect
- Underperformance over the next 5 years of the best funds
  - Mean-reversion effect
  - See e.g. The best, the worst and the ugly in The Economist (2012)


Place your bets
Average total returns* when selecting funds from best/worst-performing sector of previous years
1983-2012, %
Best-performing
Worst-performing
1-year return when selection based on performance over the past:
0
10
20
30
40
50
60
70
80
1 year
3 years
5 years
3-year return:
1 year
3 years
5-year return:
1 year
5 years
Source: Lipper
*British mutual funds

Looking for Skill by Bootstrapping

- An alternative to looking for persistence as evidence of skill is to compare the distribution of alpha in the data to the distribution we would see if true alpha is zero and all of the measured alpha is due to luck
  - Fama and French (2010, JF) simulate 10,000 bootstrap simulations for the cross-section of mutual funds under the null that true alpha is zero (same properties as actual fund returns except that alpha is set to zero)
- After fees (on net returns), the simulated data look a lot like the actual data: if there are managers skilled enough to cover costs, this gets completely undone by the mass of unskilled managers who can't cover costs

Time-varying betas

Time-varying betas

Ang and Kristensen (2012)

- Asset's expected return should be zero after controlling for asset's systematic factor exposure
- However, we typically assume constant loadings
- Betas change over time!
- We have done some time-vary betas:

  Remember Lecture 4 on volatility-based strategies?
- Ang and Kristensen (2012)
  - Methodology to estimate time-vary alphas and betas
  - Distinction between long-run and short-run estimates
  - Kernel-weighted ordinary least squares
  - Asymptotic distributions

Time-varying betas

Conditional Factor Model

Return of stock k at time t_i:

$$R_{k,i} = \alpha_k(t_i) + \beta_k(t_i)' f_i + \omega_{kk}(t_i) z_{k,i}$$

In matrix notation:

$$R_i = \alpha(t_i) + \beta(t_i)' f_i + \Omega^{1/2}(t_i) z_i$$

where

- R_i is M by 1
- $\alpha(t_i)$ is M by 1
- $\beta(t_i)$ is J by M
- f_i is J by 1
- $\Omega(t_i)$ is M by M
- z_i is M by 1: $\mathbb{E}_i[z_i] = 0$ and $\mathbb{E}_i[z_i z_i'] = I_M$

Time-varying betas

Conditional Factor Model

We want to estimate:

$$\alpha(t_i)$$

and

$$\beta(t_i)$$

Time-varying betas

Conditional Factor Model

Definition of long-run alphas and betas

$$\alpha_{LR,k} \equiv \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \alpha_k(t_i)$$

and

$$\beta_{LR,k} \equiv \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \beta_k(t_i)$$

Remember that: $0 < t_1 < t_2 < \ldots < t_n < T$

Time-varying betas

Conditional estimators

Definition of time-varying alphas an betas

$$\left[\alpha(t_i), \beta(t_i)'\right]' = \Lambda^{-1}(t_i)\mathbb{E}_i\left[X_i R_i'\right]$$

where

$$X_i = [1, f_i']'$$

and

$$\Lambda(t_i) = \mathbb{E}_i\left[X_i X_i'\right]$$

This is an OLS conditional on information available at time t_i

Time-varying betas

Conditional estimators

Local least squares estimator for any time $0 \leq t \leq T$:

$$\left[\hat{\alpha}(t), \hat{\beta}(t)'\right]' = \arg\min_{(\alpha,\beta)} \sum_{i=1}^{n} K_{h_k T}(t_i - t)\left(R_{k,i} - \alpha - \beta' f_i\right)^2$$

where

$$K_{h_k T}(z) = \frac{K\left(\frac{z}{h_k T}\right)}{h_k T}$$

$K(\cdot)$ is a kernel and $h_k > 0$ is a bandwidth

Intuition: kernel defines which obs. are more 'important'

- What happen is $K(z)$ higher around $z = 0$?
- What happen is h_k is large or too small?

Time-varying betas
Conditional estimators

Optimal kernel-weighted least squares:

$$\left[\hat{\alpha}(t), \hat{\beta}(t)'\right]' = \left[\sum_{i=1}^{n} K_{h_k T}(t_i - t) X_i X_i'\right]^{-1} \left[\sum_{i=1}^{n} K_{h_k T}(t_i - t) X_i R_{k,i}\right]$$

and they use a Gaussian density as kernel

$$K(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$$

How to optimally choose the bandwidth?

Time-varying betas

Data generating process

Let $\Delta = t_i - t_{i-1}$. Discrete version of continuous-time model:

$$\Delta s_i = \alpha(t_i) + \beta(t_i)'\Delta F_i + \Sigma^{1/2}(t_i)\sqrt{\Delta} z_i$$

where Δs_i is log change in prices excess of the risk-free asset

$$\Delta F_i = \mu_F(t_i)\Delta + \Lambda_{FF}^{1/2}\sqrt{\Delta} u_i$$

Estimators:

$$\mathbb{E}\left[\hat{\beta}(t)\right] \simeq \beta(t) + (hT)^2 \beta^{(2)}(t)$$

$$\mathrm{Var}\left(\hat{\beta}(t)\right) \simeq \frac{1}{nh}\kappa_2 \Lambda_{FF}^{-1}(t) \otimes \Sigma(t)$$

where h is a common bandwidth

- Trade-off between bias and variance

Time-varying betas

Theorem 1: betas

Asymptotics :

$$\sqrt{nh}\left(\hat{\beta}(t) - \beta(t)\right) \sim N\left(0, \kappa_2 \Lambda_{FF}^{-1}(t) \otimes \Sigma(t)\right)$$

Sample equivalents:

$$\hat{\Lambda}_{FF}^{-1}(t) \text{ and } \hat{\Sigma}(t)$$

Time-varying betas

Alphas

Estimators:

$$\mathbb{E}\left[\hat{\alpha}(t)\right] \simeq \alpha(t) + (hT)^2 \alpha^{(2)}(t)$$
$$\mathrm{Var}\left(\hat{\alpha}(t)\right) \simeq \frac{1}{Th} \kappa_2 \Sigma(t)$$

No Asymptotics! As $Th \to 0$, the variance explodes..

- Informally:

  $$\sqrt{nh}\left(\hat{\alpha}(t) - \alpha(t)\right) \sim N\left(0, \kappa_2 \Sigma(t)\right)$$

  in large samples
- Solution: normalization assumption

$$\alpha(t) = a(t/T) \text{ and } \Sigma(t) = S(t/T)$$

Time-varying betas

Long-run alphas and betas

Estimators:

$$\hat{\alpha}_{LR,k} = \frac{1}{n}\sum_{i=1}^{n}\hat{\alpha}_k(t_i)$$

$$\hat{\beta}_{LR,k} = \frac{1}{n}\sum_{i=1}^{n}\hat{\beta}_k(t_i)$$

Theorem 2:

$$\sqrt{T}\left(\hat{\alpha}_{LR} - \alpha_{LR}\right) \sim N(0, \Sigma_{LR,\alpha\alpha})$$

$$\sqrt{n}\left(\hat{\beta}_{LR} - \beta_{LR}\right) \sim N(0, \Sigma_{LR,\beta\beta})$$

Sample equivalents:

$$\hat{\Sigma}_{LR,\alpha\alpha} \text{ and } \hat{\Sigma}_{LR,\beta\beta}$$

Now we can test if alphas are zero!

Time-varying betas

Long-run alphas and betas

- We can test if long-run alphas are zero!

  Wald-type of test
- We can also test if alphas and betas are constant over time

  Theorem 3
- But we still need to choose the bandwidth..

Time-varying betas

Bandwidth

How to choose the optimal bandwidth?

$$h^*_{\beta,k} \qquad \text{and} \qquad h^*_{\alpha,k}$$

Two-step method proposed

- Step 1:
  - Assume variance-covariances matrices to be constant,
  - alphas and betas to be polynomials,
  - estimate polynomial parameters, and
  - compute the 'optimal' bandwidth
- Step 2
  - Use the bandwidth from step 1, and compute time-varying betas and alphas
  - Calculate another bandwidth: second-pass bandwidth
  - LR bandwidth is further scaled down

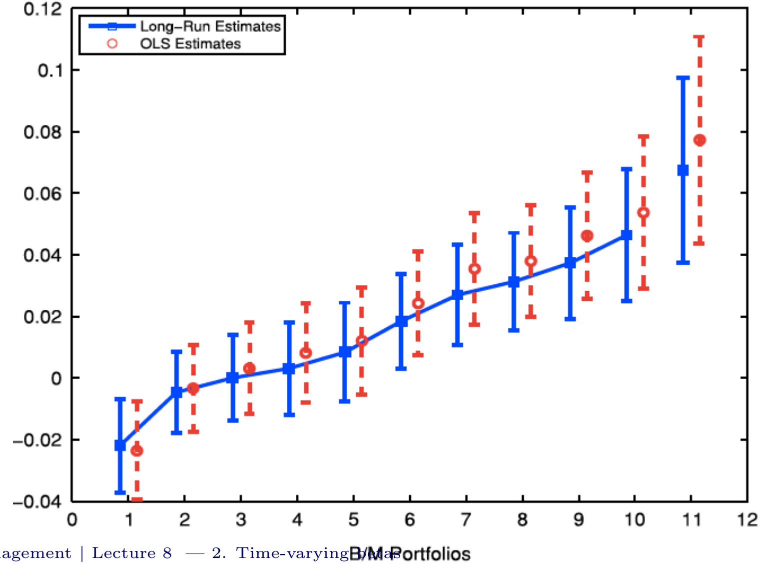
Time-varying betas

Data

- Look at BM sorted portfolios
- Alphas do not change much over time
- but Betas change over time

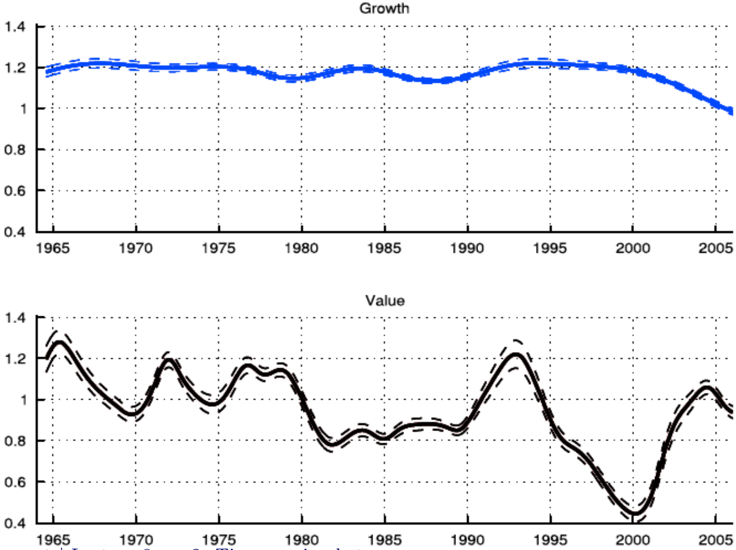
Time-varying betas

Fig 1: Long-run alphas vs. OLS alphas in BM-sorted portfolios



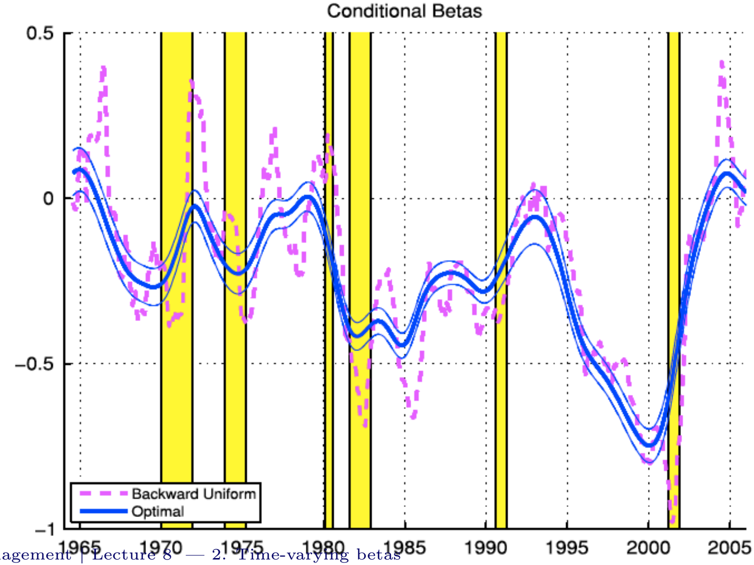
Time-varying betas

Fig 2: Time-varying betas for growth and value stocks



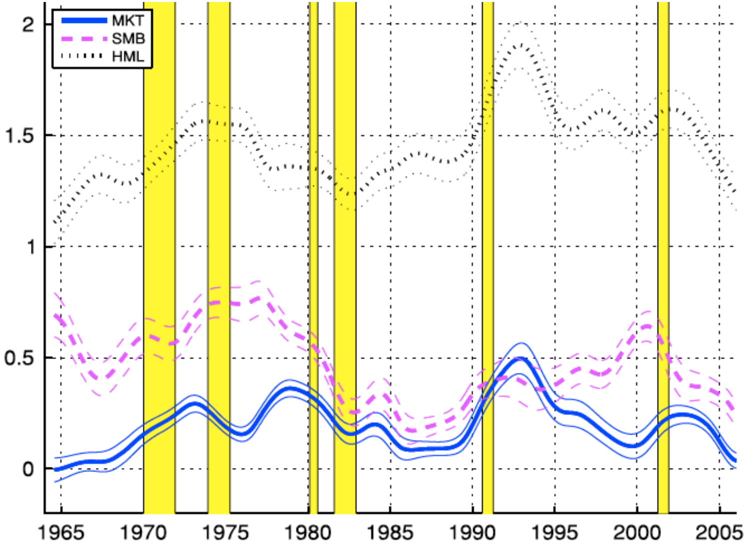
Time-varying betas

Fig 3: Time-varying betas for value strategy



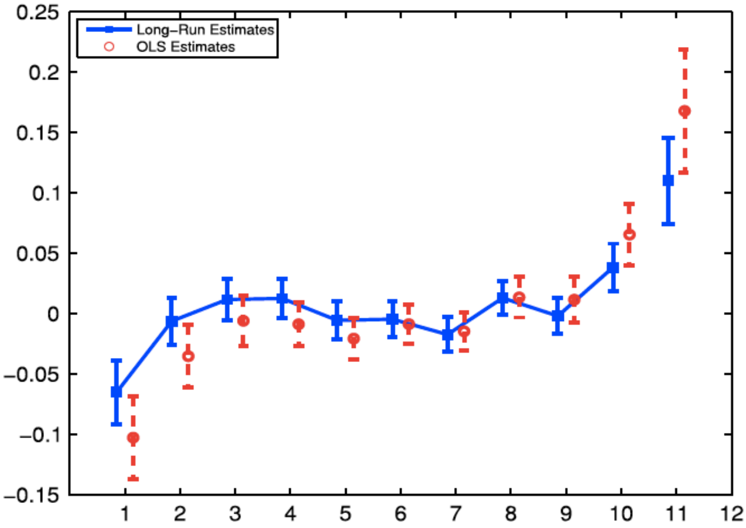
Time-varying betas

Fig 4: Time-varying FF loadings for value strategy



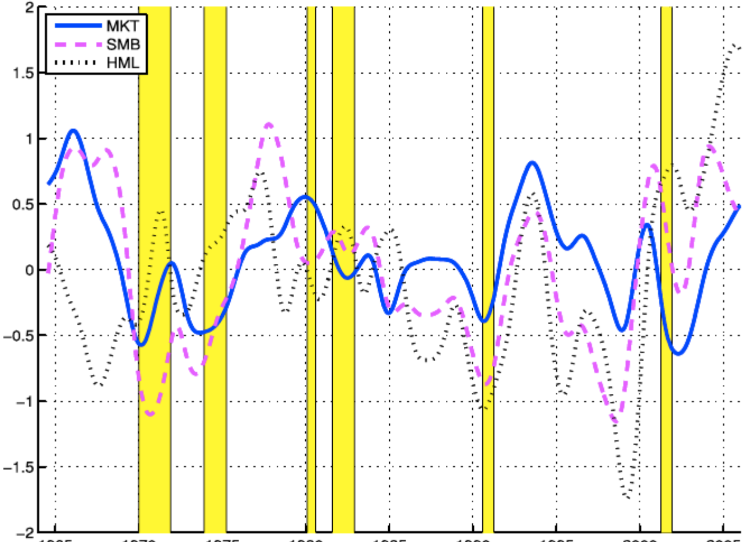
Time-varying betas

Fig 5: Long-run alphas vs. OLS alphas in momentum-sorted portfolios



Time-varying betas

Fig 6: Time-varying FF loadings for momentum strategy



Additional Material: Time-varying prices of risk

Time-varying prices of risk: Set-up

- K state variables following an vector autoregressive process

$$X_{t+1} = \mu + \Phi X_t + v_{t+1}, \quad t = 0, \ldots, T-1,$$

- Partition the state variables into three categories:
  - risk factor only: $X_{1,t}$
  - risk and price of risk factor: $X_{2,t}$
  - price of risk factor only: $X_{3,t}$

$$C_t = \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix}, \quad F_t = \begin{bmatrix} X_{2,t} \\ X_{3,t} \end{bmatrix}, \quad u_t = \begin{bmatrix} v_{1,t} \\ v_{2,t} \end{bmatrix}$$

- Assumptions:

$$\mathbb{E}\left[v_{t+1} \middle| \mathcal{F}_t\right] = 0, \quad \mathbb{V}\left[v_{t+1} \middle| \mathcal{F}_t\right] = \Sigma_{v,t}$$

Time-varying prices of risk: Set-up

- Using excess returns,
- Assume existence of a pricing kernel:

$$\mathbb{E}\left[M_{t+1}R_{i,t+1}\middle|\mathcal{F}_t\right]=0$$

This is a very general assumption

see Cochrane's book (Asset Pricing, chapters 4 and 6)

- Assume linear pricing kernel:

$$\frac{M_{t+1}-\mathbb{E}\left[M_{t+1}\middle|\mathcal{F}_t\right]}{\mathbb{E}\left[M_{t+1}\middle|\mathcal{F}_t\right]}=-\lambda_t'\Sigma_{u,t}^{-1/2}u_{t+1}$$

- Assume affine prices of risk:

$$\lambda_t=\Sigma_{u,t}^{-1/2}\left(\lambda_0+\Lambda_1F_t\right)$$

Time-varying prices of risk: Set-up

- All this implies

$$
\begin{aligned}
\mathbb{E}\left[R_{i,t+1}|\mathcal{F}_t\right] &= -\frac{\mathbb{C}[M_{t+1}, R_{i,t+1}|\mathcal{F}_t]}{\mathbb{E}\left[M_{t+1}|\mathcal{F}_t\right]} \\
&= \lambda_t' \Sigma_{u,t}^{-1/2} \mathbb{C}\left[u_{t+1}, R_{i,t+1}|\mathcal{F}_t\right] \\
&= (\lambda_0 + \Lambda_1 F_t)' \Sigma_{u,t}^{-1} \mathbb{C}\left[C_{t+1}, R_{i,t+1}|\mathcal{F}_t\right]
\end{aligned}
$$

- Some re-writing

$$
\mathbb{E}\left[R_{i,t+1}|\mathcal{F}_t\right] = \beta_{i,t}' (\lambda_0 + \Lambda_1 F_t)
$$

where

$$
\beta_{i,t} = \Sigma_{u,t}^{-1} \mathbb{C}\left[C_{t+1}, R_{i,t+1}|\mathcal{F}_t\right]
$$

Time-varying prices of risk: Set-up

- Hence, we can write realized returns as

$$R_{i,t+1} = \beta'_{i,t}(\lambda_0 + \Lambda_1 F_t) + (R_{i,t+1} - \mathbb{E}[R_{i,t+1}|\mathcal{F}_t])$$

  where returns can be decomposed as follows

$$R_{i,t+1} - \mathbb{E}[R_{i,t+1}|\mathcal{F}_t] = \gamma'_{i,t}(C_{t+1} - \mathbb{E}[C_{t+1}|\mathcal{F}_t]) + e_{i,t+1} = \gamma'_{i,t} u_{t+1} + e_{i,t+1}.$$

  $e_{i,t+1}$ is a cond. orthogonal to innovations in C_{t+1}, and

$$\gamma_{i,t} = \Sigma_{u,t}^{-1} \mathbb{C}[C_{t+1}, R_{i,t+1}|\mathcal{F}_t] = \beta_{i,t}$$

- Finally

$$R_{i,t+1} = \beta'_{i,t}(\lambda_0 + \Lambda_1 F_t) + \beta'_{i,t} u_{t+1} + e_{i,t+1}$$

Time-varying prices of risk: Constant betas

- Constant betas (matrix notation)

$$R = B\lambda_0 \iota_T' + B\Lambda_1 F_- + BU + E$$
$$X = \mu + \Phi X_- + V$$

- We can write more generally as

$$R = A_0 \iota_T' + A_1 F_- + BU + E = A\tilde{Z} + E,$$

where $\tilde{Z} = [\iota_T \mid F_-' \mid U']'$ and

$$A_0 = B\lambda_0, \quad A_1 = B\Lambda_1, \quad A = [A_0 \mid A_1 \mid B]$$

- For some positive definite weighting matrix, we have

$$\lambda_0 = (B'WB)^{-1} B'WA_0, \quad \Lambda_1 = (B'WB)^{-1} B'WA_1$$

and the regression counterpart is

$$\hat{\lambda}_{0,\text{ols}} = \left(\hat{B}_{\text{ols}}'\hat{B}_{\text{ols}}\right)^{-1} \hat{B}_{\text{ols}}'\hat{A}_{0,\text{ols}}, \quad \hat{\Lambda}_{1,\text{ols}} = \left(\hat{B}_{\text{ols}}'\hat{B}_{\text{ols}}\right)^{-1} \hat{B}_{\text{ols}}'\hat{A}_{1,\text{ols}}$$

Time-varying prices of risk: Constant betas

Theorem 1

- Asymptotic distribution (ols) at $T \to \infty$

$$\sqrt{T}\,\text{vec}\left(\hat{\Lambda}_{\text{ols}} - \Lambda\right) \xrightarrow{d} \mathcal{N}(0, \mathcal{V}_\Lambda)$$

where

$$\mathcal{V}_\Lambda = \left(\Upsilon_{FF}^{-1} \otimes \Sigma_u\right) + \mathcal{H}_\Lambda(B,\Lambda)\mathcal{V}_{\text{rob}}\mathcal{H}_\Lambda(B,\Lambda)',$$

$$\Upsilon_{FF} = \text{p}\lim_{T\to\infty} \tilde{F}_{-}\tilde{F}'_{-}/(T-1),\ \tilde{F}_{-} = \left[\iota_T \,\vdots\, F'_{-}\right]',\ \textit{and}$$

$$\mathcal{H}_\Lambda(B,\Lambda) = \left[\left(I_{(K_F+1)} \otimes (B'B)^{-1}B'\right) \,\vdots\, -\left(\Lambda' \otimes (B'B)^{-1}B'\right)\right]$$

Time-varying prices of risk: empirical application

- Test assets:
  - Ten-size sorted portfolios
  - Seven Constant maturity treasury portfolios:

    1, 2, 5, 7, 10, 20, 30 years
- Risk factors (C):
  - Market portfolio (CRSP)
  - SMB factors
  - TSY10: constant maturity ten-year Treasury yield
- Prices of risk factors (F):
  - TSY10: constant maturity ten-year Treasury yield
  - TERM: term spread between ten-year vs. three-month Tbill
  - DY: dividend yield on the S&P 500

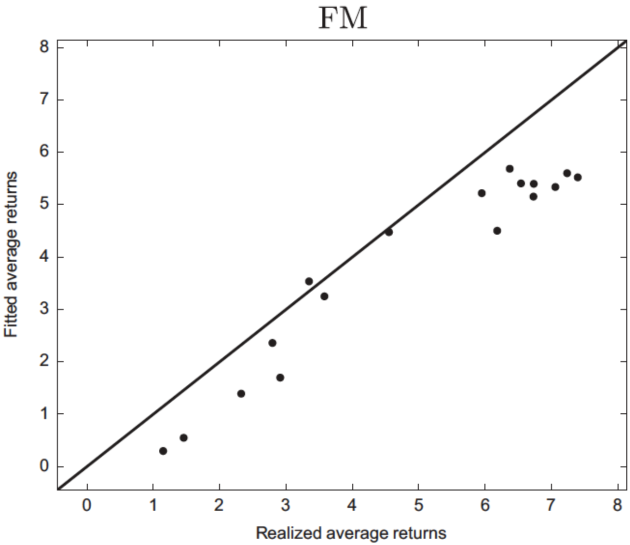
Time-varying prices of risk: empirical application

Table 2

	λ_0	TSY10	TERM	DY	$\bar{\lambda}$
Panel A: Time-varying betas					
MKT	0.062***	−0.184***	0.302***	0.014***	6.797**
	(0.017)	(0.058)	(0.088)	(0.004)	(2.785)
SMB	0.054***	−0.194***	0.099	0.011***	3.565
	(0.013)	(0.044)	(0.066)	(0.003)	(2.690)
TSY10	0.004***	−0.014***	−0.046***	0.001**	−0.359
	(0.001)	(0.005)	(0.007)	(0.000)	(0.229)
Panel B: Constant betas (OLS)					
MKT	0.063**	−0.187*	0.301**	0.014**	6.067***
	(0.028)	(0.098)	(0.147)	(0.006)	(1.487)
SMB	0.054**	−0.192***	0.093	0.011**	3.023
	(0.022)	(0.073)	(0.108)	(0.005)	(2.336)
TSY10	0.004	−0.013	−0.050***	0.001	−0.386***
	(0.002)	(0.008)	(0.012)	(0.001)	(0.085)

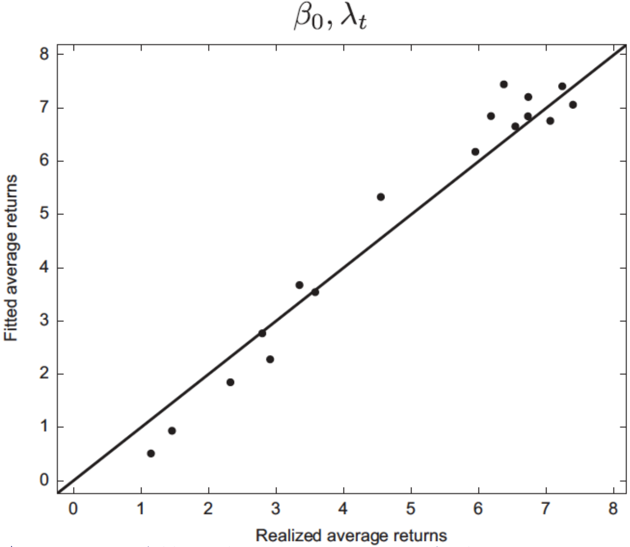
Time-varying prices of risk: empirical application

Figure 2



Time-varying prices of risk: empirical application

Figure 2



Time-varying prices of risk: empirical application

Figure 2