Quantitative Asset Management

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Asness, Frazzini, and Pederson (2012)

Leverage aversion and risk parity.

Connection with MVE portfolios

Minimum-variance portfolios: mean-variance efficient portfolios under the assumption that all expected returns are equal.

Risk-parity portfolios: mean-variance efficient portfolios assuming that all assets are uncorrelated and have the same expected return.

Equal-weighted portfolios: mean-variance efficient portfolios under the assumption that all expected returns, variances and co-variances are equal across assets.

Traditional Asset Allocation

- ▶ Mean-variance analysis puts a lot of stock in our ability to accurately measure all of the variances, co-variances of returns as well as the expected returns
  - ▶ Results are very sensitive to small changes in assumptions
  - ▶ Capital market assumption better be right!
    - See AQR's 2022 Capital Market Assumptions for Major Asset Classes
- ▶ Risk Parity Investing:
  - ▶ Takes a more agnostic approach.
  - ▶ Puts changes in volatility front and center

Traditional Asset Allocation

- ▶ Mean-variance analysis prescribes a 60/40 equity/bonds allocation
- ▶ However, most of the variation in returns is driven by the equity component, simply because stocks are much more volatile.
- ▶ So, at the end of the day, diversification gains are limited..
- ▶ Novel approach tries to avoid this: risk parity investing

What's going on?

- ▶ Lots of reasons for the empirical failure of mean-variance analysis (see previous lecture)
- ▶ In addition, high volatility assets have tended to underperform low volatility assets
  - ▶ That explains why the minimum variance portfolio typically does very well!
- ▶ Risk parity investing will exploit this fact

Risk Parity Investing

- ▶ Try to equalize the risk contribution of all asset categories
- ▶ Now we're diversified in terms of risk (rather than in terms of dollar investments)
- ▶ We'll invest more than 40% in bonds because bonds have much lower volatility than equities
- ▶ Risk is balanced across asset classes, but..
- ▶ Average return is lower
- ▶ Investors use leverage to increase the average return

Constructing Risk-parity portfolios

- ▶ "Risk Parity" is a portfolio that targets equal risk allocation across the available instruments

- ▶ To construct a risk-parity portfolio, we estimate volatilities of all the available asset classes and set the portfolio weight equal to:

$$w_{i,t} = k_t \sigma_{i,t}^{-1}, \quad i = 1, \dots, n$$

  - ▶ If we want an unlevered portfolio, we choose $k_t = 1/\sum_i \sigma_{i,t}^{-1}$
  - ▶ If we want a levered portfolio, we simply choose a constant k over time; delivers constant volatility in each asset class

- ▶ Rebalancing is monthly

Two assets

- ► Example with two assets:
    - ► $w_{1,t} = k_t \sigma_{1,t}^{-1}$
    - ► $w_{2,t} = k_t \sigma_{2,t}^{-1}$
    - ► In case without leverage, we set $k_t = \frac{1}{\sigma_{1,t}^{-1} + \sigma_{2,t}^{-1}}$
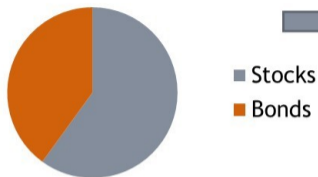   - ► In case with leverage, we choose constant k and you short the risk-free $(w_{1,t} + w_{2,t} - 1)$

Levered Risk Parity Portfolio

► At the end of each calendar month, we set the portfolio weight in each asset class equal to the inverse of its volatility, estimated using 3-year monthly excess returns up to month $t - 1$, and

► these weights are multiplied by a constant to match the ex-post realized volatility of the Value-Weighted benchmark.

Traditional Portfolios

- ► "Value-Weighted Portfolio" is a market portfolio weighted by total market capitalization and rebalanced monthly to maintain value weights.
- ► "60-40" is a portfolio that allocates 60% in stocks and 40% in bonds, rebalanced monthly to maintain constant weights.

Traditional Portfolio Allocation → **Risk Allocation**
- Stocks
- Bonds

Risk Parity Portfolio Allocation → **Risk Allocation**
- Stocks
- Bonds

Risk Parity Performance

	Excess Return	t-Stat. of Excess Return	Alpha	t-Stat. of Alpha	Volatility	Sharpe Ratio	Skewness	Excess Kurtosis
A. Long sample (U.S. stocks and bonds, 1926–2010)								
CRSP stocks	6.71%*	3.18			19.05%	0.35	0.18	7.51
CRSP bonds	1.56*	4.28			3.28	0.47	−0.01	4.37
Value-weighted portfolio	3.84*	2.30			15.08	0.25	0.37	13.09
60/40 portfolio	4.65*	3.59			11.68	0.40	0.20	7.46
RP, unlevered	2.20*	4.67	1.39%*	4.44	4.25	0.52	0.05	4.58
RP	7.99*	4.78	5.50*	4.30	15.08	0.53	−0.36	1.92
RP minus value-weighted	4.15*	2.95	5.50*	4.30	12.69	0.33	−0.79	8.30
RP minus 60/40	3.34*	2.93	3.76*	3.33	10.31	0.32	−0.61	5.04

Sample: 1926-2010

Risk Parity Performance: including recent sample

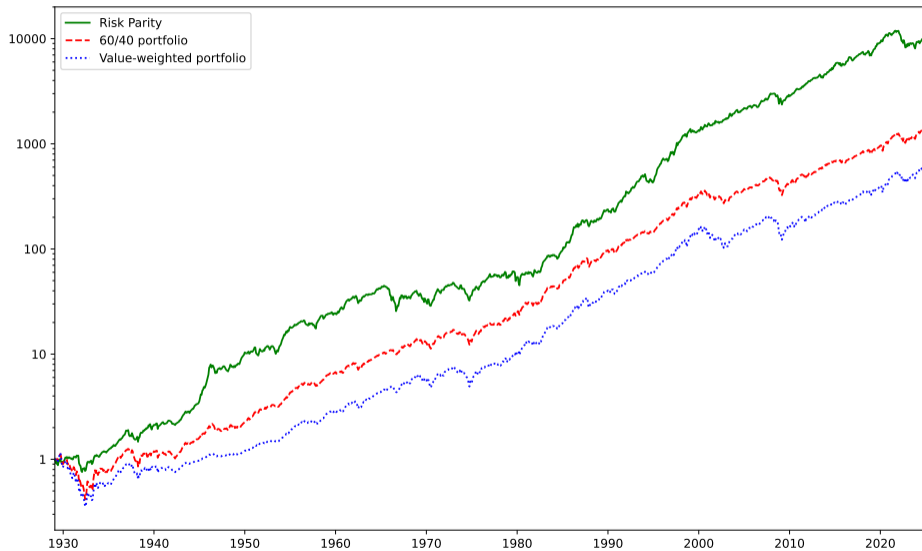|	Excess Return	t-stat	Volatility	Sharpe Ratio	Skewness	Excess Kurtosis
CRSP stocks	7.57	4.00	18.56	0.41	0.16	7.66
CRSP bonds	1.19	3.99	2.92	0.41	0.17	3.50
Value-weighted portfolio	4.23	3.51	11.83	0.36	-0.57	4.36
60/40 portfolio	5.02	4.34	11.32	0.44	0.18	7.41
RP, unlevered	2.06	5.51	3.66	0.56	0.00	2.58
RP	6.77	5.61	11.83	0.57	-0.40	2.06
RP minus value-weighted	2.54	2.72	9.12	0.28	0.18	3.57
RP minus 60/40	1.75	1.99	8.64	0.20	-0.57	6.31

Sample from 01/31/1929 to 12/31/2024

Risk Parity Performance: ONLY recent sample

	Excess Return	t-stat	Volatility	Sharpe Ratio	Skewness	Excess Kurtosis
CRSP stocks	6.59	2.08	15.86	0.42	-0.55	1.04
CRSP bonds	1.06	1.70	3.13	0.34	-0.11	0.58
Value-weighted portfolio	4.45	1.89	11.74	0.38	-0.68	1.14
60/40 portfolio	4.38	2.31	9.46	0.46	-0.52	1.05
RP, unlevered	2.04	2.99	3.41	0.60	-0.55	1.99
RP	7.62	3.25	11.74	0.65	-0.61	1.28
RP minus value-weighted	3.18	1.63	9.78	0.33	0.42	1.24
RP minus 60/40	3.25	1.92	8.46	0.38	0.31	0.91

Sample from 01/31/2000 to 12/31/2024

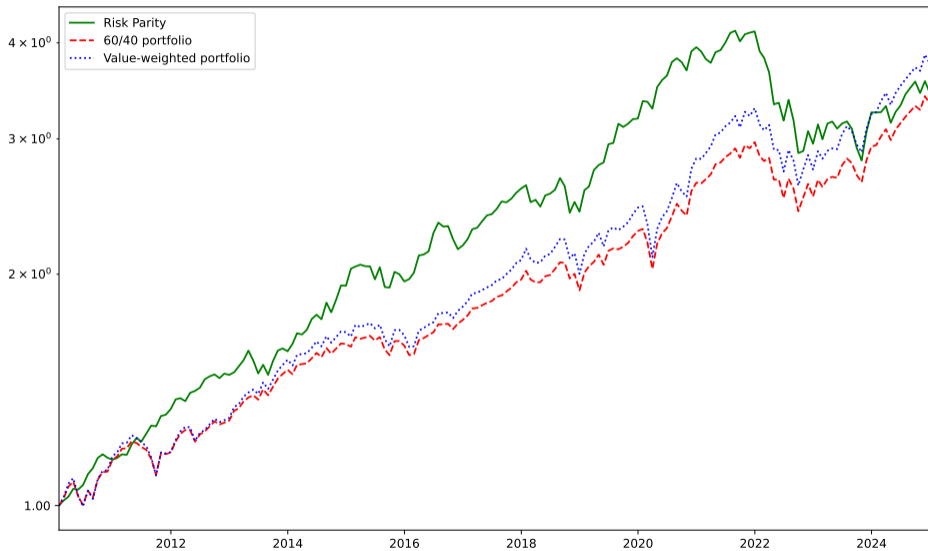# Risk Parity Performance: ONLY recent sample

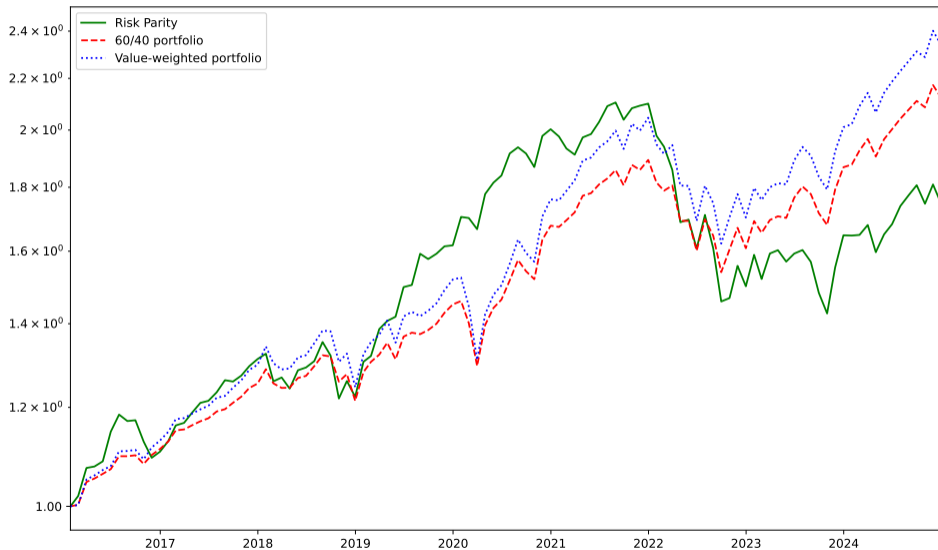|	Excess Return	t-stat	Volatility	Sharpe Ratio	Skewness	Excess Kurtosis
CRSP stocks	11.66	3.03	14.88	0.78	-0.36	0.88
CRSP bonds	0.50	0.58	3.30	0.15	-0.07	0.47
Value-weighted portfolio	8.11	3.01	10.44	0.78	-0.41	0.79
60/40 portfolio	7.19	3.09	9.03	0.80	-0.32	0.79
RP, unlevered	2.29	2.40	3.69	0.62	-0.57	2.00
RP	7.69	2.85	10.44	0.74	-0.55	0.86
RP minus value-weighted	-0.43	-0.21	8.02	-0.05	0.67	0.79
RP minus 60/40	0.49	0.26	7.21	0.07	0.51	0.37

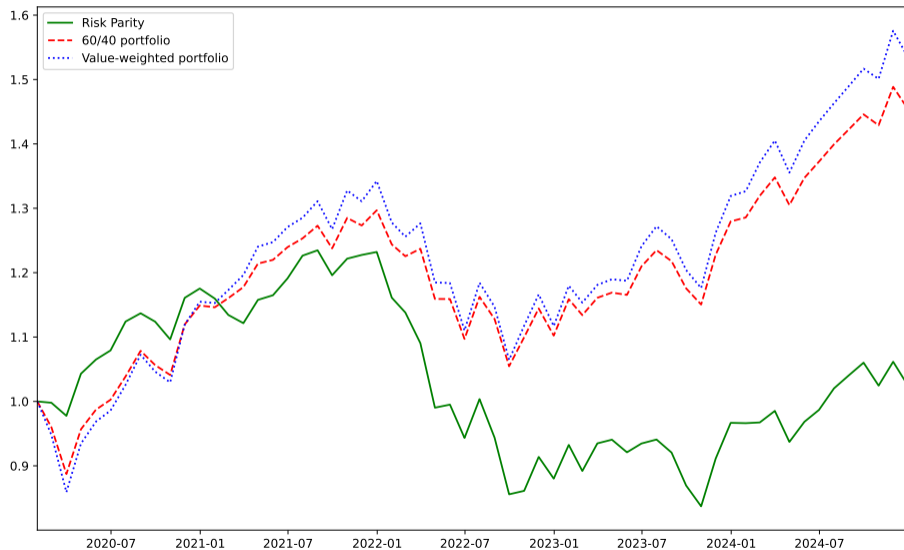Sample from 01/31/2010 to 12/31/2024

Replication
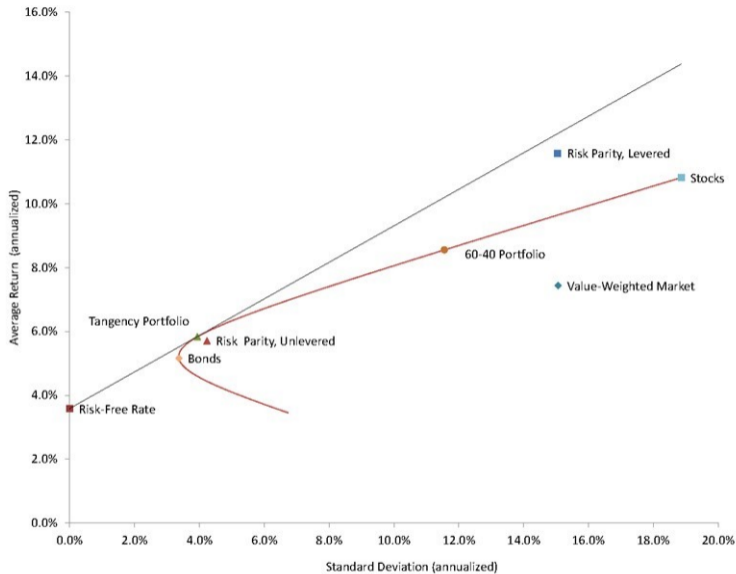
Replication post 2010
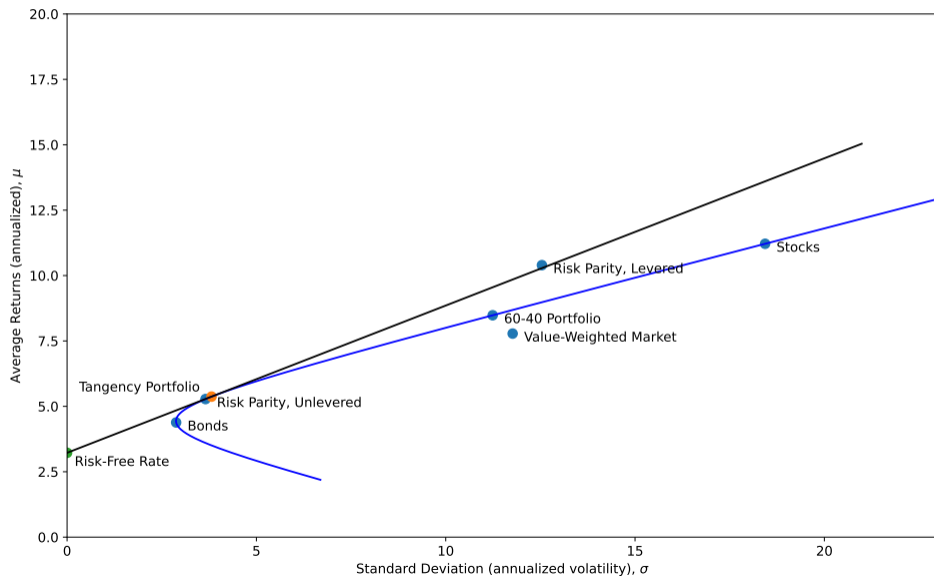
Replication post 2016

Replication post 2020

Sample: 1926-2010 (paper sample)

Replication (recent data included)

Rationale

- Nothing wrong with large risk exposure to equities, unless of course per unit risk compensation is too low in the stock market, relative to what we see in other asset markets (such as the bond market!)
    - Low SR's in stock market
    - Higher SR's in bond market
    - Explains why the value-weighted market portfolio is far away from the tangency point
        - Market is not Mean-Variance Efficient
- That's the rationale behind risk-parity investing

What's behind this?

Leverage constrains

- ► Some investors cannot move up (northeast) along the CML (capital market line) because they are leverage constrained

- ► Instead, leverage-constrained investors will overweight riskier assets (substitute for leverage), driving up the price of riskier assets

- ► This invalidates the CAPM, which assumes everyone invests on the CML in the mean-volatility diagram

- ► Safer assets are underweighted by leverage-constrained investors and are underpriced as a result

- ► The mean-variance efficient portfolio will overweight safer assets

Leverage-constrained investors?

- ► Evidence of Leverage-constrained investors
  - ► Some investors are not allowed to borrow:
    - ► Mutual funds
    - ► Pension funds
  - ► Mutual funds provide asset allocations for low-to-high-risk-tolerant investors
    - ► High-risk-tolerant investors are told to concentrate in equities
  - ► Embedded leverage in ETFs

Caveats (1)

- ► RP does very well in sample of secularly declining long-term interest rates 1980-2012
- ► RP completely ignores valuations; only looks at volatility
  - ► Volatility tends to be low when prices are high
  - ► Pro-cyclical bias in investing.

Caveats (2)

- ► Lots of leverage required
- ► Different investment technology
  - ► Risks associated with high leverage investing
    - ► LTCM (Long Term Capital Management) is a great example of the risks posed by highly levered portfolios during financial crises.
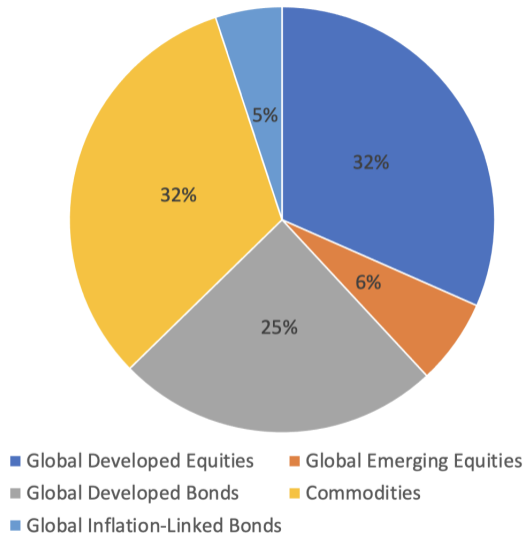   - ► This risk is not reflected in standard volatility measures.

Leverage requires a different technology: 'Getting leverage requires getting financing, using derivatives, and establishing counterparty relations' Asness, Frazzini and Pedersen (2012, Financial Analysts Journal)

- ► We ignored the actual costs associated with leverage in the exercise
  - ► Financing spreads
  - ► Costs of deleveraging

AQR Risk Parity Fund

- On Dec. 2017
  - Net Asset Value: $ 478,472,753
  - Total Exposure: $ 1,535,088,754
  - Implied Leverage: 3.21
- On Nov. 2018 (Bloomberg article: link)
  - Net Asset Value: approx. $ 344M
  - Big losses and redemption in recent years...
- On April 2020 (from AQR website)
  - Net Asset Value: $ 130 MM
  - Can-Bridgewater, AQR Risk Parity Funds Hang in There?
- On April 2021 (from AQR website)
  - Net Asset Value (AQRIX): $ 132 MM
  - Net Asset Value (QRMIX):$86MM
- 2022: https://funds.aqr.com/fund-finder?assetclass=Multi-Asset

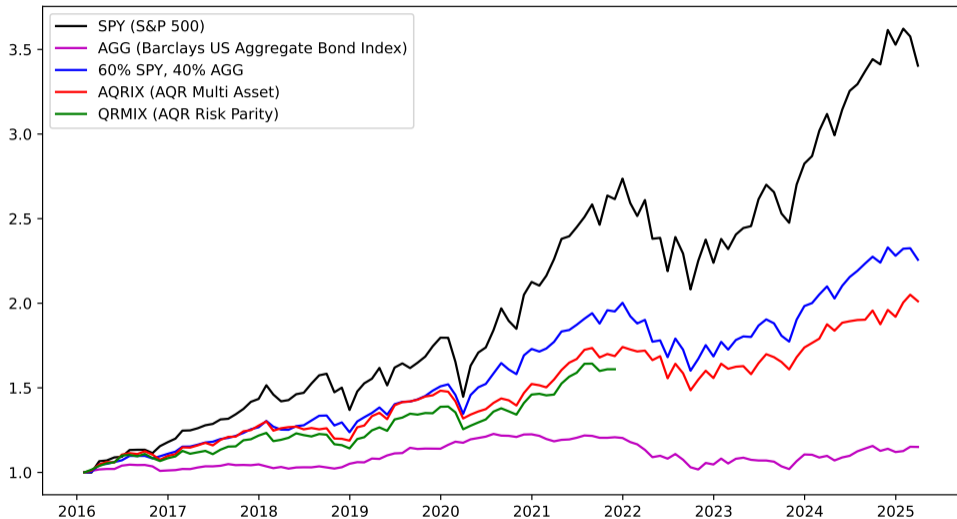# Exposure Allocation (QRMIX in Dec 2020)
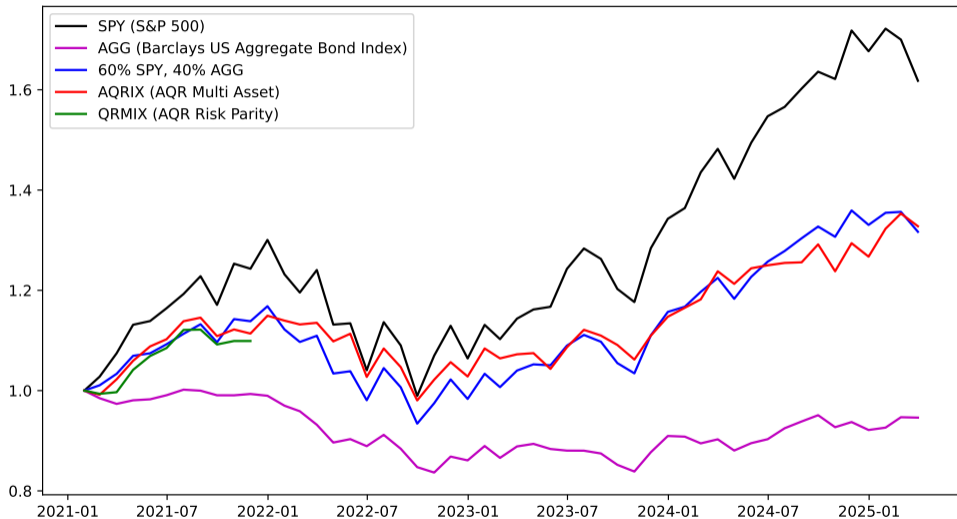

Source: AQR Risk parity Fund

AQR Risk Parity Fund Derivatives positions

- ► Equity: futures
- ► Fixed income: bond futures and interest rate swaps
- ► Currency: forwards and futures
- ► Commodities: forwards and futures
- ► Credit: Credit Default Swaps

- ► See AQR's risk parity fund holdings: here

$1 invested in Jan 2016: SPY, AGG, 60/40, and AQR risk parity funds

Legend:
- SPY (S&P 500)
- AGG (Barclays US Aggregate Bond Index)
- 60% SPY, 40% AGG
- AQRIX (AQR Multi Asset)
- QRMIX (AQR Risk Parity)

$1 invested in Jan 2021: SPY, AGG, 60/40, and AQR risk parity funds

Legend:
- SPY (S&P 500)
- AGG (Barclays US Aggregate Bond Index)
- 60% SPY, 40% AGG
- AQRIX (AQR Multi Asset)
- QRMIX (AQR Risk Parity)

Chaves, Hsu, Li, and Shakernia (2011)

Risk Parity Portfolio vs. Other Asset Allocation Heuristic Portfolios

Risk Parity with more asset classes

Risk Parity vs. Other Portfolio Heuristics (with Nine Asset Classes), January 1980–June 2010

	Excess Return over T-bill	Volatility	Sharpe Ratio
60/40 S&P 500/BarCap Agg	5.1%	10.1%	0.50
U.S. Pension Model Portfolio (with 60/40 anchor)	5.1%	9.8%	0.52
Risk Parity Portfolio	3.8%	7.5%	0.51
Equal Weighting	4.5%	8.8%	0.51
Minimum Variance Weighting	1.6%	6.6%	0.24
Mean–Variance Optimal Weighting	4.4%	10.3%	0.43

Risk Parity in Subsamples

Subsample Analysis of Sharpe Ratios: Risk Parity vs. Other Portfolio Heuristics (with Nine Asset Classes), January 1980–June 2010

	Full Sample: Jan. 1980– Jun. 2010	Jan. 1980– Dec. 1989	Jan. 1990– Dec. 1999	Jan. 2000– Dec. 2009
60/40 S&P 500/BarCap Agg	0.50	0.56	0.99	0.04
U.S. Pension Model Portfolio (with 60/40 anchor)	0.52	0.63	0.89	0.15
Risk Parity Portfolio	0.51	0.39	0.69	0.54
Equal Weighting	0.51	0.49	0.64	0.48
Minimum Variance Weighting	0.24	−0.02	0.28	0.49
Mean–Variance Optimal Weighting	0.43	0.60	0.56	0.18

Take-away

Mean-variance analysis is useful but has significant drawbacks when it comes to implementation out-of-sample

Mean-variance analysis ignores tail risk (or assumes investors do not care about tail risk)

Alternatives have been proposed.