Week 10: American Options and the Limits to Arbitrage

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I. Early Exercise and American Options

- Many options in the real world allow early exercise.
- The concept of early exercise makes it hard to obtain analytical solutions
- We first show how to deal with early exercise conceptually and then numerically

A. Some basic properties of calls (C) and Puts (P)

- Options never have negative value
  - They give the holder a right, but not an obligation

$$C \geq 0$$
$$P \geq 0$$

- Call option price is always less than the price of the underlying asset

$$\boxed{C \leq S}$$

  - Can think of S as a call option with a strike of zero

- For a call option on a stock that doesn't pay a dividend over the life of the option

$$\boxed{C \geq S - \frac{K}{(1+r)^{\tau}}}$$

where τ is the time remaining until expiration:

$$\boxed{\tau = T - t}$$

- Why? Compare the following two strategies

Strategy	Description	Payoff at T
1	Buy a call option struck at K	$\max(0, S_T - K)$
2	Borrow $\frac{K}{(1+r)^{\tau}}$ in T-Bills	$-K$
	Buy the underlying stock	S_T

- Hold both portfolios until expiration
  - The payoff from **Strategy 1** is at least as high as the payoff from **Strategy 2:**
    - If $S_T > K$, the call is exercised. Both strategies generate a payoff of $S_T - K$
    - If $S_T < K$, the call expires worthless. The stock is worth S_T which is less than the face value K of your debt
  - Given the same or higher payoffs, **Strategy 1** must be worth at least as much as **Strategy 2** *today*, therefore

$$C \geq S - \frac{K}{(1+r)^{\tau}}$$

- We just proved the **Merton bounds** for the call price

$$\boxed{S \geq C \geq S - \frac{K}{(1+r)^{\tau}}}$$

1. American call (no dividends)

- It is *never* optimal to exercise an American call option early on a non-dividend paying stock, when interest rates are positive

$$C \geq S - \frac{K}{(1+r)^{\tau}} > S - K$$

  (the first inequality comes from the Merton bounds)

- $S - K$ is what you would get today by exercising the option
- But the value of the call is higher than this
  - So don't exercise
- If you never exercise early an American call on a **non-dividend paying stock**, how does it compare to the European call?

$$\boxed{\text{Price (American call)} = \text{Price (European call)}}$$

Another way to think about the early exercise of options

- Suppose you consider exercising the call option today, and compare it to a wait-and-see strategy
  - You pay K for the stock today rather than at maturity
    - So you lose the **time value** of the strike price K
  - If you exercise today, you cannot un-exercise if the price falls later
    - So you lose the **option value** (which you paid for when you paid the option premium)
  - However, you get the stock earlier. Isn't this valuable compared to holding the stock until maturity?
    - Not if it's not paying dividends!

2. American call (with dividends)

- It *can* be optimal to exercise an American call early
  - When might you want to do so? Right before the payment of a dividend
  - But the dividend has to be more valuable than the option value and time value combined
  - This can happen when: the dividend is large, time to maturity is small, interest rate is low, or the option is deep ITM (stock price is high)

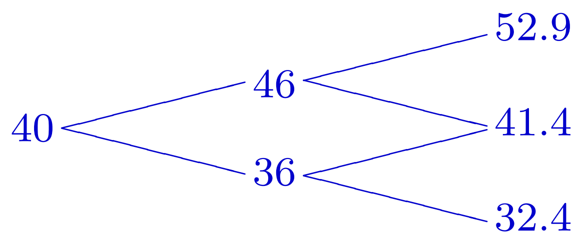
3. American Put

- Exercising an American Put *may* be optimal whether the stock pays a dividend or not
  - You would lose the option value by exercising early. I.e. you can't sell at a higher price if the market subsequently rebounds
  - You also give up any dividend stream
  - But you would gain the time value, because you get K sooner rather than later
  - The gain from early exercise could exceed the loss when: the option is deep ITM (stock price is low), dividends are small, and/or interest rates are high

Using Biomial Trees to Numerically price American Options

Problem

- You are considering HPQ stock over a period of 2 years. Today HPQ trades at \$40. Assume that every year HPQ can go up by 15% or down by 10%:



  The interest rates are stable at 5%

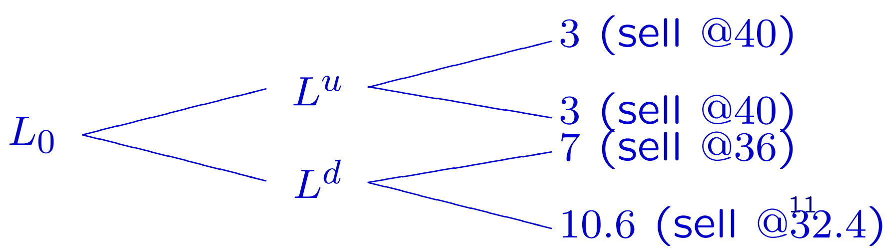
- Find the value of an American **look-back** put option on HPQ, with a strike of \$43, and which expires in 2 years
  - A look-back put uses the *minimum* share price that has occurred over the life of the option

Solution

- First, compute the risk-neutral probabilities of the "up" and "down" movements on HPQ

$$
\begin{aligned}
q^u &= \frac{S_0\,(1 + r_f) - S^d}{S^u - S^d} = \frac{40 \times 1.05 - 36}{46 - 36} = 0.6 \\
1 - q^u &= 0.4
\end{aligned}
$$

- Consider first the *European* look-back put
  - Its payoff is determined by the difference between \$43 (the strike price) and the minimum stock price along the path
  - Since the payoff depends on the price path, we have to separate the paths in the binary tree



- We can use the risk-neutral probability $q^u = 0.6$ to compute L^u, L^d:

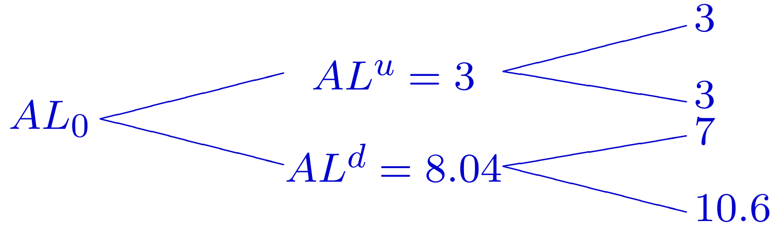
$$L^u = \frac{0.6 \times \$3 + 0.4 \times \$3}{1.05} = \$2.86$$
$$L^d = \frac{0.6 \times \$7 \ + \ 0.4 \times \$10.6}{1.05} = \$8.04$$

- But the *American* look-back gives us the option to exercise early, at the "up" or "down" nodes
  - Exercising early at the "up" node gives a payoff of $\$43 - \$40 = \$3$, which is more than $L^u = \$2.86$ (the value of the look-back put if one doesn't exercise)
  - Exercising early at the "down" node gives a payoff of $\$43 - \$36 = \$7$, which is less than $L^d = \$8.04$

- So the American look-back put has payoffs

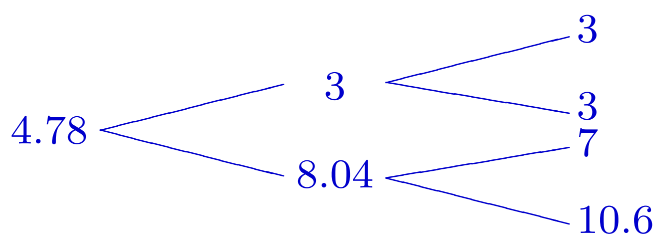
$$AL^u = \max\{2.86, 3\} = \$3, \qquad AL^u = \max\{7, 8.04\} = \$8.04$$

- So far we have the following tree for the American look-back put option



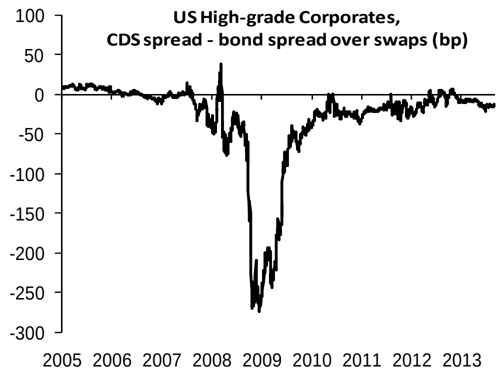
- The payoff at t=0 from not exercising is $\frac{0.6\times\$3+0.4\times\$8.04}{1.05} = \$4.78$. Exercising early gives a (lower) payoff of $\$43 - \$40 = \$3$

- Thus we get the following payoff tree for the American look-back

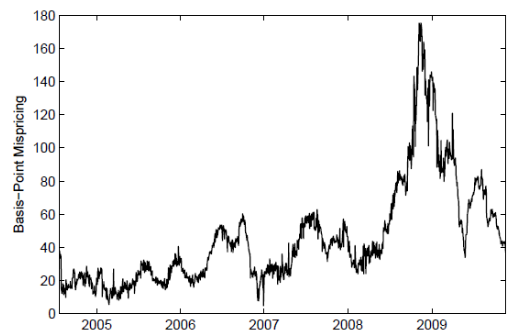


- So one should exercise early only at $t = 1$, at the "up" node.

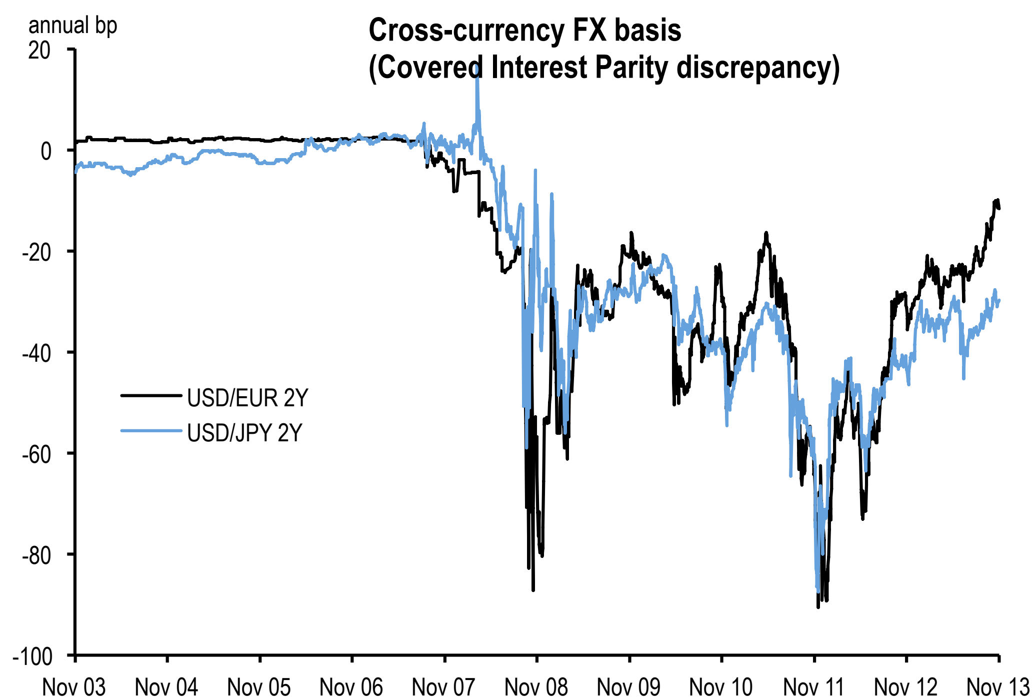
Case Study: Arbitrage in the markets



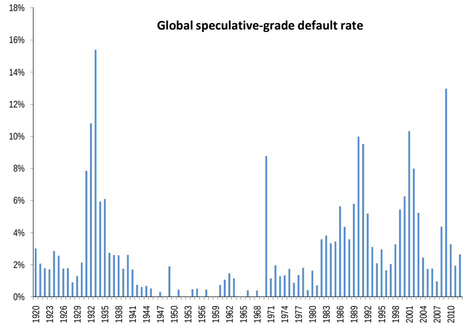
Wtd. Average TIPS–Treasury mispricing, notional-weighted across bond pairs in sample (bp)



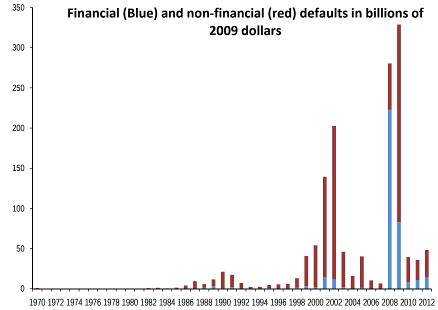
Source: *Why does the Treasury issue TIPs? The TIPS– Treasury bond puzzle*, Fleckenstein, Longstaff and Lustig, Sep. 2010



- Default rates in speculative grades can be quite large and spike during financial crises and recessions.



Source: Moody's



Source: Moody's

II. Limits of Arbitrage

In practice,

- Arbitrage opportunities often involve derivatives

- Take up lots of balance sheet due to offsetting positions

  $\Longrightarrow$ *Counterparty risk*

- One has to be able to hold the position to maturity to realize the cashflow difference
  - This is a problem for those using leverage or facing redemptions

"The market can stay irrational longer than you can stay solvent" John Maynard Keynes, 1920

Abstract Example:

- Suppose besides the usual **green** dollar, as an experiment the Fed decides to also issue **red** and **blue** dollars
- At the end of three years, the blue and red dollars will be retired at face value in exchange for green dollars
- Suppose that investors like the **red $** and push it to $1.10 (**green $**), but dislike the **blue $** which trades for $0.90
- You have $100,000. How can you take advantage of this mispricing?
- Buy the cheap **blue $** and short the expensive **red $**
  - E.g. if you short 100k **red $**, you get $110k
  - And for $90k you can then buy 100k **blue $**
  - This means you pocket $20k today for sure
  - And do nothing for 3 years, when your long position in **blue $** covers your short position in **red $**

- One would like to go long and short huge amounts of blue and red $ (some people are greedy!)
- Are there any risks/limits in executing this strategy?
- Yes, you need a *margin account* for the short position!
- You keep the proceeds ($110k) in the margin account, but you also need an *initial margin requirement* of 50% = $55k

Shorted Quantity	Price/Unit	Short Sale Value	Initial Margin Requirement	Total Margin Requirement
100,000	$1.10	$110,000	$55,000	$165,000

- After that, if the price of the red $ increases, you need to make sure that you keep a *maintenance margin requirement* of 30% of the short sale value (i.e. of the market value of the shorted red $)
- Assume that there are no lending fees

- As the price of the red dollar increases further, you might get margin calls:

Shorted Quantity	Price/Unit	Short Sale Value	Maintenance Margin Requirement	Total Margin Requirement	Margin Call
100,000	\$1.20	\$120,000	\$36,000	\$156,000	\$0
100,000	\$1.30	\$130,000	\$39,000	\$169,000	\$4000
100,000	\$1.40	\$140,000	\$42,000	\$182,000	\$17,000
100,000	\$1.80	\$180,000	\$54,000	\$234,000	\$69,000

- So for example, if the market becomes even more irrational and the price of the red \$ rises to \$1.80, then you get a margin call for \$69k
- Since you already spent \$55k for the initial margin requirement, and your initial capital was of only \$100k, you cannot afford to pay \$69k, so you become insolvent!
- *"Markets can remain irrational for longer than you can remain solvent!"* (J.M. Keynes)

- What else can happen other than price going against you?
  1. Suppose you want to unwind your position
     * Buy the **red \$** and sell the **blue \$**
     * Gap can become even wider, worsening your position
     * Moreover, typically the **blue \$** is *illiquid* (since people don't want it), which makes the gap even worse!
  2. What if other people hold positions similar to yours?
     * They might want to unwind their positions before you, which makes your position worse
  3. What if other traders find out about your positions?
     * They will take the opposite positions to yours (buy the **red \$** and sell the **blue \$**) and you get margin calls
     * As you liquidate, your position gets worse, and theirs gets better $\Longrightarrow$ **predatory trading**
  4. When spread widens (you make losses), investors also pull money out, which may force you to liquidate