Investments

Topic 3: Fixed Income Management

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Overview of Topic 3

1. Forward rates
2. Interest Rate Movements and Bond Prices
3. Performance Profile
4. Duration
5. Managing Interest Rate Risk
6. Convexity

1. Forward Rates

Predictability of Bond Returns

- Predictability = Time variation in expectation

- Long-maturity bonds return more, on average, than short-maturity bonds.
  - 5-year bonds have returned 1.3% (6.00 vs 4.7) more per year than 1-year bonds, over 1953-2020 in US.

- Excess return of long-maturity bonds is positively related to slope of term structure.
  - Positive slope predicts outperformance of long-maturity bonds.
  - Negative slope predicts underperformance of long-maturity bonds.

- $\Rightarrow$ Bond returns exhibit some predictability.

Forward rates

- Invest today: today's spot rates.

- Invest next year:
  - Wait until next year, and invest at next year's spot rates.
  - Invest at forward rates, guaranteed today.

- **Forward rates** are rates that we can guarantee today for investing in the future.

Obtaining Forward Rates from Spot Rates

- Forward rates can be derived from today's spot rates.

- Invest \$1 for $t + T$ years.
  - Invest at the $t + T$-year spot rate:
$$(1 + r_{t+T})^{t+T}.$$
  - Invest at the t-year spot rate, and reinvest at the forward rate ${}_tf_T$ between years t and $t + T$:
$$(1 + r_t)^t(1 + {}_tf_T)^T.$$

- Both amounts are guaranteed today $\Rightarrow$ They must be equal. If not equal $\Rightarrow$ Arbitrage.
$$(1 + {}_tf_T)^T = \frac{(1 + r_{t+T})^{t+T}}{(1 + r_t)^t}.$$

Obtaining Forward Rates from Spot Rates: An Example

- Suppose that the 1-year spot rate is 5%, the 2-year spot rate is 4.5%, and the 3-year spot rate is 4%. Which forward rates can you compute based on this information?
- Forward rate between years 1 and 2:

$$1 + {}_1f_1 = \frac{(1 + r_2)^2}{1 + r_1} \Rightarrow {}_1f_1 = 4.0\%.$$

- Forward rate between years 2 and 3:

$$1 + {}_2f_1 = \frac{(1 + r_3)^3}{(1 + r_2)^2} \Rightarrow {}_2f_1 = 3.0\%.$$

- Forward rate between years 1 and 3:

$$(1 + {}_1f_2)^2 = \frac{(1 + r_3)^3}{1 + r_1} \Rightarrow {}_1f_2 = 3.5\%.$$

Use of Forward Rates

- Expectations theory: Forward rates coincide with expected future spot rates.
  - Forward rate ${}_tf_T$ coincides with expected T-year rate that will prevail in year t.

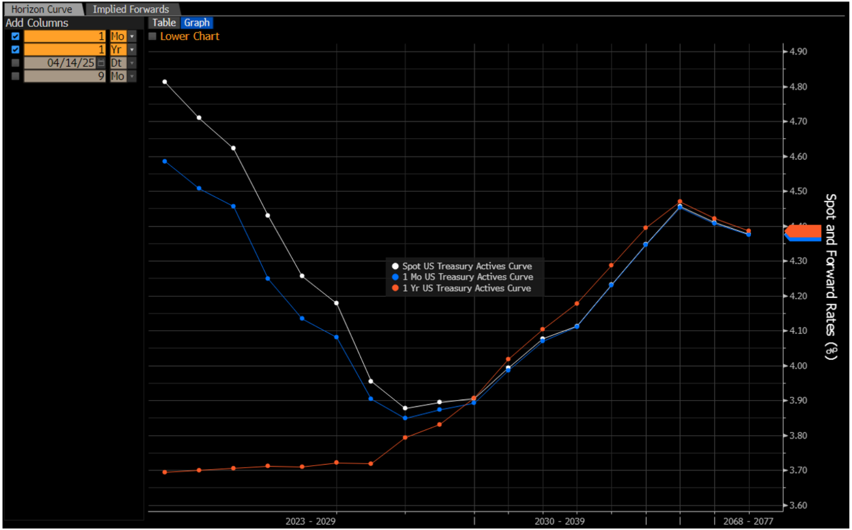
$$ {}_tf_T = E_0[r_T(t)]. $$

  - Example: Suppose that $r_1 = 5\%$ and $r_3 = 4\%$. According to expectations theory, market expects two-year spot rate one year from now to be ${}_1f_2 = 3.5\%$.
- Testing the expectations theory:

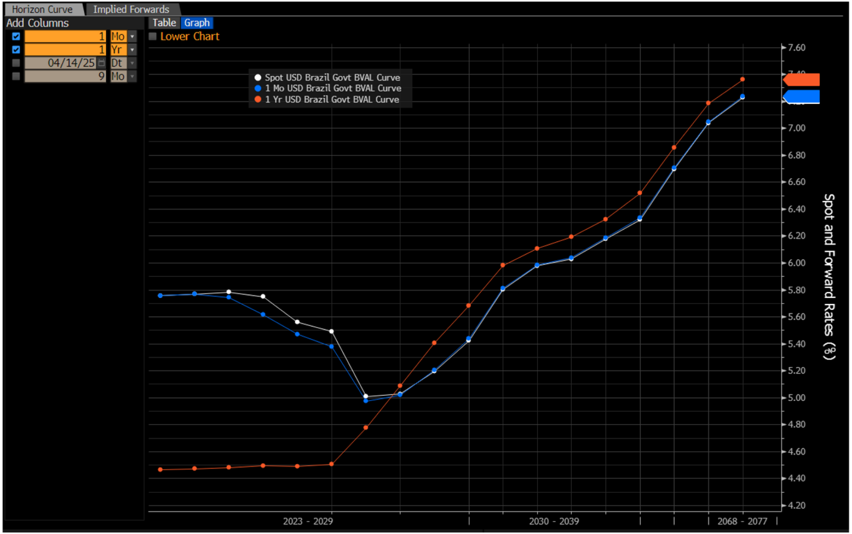
$$ r_1(t+1) - r_1(t) = \underbrace{\alpha}_{0} + \underbrace{\beta}_{1}({}_1f_1(t) - r_1(t)) + \varepsilon $$

	β	R^2
U.S.	.55	2.8%
Japan	.55	8.6 %

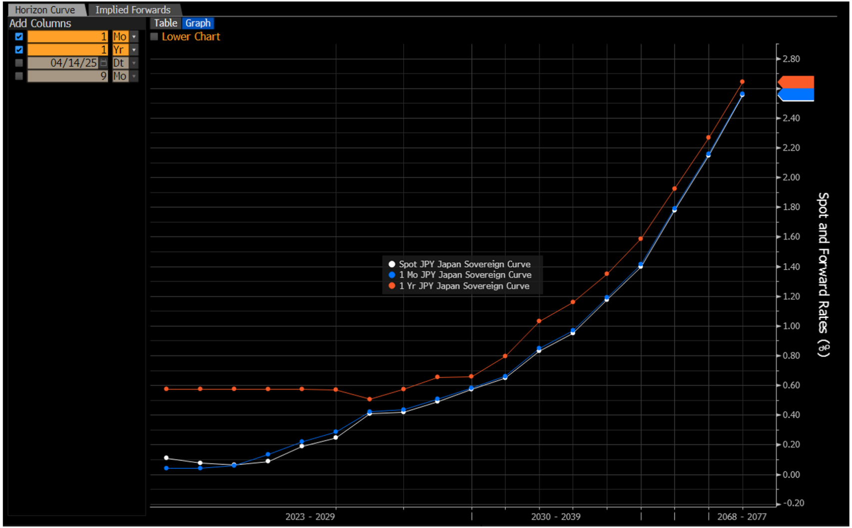
Forward curves: US



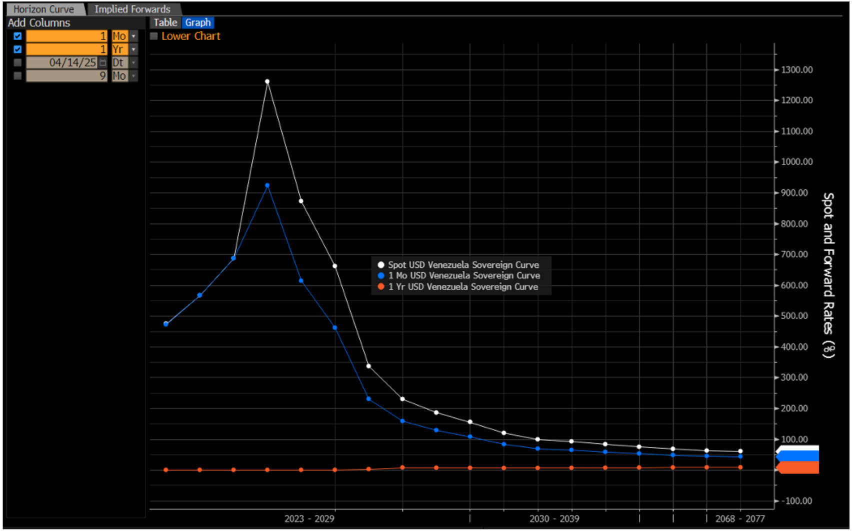
Forward curves: Brazil



Forward curves: Japan



Forward curves: Venezuela



2. Interest Rate Movements and Bond Prices

Interest Rate Movements and Bond Prices

- Interest rates move substantially over time.

- Main factors driving interest rate movements:
  - Monetary policy.
  - Expectations of future inflation.
  - Bond demand and supply.

First Basic Fact

- Bond prices are sensitive to interest rate movements. They go down when interest rates go up, and vice-versa.

$$\boxed{\text{Bond prices are negatively related to interest rates}}$$

- Intuition 1: Bond prices are the PV of bonds' cash flows. If interest rates go up, cash flows are discounted more heavily, and the PV goes down.
- Intuition 2: Suppose that interest rates go up. Investors will sell bonds in order to invest using the higher interest rates. Therefore, bond prices will decrease.

Second Basic Fact

- Prices of long term bonds are more interest rate sensitive than prices of intermediate term bonds.

The interest rate sensitivity of bond prices increases with maturity

- Intuition:
  - The cash flows of long term bonds are farther in the future, and are more heavily discounted than the cash flows of shorter term bonds.
  - Therefore, the PV of the cash flows is more interest rate sensitive for long term bonds than for shorter term bonds.

Measuring and Managing Interest Rate Risk

- We showed that interest rate risk is important, and impacts bond prices.
- Many market participants need to measure and manage interest rate risk.
  - Bond mutual funds.
  - Banks.
  - Pension funds.
  - Insurance companies.
- The objective of this Lecture is to develop tools for measuring and managing interest rate risk. The main tools are duration and convexity.
- We first develop the tools, and then explain how they can be used.

3. Performance Profile

Performance Profile

- To define duration and convexity, we first need to define an important curve, the performance profile.
- For simplicity, we assume that interest rates are independent of maturity. This means that the term structure is **flat**.
- With a flat term structure, interest rate movements can only be parallel shifts in the term structure.

Defining the Performance Profile

- Consider a bond with annual coupon rate $c\%$ and T years to maturity.
- Consider an investment in the bond, worth \$1 at the current interest rate r. Determine value of the investment when interest rate moves to $r + \Delta r$.
  - Price of the bond at r:

$$P(r) = \frac{c}{1+r} + \frac{c}{(1+r)^2} + \cdots + \frac{100+c}{(1+r)^T}.$$

  - Units of the bond that can be bought with \$1 at r:

$$\frac{1}{P(r)}.$$

  - Value of the investment at $r + \Delta r$:

$$P(r+\Delta r)\frac{1}{P(r)}.$$

Defining the Performance Profile (cont'd)

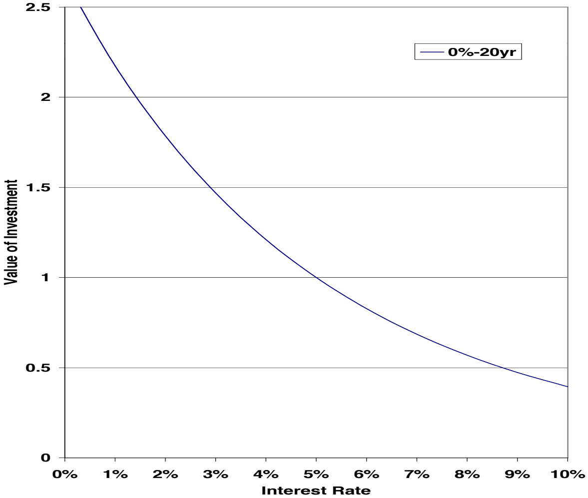
- We can plot the value of the investment as a function of the interest rate

- This curve is the **performance profile** of the bond.

- Duration and convexity are based on the performance profile.
  - Duration is related to the slope (first derivative) of the performance profile.
  - Convexity is related to the curvature (second derivative) of the performance profile.

Example of a Performance Profile

Zero-coupon bond with 20 years to maturity. Current interest rate $r = 5\%$.

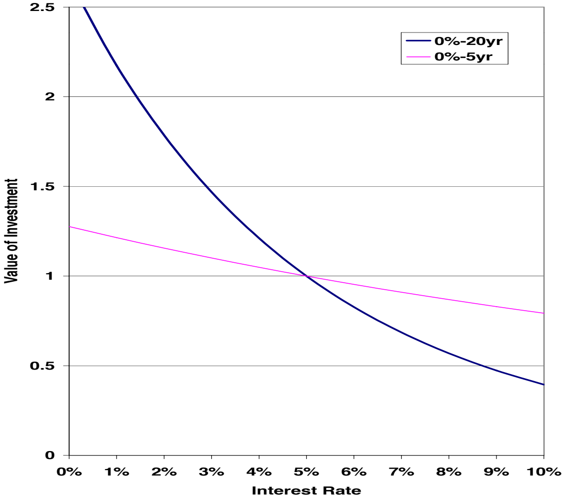


Conclusions

- The performance profile is <u>downward sloping</u>: The value of the investment goes down when interest rates go up.
- The performance profile is <u>convex</u>: The value goes down at a decreasing rate.
- These conclusions hold for any standard bond.

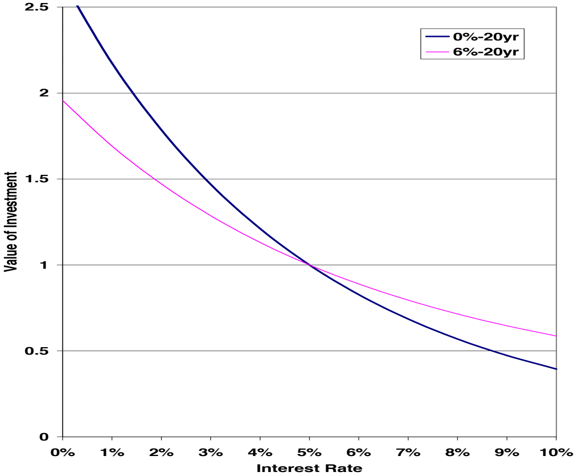
Effect of Maturity

Two zero-coupon bonds with 5 and 20 years to maturity. Current interest rate $r = 5\%$.



Effect of Coupon

Two bonds with annual coupon rates 0% and 6%, and 20 years to maturity. Current interest rate $r = 5\%$.



Conclusions

- The slope of the performance profile increases with maturity.
- The slope of the performance profile decreases with coupon rate.

4. Duration

Duration

- Duration is related to the slope of the performance profile, at the current interest rate r.

- Three concepts of duration:
  - Macaulay duration (D).
  - Modified duration (D^*).
  - Dollar duration ($DV01$).

Macaulay Duration

- Consider a bond with annual coupon rate $c\%$ and T years to maturity.
- The **Macaulay duration** (D) of the bond is

$$D = \sum_{t=1}^{T} w_t \cdot t,$$

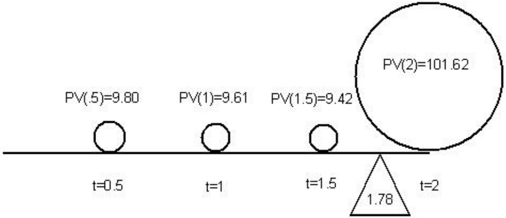
where

$$\begin{aligned} w_t &= \frac{c}{(1+r)^t} \frac{1}{P} \quad \text{for} \quad t = 1, .., T-1 \\ w_T &= \frac{100+c}{(1+r)^T} \frac{1}{P} \end{aligned}$$

and P is the bond price.
- The Macaulay duration is a weighted average of the years in which the bond pays cash flows.
- The weight of a given year is the PV of that year's cash flow divided by the PV of all cash flows. (The latter PV is the price of the bond).

Macaulay Duration: Intuition

- The Macaulay duration can be interpreted as a "center of gravity". It is the year around which the bond's discounted cash flows are "balanced".



Macaulay duration for 2 year 20% bond = 1.78yrs
(semi-annual coupon 20%, yield 3.9605% continuously-compounded)

- The Macaulay duration is, in a sense, the effective maturity of the bond.
- We have not yet linked the Macaulay duration to the slope of the performance profile.
- However, we can expect a link, since Macaulay duration is a measure of maturity, and the slope of the performance profile depends on maturity.

Macaulay Duration: Example

Bond with annual coupon rate 6% and 20 years to maturity. Current interest rate $r = 5\%$.

coupon rate	*6.0%*
term structure flat at	*5.0%*
bond's face value	*100.0*

Year	*CF*	*PV(CF)*	*weight*	*weight*time*
1	6	5.71	0.051	0.05
2	6	5.44	0.048	0.10
3	6	5.18	0.046	0.14
4	6	4.94	0.044	0.18
5	6	4.70	0.042	0.21
6	6	4.48	0.040	0.24
7	6	4.26	0.038	0.27
8	6	4.06	0.036	0.29
9	6	3.87	0.034	0.31
10	6	3.68	0.033	0.33
11	6	3.51	0.031	0.34
12	6	3.34	0.030	0.36
13	6	3.18	0.028	0.37
14	6	3.03	0.027	0.38
15	6	2.89	0.026	0.38
16	6	2.75	0.024	0.39
17	6	2.62	0.023	0.40
18	6	2.49	0.022	0.40
19	6	2.37	0.021	0.40
20	106	39.95	0.355	7.10
		112.46	1.00	**12.62**

Macaulay Duration: Extension

In reality, the yield curve is not flat. Simply replace r by YTM y
So the procedure is:

1. Compute YTM of a given bond
2. Compute Macaulay Duration:

$$
\begin{aligned}
D &= \sum_{t=1}^{T} w_t \cdot t, \\
w_t &= \frac{c}{(1+y)^t} \frac{1}{P} \text{ for } t = 1, .., T-1 \\
w_T &= \frac{100+c}{(1+y)^T} \frac{1}{P}
\end{aligned}
$$

Macaulay Duration: Properties

- For a zero-coupon bond with T years to maturity, $D = T$.
- D increases with time to maturity.
- D decreases with coupon rate.

Modified Duration

- Consider a bond with annual coupon rate $c\%$ and T years to maturity.
- The **modified duration** (D^*) of the bond is

$$D^* = \frac{D}{1+r}.$$

or, taking the shape of the yield curve into account,

$$D^* = \frac{D}{1+y}.$$

- Fact: The slope of the performance profile, at the current interest rate r, is

$$D^* = -\frac{1}{P_T}\frac{\partial P_T}{\partial y_T}$$

.

Dollar Duration

- The **dollar duration** ($DV01$) of the bond is

$$DV01 = -\frac{1}{10,000}\frac{\partial P_T}{\partial y_T}.$$

  The division by $10,000$ is added to express $DV01$ as a \$ change in price per 1 basis point change in yield.

- Ignoring the $10,000$, $DV01 = D^* \cdot P_T$

- This fact establishes that the various concepts of duration are linked to the slope of the performance profile.

Using Duration: A Preview

- So far:
  - we defined D, D^*, and $DV01$
  - we showed that the slope of the performance profile, at the current interest rate r, is D^*
- But how can we use duration?
- Consider an investment in the bond, which is worth P at the current interest rate r.
- We can use duration to approximate the change in the investment's value as interest rates change.

Using Duration: A Preview (cont'd)

- We can approximate the change in the investment's value, using the slope of the investment's performance profile.
  - The slope of the performance profile of a \$1 investment is D^*.
  - Therefore, the slope of the performance profile of an investment worth P is PD^*.

- For a small interest rate movement Δr, the change in the investment's value is approximately

$$\Delta P \approx -P \cdot D^* \Delta r = -DV01 \Delta r.$$

Example

Bond with annual coupon rate 6% and 20 years to maturity. Current interest rate $r = 5\%$.

- Price of the bond is $P(r) = 112.46$.
- Macaulay duration is 12.62, and modified duration is $12.62/(1 + 5\%) = 12.02$.
- Suppose that interest rate goes up to $r + \Delta r = 5.1\%$.
    - Price of the bond becomes $P(r + \Delta r) = 111.12$.
      Exact change in bond price is

$$\Delta P = 111.12 - 112.46 = -1.34.$$

    - Approximate change in bond price is

$$\begin{aligned} \Delta P &\approx -P \cdot D^* \Delta r \\ &= (-112.46) \times (12.02) \times (0.1\%) = -1.35. \end{aligned}$$

Example (cont'd)

- Now suppose the interest rate goes down to $r + \Delta r = 4.9\%$.

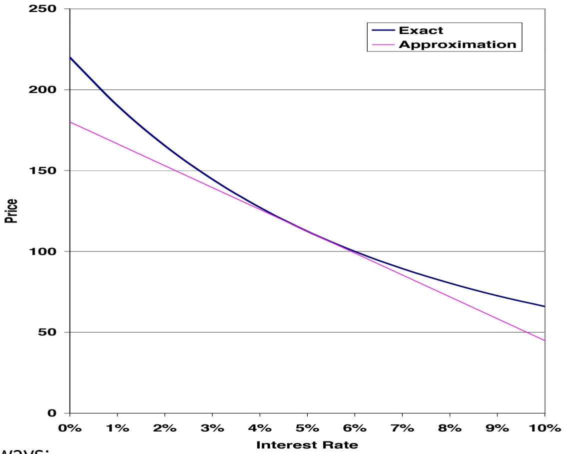
$$\begin{aligned} \text{Exact change:} \quad & \Delta P = 1.36; \\ \text{Approximate change:} \quad & \Delta P \approx 1.35 \end{aligned}$$

- *For small shifts in the term structure, duration provides a good approximation to the actual change.*
- Now suppose the interest rate change is $\Delta r = 0.5\%$ (up to 5.5% or down to 4.5%).

$$\begin{aligned} \text{Exact change:} \quad & \Delta P = -6.49 \text{ for an upward shift,} \\ & \Delta P = 7.05 \text{ for a downward shift;} \\ \text{Approximate change:} \quad & \Delta P \approx -6.76 \text{ for an upward shift,} \\ & \Delta P \approx 6.76 \text{ for a downward shift.} \end{aligned}$$

- *For large shifts in the term structure, duration provides imprecise approximation to the actual change.*

Example (cont'd)



- Duration model always:
  - understates the capital gain if interest rates go down
  - overstates the capital loss if interest rates go up.
- Approximation error depends on the curvature of the performance profile.

5. Managing Interest Rate Risk

Managing Interest Rate Risk

The net worth of many market participants is sensitive to interest rate movements.

- Bond Mutual Funds
- Banks
  - Assets: Loans. Long term.
  - Liabilities: Demand deposits. Short term.
  - Net worth decreases when interest rates go up.
- Pension funds:
  - Assets: Fixed-income securities, stocks, etc.
  - Liabilities: Future pensions to be paid to employees. Long term.
  - If assets consist of short maturity bonds, net worth decreases when interest rates go down.
- Insurance companies:
  - Similar asset and liability structure to pension funds.

Why Hedge? A Preview

- Hedgers are interested in reducing a risk that they already face.

  Example: An investment bank sold (short) a complicated (fixed-income) instrument to a client.

  Client:

  - bears the instrument-specific risk

  Investment bank:

  - owes the instrument (short position)
  - bears the counterparty risk

- Market participants may be averse to big losses.

Risk Management Approaches

Risk managers may want to:

- Lock into a known price/rate via a forward contract
- Fully offset (hedge) all existing risk
  - balance sheet immunization
- Approximately offset all existing risk
  - duration and duration-convexity models
- Control the downside
  - Portfolio Insurance: under no circumstances can the value of a portfolio drop below the floor \$K.
  - Value-at-Risk: the probability that the value of a portfolio drops below the floor \$K may not exceed 1%.

Using Duration

- Market participants need to <u>measure</u> the sensitivity of their net worth to interest rate movements.

- Duration provides a simple, convenient measure.

- Market participants can sometimes control (or <u>manage</u>) the interest rate sensitivity of their net worth. They can do this by changing the composition of their assets or liabilities.

- Duration is a useful tool for managing interest rate risk.

Managing Interest Rate Risk: An Example

You are the CFO of an insurance company. Your liabilities consist of four \$20M payments in 14, 15, 17, and 18 years. Your assets consist of \$35M. You are uncertain about future interest rates, and would like to immunize your net worth (i.e., make it insensitive to interest rate movements).

What should you do?

A Dangerous Choice

Suppose you keep the C=\$35M in cash.

- Suppose the current interest rate is 6% (i.e, the term-structure is flat at 6%).
  - The PV of your liabilities is
$$L = \frac{20M}{(1.06)^{14}} + \frac{20M}{(1.06)^{15}} + \frac{20M}{(1.06)^{17}} + \frac{20M}{(1.06)^{18}} = 31.63M.$$
  - Your net worth is
$$C - L = 35M - 31.63M = 3.37M.$$
- Suppose the interest rate moves to 5%.
  - The PV of your liabilities becomes
$$L = \frac{20M}{(1.05)^{14}} + \frac{20M}{(1.05)^{15}} + \frac{20M}{(1.05)^{17}} + \frac{20M}{(1.05)^{18}} = 36.76M.$$
  - Your net worth becomes
$$C - L = 35M - 36.76M = -1.76M.$$

Immunization Using Duration

Suppose you can invest in two zero-coupon bonds with maturities 10 and 20 years. Can you choose investments so that your net worth is insensitive to small interest rate movements? Approach: Choose investments so that the approximate changes in assets and liabilities, computed using duration, are equal. Liabilities:

- The liabilities are not a bond, but their duration can be computed by the same formula as for a bond:

$$D = w_{14}14 + w_{15}15 + w_{17}17 + w_{18}18,$$

  where

$$w_t = \frac{20M}{(1.06)^t}\frac{1}{31.63M}.$$

  We get $D = 15.85$, $D^* = 14.96$, and $DV01 = 0.0473M$

- The approximate change in the value of liabilities is

$$\Delta L \approx -DV01\Delta r = -0.0473 \times \Delta r.$$

Immunization Using Duration (cont'd)

- The idea is that you select some of your assets in such a way that a parallel shift in the yield curve does not affect your net worth
- Suppose we can trade x units of the 20-year ZCB:
  - At current interest rates, $P_{20} = 1.06^{-20} = 0.312$
  - Durations: $D = 20$, $D^* = 20/1.06 = 18.87$, $DV01 = P_{20}D^*/10000 = 0.00059$
  - Immunized portfolio: $P\&L = DV01_L \Delta y_L - x \cdot DV01_{20} \Delta y_{20}$
- Parallel shift means $\Delta y_L = \Delta y_{20} = \Delta r$ and $P\&L = 0$
- Then

$$x = \frac{DV01_L}{DV01_{20}} = 80.40M$$

  and the value of the bond position is $\$25.01M$
- Given the current value of assets of $\$35M$, this is affordable, but the rest cannot be invested in interest-rate-sensitive assets, aka no full immunization
- Alternatively, use two bonds and ensure that the value of their position adds up to $\$35M$

6. Convexity

Convexity

- We use convexity (in addition to duration) in order to better approximate the change in an investment's value.
- Convexity is related to the curvature of the performance profile, at the current interest rate r.
- The **convexity** (C) of a bond with annual coupon rate $c\%$ and T years to maturity is

$$C_T = \frac{1}{P_T}\frac{\partial^2 P_T}{\partial y_T^2} = \frac{1}{(1+y_T)^2}\sum_{t=1}^{T} w_t t(t+1) \underset{\text{ZCB}}{=} \frac{T(T+1)}{(1+y_T)^2},$$

  where the weights w_t are as in the definition of Macaulay duration.
- <u>Fact:</u> The curvature of the performance profile, at the current interest rate r, is C.
- For a small interest rate movement Δr, the change in an investment's value is approximately

$$\Delta P \approx -PD^* \Delta r + \frac{1}{2} PC(\Delta r)^2.$$

Convexity Correction

We can plot the exact price, the approximate price obtained using duration, and the approximate price obtained using duration and convexity.