Investments

Topic 7: CAPM

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Overview of Topic 7

1. Terminology.
2. Regression and Beta.
3. The CAPM.
4. The CAPM's Basic Insight.
5. CML and SML.
6. Uses of the CAPM.
7. Statistical Tests of the CAPM.

1. Terminology

Terminology

We assume that there are N risky assets (stocks) and a riskless asset (T-bill).

- The **excess return** of asset n is the asset's return minus the return on the riskless asset,
$$R_n - R_f.$$
- The **expected excess return** of asset n is the asset's expected return minus the return on the riskless asset,
$$E(R_n) - R_f.$$

Market Portfolio

- The **market portfolio** is the value-weighted portfolio of the N risky assets.
  - The market value (market capitalization) of asset n is $P_n s_n$, where P_n is the price of one share and s_n the total number of shares.
  - The market value of the market portfolio is

$$\sum_{n=1}^{N} P_n s_n.$$

  - The weight of asset n in the market portfolio is

$$\frac{P_n s_n}{\sum_{n=1}^{N} P_n s_n}.$$

  - We denote by R_M the return on the market portfolio.
- The **market risk premium** is the expected excess return of the market portfolio,

$$E(R_M) - R_f.$$

2. Regression and Beta

Regression and Beta

- Consider two random variables X and Y.
- Fact: We can write Y as
$$Y = \alpha + \beta X + \epsilon.$$
  - α, β: constants,
  - ϵ: random variable, such that
$$Cov(X, \epsilon) = 0 \quad \text{and} \quad E(\epsilon) = 0.$$
- Variation in Y is decomposed into
  - βX: variation that can be "explained" by X.
  - ϵ: "unexplained" variation.
- Fact: β is given by
$$\beta = \frac{Cov(X, Y)}{V(X)}.$$

The Regression Equation

- Equation
$$Y = \alpha + \beta X + \epsilon$$
is the **regression equation**.

- It is a useful way of thinking about how Y is generated.

- Mechanism:
  - α and β are given.
  - For each "observation", an X and an ϵ are randomly drawn.
  - Y is then determined according to regression equation.

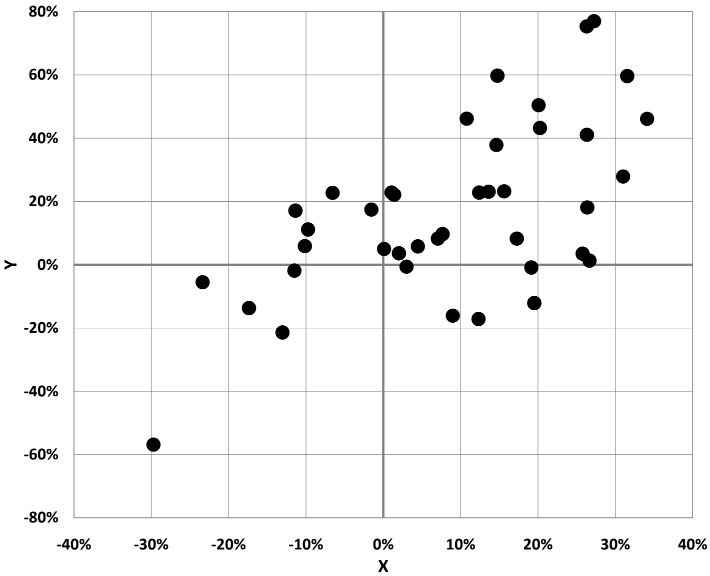
Regression

- In general, we do not know α and β.

- However, we can estimate them using regression.

- Regression:
  - We consider the scatterplot of Y vs. X.
  - We fit the "best" line to the scatterplot.
  - Suppose that line is
$$Y = a + bX.$$
  - a is estimate for α, and b is estimate for β.

A Scatterplot



Regression Output

Regression output gives:

- Estimate for α, we call it a.
  - Standard error of estimate a, we call it s_a.
- Estimate for β, we call it b.
  - Standard error of estimate b, we call it s_b.
- Estimate for standard deviation of ϵ denoted $\sigma(\epsilon)$, we call it $s(\epsilon)$.
- R-Squared.

$$\begin{aligned}
\text{R-squared} &= \frac{\text{Explained Variance}}{\text{Explained Variance} + \text{Unexplained Variance}} \\
&= \frac{V(bX)}{V(bX) + V(\epsilon)} \\
&= \frac{V(bX)}{V(bX) + s(\epsilon)^2}.
\end{aligned}$$

Regression and Asset Returns

- When studying asset returns, we assume that
  - X is excess return of market portfolio, $R_M - R_f$,
  - Y is excess return of asset n, $R_n - R_f$.

- Regression equation is

$$R_n - R_f = \alpha_n + \beta_n(R_M - R_f) + \epsilon_n.$$

- Variation in returns of asset n is decomposed into
  - $\beta_n(R_M - R_f)$: **Systematic risk**, i.e., risk that is perfectly correlated with the market portfolio.
  - ϵ_n: **Idiosyncratic risk**, i.e., risk that is uncorrelated with the market portfolio.

Asset Characteristics

Three characteristics of an asset:

- Alpha, α_n.
- Beta, β_n.
- Sigma, $\sigma(\epsilon_n)$.

Beta

- Beta:
$$R_n - R_f = \alpha_n + \boxed{\beta_n}(R_M - R_f) + \epsilon_n.$$
- Measures the asset's sensitivity to market movements.
- If the return on the market portfolio is higher by 1%, then the return on asset n is higher by β_n (holding all else equal).
- Beta is given by
$$\beta_n = \frac{Cov(R_n, R_M)}{V(R_M)}.$$

Alpha and Sigma

- Alpha:
$$R_n - R_f = \boxed{\alpha_n} + \beta_n(R_M - R_f) + \epsilon_n.$$
- Measures the asset's attractiveness.

- Sigma:
$$R_n - R_f = \alpha_n + \beta_n(R_M - R_f) + \boxed{\epsilon_n}\,.$$
- Sigma is the standard deviation of ϵ_n. It measures the asset's idiosyncratic risk.

Regression: Example

IBM vs. CRSP market portfolio. Monthly returns 1/1990 - 9/2022.

```
import pandas as pd
import statsmodels.api as sm
df = pd.read_csv("l7reg2022.csv") # Load the data
MktRF = sm.add_constant(df['MktRF']) #adding a constant
model = sm.OLS(df['IBMminusRF'], MktRF).fit()
modelrss = model.resid.std(ddof=MktRF.shape[1]) #model residual standard error
model.summary() # Printing model summary
```

OLS Regression Results

Dep. Variable:	IBMminusRF	**R-squared:**	0.231
Model:	OLS	**Adj. R-squared:**	0.230
Method:	Least Squares	**F-statistic:**	117.8
Date:	Sat, 12 Nov 2022	**Prob (F-statistic):**	3.68e-24
Time:	15:03:53	**Log-Likelihood:**	-1307.8
No. Observations:	393	**AIC:**	2620.
Df Residuals:	391	**BIC:**	2627.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	0.1915	0.345	0.555	0.579	-0.487	0.869
MktRF	0.8351	0.077	10.852	0.000	0.684	0.986

Regression Output: Example

- Estimate for alpha (monthly): $a_n = 0.19$.
  - Standard error of estimate $s_a = 0.34$.
- Estimate for beta: $b_n = 0.84$.
  - Standard error of estimate $s_b = 0.08$.
- R-Square: 23.1%.

Expected Return

- Regression equation is

$$R_n - R_f = \alpha_n + \beta_n(R_M - R_f) + \epsilon_n.$$

- Taking expectations, we get

$$E(R_n) - R_f = \alpha_n + \beta_n(E(R_M) - R_f).$$

- Expected excess return of asset n depends on
  - Alpha.
  - Beta.
  - Market risk premium.

- Can we obtain some insight on these?

3. The CAPM

The CAPM

- The CAPM is a theoretical model which provides insight on assets' expected returns.

- Assumptions:
  - Markets are perfect, in particular
    - Trading of assets is costless (including short sales).
    - Investors have the same information (beliefs).

  - There are N risky assets and a riskless asset.

  - Investors care only about mean and variance.

  - Investors have a one-period horizon.

Asset Demand

- We first consider the demand for the assets.

- A single investor:
  - Cares only about mean and variance.
  - Chooses a portfolio on the portfolio frontier.
  - Portfolio is a combination of tangent portfolio and riskless asset.
    - Very risk-averse: Portfolio closer to riskless asset.
    - Not very risk-averse: Portfolio closer to tangent portfolio, or even above tangent portfolio.

- Investors as a group:
  - Demand is a combination of tangent portfolio and riskless asset.

Asset Supply

- We next consider the supply of the assets.

- Supply is $\sum_{n=1}^{N} P_n s_n$ dollars of market portfolio, and the riskless asset.

Market Equilibrium

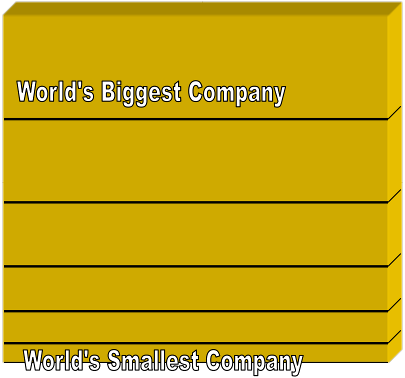
- In market equilibrium, demand equals supply.
- In particular:

$$\boxed{\text{Tangent portfolio coincides with market portfolio}}$$

- Example:
  - Suppose that weight of GE is 1% in market portfolio and only 0.7% in tangent portfolio.
  - Supply of GE exceeds demand.
  - Price of GE has to fall.
    - Weight of GE in market portfolio decreases.
    - Weight of GE in tangent portfolio increases.
  - Price has to fall until weights become equal.

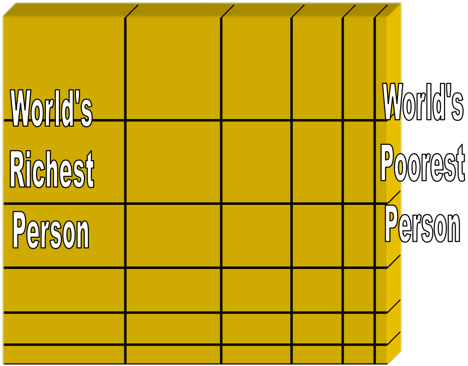
Market Portfolio as a Cake

- Let's think about market portfolio as a square cake
- All risky assets (stocks) will be viewed as slices of this cake
- The cake will be sliced
  - Horizontally
  - Varying width of slices will be proportional to market capitalization of each stock



Equilibrium Again

- All investors collectively hold all assets (demand=supply)
- For every borrower at risk-free rate there is a lender
  - Then all riskless positions net-out across all investors
- We have already observed that everyone will hold the same tangency portfolio
- Given these observations we can now slice the cake vertically according to the wealth of the investor



The Market Portfolio

- Note that no matter what is the investors wealth, each one has the same proportion invested in each of the stock (the tangency portfolio)

- Looking at the picture, we can compute this proportion: it is the same as proportion of each company in the market portfolio

$$w_i = \frac{\text{total dollar value of asset } i}{\text{total dollar value of all risk securities}}$$

- The tangency portfolio coincides with the Market portfolio

An Important Property of the Tangent Portfolio

- Suppose that we hold the TP, and decide to
  - increase the weight of a risky asset n
  - decrease the weight of the riskless asset (by the same amount).
- The change in expected return is

$$\frac{dE(R)}{dw_n} = E(R_n) - R_f.$$

- The change in variance is

$$\begin{aligned} \frac{dV(R)}{dw_n} &= 2\left(w_n V(R_n) + \sum_{m \neq n} w_m Cov(R_n, R_m)\right) \\ &= 2Cov(R_n, R^*), \end{aligned}$$

  where R^* denotes the return on the tangent portfolio.
- Notice that the change in variance involves the covariance of asset n with the tangent portfolio, and not the variance of asset n.

Measuring Asset Risk

- When the asset is examined in isolation:

  Variance of asset return.

- When the asset is examined as part of a portfolio:

  $\boxed{\begin{array}{c}\text{Covariance between asset return}\\ \text{and return on the portfolio}\end{array}}$

The Buck for the Bang Ratio

- Define the buck for the bang ratio as the ratio of the change in expected return (buck) to the change in variance (bang).
- This ratio is

$$\frac{E(R_n) - R_f}{2Cov(R_n, R^*)}.$$

- The important property of the TP is that this ratio is independent of the particular asset n.
- Intuition (next slide for details):
  1. Suppose that the buck to the bang ratio is higher for asset n than for asset m.
  2. Then, by buying n and selling m, we can decrease variance of TP, holding expected return constant
- In market equilibrium, tangent portfolio coincides with market portfolio.
- Therefore, $R^* = R_M$ in the formula above

Why the Bang for the Buck Ratio is Equal for all Assets

Suppose that the buck for the bang ratio for asset n (BB_n) is greater than for asset m (BB_m). We will show that we can decrease the variance of the TP, holding expected return constant. This will be a contradiction since the TP is a frontier portfolio.

Consider an increase in the weight of asset n (accompanied by an equal decrease in the weight of the riskless asset), such that the expected return of portfolio increases by dE. The variance of the portfolio will increase by dV_n such that

$$\frac{dE}{dV_n} = BB_n \Rightarrow dV_n = \frac{dE}{BB_n}.$$

Consider also a decrease in the weight of asset m, such that expected return of the portfolio decreases by dE. The variance of the portfolio will decrease by $dV_m = dE/BB_m > dE/BB_n = dV_n$.

The CAPM

- Since the buck for the bang ratio is the same for all assets, it is also the same for all portfolios.

- Therefore, it is the same for asset n and the market portfolio, i.e.,

$$\frac{E(R_n) - R_f}{2Cov(R_n, R_M)} = \frac{E(R_M) - R_f}{2V(R_M)}.$$

- This equation implies that

$$E(R_n) - R_f = \frac{Cov(R_n, R_M)}{V(R_M)}(E(R_M) - R_f).$$

- This is the CAPM.

The CAPM, Beta, and Alpha

- Recall that

$$\beta_n = \frac{Cov(R_n, R_M)}{V(R_M)}.$$

- Therefore, the CAPM is

$$E(R_n) - R_f = \beta_n(E(R_M) - R_f).$$

- Recall that the regression equation implies that

$$E(R_n) - R_f = \alpha_n + \beta_n(E(R_M) - R_f).$$

- Therefore, the CAPM says that $\alpha_n = 0$.

4. The CAPM's Key Insight

The CAPM's Key Insight

- The CAPM is
$$E(R_n) - R_f = \beta_n(E(R_M) - R_f).$$
- An asset's expected return depends on the asset's risk
  - through the asset's beta (systematic risk),
  - and not through the asset's sigma (idiosyncratic risk).
- Key insight:

$$\boxed{\begin{array}{c}\text{It is systematic risk and not idiosyncratic risk}\\ \text{that is priced in the market}\end{array}}$$

In other words:

$$\boxed{\begin{array}{c}\text{The relevant measure of asset risk}\\ \text{is beta and not the variance}\end{array}}$$

Intuition

- Suppose that an asset has zero beta. The CAPM implies that it has the same expected return as the riskless asset.
  - Intuition: The asset's risk is only idiosyncratic and can be diversified. The asset does not contribute to portfolio risk.
- Suppose that an asset has positive beta. The CAPM implies that it has higher expected return than the riskless asset.
  - Intuition: The asset increases portfolio risk.
- Suppose that an asset has negative beta. The CAPM implies that it has lower expected return than the riskless asset.
  - Intuition: The asset reduces portfolio risk.

Linearity

- The CAPM is
$$E(R_n) - R_f = \beta_n(E(R_M) - R_f),$$
and implies that an asset's expected return depends on risk only through beta.

- It also implies that the asset's expected excess return is linear in beta.

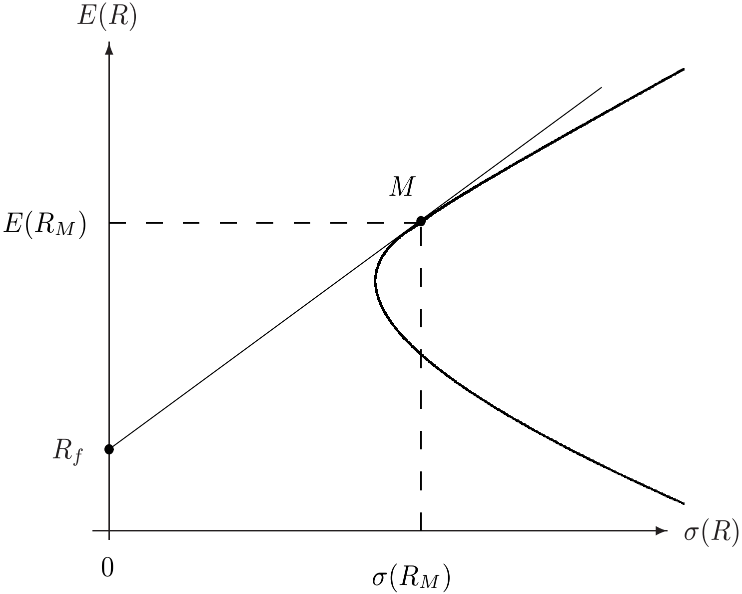
- For instance, if beta is 2, then the asset's expected excess return is twice the market risk premium.

5. CML and SML

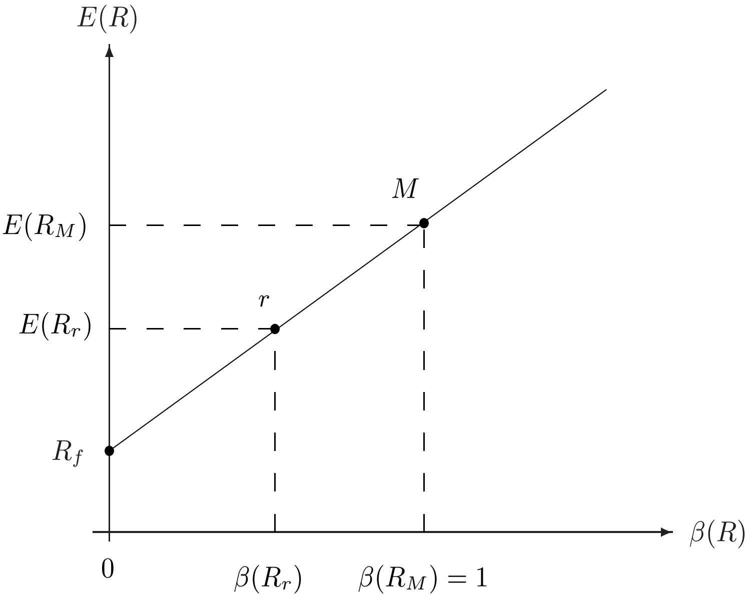
CML and SML

- Two lines which illustrate the CAPM are:
  - the Capital Market Line (CML),
  - the Security Market Line (SML).

The Capital Market Line (CML)



The Security Market Line (SML)

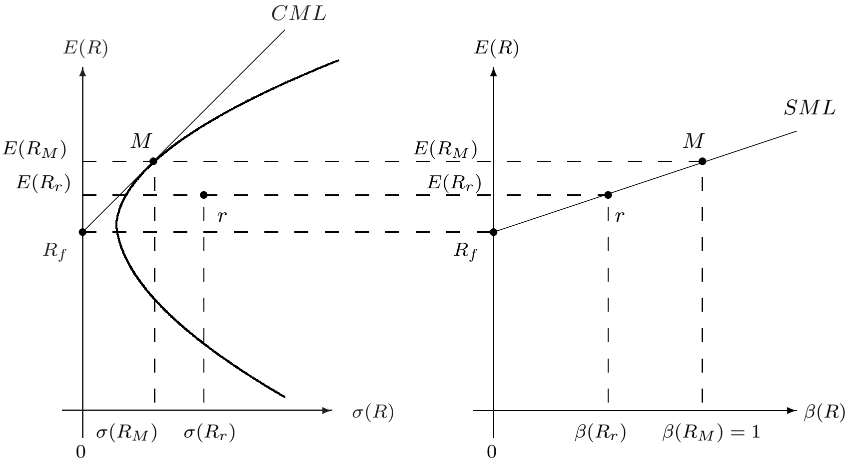


CML vs. SML

- The CML:
  - is in the standard deviation/expected return space.
  - contains only the frontier portfolios.

- The SML:
  - is in the beta/expected return space.
  - contains all portfolios (according to the CAPM).

CML vs. SML: Graph



6. Uses of the CAPM

Uses of the CAPM

- Summary so far:
  - Objective: Obtain some insight on stocks' expected returns.
  - Regression equation

    $$R_n - R_f = \alpha_n + \beta_n(R_M - R_f) + \epsilon_n,$$

    implies that

    $$E(R_n) - R_f = \alpha_n + \beta_n(E(R_M) - R_f).$$

  - CAPM is

    $$E(R_n) - R_f = \beta_n(E(R_M) - R_f),$$

    i.e., $\alpha_n = 0$.
- But how can we use the CAPM?

Uses of the CAPM

- Valuation.
  - Valuation of stocks.
  - Valuation of firms' investments.
  - CAPM provides a risk-adjusted discount rate for the Present Value calculation.
- Performance evaluation.
- Portfolio selection.
  - Estimating expected returns
  - Estimating covariances: This was Sharpe's original intent. Why? 100 assets require estimates of 4950 correlations. In 1950's computers couldn't do it.
- Option pricing: "I applied the CAPM to every moment in an option's life, for every stock price and option value" (Fischer Black)

Portfolio Selection in a CAPM World

- Suppose that we:
  - Estimate the betas of all stocks.
  - Assume that stocks' expected returns are given by the CAPM.
  - Care only about mean and variance.

- What is our optimal portfolio?

Portfolio Selection in a Non-CAPM World

- Portfolio selection in a CAPM world is straightforward.
- The CAPM is still useful, however, because it can guide portfolio selection in a non-CAPM world.
- Suppose that we:
  - Are confident that expected returns of a few stocks are not given by the CAPM.
  - Assume that expected returns of all other stocks are given by the CAPM.
- How would we choose our portfolio?

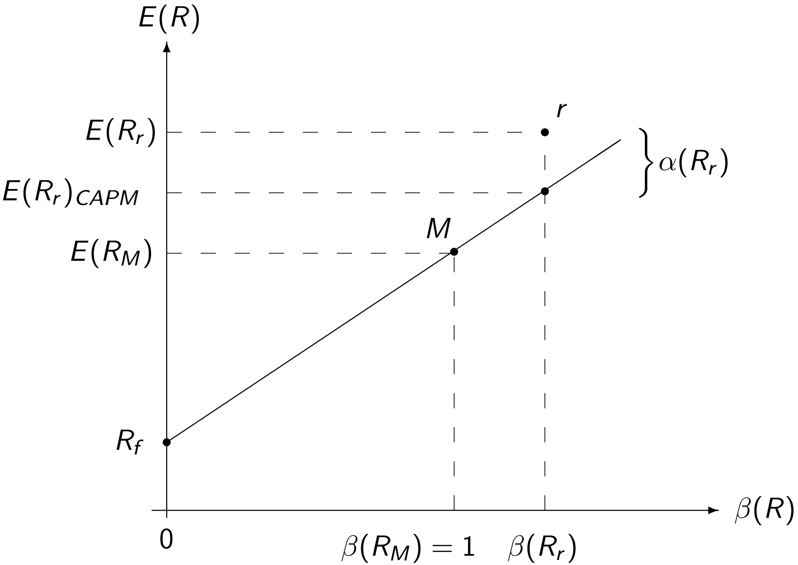
Risk Adjustment

- Compared to its weight in the market portfolio, a stock should get greater weight if
  - its expected return is greater than that given by the CAPM,
  - i.e., if its alpha is positive.

- The CAPM is useful because it provides a benchmark to which we can compare a stock's expected return.

- In other words, the CAPM tells us how to <u>adjust</u> the stock's expected return in order to account for risk.

Back to the SML



7. Statistical Tests of the CAPM

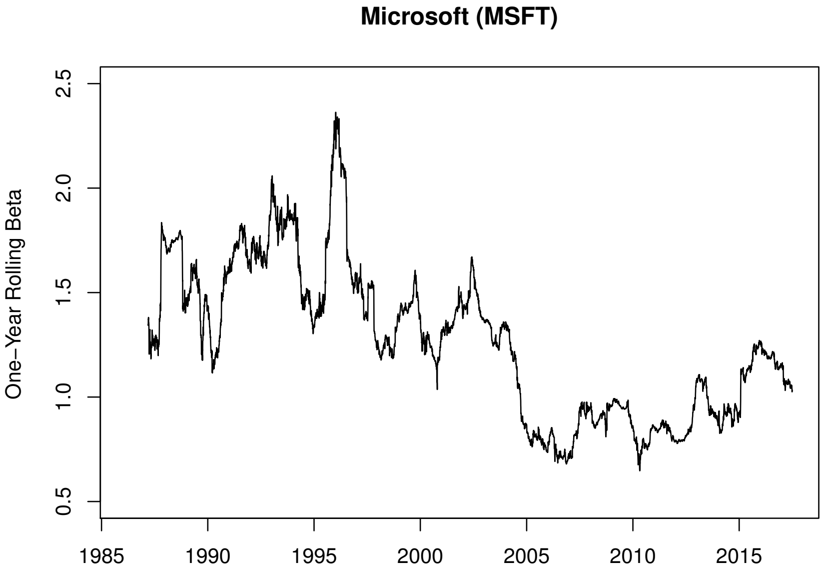
Statistical Tests of the CAPM

- The CAPM is
$$E(R_n) - R_f = \beta_n(E(R_M) - R_f).$$
- Implications:
  1. Expected excess returns are linear in beta.
  2. Slope of the line is market risk premium.
  3. Expected returns depend only on beta.

A Lot of Data Issues

- Which frequency of data?
  - Monthly, weekly, daily?
- How much data?
  - 2 years, 5 years, 10 years?
- What is the market?
  - S&P 500? Wilshire 5000?
- What is the risk free rate?
  - Treasury bill rate?
- How do betas change with time?
  - Does the relationship between the market and stock returns change over time? How serious are these changes?
  - To assess this, compute rolling estimates betas: Every week, use last 1 year of data and re-run the regressions.

Time-Variation in Beta



Remedies

- Can Microsoft's beta move that much?
- Does the past tell us anything about the future?
- How to get more plausible numbers (without throwing out the data)?
- Improving the estimates:
  - Merrill Lynch "adjusts" βs
  - BARRA predicts βs

The ML Beta

- Use our views to improve estimates

- Suppose the data gives a β_s (s for sample), but you think that beta is β_p (p for personal view).

- Form a weighted average:

$$\beta = w_p \beta_p + (1 - w_p)\beta_s,$$

  where w_p is weight you place on your personal view.

- Merrill Lynch takes $\beta_p = 1$, and $w_p = 1/3$.
  - Why? $\beta = 1$ is the average of all betas (market's beta)

- You could use any other combinations you want.

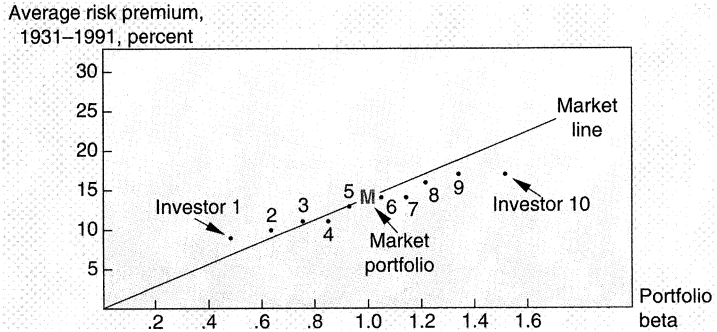
The BARRA Beta

- Can they be predicted? Yes. Run regressions
- Let β_t be today's beta.
- Estimate new beta using past betas: regression

$$\beta_{t+1} = c_0 + c_1\beta_t + \varepsilon_{t+1}$$

- BARRA (sells investment advice) argues this is better than traditional ways of estimating betas.
- Easy to change βs. What do they mean now?

Implications 1 and 2



Source: Black, Fischer, 1993, Beta and return, *The Journal of Portfolio Management*.

Conclusions:

- Expected excess returns are approximately linear in beta.
- Slope of the line is smaller than market risk premium.

Implication 3

Do expected returns depend on other factors, in addition to beta?

- They do not seem to depend on sigma (idiosyncratic risk).

- They seem to depend on
  - Size.
  - Liquidity.
  - Value.
  - Momentum.

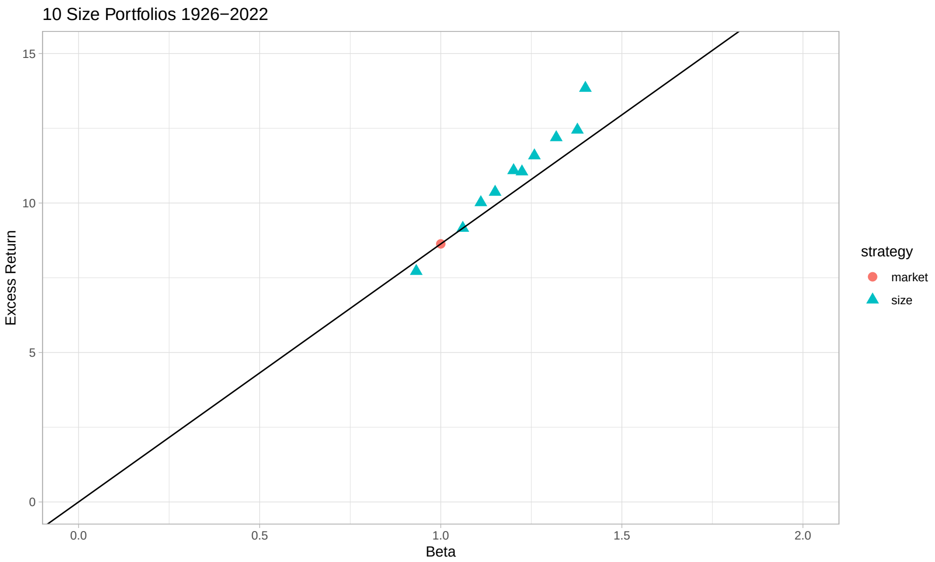
Size and Liquidity Effects

- **Size effect:** Expected returns of small stocks exceed those of large stocks (holding beta equal).
- **Liquidity effect:** Expected returns of low-liquidity stocks exceed those of high-liquidity stocks (holding beta equal).
- The two effects are related because liquidity increases with size.
- Yet, they are not the same effect.

Size effect: Portfolio construction

- Let's look at all the stocks in CRSP
- Sort stocks into deciles based on market cap at the end of June each year
- Portfolios are held fixed throughout year
- The portfolios for July of year t to June of $t + 1$ include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for June of year t
- Source: Kenneth French's data library

Puzzles: Size effect



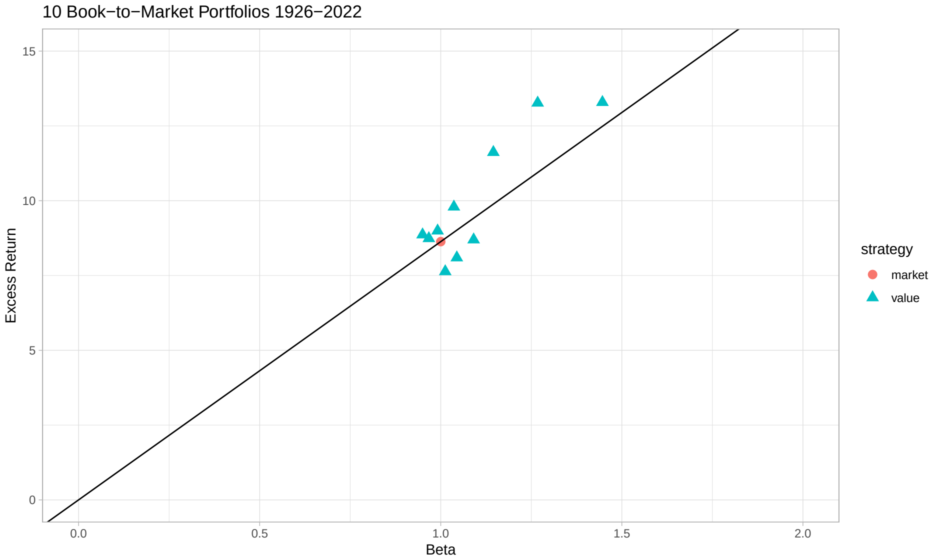
Value Effect

- Compare price to accounting measures, e.g., book value or earnings.
  - High book-to-market ratio: Value stocks.
  - Low book-to-market ratio: Growth stocks.
- **Value effect:** Expected returns of value stocks exceed those of growth stocks (holding beta equal).
- **Reversal effect:** Expected returns of stocks with long history of underperformance exceed those with long history of overperformance (holding beta equal).
- The two effects are related.
  - Long history of underperformance $\Rightarrow$ High book-to-market ratio.
  - Long history of overperformance $\Rightarrow$ Low book-to-market ratio.

Value effect: portfolio construction

- Let's look at all the stocks in CRSP
- Sort stocks into deciles based on book-to-market ratios at the end of June each year; high book-to-market stocks are 'value' stocks
- Portfolios are held fixed throughout year
- The portfolios for July of year t to June of $t+1$ include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for June of year t

Puzzles: Value effect



Momentum Effect

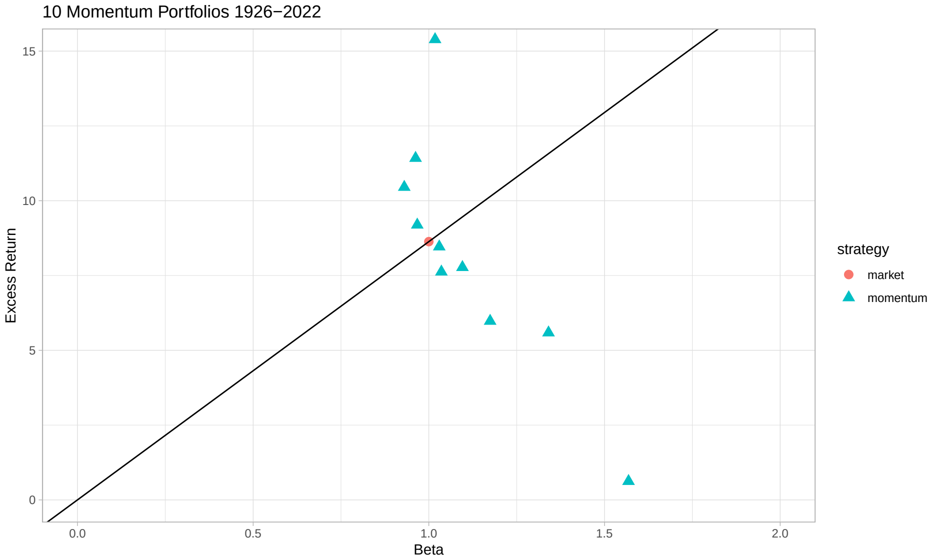
- **Momentum effect:** Expected returns of stocks with short history of overperformance exceed those with short history of underperformance (holding beta equal).

- Momentum is not inconsistent with Reversal/Value.
  - Momentum: Performance over recent history (3 months-1 year) is expected to continue.
  - Reversal: Performance over longer history (3-5 years) is expected to reverse.

Momentum effect: portfolio construction

- Let's look at all the stocks in CRSP
- Sort stocks into deciles based on prior return (-2 to -12) deciles at the end of June each year; high prior return stocks are 'momentum' stocks
- Portfolios are held fixed throughout year
- The portfolios for July of year t to June of $t+1$ include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for June of year $t$and a stock must have a price for the end of month $t-13$ and a good return for $t-2$.

Puzzles: Momentum effect



Possible Explanations

- Value, momentum, size and liquidity effects are important challenges to the CAPM.

- Possible explanations.
  - Risk.
  - Irrationality.
  - Frictions.

Risk

- Assume multiple risk factors.

- Example: Value effect can be explained if
  - There is an additional factor to the market portfolio, carrying a positive risk premium.
  - Value stocks have higher beta with respect to that factor than growth stocks.

- Empirical multi-factor models: Factors are
  - Market portfolio.
  - Value portfolio (HML).
  - Momentum portfolio (WML).
  - Size portfolio (SMB).

- What is economic interpretation of these factors?

Irrationality

- Assume that investors process information incorrectly.
- Example: Value effect can be explained if
  - Investors are too optimistic about future earnings of some stocks (overpricing them) and too pessimistic about future earnings of other stocks (underpricing them).
- Example: Momentum effect can be explained if
  - Optimism/pessimism builds gradually...
  - ... and this is not anticipated by rational investors.
- Do biases aggregate?
- Rational investors must have limited capital.

Frictions

- Assume that investors are rational but there are frictions arising because of
  - Transaction costs and illiquidities.
  - Delegation of portfolio management and agency problems.

- Example: Value and momentum effects can be explained if
  - Investors invest through asset managers.
  - Following a manager's poor performance, investors update negatively on manager's ability and withdraw funds gradually.
  - Manager sells stocks following the withdrawals.

- Investors not subject to the frictions must have limited capital.

Summary

- The CAPM is a simple and intuitive model, used in practice.
- Statistical tests:
  - The CAPM does OK but not great.
  - A four-factor model with market portfolio, value, momentum and size does better than the CAPM.
  - However, statistical tests are subject to measurement and data mining issues.
- One (popular) view: While the CAPM may not be perfect, it is unlikely that a truly perfect model will be found in the foreseeable future. In that sense, the CAPM may be good enough, especially relative to the cost of implementing a more sophisticated model.