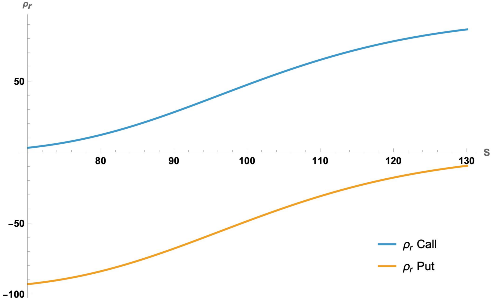
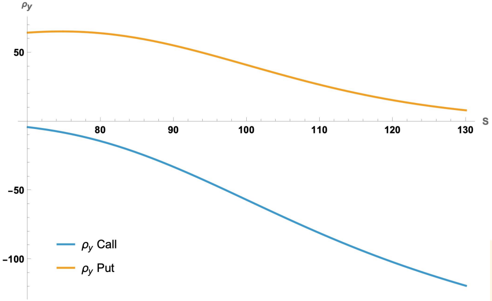
4.2. Sensitivities (3)

- Ordinarily, one proceeds from Δ_S to Γ_{SS}, but there is a close relationship between deltas and rhos.
- **Interest rate rho**:

$$\rho = \rho_r \text{ (or } \rho_B) \doteq \frac{\partial\{C,P\}}{\partial r} = \partial_r\left\{\phi\left[Se^{-yT}\mathcal{N}(\phi z_+) - Ke^{-rT}\mathcal{N}(\phi z_-)\right]\right\} = \phi\, K\, T\, e^{-rT}\mathcal{N}(\phi z_-)$$





- **Yield rho** (formerly sometimes nu = ν):

$$\rho_y \text{ (or } \rho_S) \doteq \frac{\partial\{C,P\}}{\partial y} = \partial_y\left\{\phi\left[Se^{-yT}\mathcal{N}(\phi z_+) - Ke^{-rT}\mathcal{N}(\phi z_-)\right]\right\} = -\phi\, S\, T\, e^{-yT}\mathcal{N}(\phi z_+)$$

- Alternatively, if we define rate/yield vector $\underline{\mathbf{r}}$ with, e.g., $r_0 = r$ and $r_1 = y$ (corresponding to assets in $\underline{\mathbf{S}}$), then rho vector can be defined as: $\underline{\rho} \doteq \nabla_{\underline{\mathbf{r}}}\{C,P\}$ with $\rho_i \doteq \partial\{C,P\}/\partial r_i$
- Some firms use scaled rho, e.g. $0.01\frac{\partial\{C,P\}}{\partial r}$ or $0.0001\frac{\partial\{C,P\}}{\partial r}$. Conventions vary...