Lecture 7b
ARIMA models and intro to cointegration

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Outline

1. ARIMA models
   - The "I" stands for integrated
2. Introduction to cointegration
   - Pairs trading

ARIMA models

ARIMA models

We have learned all about ARMA models

- But, sometimes you will encounter ARIMA models

Example:

$$
\begin{aligned}
p_t &= p_{t-1} + x_t, \\
x_t &= \phi x_{t-1} + \varepsilon_t, \quad |\phi| < 1
\end{aligned}
$$

1. x_t is stationary
2. p_t is not

BUT: if we first difference p_t we get a stationary process: $\Delta p_t = x_t$

- If we only have to difference once to achieve stationarity we say p_t is integrated of order 1
- If we have to difference n times, we say p_t is integrated of order n

ARIMA models in practice

Say you estimate an ARIMA(1,1,1) on p_t

- Step 1: difference the series, $\Delta p_t = x_t$
- Step 2: estimate an ARMA(1,1) on Δp_t (which will give you a statistical zero on MA term as Δp_t is an AR(1) in our example)

So, why use ARIMA models at all?

- We should just first make sure we have stationary series and then estimate ARMA
- But, sometimes we want to forecast the price in the future, or level of GDP, etc.
- The ARIMA setup adds this to the ARMA estimation (see next slide)

Forecasting with ARIMA models

We can calculate

$$
\begin{aligned}
E_t\left(p_{t+1}\right) &= p_t + E_t\left(x_{t+1}\right) \\
E_t\left(p_{t+2}\right) &= E_t\left(p_{t+1}\right) + E_t\left(x_{t+2}\right) \\
&= p_t + E_t\left(x_{t+1}\right) + E_t\left(x_{t+2}\right), \\
& \quad etc.
\end{aligned}
$$

Thus: we use the ARMA model to forecast the stationary component, and then use $\Delta p_t = x_t$ to forecast the price level

Note that this can only be done conditionally, as the unconditional price level is nonstationary (so unconditional mean and variance do not exist)

Introduction to cointegration

Using prices in estimation

So, can I ever estimate models with nonstationary variables without differencing?

- Yes, if you have cointegration

Let $z_t = ap_t + bx_t + \varepsilon_t$, where ε_t is White noise

- z_t is nonstationary since p_t is nonstationary
- But, $z_t - ap_t$ is stationary:

$$\begin{aligned} z_t - ap_t &= ap_t + bx_t + \varepsilon_t - ap_t \\ &= bx_t + \varepsilon_t. \end{aligned}$$

We say z_t and p_t are cointegrated with cointegrating vector $[1, -a]$.

- They share a common stochastic trend, which can be eliminated through judicious weighting

Examples from economics and finance

In economics, we can often use theory to find the cointegrating vector

1. Prices and earnings must share a long-run trend. Thus, it is reasonable to think that the log of price and the log of earnings have cointegrating vector $[1, -1]$. In other words, the price earnings ratio $\ln P_t / E_t = p_t - e_t$ is stationary!
2. The same argument can be applied to the price-dividend, the book-to-market, and the price-to-rent ratios, for instance

Another concept: generalized differencing

- You don't need to always take a first difference. Any moving average with weights that sum to 1 will do
- For instance,

$$p_t - \frac{1}{K}\sum_{k=1}^{K} p_{t-k} \quad \text{is stationary}$$

  Prove this yourself!

Pairs trading

Pairs trading refers to the idea that two assets (e.g., stocks) “move together” in the long-run

- Thus, if prices are currently “far apart” they are likely to converge in the future
- So, buy relatively low-priced asset, short relatively high-priced asset, reap high future returns.

But, are the prices really cointegrated?

- I.e., can we find a coefficient a such that $p_{1,t} - a \times p_{2,t}$ is stationary?

Often: probably not

- E.g., maybe Apple and Google have prices that in some sense are likely to converge
- But, unlikely they are cointegrated (move together in the long-run)
- For instance, what if in the future one of the companies go bankrupt and the other doesn’t?

Pairs trading (cont'd)

Avoiding this issue

- Transform prices to stationary quantities
- Make sure transform preserves long-run idea
- E.g.: price versus 5-year moving average price, or price-to-book

The simplest model:

$$r_{1,t+1} - r_{2,t+1} = \alpha + \beta \left(\tilde{p}_{1,t} - \tilde{p}_{2,t} \right) + \varepsilon_{t+1},$$

where, for example, $\tilde{p}_{j,t} = p_{j,t} - \frac{1}{K} \sum_{k=1}^{K} p_{j,t-k}$, where K corresponds to 5 years

- A negative β implies that one should short asset 1 and long asset 2 when $\tilde{p}_{1,t} > \tilde{p}_{2,t}$
- Of course, you could consider a more complicated model to predict the return differential

More on cointegration in the Note on Cointegration on BruinLearn (optional, we don't have time)