

# Lecture 1: Time Series Properties of Stock Market Returns

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Winter 2025

# Overview of Lecture 1

## Time series properties of stock market returns

- Moments
- Distributions
- The failure of a simple benchmark model of returns
- Risk metrics
- Estimating means and variances

# Prices and Returns: A Brief Review

$P_t$  denotes the price of an asset at time  $t$

$D_t$  denotes the dividend of an asset at time  $t$

**simple gross return:**

$$1 + R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

**simple net return:**

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1$$

Where  $R_{t+1}$  is short hand for  $R_{t,t+1}$ ; the return from holding the asset from time  $t$  to time  $t + 1$

# Multi-Period Returns

suppose we buy an asset at  $t$ , we reinvest all of the dividend payments in the same risky asset and we sell at  $t + k$

suppose  $k = 2$

$$1 + R_{t,t+2} = \frac{P_{t+1} + D_{t+1}}{P_t} \times \frac{P_{t+2} + D_{t+2}}{P_{t+1}} = (1 + R_{t+1}) \times (1 + R_{t+2})$$

then the  $k$ -period return is:

$$1 + R_{t,t+k} = (1 + R_{t+1}) \times (1 + R_{t+2}) \times \dots \times (1 + R_{t+k})$$

# Log Returns

**continuously compounded** or log return:

$$r_t = \log(1 + R_t) = \log\left(\frac{P_t + D_t}{P_{t-1}}\right),$$

log returns are easy to work with, especially over longer holding periods, because they're additive!

multi-period log return:

$$r_{t,t+k} = \log(1 + R_{t,t+k}) = r_{t+1} + r_{t+2} + \dots + r_{t+k}$$

statistically, it's easier to work with log returns because of additivity

## A simple example

Buy a stock at \$85 and sell at \$87.50

the **gross return** is:

$$1 + R_{t+1} = \frac{87.5}{85} = 1.0294$$

the **simple net return** is:

$$R_{t+1} = \frac{87.5}{85} - 1 = 0.0294$$

**continuously compounded** or log return:

$$r_t = \log\left(\frac{87.5}{85.0}\right) = 0.0289$$

## Excess returns

Often we are interested in *excess returns*

- That is, returns in excess of some reference return

The standard definition is returns in excess of the risk-free rate:

$$R_{i,t}^e = R_{i,t} - R_{f,t}$$

- Here,  $i$  refers to the asset (e.g., AAPL stock),  $f$  refers to a risk-free rate such as the 1-month T-bill, and  $t$  is time

Example: market return in month  $t$  was 6.5%, risk-free rate was 0.2%.

- The monthly excess return was then 6.3%.
- If the market return instead was  $-6.5\%$ , the excess return would be  $-6.7\%$

The average excess return as defined above is referred to as the *Risk Premium*

$$\text{Risk Premium} = E[R_{i,t}^e]$$

where  $E[\cdot]$  refers to the *expectations operator* (expected value)

- A long sample average is an unbiased estimate of the expected value

# Stylized facts

a **stylized fact** is something that is generally true but not always.

- Let's look at some stylized facts about stock market returns
- We will use the simple log-normal model as a benchmark

$$r_t \sim N(\mu, \sigma^2)$$

- Let's see how well it does at fitting:
  - ▶ daily returns
  - ▶ monthly returns
- In what ways are assumptions in the benchmark model violated?

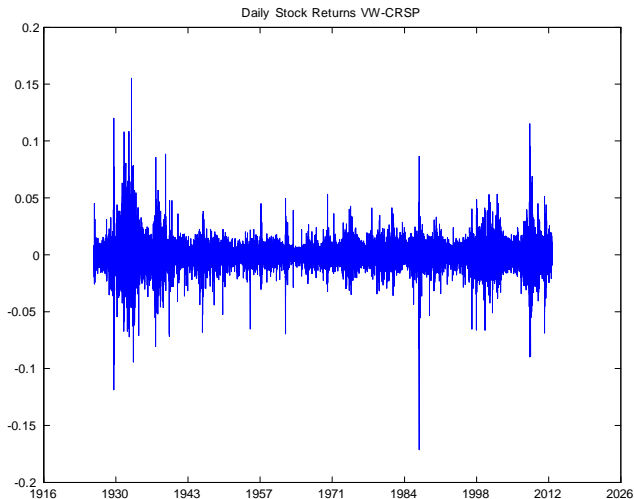
# First: A brief review of the log-normal distribution

Gross returns  $(1 + R_t)$  are lognormally distributed if the natural logarithm  $(r_t)$  is normally distributed

- Since,  $1 + R_t = e^{r_t}$  this implies gross returns can never go negative
  - ▶ Limited liability!
- If gross returns are lognormal, then cumulative gross returns are also lognormal
  - ▶ Easiest to see using sums of the log returns. A sum of normally distributed variables is also normal
  - ▶ E.g., if returns are i.i.d. (*identically, independently distributed*; in this case uncorrelated normal with mean  $\mu$  and variance  $\sigma^2$ )

$$r_{t,t+k} = r_{t+1} + r_{t+2} \dots + r_{t+k} \sim N(k\mu, k\sigma^2)$$

# Daily stock market returns



Daily log returns for CRSP Value-Weighted market index 01/1925 - 12/2012

# Daily Log-Normal Returns

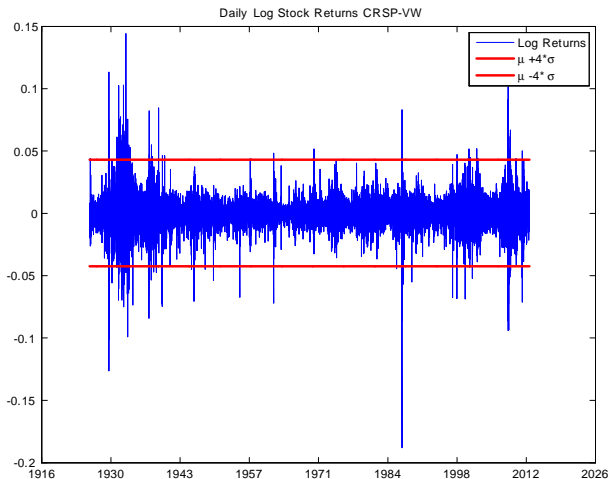
if returns are log-normal, we should not see any returns greater than  $4\text{-}\sigma$ 's:

$$\Pr\left(\frac{r_t - \mu}{\sigma} < -4\right) \approx 0$$

$$\Pr\left(\frac{r_t - \mu}{\sigma} > 4\right) \approx 0$$

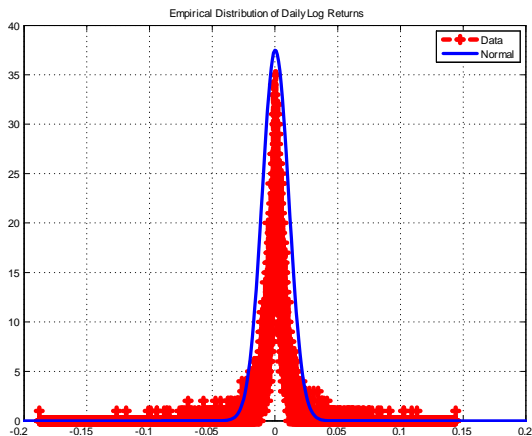
- but we see plenty of those in the data...(see next slide)
- returns are not lognormal (at least not with a constant variance!)

# Daily Stock Market Returns



Daily log returns for CRSP Value-Weighted market index 01/1925 - 12/2012  
 $\mu$  is 0.0346%;  $\sigma$  is 1.07%

# Empirical Distribution of Daily Stock Market Returns



Empirical and Normal density for daily log returns

- Daily returns have *fat tails*.
  - ▶ The empirical distribution is *leptokurtic*, meaning the *excess kurtosis* is greater than zero.
- Is it *skewed*? Let's first remind ourselves what these moments are...

## Quick reminder of "moments"

- The  $j$ 'th moment of a random variable  $X$  is

$$m'_j = E[X^j] = \int_{-\infty}^{\infty} x^j f(x) dx$$

- The  $j$ 'th *central* moment of a random variable  $X$  is

$$m_j = E[(X - \mu_X)^j] = \int_{-\infty}^{\infty} (x - \mu_X)^j f(x) dx$$

- ① The first moment is the mean,  $\mu_X$
- ② The second central moment is the variance of  $X$

$$m_2 = E[(X - \mu_X)^2] = \int_{-\infty}^{\infty} (x - \mu_X)^2 f(x) dx$$

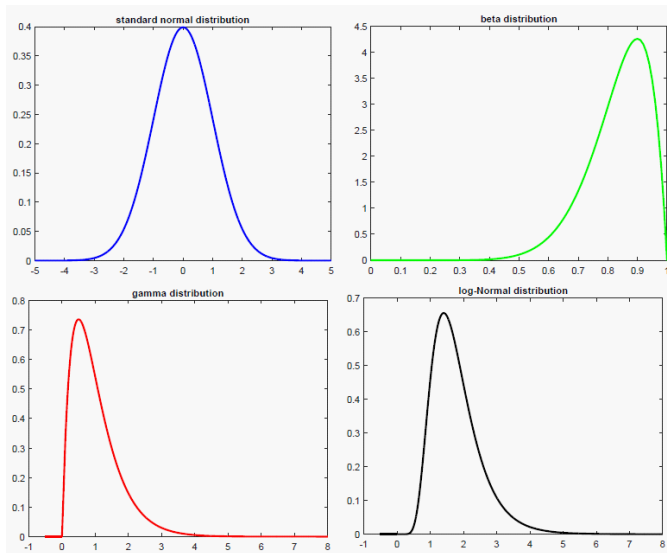
# Skewness

the **skewness** of a random variable  $X$  is defined as the third central moment normalized by the standard deviation:

$$S(X) = E \left[ \frac{(X - \mu_X)^3}{\sigma^3} \right]$$

- Skewness is a measure of the degree of asymmetry of a distribution.
  - ▶ If the left tail (tail at small end of the distribution) is more pronounced than the right tail (tail at the large end of the distribution), the function is said to have negative skewness.
  - ▶ If the reverse is true, it has positive skewness.
  - ▶ If the two are equal, it has zero skewness.
  - ▶ The Normal distribution is symmetric and therefore has zero skewness.

# Skewness: Examples



# Kurtosis

the **kurtosis** of a random variable  $X$  is defined as the fourth central moment normalized by the standard deviation:

$$K(X) = E \left[ \frac{(X - \mu_X)^4}{\sigma^4} \right]$$

- Kurtosis is a measure of the heaviness of the tails of the distribution.
  - ▶ The Normal distribution has kurtosis of 3.
  - ▶ **Excess Kurtosis** is kurtosis in excess of 3,  $K(X) - 3$

# Student's $t$ distribution

Let  $X \sim t(\mu, \sigma^2, \nu)$

- The parameter  $\nu$  is the **degrees of freedom**
- The mean, variance, skewness, and kurtosis of this distribution are:
  - ▶  $E[X] = \mu$  if  $\nu > 1$
  - ▶  $V[X] = \sigma^2$  if  $\nu > 2$
  - ▶  $S[X] = 0$  if  $\nu > 3$
  - ▶  $K[X] = 3 + \frac{6}{\nu-4}$  if  $\nu > 4$
- The Student's  $t$  distribution has heavier tails than a Normal
- as  $\nu \rightarrow \infty$ , the Student's  $t$  distribution converges to a Normal
- The  $t$ -distribution arises naturally in Bayesian statistics.  $r_t = \sigma \epsilon_t$ , where  $\sigma \sim IG(a, b)$  implies that the distribution of  $r_t = \int \sigma \epsilon_t d\sigma$  is a  $t$ -distribution.  
Lesson: parameter uncertainty generates fat tails.

## Example: The Bernoulli distribution

Consider a Bernoulli distribution

- the probability mass function is

$$P(x) = \begin{cases} 1-p & \text{if } x = 0 \\ p & \text{if } x = 1 \end{cases}$$

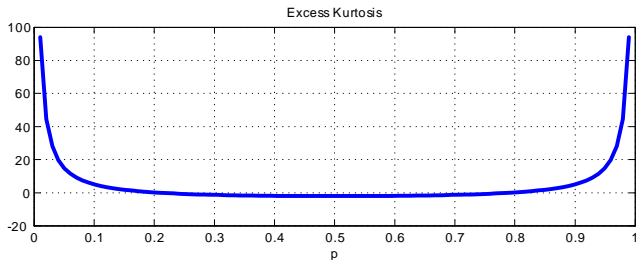
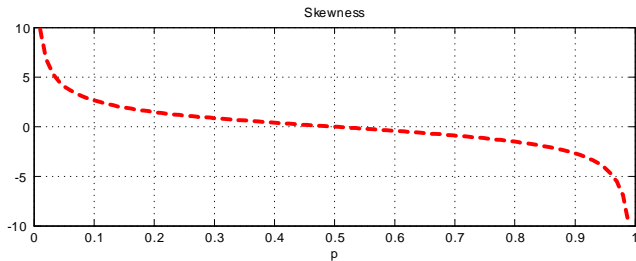
- Example: flip a coin
- The variance, skewness, and excess kurtosis

$$V(X) = p(1-p)$$

$$S(X) = \frac{1-2p}{\sqrt{p(1-p)}}$$

$$K(X) - 3 = \frac{6p^2 - 6p + 1}{p(1-p)}$$

# Example: The Bernoulli distribution



# Skewness Test

- Suppose we have data on asset log returns  $\{r_1, r_2, \dots, r_T\}$
- Test whether series is skewed with the following null hypothesis

$$H_0: S_r = 0$$

- Compute the following **t-ratio** or **t-statistic**:

$$t = \frac{\hat{S}(r)}{\sqrt{6/T}}$$

- Reject the null hypothesis at the 5% level if  $|t| > 1.96$  if the sample is large
  - ▶ under the null,  $t$  is asymptotically (as  $T \rightarrow \infty$ ) standard normal

# Kurtosis Test

- Suppose we have data on asset log returns  $\{r_1, r_2, \dots, r_T\}$
- Test whether series has fat tails with the following null hypothesis

$$H_0: K_r - 3 = 0$$

- Compute the following **t-ratio** or **t-statistic**:

$$t = \frac{\hat{K}(r) - 3}{\sqrt{24/T}}$$

- Reject the null hypothesis at the 5% level if  $|t| > 1.96$  if the sample is large
  - ▶ under the null,  $t$  is asymptotically (as  $T \rightarrow \infty$ ) standard normal

# Jarque and Bera (1980) Normality Test

we can combine the skewness and kurtosis test to test normality.

- the Jarque-Bera test statistic is given by:

$$JB = \frac{\hat{S}(r)^2}{6/T} + \frac{(\hat{K}(r) - 3)^2}{24/T}$$

which is  $\chi^2$ -distributed with 2 degrees of freedom

- let  $CHI_\alpha$  denote the  $(1 - \alpha) \times 100^{th}$  quantile of  $\chi^2(2)$
- reject null if  $JB > CHI_\alpha$ 
  - ▶ the sum of two squared standard normals is  $\chi^2$  with 2 degrees of freedom

# Sample moments of daily stock market returns

Portfolio	S&P500	CRSP – VW	CRSP – EW
Daily log Returns			
<i>std</i>	1.03	1.07	1.06
<i>skewness</i>	-1.03	-0.44	-0.07
<i>t – test</i>	[-63.92]	[-27.41]	[-4.59]
<i>kurtosis</i>	30.53	20.42	26.86
<i>t – test</i>	[852.74]	[539.50]	[739.05]
<i>JB – test</i>	731247.51	291810.13	546216.30
<i>p-value JB</i>	0.00	0.00	0.00

Sample Moments of log returns (in percentage points). Daily data. Sample is 01/1925 - 12/2012. T-tests in brackets.

- how does the *t*-test work?

- ▶ set  $\alpha = 0.05$  (this is referred to as the significance level)
- ▶ reject the null hypothesis at the  $\alpha = 0.05$  significance level if  $|t| > 1.96$
- ▶ this is a two-sided test

# Sample moments of daily stock market returns

Portfolio	S&P500	CRSP – VW	CRSP – EW
Daily log Returns			
<i>std</i>	1.03	1.07	1.06
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Sample Moments of Daily log returns (in percentage points). Daily data. Sample is 01/1925 - 12/2012. T-tests in brackets.

- how does the  $\chi^2$ -test work?
  - ▶ set  $\alpha = 0.05$  (this is referred to as the significance level)
  - ▶ let  $\text{CHI}_\alpha = 5.99$  denote the 95-th quantile of the  $\chi^2(2)$
  - ▶ reject null  $H_0$  if  $\text{JB} > \text{CHI}_\alpha = 5.99$  which is the value of the  $\chi^2(2)$
  - ▶ this is a one-sided test

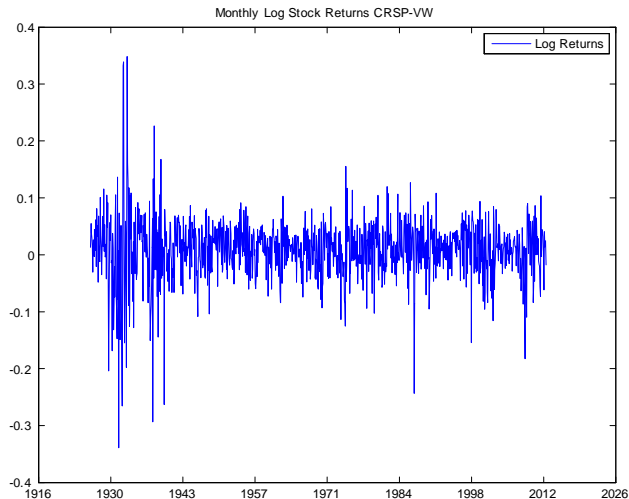
# Summary: Daily Log Stock Market Returns

Consider the simple log-normal model as a benchmark

$$r_t \sim N(\mu, \sigma^2)$$

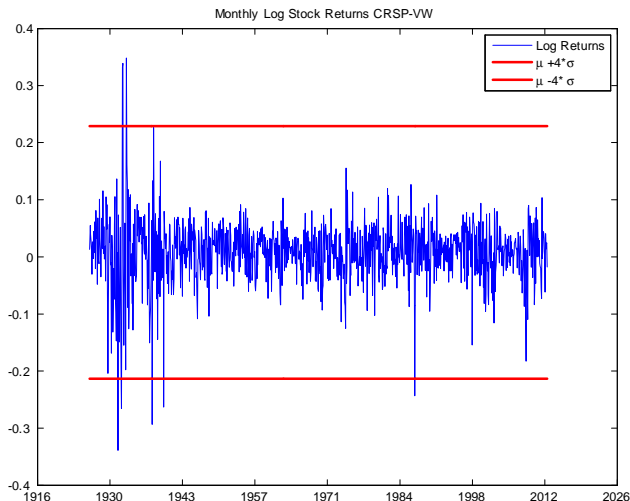
- Not a good model for daily log-returns  $r_t$ !
  - ▶ data is negatively skewed
  - ▶ data has high kurtosis: very heavy tails!
  - ▶  $\implies$  prices move too much from day-to-day to be normally distributed
  - ▶ From the time series plots, the variance  $\sigma^2$  does not appear to be the same each time period.
  - ▶  $\implies$  time-varying variance  $\sigma_t^2$
  - ▶ Clearly, the **identically distributed** assumption fails.

# Monthly stock market returns



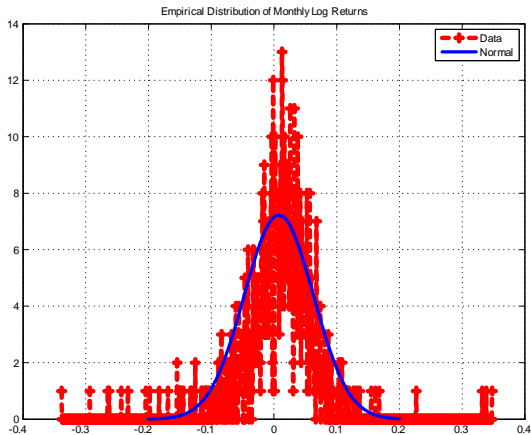
Monthly log returns for CRSP Value-Weighted market index 01/1925 - 12/2012

# Monthly stock market returns



Monthly log returns for CRSP Value-Weighted market index 01/1925 - 12/2012  
 $\mu$  is 0.77%;  $\sigma$  is 5.52%

# Empirical Distribution of Monthly Stock Market Returns



Empirical and Normal density for monthly log returns

# Sample moments of monthly stock market returns

<i>Portfolio</i>	<i>S&amp;P500</i>	<i>CRSP – VW</i>	<i>CRSP – EW</i>
Monthly log Returns			
<i>std</i>	5.52	5.53	6.73
<i>skewness</i>	−0.53	−0.46	0.03
<i>t – test</i>	[−6.91]	[−6.00]	[0.44]
<i>kurtosis</i>	10.72	10.73	13.55
<i>t – test</i>	[50.78]	[50.81]	[69.36]
<i>JB – test</i>	[2626.42]	[2617.71]	[4811.08]
<i>p-value JB</i>	0.00	0.00	0.00

Sample Moments of monthly log returns (in percentage points). Sample is 01/1925 - 12/2012. T-tests in brackets.

# Summary: Monthly Log Stock Market Returns

Consider the simple log-normal model as a benchmark

$$r_t \sim N(\mu, \sigma^2)$$

- Monthly log returns  $r_t$  are closer to this benchmark model, but still not great
  - ▶ data is negatively skewed
  - ▶ kurtosis  $> 3$ : still heavier tails than the Normal
  - ▶ the variance  $\sigma^2$  still appears to vary over time, though not as much as with daily data
  - ▶ the **identically distributed** assumption still fails.

# Tail Risk

- you can buy and hold the market portfolio: buy a market index fund.
- even with a market index, you are exposed to tail risk.
  - ▶ evidence of tail risk in passively managed stock market index
- even more evidence of tail risk in actively managed portfolios
  - ▶ writing out-of-the-money put options: excess kurtosis of 16.64 reported by Broadie, Chernov, and Johannes (2009).
  - ▶ currency carry trade: Brunnermeier, Nagel, and Pedersen (2009)
  - ▶ momentum portfolios in stocks: Daniel, Jagannathan, and Kim (2012)
  - ▶ other hedge fund strategies look like writing out of the money put options Jurek and Stafford (2015)

# Long and short positions

**long position:** buy an asset and hold it for  $n$  periods

**short position:** sell an asset you do not have (by borrowing it) and then buy it back after  $n$  periods

# Zero-cost portfolios

**zero-cost portfolios:** start with \$0 and build a portfolio

for example, we borrow \$1 at the risk-free rate and invest it in the risky asset;  
that's a self-financing portfolio

you borrow \$1 in the risk-free asset and go long in asset  $i$ , earning a return:

$$R_{i,t}^e = R_{i,t} - R_{0,t}$$

this is referred to as an **excess return**

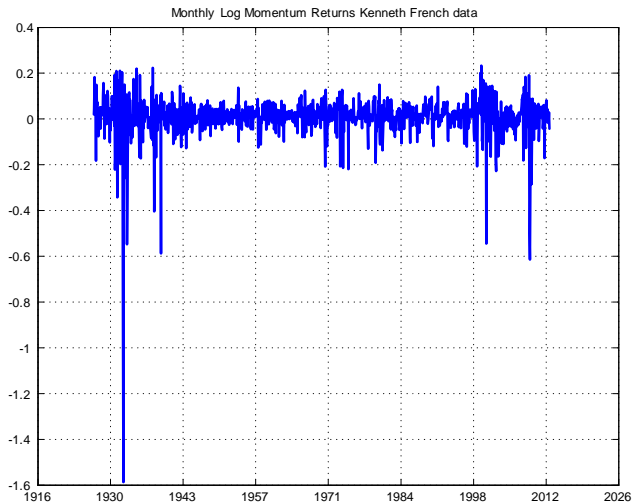
but you could short other assets too, not just the risk-free asset

- you short \$1 of asset  $j$  and you go long in \$1 of asset  $i$ , earning a return:

$$R_{ij,t}^e = R_{i,t} - R_{j,t}$$

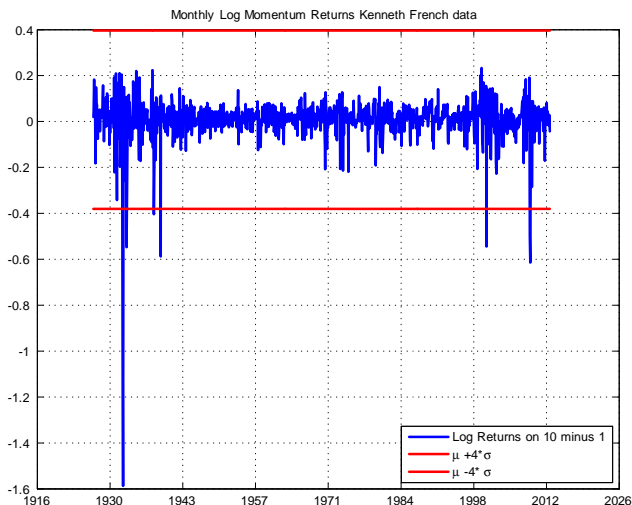
- E.g., Momentum: long 10th decile prior return sorted portfolio, short 1st decile prior return sorted portfolio

# Monthly Momentum Stock Returns



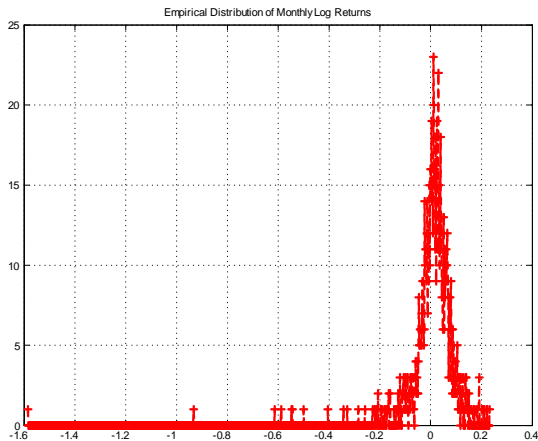
Monthly log returns - Kenneth French 10 momentum portfolios (10-minus-1) .  
01/1927-7/2015.  $\hat{K}$  is 75.92.  $\hat{S}$  is -5.924.

# Monthly Momentum Stock Returns



Monthly log returns - Kenneth French 10 momentum portfolios (10-minus-1) .  
01/1927-7/2015.  $\mu$  is 0.84%.  $\sigma$  is 9.37%.

# Empirical Distribution of Monthly Log Momentum Returns



Empirical density for monthly log returns

# Portfolio Theory: Mean-Variance Case

Classic portfolio theory: investors trade off mean against variance when choosing the efficient portfolio (see intro slides)

- makes sense when returns are normally distributed.
- mean and variance are sufficient statistics for the normal distribution
  - ⇒ Sharpe ratio is sufficient performance statistic: the efficient portfolio is the one with the highest possible *Sharpe ratio*

# Sharpe Ratio: Sharpe (1965)

- the Sharpe Ratio is defined as the expected return of an asset  $R_{it}$  relative to a benchmark asset  $R_{jt}$  divided by the standard deviation of  $R_{it}$

$$SR(R_{it}) = \frac{E[R_{it} - R_{jt}]}{Std(R_{it} - R_{jt})}$$

- historically, the benchmark asset is the risk-free rate  $R_{ft}$ .
- the Sharpe Ratio measures the 'reward' per unit 'risk' of an asset.
- the Sharpe ratio is simple & intuitive but does not fully reflect risk if higher order moments matter.

# Sharpe Ratio and Leverage

An important and useful property of the Sharpe Ratio is that it is not affected by *leverage*

- Leverage refers to borrowing or lending money in combination with your risky asset investment

Let's go through an example of how leverage affects portfolio risk and return

- Assume the stock market has expected return 7% and standard deviation of 18%.
- Assume the risk-free rate is 1% with a standard deviation of 0% (it's known today)

What is the Sharpe ratio of the market portfolio?

$$SR(R_m) = \frac{E[R_m - R_f]}{\sigma(R_m - R_f)} = \frac{7\% - 1\%}{18\%} = \frac{1}{3}.$$

## Sharpe Ratio and Leverage: continued

Consider an investor that has \$100. She chooses to borrow \$50 at the risk-free rate and put \$150 in the stock market.

- What is the wealth of this investor?  $\$100 = \$150 - \$50$ .
- What is the portfolio weight in the risky asset?  $1.5 = \$150 / \$100$ .
- What is the portfolio weight in the risk-free asset?  $-0.5 = -\$50/\$100$
- What is the portfolio expected excess return?

$$\begin{aligned}E[R_p] - E[R_f] &= 1.5 \times E[R_m] - 0.5 \times E[R_f] - E[R_f] \\&= 1.5 \times E[R_m - R_f] = 1.5 \times 6\%\end{aligned}$$

- What is the portfolio standard deviation?

$$\begin{aligned}\sigma(R_p) &= \sqrt{(1.5)^2 (18\%)^2 + (-0.5)^2 (0)^2 - 1.5 \times 0.5 \times 0} \\&= 1.5 \times 18\%\end{aligned}$$

- What is the portfolio Sharpe ratio?

$$SR(R_p) = \frac{E[R_p - R_f]}{\sigma(R_p - R_f)} = \frac{1.5 \times 6\%}{1.5 \times 18\%} = \frac{1}{3}$$

# Leverage, defaults, and margins

Once an investor takes on leverage (as in our previous example), they can lose more money than they had to begin with

- Thus, whomever is lending them money may not get all their money back (default)!

Example: Let's say, continuing the previous example, the market return is -70%. Now the investors wealth before paying back \$50 loan is:

$$\$150 \times (1 - 70\%) = \$45.$$

They borrowed \$50 and owe \$50.5 since the interest rate was 1%. Thus, they have \$5.5 too little to pay off loan and are in default.

## Leverage, defaults, and margins

In order to avoid this, it is often the case that counterparties will demand some form of margin along with marking-to-market

- A broker can say, if you invest \$100 I will lend you another \$50 so you can invest \$150 in the market portfolio, but I will not allow any more borrowing.
- If the broker lends you more, the likelihood of default increases. In our example, we had to have market fall about 70%. That's not very likely over a short time frame. What if the broker lends you \$900? Now, how much does the market have to fall? Only about 10%. That's not that unusual.

Marking-to-market means your account is assessed for its value, typically daily, and the loan amount you are given depends on this value in a dynamic fashion.

- The broker gives you a 50% margin as in our initial example. If the total value of your position decreases so that what you owe is more than, say 55% of your net worth (assessed by taking the current market value of the position), the broker can demand you immediately pay off some of the loan so you again are within the 50% margin.

Big take-away: if you have a position that in the end would have been profitable, any intermediate losses might demand a lot of cash on hand to keep up your margin requirement.

# Alternatives to mean-variance analysis

standard mean-variance analysis assumes agents only care about the first two moments

- if returns are heavy tailed, we cannot simply rely on the Sharpe Ratio as only measure of performance
- some strategies deliver  $SRs > 1$ , but this comes with lots of tail risk.
- Reporting Skewness and Kurtosis of returns also useful

In practice, Maximum Drawdown is a popular alternative

# Drawdown

the **drawdown** at time  $\tau$  is defined as the decline from the peak (the running maximum) of asset value  $X_\tau$ :

$$DD(\tau) = \max_{t \in (0, \tau]} [X_t] - X_\tau$$

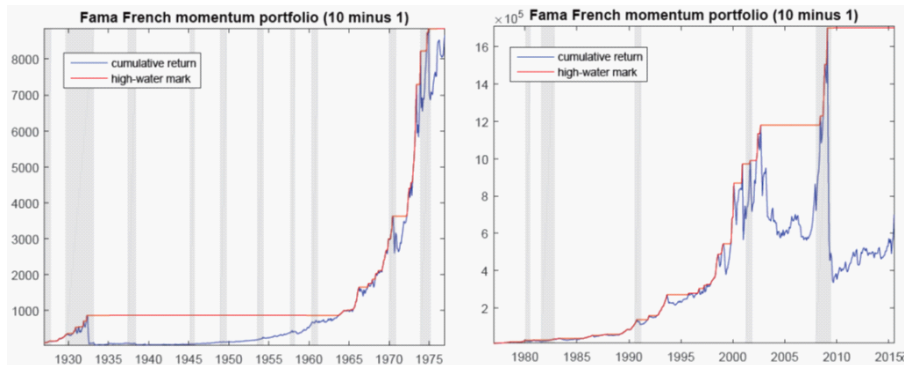
the **drawdown** can (equivalently) be calculated as

$$DD_t = \frac{HWM_t - P_t}{HWM_t}$$

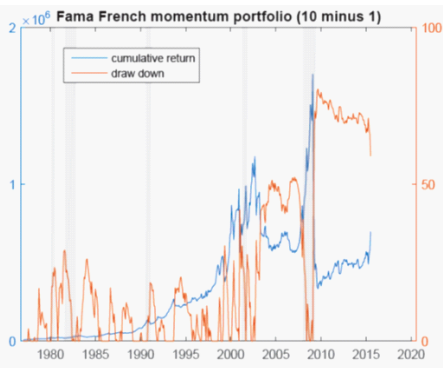
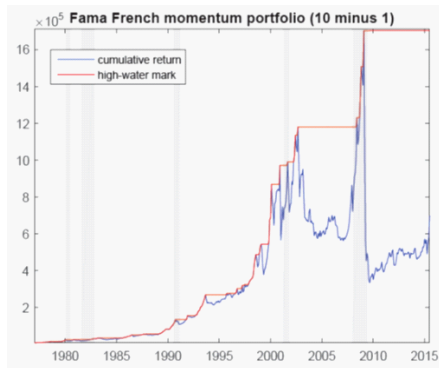
where  $HWM_t$  is the the asset's peak or **high water mark**

- the **high water mark** is the highest price the asset ever had.

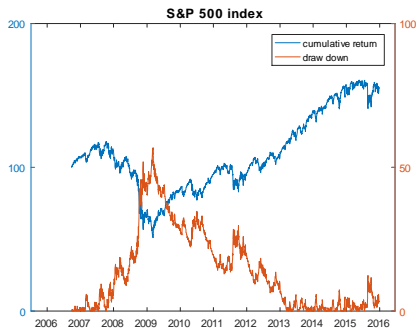
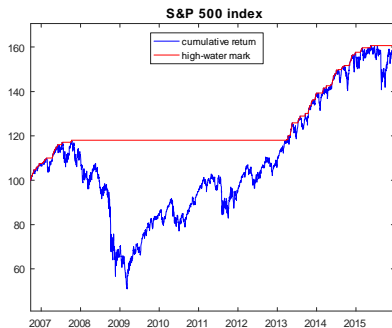
## Example: high water marks



# Example: drawdowns



# Example: drawdowns for the S&P500 index



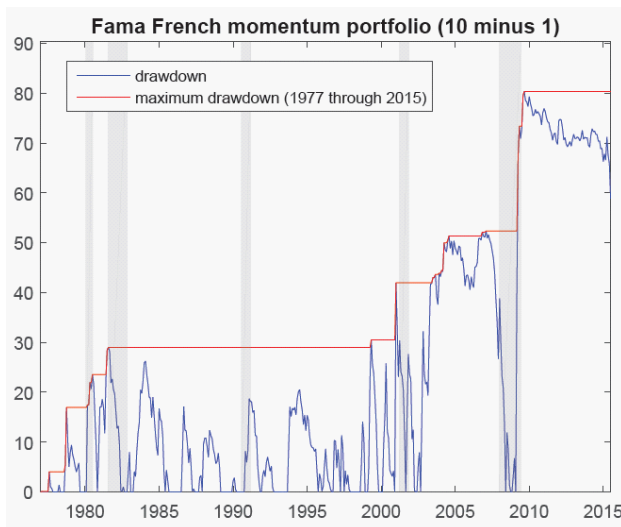
# Maximum Drawdown

a commonly used measure in alternative investments is the maximum drawdown.

- the **maximum drawdown** is defined as the largest drawdown over the entire history:

$$MDD(T) = \max_{t \in (0, T]} \left[ \max_{\tau \in (0, t]} [X_t] - X_\tau \right]$$

## Example: Maximum Drawdown



Monthly log Returns - Kenneth French 10 momentum portfolios (10-minus-1) .  
01/1927-7/2015.

# Introduction to estimating means and variance

The baseline mean-variance model requires estimates of means and variances

In the following we make an important point:

- ① Means of volatile series are really hard to estimate with sufficient precision
- ② Variances and covariances are much easier to estimate if we have (relatively) high frequency data available

# The Equity Premium

The equity premium is the return that equity investors expect to earn in excess of the risk-free rate:

$$E [R_{equity} - R_f]$$

The risk-free rate  $R_f$  is the rate you are guaranteed to get in advance with certainty.

- The return on the risk-free asset is known in advance and is not a random variable.
- In the real-world, there is no such thing as a risk-free asset.
- In practice, we use ultra-short U.S. treasury-bills as a proxy (default risk is low).

# Estimating the Equity Premium: data

- collect annual stock return data: CRSP-VW index
- (NYSE-AMEX-NASDAQ)(1925-2009)
- the risk-free rate is the average 90-day T-bill rate from CRSP
- (1925-2009)
- The sample mean  $\hat{\mu}$  of the excess returns is 7.50% per annum
  - ▶ our estimate of the population moment (the equity premium)

# Estimating the Equity Premium: standard error

The sample mean  $\hat{\mu}$  of the excess returns is 7.50% per annum

- how precise is this estimate of the mean? not very..
  - ▶ the standard error on the sample mean is (assuming returns are uncorrelated across time):

$$\sqrt{\text{Var}(\hat{\mu})} \approx \frac{\hat{\sigma}}{\sqrt{T}} = 2.27\%$$

- ▶ what matters here is the span of the data. See Merton (1980).
  - ▶ e.g., using daily data will not help us pin down the mean return on an asset (the volatility will decrease at a rate  $\sqrt{T}$ )
  - ▶ *we need **longer** data sets instead.* (THIS IS AN IMPORTANT POINT!)

# Estimating the Volatility

- collect annual stock return data: CRSP-VW index
- (NYSE-AMEX-NASDAQ)(1925-2009)
- the risk-free rate is the average 90-day T-bill rate from CRSP
- (1925-2009)
- The sample standard deviation  $\hat{\sigma}$  of the excess returns is 20.98% per annum
- How precise is this estimate? Depends on how many observations for each year  $t$ . If monthly,  $h = 12$  observations per  $t$ 
  - ▶ standard error of estimated annualized return variance is

$$\sqrt{\text{Var}(\hat{\sigma}^2)} \approx \begin{cases} \frac{\hat{\sigma}^2}{\sqrt{T \times h}} = 5.24 (\%)^2 & \text{if } h = 1 \text{ (annual data)} \\ \frac{\hat{\sigma}^2}{\sqrt{T \times h}} = 1.51 (\%)^2 & \text{if } h = 12 \text{ (monthly data)} \end{cases}$$

- ▶ **High frequency** data will help us estimate the variance (see Merton, 1980; another IMPORTANT POINT).
  - ★ Also true for covariance and beta's!

# Important insights regarding estimation

- It is hard to estimate the unconditional mean of a volatile series
  - ① Need a LONG data-series, unless we have a *theory* we believe in
  - ② Recall: CAPM needs market risk premium is an input; we have to use long data series to infer its value
  - ③ In *Multi-Factor Models*, we must estimate the risk premium of each factor in a similar manner – using long data series
- Volatility (really, variances and covariances) are better estimated with higher frequency data
  - ① In theory, we could get incredibly precise estimates using tick-data
  - ② But, there are micro-structure issues (bid-ask spread, inventory effects, data errors)
  - ③ Since betas (covariance over variance) are easier to measure, this fact is useful for getting expected returns on individual stocks