Lecture 6b
Combining ARMA models with forecasting regressions: ARMA-X

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ARMA-X models

A forecasting regression:

$$y_{t+1} = \alpha + \beta' X_t + \varepsilon_{t+1},$$

where y_{t+1} is an outcome realized at time $t+1$ and X_t is a vector of predictive variables known at time t.

- The predicted value is then $E\left(y_{t+1}|X_t; \hat{\alpha}, \hat{\beta}\right) = \hat{\alpha} + \hat{\beta}' X_t$.
- The vector X_t could include, say, y_t, but it could also include other variables.

This is different from the ARMA models

- These use only lags of y and ε to predict future y
- Recall, *MA*-lags are useful if there is a latent AR-process (observed with noise) or if there are overlapping observations
- Can we combine forecasting regressions and ARMA models?

ARMA-X

Consider the ARMA-X(1,1):

$$y_{t+1} = \phi_0 + \phi_1 y_t - \theta_1 \varepsilon_t + \beta' X_t + \varepsilon_{t+1}.$$

- This model is well-specified if X_t are 'exogenous' predictors.

Very useful class of predictors!

- But: Note that since you don't have a dynamic model for the X_t variables, you can only predict y_{t+1} on period ahead:

$$\begin{aligned} E\left(y_{t+1}|y_t, y_{t-1}, ..., X_t\right) &= \phi_0 + \phi_1 y_t - \theta_1 \varepsilon_t + \beta' X_t \quad \text{(this works!)} \\ E\left(y_{t+2}|y_t, y_{t-1}, ..., X_t\right) &= \phi_0 + \phi_1 E_t\left(y_{t+1}\right) + \beta' E_t\left(X_{t+1}\right) \quad \text{(this doesn't)} \end{aligned}$$

Issue: we don't have a model for $E_t\left(X_{t+1}\right)$