

Quantitative Asset Management

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Lecture 2

1. Review: HML and SMB

Fama and French (1992, 1993) three-factor model

2. Asset Growth

Cooper, Gulen and Schill (2008, JF)

3. Profitability

Novy-Marx (2013, JFE)

4. Fama and French five-factor model

Fama and French (2015, JFE)

Size Portfolios

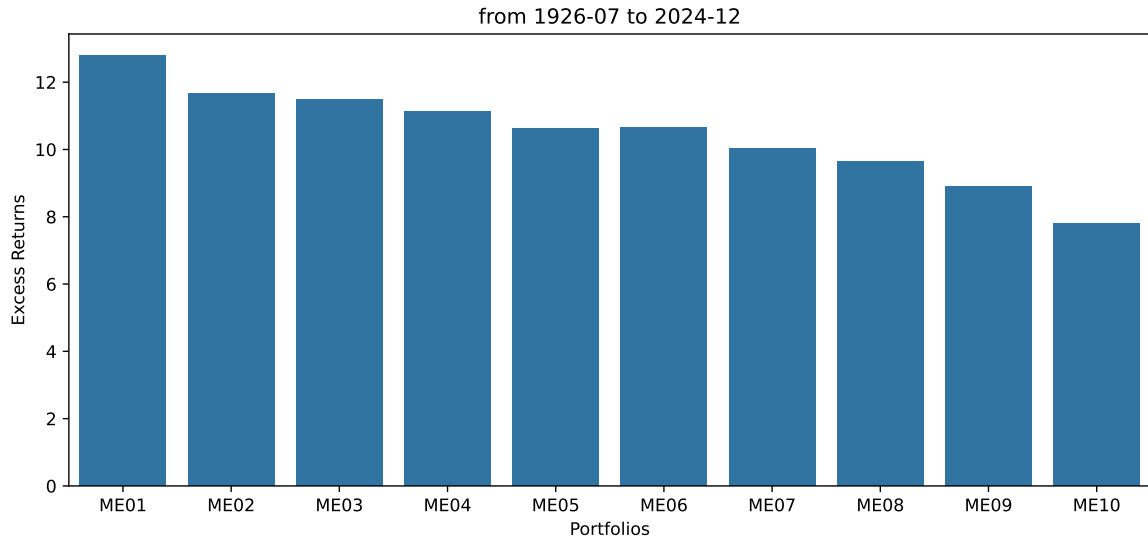
- ▶ There is a size effect in stock returns (it's not really a size effect: it's about market cap, not the actual size of the company)
- ▶ Small cap stocks earn much higher average returns than large cap stocks
- ▶ Small cap stocks do have higher betas, but the difference in betas is not large enough to explain the difference in returns

Portfolio Construction

- ▶ Let's look at all the stocks in CRSP
- ▶ Sort stocks into deciles based on market cap at the end of June each year
- ▶ Portfolios are held fixed throughout year
- ▶ The portfolios for July of year t to June of $t+1$ include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for June of t .
- ▶ NYSE Breakpoints
- ▶ Source: [Kenneth French's data library](#)

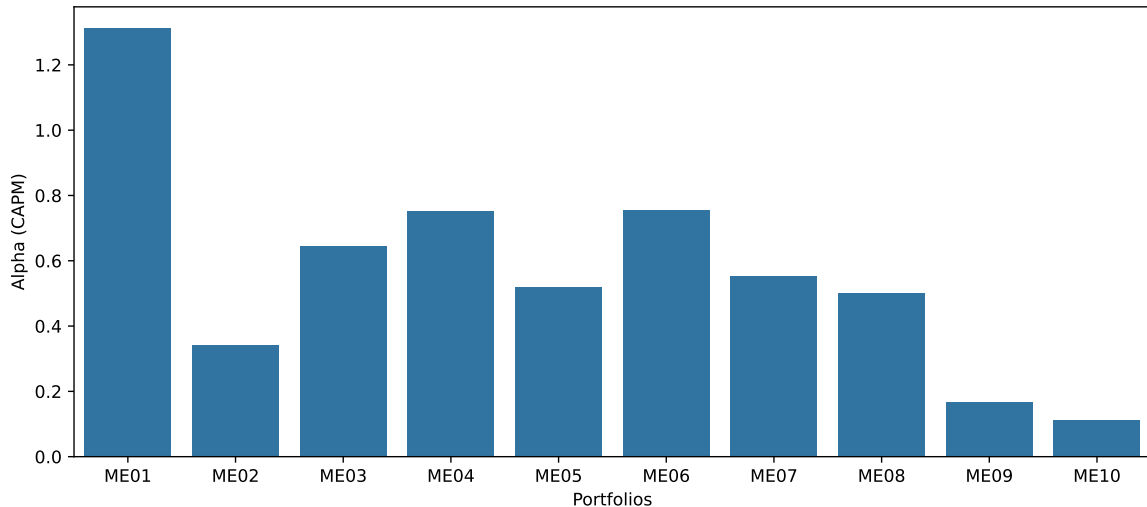
- ▶ Can also rebalance monthly

Size Effect (excess returns)

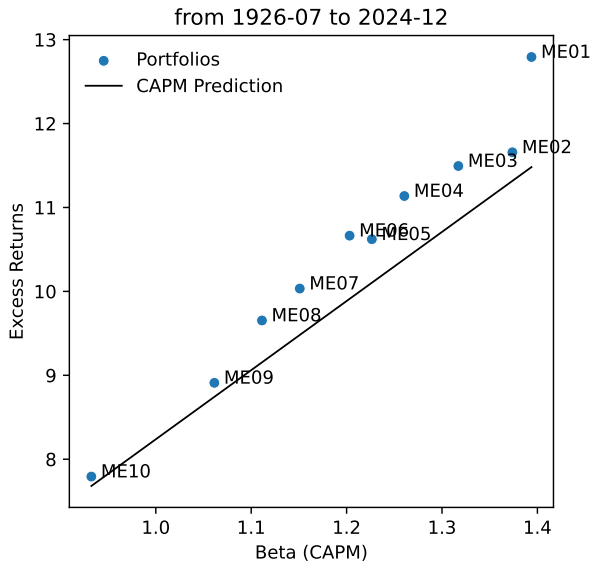


Size Effect (CAPM Prediction),

from 1926-07 to 2024-12



Size Effect (CAPM Prediction)



Beta and Returns in Size Portfolios

- ▶ Clearly, the return-beta relation goes the right direction when we look at size portfolios
- ▶ Investors do seem to be compensated for taking on beta risk by getting higher returns
- ▶ However, the relation between returns and betas is too steep in the data.
- ▶ Facts about size anomaly
 - ▶ January effect
 - ▶ Other measures of size, e.g. sales
 - ▶ Before/after 1980

Value Effect

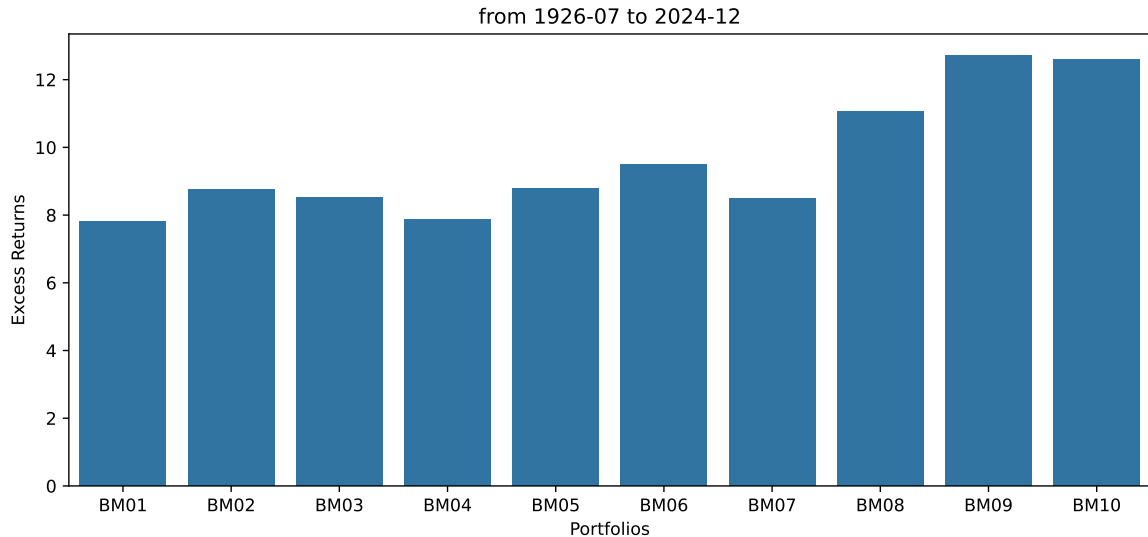
Value Effect

- ▶ There is a value effect in stock returns
- ▶ Stocks with high Book-to-Market ratios subsequently earn much higher returns than stocks with low Book-to-market ratios
- ▶ How are book values constructed?

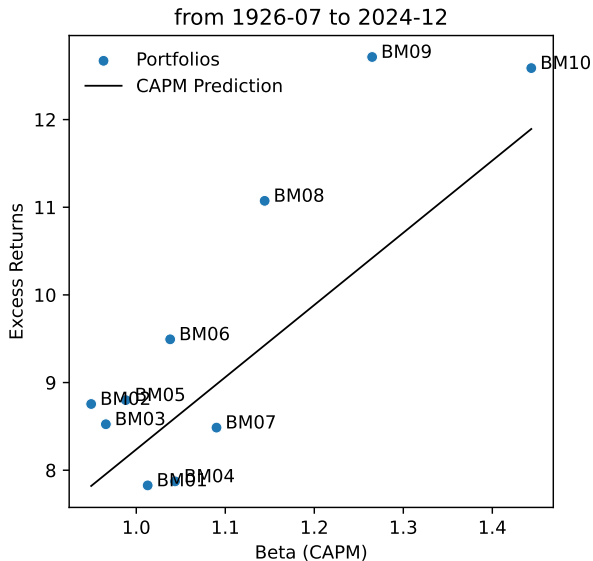
Portfolio Construction

- ▶ Let's look at all the stocks in CRSP
- ▶ Sort stocks into deciles based on book-to-market ratios at the end of June each year; high book-to-market stocks are 'value' stocks
- ▶ Portfolios are held fixed throughout year
- ▶ The portfolios for July of year t to June of $t+1$ include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for June of t .
- ▶ NYSE Breakpoints
- ▶ Source: [Kenneth French's data library](#)

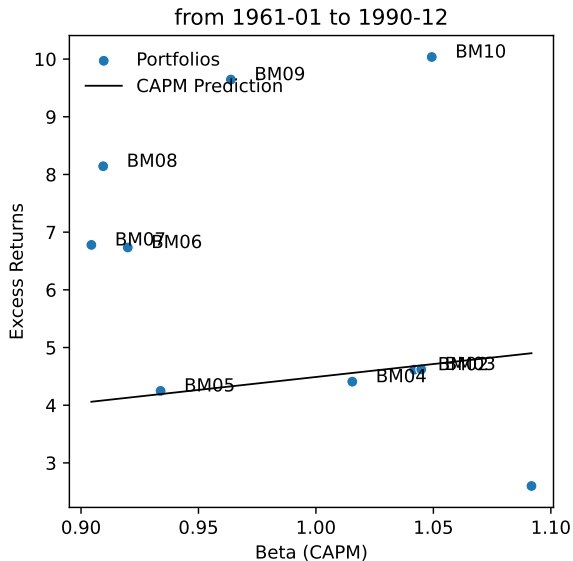
Value Effect (excess returns)



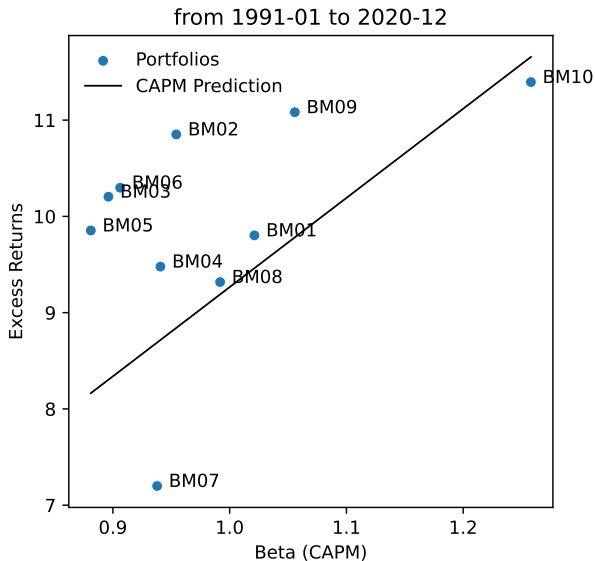
Value Effect (CAPM Prediction), full sample



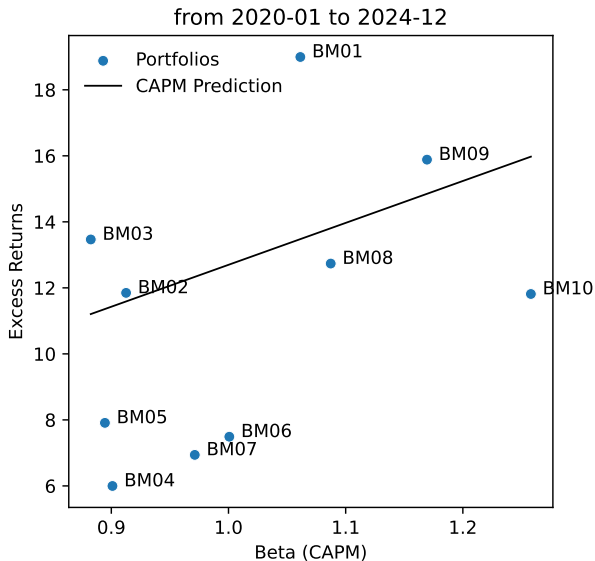
Value Effect (CAPM Prediction), 1961-1990



Value Effect (CAPM Prediction), 1991-2020



Value Effect (CAPM Prediction), 2020-



Failure of CAPM

- ▶ The CAPM did a reasonable job before 1960, but after that the relation between beta and average returns seems to completely disappear
- ▶ Anomalies documented in 1980's: Size and Value

Multi-factor Models: Risk comes in many flavors

- ▶ Before we can do performance analysis, we need a good model of risk and risk compensation in financial markets
- ▶ We used to think that priced risk comes in only a single flavor: β
- ▶ But we know now that risk comes in many flavors
 - ▶ Complicates portfolio advice
 - ▶ Makes performance analysis more challenging

Why do we expect multiple factors?

- ▶ Heterogeneity in investor sophistication:
 - ▶ Some less sophisticated investors have portfolios that are biased towards large growth firms (because these are considered to be more glamorous)
 - ▶ Then the sophisticated investors (by definition) have to overweight the small value firms in their portfolio of risky assets
 - ▶ Now, the sophisticated investors do not and cannot hold the market portfolio
 - ▶ This creates new priced factors like value and size

Why do we expect multiple factors?

- ▶ Heterogeneity in investor trading technology:
 - ▶ Some less sophisticated investors have portfolios that are biased towards high beta stocks because they do not have access to leverage
 - ▶ Then the sophisticated investors (by definition) have to overweight the low beta stocks
 - ▶ Now, the sophisticated investors do not and cannot hold the market portfolio
 - ▶ This creates new priced factors like BAB

Arbitrage Pricing Theory

- ▶ **Result:** There exist risk prices for each factor such that the expected return on any security can be stated as:

$$\mathbb{E}[R_i] = \lambda_0 + b_{i,1}\lambda_1 + b_{i,2}\lambda_2 + \dots + b_{i,L}\lambda_L, \quad \text{for } i = 1, \dots, N$$

- ▶ Very general
- ▶ No need to measure the return on the market
- ▶ The theory does not tell you which factors to use
- ▶ License to go fishing for priced risk factors...

Which Factors?

- ▶ One (ad hoc but effective) way to proceed is to use traded factors that we believe to be the main drivers of returns:
 1. Size factor
 2. Value or book-to-market factor
 3. Momentum factor
- ▶ This essentially means we are taking a shortcut: we're not trying to actually capture the sources of macroeconomic risk that are priced directly, but we're using traded factors that proxy for these macro-economic risks.

Fama-French

- ▶ Fama and French formed three portfolios of stocks sorted by book-to-market ratios and two portfolio of all stocks sorted by size (market cap)
- ▶ The intersection of these creates six portfolios:

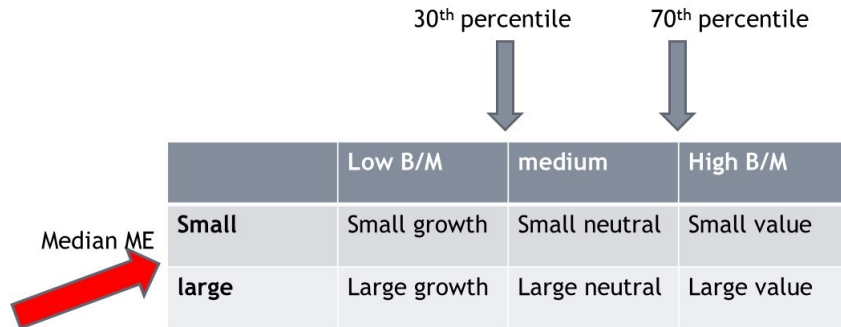
	Low B/M	medium	High B/M
Small	Small growth	Small neutral	Small value
large	Large growth	Large neutral	Large value

Two-way (double) sorted portfolios

- ▶ Two Characteristics
- ▶ Independence Sorting: thresholds are computed separately
 - ▶ Testing if both anomalies hold separately
 - ▶ If characteristics too correlated, may have few stocks in some groups (may lead to insignificant results)
- ▶ Sequential Sorting: sort on one characteristic first, then sort on the other characteristic within each group
 - ▶ testing if second characteristic anomaly holds within each group formed based on the first characteristic

Fama-French

- The intersection of these creates six portfolios



Size Factor

- SMB is the average return on a long position in the three small portfolios and a short position in the three large portfolios

$$SMB = \frac{1}{3} (\text{Small Value} + \text{Small Neutral} + \text{Small Growth}) \\ - \frac{1}{3} (\text{Large Value} + \text{Large Neutral} + \text{Large Growth})$$

Value Factor

- ▶ HML is the average return on a long position in the two value portfolios and a short position in the two growth portfolios:

$$HML = \frac{1}{2} (\text{Small Value} + \text{Large Value}) \\ - \frac{1}{2} (\text{Small Growth} + \text{Large Growth})$$

Fama and French Three-Factor Model

- Fama and French Three-factor Model for stock returns:

$$(R_i - R_f) = a_i + b_{i,m} (R_m - R_f) + b_{i,smb} R_{smb} + b_{i,hml} R_{hml} + e_i$$

- Implication for APT:

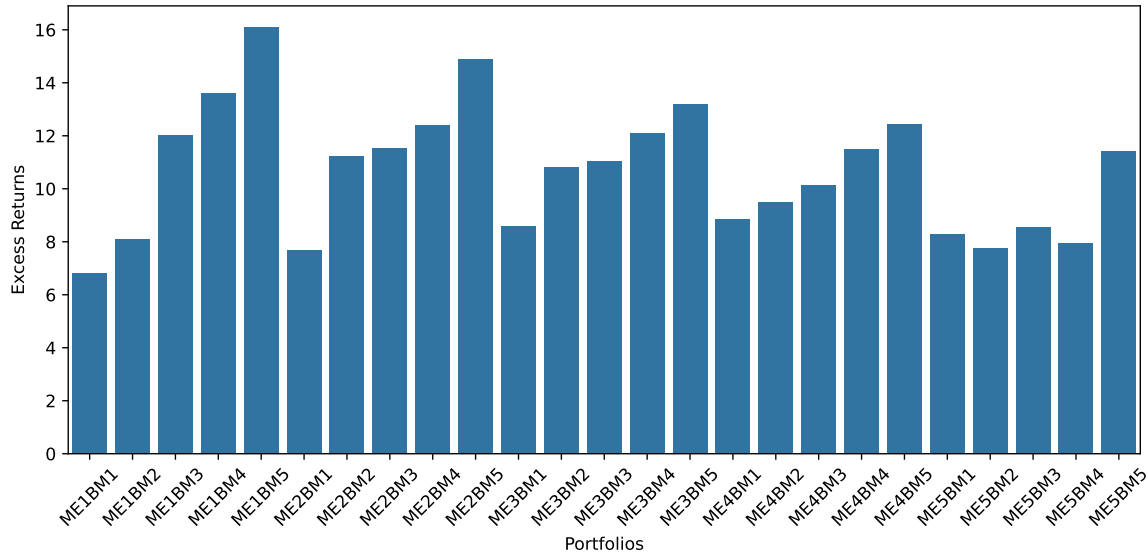
$$\mathbb{E}[R_i] = \lambda_0 + b_{i,m} \lambda_m + b_{i,smb} \lambda_{smb} + b_{i,hml} \lambda_{hml}, \quad i = 1, \dots, N$$

Test Assets

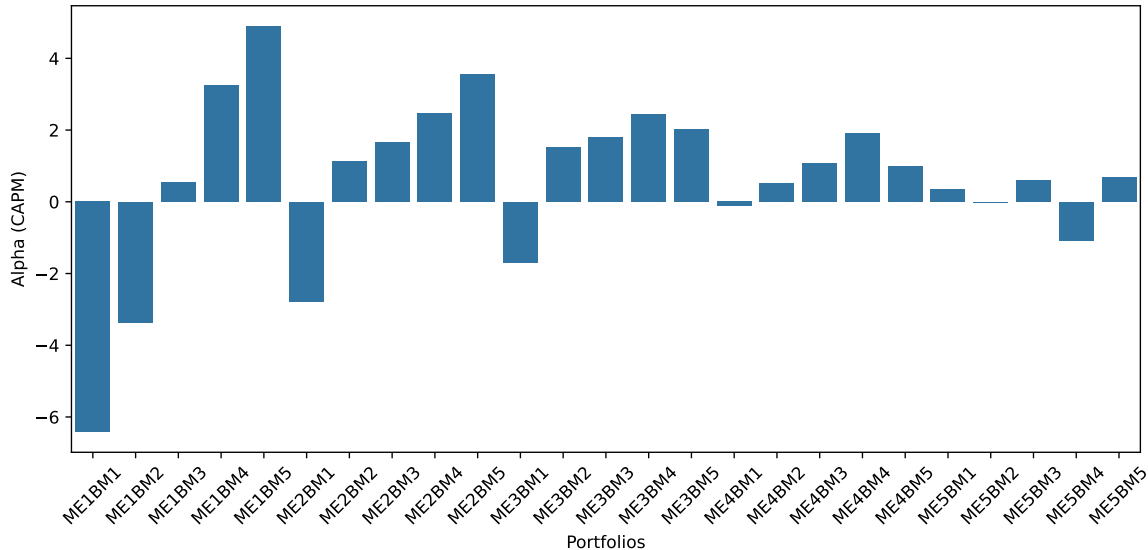
- ▶ Let's test this asset pricing model on 25 portfolios (intersection of 5 portfolios sorted by size and 5 portfolios sorted by B/M)
- ▶ To test an asset pricing model, we want to check whether it explains the returns on passively managed portfolios
 - ▶ That seems like a reasonable way to test this model
 - ▶ Once we know we have a decent model, we can use the model to do performance analysis

Book-to-Market and Size Portfolios (excess returns)

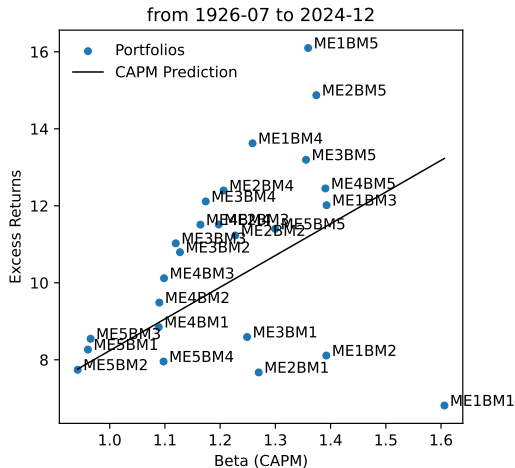
from 1926-07 to 2024-12



Book-to-Market and Size Portfolios (CAPM Prediction), from 1926-07 to 2024-12



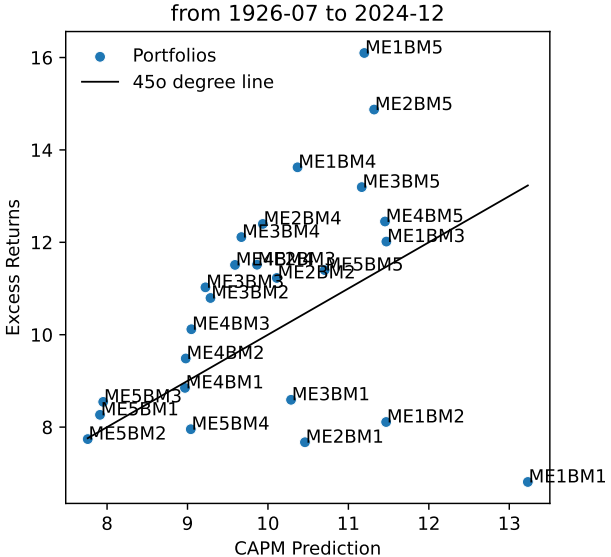
Book-to-Market and Size Portfolios (CAPM Prediction)



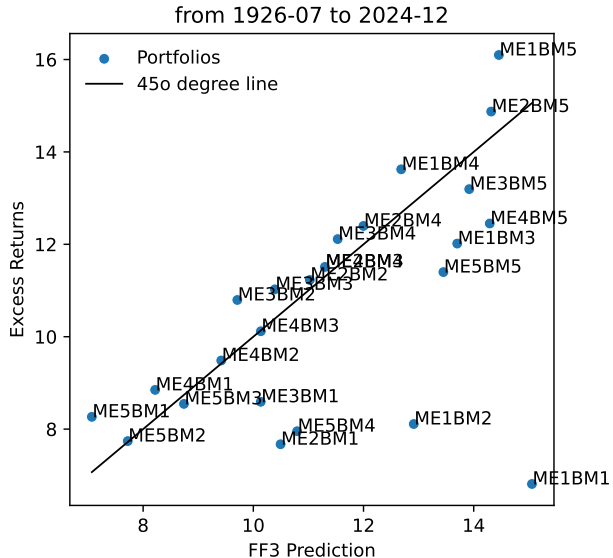
Sample: Monthly data. Annualized Returns.

Source: Kenneth French's data library.

Model Fit: CAPM



Model Fit: Fama and French three-factor model



Macro-economic Risk

- ▶ Do these factors capture macro-economic risk?
- ▶ We can check whether these factors price other portfolios (other than the ones sorted by size and book-to-market)
- ▶ Other portfolios whose returns are well explained by HML and SMB include
 1. Portfolios of stocks sorted by price/earnings
 2. Portfolios of stocks sorted by sales growth
 3. Portfolios of stocks sorted by reversals
- ▶ But these portfolios have high time series R^2

Other Test Assets

- ▶ e.g. HML and SMB cannot account for returns on portfolios of stocks sorted by momentum
- ▶ Momentum effect is really large!
- ▶ Momentum stocks do move together:
 - ▶ High momentum stocks move together with other high momentum stocks
 - ▶ Low momentum stocks move together with other low momentum stocks
- ▶ since momentum stocks co-move, adding a momentum factor eliminates the α on momentum portfolios
- ▶ Ad hoc solution: construct a momentum factor, an investment factor, a profitability factor, ...

Asset Growth

Cooper, Gulen and Schill (2008, JF)

Real Investments and Stocks Returns

Cross-sectional evidence: negative correlation between investment and future returns

- ▶ Asset expansion is typically followed by periods of lower abnormal returns
- ▶ Asset contraction is typically followed by periods of higher abnormal returns

How to measure a proxy for investment?

- ▶ Events associated with expansion/contractions
- ▶ acquisitions, equity/debt offering, loans
- ▶ spinoffs, share repurchase, debt prepayment, dividends

Larger picture: total investment!

- ▶ Asset growth

Asset Growth and Cross-Section of Stock Returns

- ▶ Cross-sectional relation: asset growth and stock returns
- ▶ Asset Growth: predictor of future stock return
 - ▶ Works even among large caps
- ▶ Not priced/explained by other determinants of the cross-section of returns
- ▶ Asset growth sorted portfolios
 - ▶ Long-short portfolio: 13% spread with 1.07 Sharpe Ratio

Asset Growth and Cross-Section of Stock Returns

Data

- ▶ NYSE, Amex, NASDAQ exchanges
- ▶ exclude financial firms (why?)
- ▶ Sample: 1963-2003
- ▶ Require at least 2 year of compustat data per firm
- ▶ Form all accounting variables at end of June of year t from fiscal year $t - 1$.
- ▶ Rebalance at the end each June
- ▶ Main variable: asset growth (ASSETG)

$$ASSETG_{i,t} = \frac{AT_{i,t-1} - AT_{i,t-2}}{AT_{i,t-2}}$$

using nonmissing/nonzero data only

Asset growth sorted portfolios: vw returns

Panel B.2: Value-Weighted Portfolio Average Monthly Raw Returns

Asset Growth Deciles											Spread	
YEAR	1(Low)	2	3	4	5	6	7	8	9	10(High)	(10-1)	t(spread)
−5	0.0121	0.0123	0.0117	0.0129	0.0142	0.0146	0.0165	0.0207	0.0243	0.0271	0.0150	7.55
−4	0.0114	0.0109	0.0119	0.0131	0.0128	0.0146	0.0172	0.0202	0.0288	0.0307	0.0193	9.15
−3	0.0064	0.0085	0.0100	0.0123	0.0143	0.0151	0.0157	0.0212	0.0279	0.0357	0.0292	10.92
−2	0.0062	0.0083	0.0090	0.0116	0.0135	0.0149	0.017	0.0206	0.0266	0.0396	0.0334	12.86
−1	0.0223	0.0175	0.0153	0.0146	0.0147	0.0141	0.0153	0.0177	0.0192	0.0230	0.0007	0.28
1	0.0148	0.0124	0.0122	0.0116	0.0100	0.0100	0.0102	0.0092	0.0077	0.0043	−0.0105	−5.04
2	0.0133	0.0126	0.0125	0.0101	0.0109	0.0102	0.0098	0.0097	0.0097	0.0065	−0.0068	−3.39
3	0.0169	0.0137	0.0141	0.0126	0.0102	0.0112	0.0116	0.0105	0.0116	0.0116	−0.0053	−2.82
4	0.0132	0.0107	0.012	0.0109	0.0114	0.0103	0.0103	0.0123	0.0111	0.0120	−0.0012	−0.61
5	0.0128	0.0133	0.0121	0.0123	0.0103	0.01	0.0107	0.0113	0.013	0.0126	−0.0002	−0.11
Cumulative Return												
[−5, −1]	1.0449	0.9918	1.0078	1.2375	1.3631	1.4788	1.7985	2.5321	3.9221	6.4272	5.3822	4.78
[1, 5]	1.2879	1.1133	1.1305	1.0038	0.931	0.8934	0.9352	0.9056	0.9458	0.7911	−0.4967	−4.25

Asset growth sorted portfolios: α_{FF} by size

Alphas relative to the Fama and French three-factor model

Panel C. Equal- and Value-Weighted Portfolio Fama–French Alphas in Year 1 by Size Groups

Panel C.1: Equal-Weighted Portfolio Fama–French Monthly Alphas

	Asset Growth Deciles										Spread (10-1)	$t(\text{spread})$
	1(Low)	2	3	4	5	6	7	8	9	10(High)		
All Firms	0.0076	0.006	0.0035	0.0026	0.002	0.0013	0.0006	0.0003	−0.0026	−0.0087	−0.0163	−8.33
Small size	0.0081	0.0067	0.0044	0.003	0.0027	0.0012	0.0002	0.0001	−0.0033	−0.0096	−0.0177	−9.12
Medium size	−0.0004	0.0007	0.0010	0.0010	0.0011	0.0010	0.0010	0.0002	−0.0016	−0.0064	−0.0060	−2.85
Large size	0.0044	0.0011	0.0003	0.0014	0.0005	0.0001	0.0005	0.0011	−0.001	−0.0041	−0.0086	−3.12

Panel C.2: Value-Weighted Portfolio Fama–French Monthly Alphas

	Asset Growth Deciles										Spread (10-1)	$t(\text{spread})$
	1(Low)	2	3	4	5	6	7	8	9	10(High)		
All Firms	0.0024	0.0013	0.0013	0.0017	0.0003	0.0006	0.0015	0.0013	−0.0001	−0.0046	−0.007	−3.84
Small size	0.0005	0.0020	0.0013	0.0016	0.0015	0.0007	−0.0009	−0.0006	−0.0043	−0.0109	−0.0114	−6.46
Medium size	−0.0002	0.0003	0.001	0.0005	0.0008	0.0008	0.0013	0.0004	−0.0012	−0.0057	−0.0055	−2.45
Large size	0.0052	0.0018	0.0013	0.0019	0.0003	0.0005	0.0018	0.0015	0.0008	−0.0028	−0.0081	−2.91

Profitability

Novy-Marx (2013, JFE)

Gross profitability

Gross profitability and the cross-section of stocks returns

- ▶ Same power as book-to-market ratio
- ▶ Value: long cheap and short expensive assets
- ▶ Profitability: long productive and short unproductive assets

- ▶ More profitable firms earn significantly higher average return than unprofitable firms

Data

- ▶ Gross profitability: clean measure of economic profits

$$\begin{aligned}\text{Gross Profitability} &= \frac{[\text{Total Revenue}] - [\text{Cost of goods sold}]}{[\text{Total Assets}]} \\ &= \frac{REV T - COGS}{AT}\end{aligned}$$

Sorted Portfolios

Profitability versus Value

Table 2

Excess returns to portfolios sorted on profitability.

This table shows monthly value-weighted average excess returns to portfolios sorted on gross profits-to-assets $[(REVT - COGS)/AT]$, employing NYSE breakpoints, and results of time series regressions of these portfolios' returns on the Fama and French factors [the market factor (MKT), the size factor small-minus-large (SMB), and the value factor high-minus-low (HML)], with test-statistics (in square brackets). It also shows time series average portfolio characteristics [portfolio gross profits-to-assets (GP/A), book-to-market (B/M), average firm size (ME, in millions of dollars), and number of firms (n)]. Panel B provides similar results for portfolios sorted on book-to-market. The sample excludes financial firms (those with one-digit standard industrial classification codes of six) and covers July 1963 to December 2010.

Portfolio	r^e	Alphas and three-factor loadings				Portfolio characteristics			
		α	MKT	SMB	HML	GP/A	B/M	ME	n
Panel A: Portfolios sorted on gross profits-to-assets									
Low	0.31	−0.18	0.94	0.04	0.15	0.10	1.10	748	771
	[1.65]	[−2.54]	[57.7]	[1.57]	[5.87]				
2	0.41	−0.11	1.03	−0.07	0.20	0.20	0.98	1,100	598
	[2.08]	[−1.65]	[67.5]	[−3.13]	[8.51]				
3	0.52	0.02	1.02	−0.00	0.12	0.30	1.00	1,114	670
	[2.60]	[0.27]	[69.9]	[−0.21]	[5.42]				
4	0.41	0.05	1.01	0.04	−0.24	0.42	0.53	1,114	779
	[1.94]	[0.83]	[70.6]	[1.90]	[−11.2]				
High	0.62	0.34	0.92	−0.04	−0.29	0.68	0.33	1,096	938
	[3.12]	[5.01]	[58.3]	[−2.03]	[−12.3]				
High−low	0.31	0.52	−0.03	−0.08	−0.44				
	[2.49]	[4.49]	[−0.99]	[−2.15]	[−10.8]				

Fama and French five-factor model

Fama and French (2015, JFE)

Fama and French five-factor model

FF five-factor model adds investment and profitability:

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t + c_iCMA_t + e_{it}.$$

- ▶ SMB: small minus big (market equity)
- ▶ HML: high minus low (book-to-market)
- ▶ RMW: robust minus weak (profitability)
- ▶ CMA: conservative minus aggressive (investment)

Fama and French five factors

Joint sort on size, BM, inv. (asset growth), and profitability!?

- ▶ Easily get many poorly diversified portfolios
- ▶ Need fewer portfolios

Fama and French five factors

Table 3

Construction of *Size*, *B/M*, profitability, and investment factors.

We use independent sorts to assign stocks to two *Size* groups, and two or three *B/M*, operating profitability (*OP*), and investment (*Inv*) groups. The VW portfolios defined by the intersections of the groups are the building blocks for the factors. We label these portfolios with two or four letters. The first always describes the *Size* group, small (*S*) or big (*B*). In the 2×3 sorts and 2×2 sorts, the second describes the *B/M* group, high (*H*), neutral (*N*), or low (*L*), the *OP* group, robust (*R*), neutral (*N*), or weak (*W*), or the *Inv* group, conservative (*C*), neutral (*N*), or aggressive (*A*). In the $2 \times 2 \times 2 \times 2$ sorts, the second character is *B/M* group, the third is *OP* group, and the fourth is *Inv* group. The factors are *SMB* (small minus big), *HML* (high minus low *B/M*), *RMW* (robust minus weak *OP*), and *CMA* (conservative minus aggressive *Inv*).

Sort	Breakpoints	Factors and their components
2×3 sorts on <i>Size</i> and <i>B/M</i> , or <i>Size</i> and <i>OP</i> , or <i>Size</i> and <i>Inv</i>	Size: NYSE median	$SMB_{B/M} = (SH + SN + SL)/3 - (BH + BN + BL)/3$ $SMB_{OP} = (SR + SN + SW)/3 - (BR + BN + BW)/3$ $SMB_{Inv} = (SC + SN + SA)/3 - (BC + BN + BA)/3$ $SMB = (SMB_{B/M} + SMB_{OP} + SMB_{Inv})/3$
	<i>B/M</i> : 30th and 70th NYSE percentiles <i>OP</i> : 30th and 70th NYSE percentiles <i>Inv</i> : 30th and 70th NYSE percentiles	$HML = (SH + BH)/2 - (SL + BL)/2 = [(SH - SL) + (BH - BL)]/2$ $RMW = (SR + BR)/2 - (SW + BW)/2 = [(SR - SW) + (BR - BW)]/2$ $CMA = (SC + BC)/2 - (SA + BA)/2 = [(SC - SA) + (BC - BA)]/2$
2×2 sorts on <i>Size</i> and <i>B/M</i> , or <i>Size</i> and <i>OP</i> , or <i>Size</i> and <i>Inv</i>	Size: NYSE median	$SMB = (SH + SL + SR + SW + SC + SA)/6 - (BH + BL + BR + BW + BC + BA)/6$ $HML = (SH + BH)/2 - (SL + BL)/2 = [(SH - SL) + (BH - BL)]/2$ $RMW = (SR + BR)/2 - (SW + BW)/2 = [(SR - SW) + (BR - BW)]/2$ $CMA = (SC + BC)/2 - (SA + BA)/2 = [(SC - SA) + (BC - BA)]/2$
	<i>B/M</i> : NYSE median <i>OP</i> : NYSE median <i>Inv</i> : NYSE median	
$2 \times 2 \times 2 \times 2$ sorts on <i>Size</i> , <i>B/M</i> , <i>OP</i> , and <i>Inv</i>	Size: NYSE median	$SMB = (SHRC + SHRA + SHWC + SHWA + SLRC + SLRA + SLWC + SLWA)/8$ $- (BHRC + BHRA + BHWC + BHWA + BLRC + BLRA + BLWC + BLWA)/8$ $HML = (SHRC + SHRA + SHWC + SHWA + BHRC + BHRA + BHWC + BHWA)/8$ $- (SLRC + SLRA + SLWC + SLWA + BLRC + BLRA + BLWC + BLWA)/8$ $RMW = (SHRC + SHRA + SLRC + SLRA + BHRC + BHRA + BLRC + BLRA)/8$ $- (SHWC + SHWA + SLWC + SLWA + BHWC + BHWA + BLWC + BLWA)/8$ $CMA = (SHRC + SHWC + SLRC + SLWC + BHRC + BHWC + BLRC + BLWC)/8$ $- (SHRA + SHWA + SLRA + SLWA + BHRA + BHWA + BLRA + BLWA)/8$
	<i>B/M</i> : NYSE median	
	<i>OP</i> : NYSE median	
	<i>Inv</i> : NYSE median	

Fama and French five-factor model

Test whether intercept is zero for all test assets

$$R_{it} - R_{Ft} = a_i + b_i(R_{Mt} - R_{Ft}) + s_iSMB_t + h_iHML_t + r_iRMW_t \\ + c_iCMA_t + e_{it}.$$

- ▶ Test assets used:
 - ▶ 25 size-BM (5x5)
 - ▶ 25 Size-OP (5x5)
 - ▶ 25 Size-Inv (5x5)
 - ▶ 32 Size-BM-OP (2x4x4)
 - ▶ 32 Size-BM-Inv (2x4x4)
 - ▶ 32 Size-OP-Inv (2x4x4)
- ▶ Results in Table 5

Fama and French five-factor model

Remarks

- ▶ Five-factor model performs better than 3FF
- ▶ 5FF still has a hard time explaining:
 - ▶ small-low OP portfolio
 - ▶ small-high Inv portfolio
 - ▶ small-growth portfolio
- ▶ Results are robust to how factors are constructed
- ▶ Benchmark: 5FF as in Ken French website (2x3)