

Quantitative Asset Management

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Lecture 6: Equity Return Predictability

1. Overview: discount rates

John H Cochrane (2011, JF Presidential address)

2. Forecasting stock returns

John Y Campbell and Samuel B Thompson (RFS, 2007)

Ivo Welch and Amit Goyal (2007, RFS)

3. (additional material) Forecasting stock returns under economic constraints

Davide Pettenuzzo, Allan Timmermann, and Rossen Valkanov (JFE, 2014)

Return Predictability

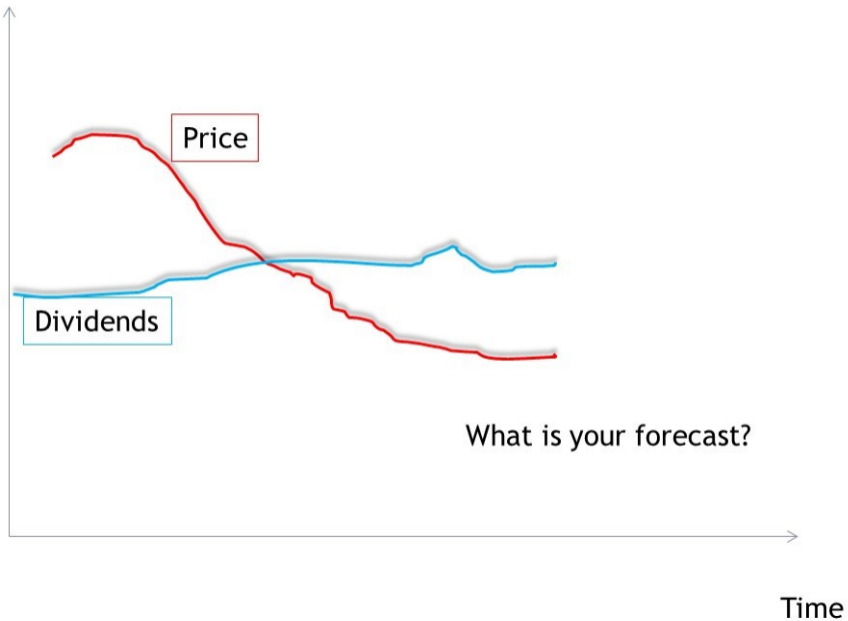
- ▶ We are now going to take a closer look at stock return predictability.
- ▶ Returns are said to be predictable whenever investors have any information in their information set today that helps to predict future returns.
- ▶ This could include past returns, but it also could include other information (such as the dividend yield, interest rates, bond yields, etc..)
- ▶ Whenever returns are predictable, this can be exploited by timing the market. Just rebalancing is no longer the optimal strategy.

Return Predictability: Chasing Yields works!

1. Stocks: high dividend yields predict high returns
2. Treasury bonds: high yields predict high returns (rather than higher short term interest rates)
3. Corporate bonds: high yields predict high returns (rather than defaults)
4. Currencies: high yields predict high returns
5. Real Estate: high yields predict high returns (not low rental prices)

1. Stock Markets

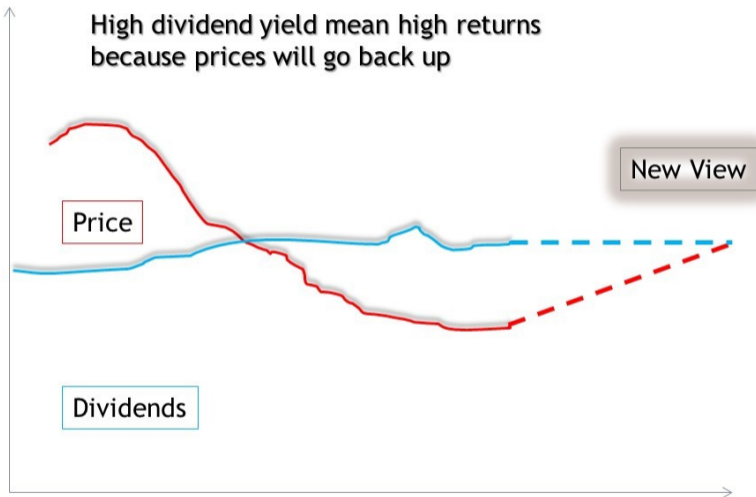
- ▶ High dividend yields or low price/dividend ratios predict high returns
 - ▶ Prices will go up in order to bring the price/dividend ratio back up to its long run steady-state value
 - ▶ Cash flows will not adjust
- ▶ This creates room for market timing: when price/dividend ratios are low, we can expect high stock returns over the next (say) 5 years





Low subsequent dividend growth pushes the dividend yield Back down

High dividend yield mean high returns
because prices will go back up



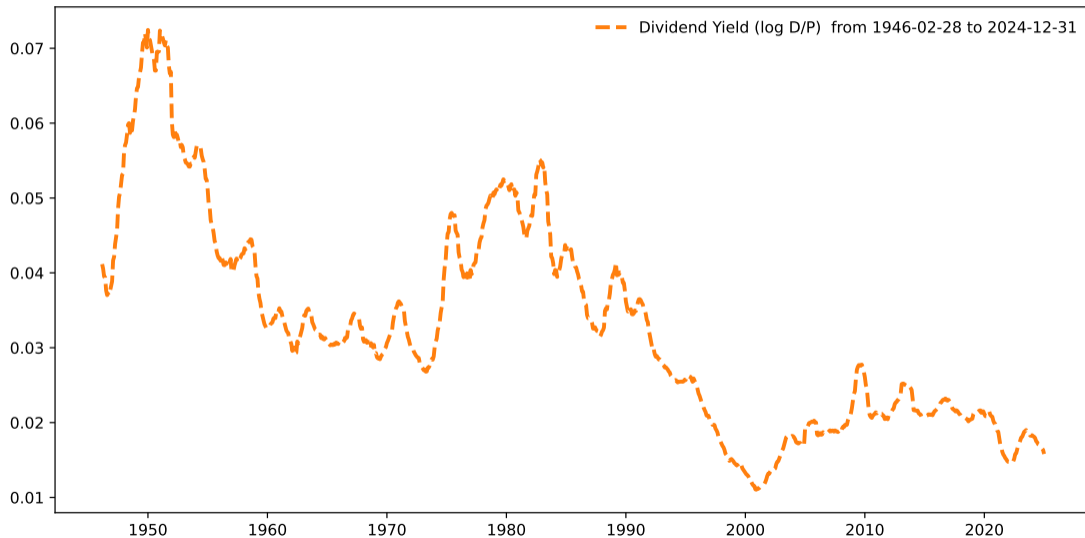
Time

Stock Return Predictability

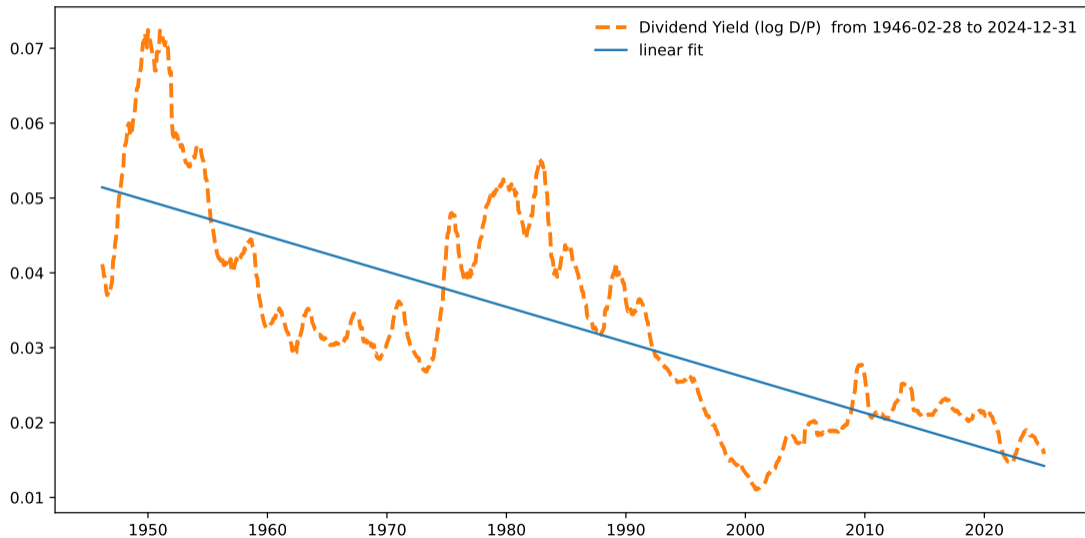
Stock Return Predictability

- ▶ Let's take a look at the dividend/yield data for the U.S.
- ▶ We'll use the dividend yield for the VW-CRSP Index (includes Amex, NYSE and NASDAQ) available on CRSP
- ▶ This index essentially includes all publicly traded stocks

Dividend Yield



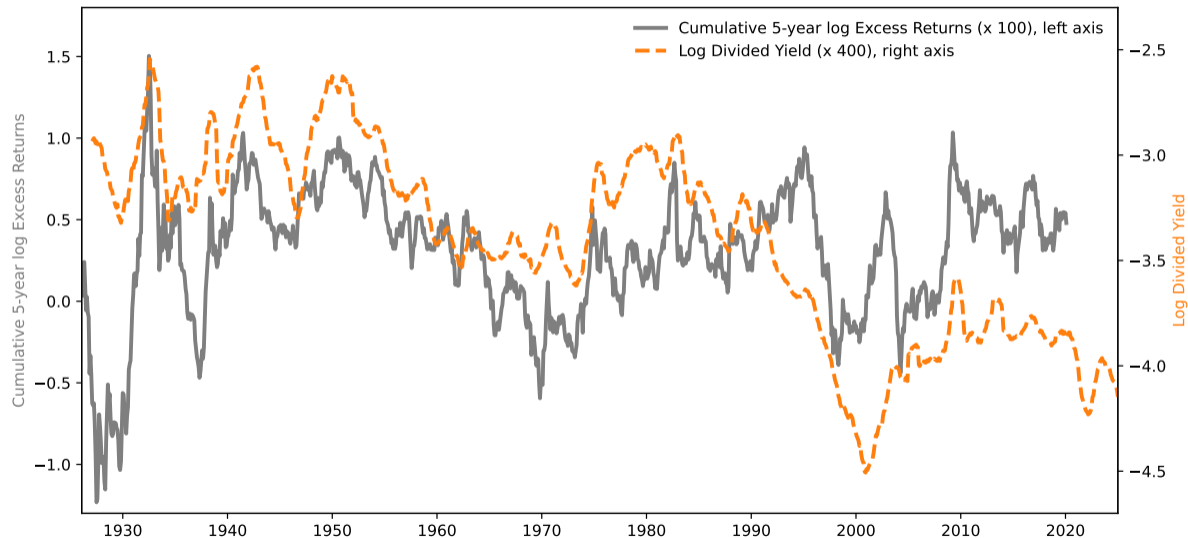
Dividend Yield



Dividend Yield

- ▶ The U.S. dividend yield has been trending down secularly since the early 80's
 - ▶ This may be partly due to changes in the payout behavior of U.S. corporations (switch to repurchases etc.)
- ▶ But it turns out there is a tight link between the dividend yield and future returns
- ▶ We compare the dividend yield $\log \frac{d_t}{p_t}$ and returns in the future $\log R_{t \rightarrow t+k} - \log R_{t \rightarrow t+k}^f$
- ▶ Do you know how to compute dividend yield from cum- and ex- divided returns?

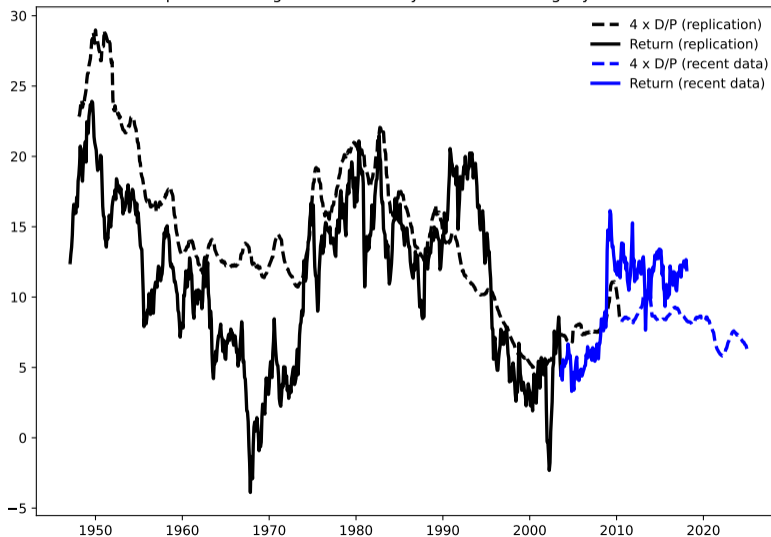
Dividend Yield and Future Returns



Dividend Yield and Future Returns

Replicating Figure 1 in John Cochrane's 2010 Presidential Address for the AFA

Replication of Figure 1: Dividend yield and following 7-year return



Regression Results

$$R_{t \rightarrow t+k}^e = a + b \times \frac{D_t}{P_t} + \varepsilon_{t+k}$$

Horizon k	b	$t(b)$	R^2	$\sigma[E_t(R^e)]$	$\frac{\sigma[E_t(R^e)]}{E(R^e)}$
1 year	3.8	(2.6)	0.09	5.46	0.76
5 years	20.6	(3.4)	0.28	29.3	0.62

Table 1 from Cochrane (2010)

- ▶ Large effects of dividend yield on returns:
 - ▶ One percentage point increase in dividend yield forecasts near four percentage points in return
- ▶ 5.46 percent of variation from fitted value!
 - ▶ This is comparable to the equity premium
- ▶ Note that the R^2 increases with the forecasting horizon:
 - ▶ Very hard to predict stock returns between now and the same time tomorrow
 - ▶ Much easier to predict stock returns between now and the same time 5 years from now.

Regression Results: extended sample

$$\log R_{t \rightarrow t+k} - \log R_{t \rightarrow t+k}^f = a + b \log \frac{d_t}{p_t} + \varepsilon_t$$

	period	b	R ²
k = 10 years	1927-2024	0.49	0.18
k = 5 years	1927-2024	0.23	0.06
k = 1 years	1927-2024	0.08	0.03

Interpretation

- ▶ Large effects of dividend yield on returns
- ▶ If you see a 100 bps increase in the dividend yield, then returns will be 23 bps higher over the next 5 years.
- ▶ Higher R^2 with longer horizons: better model fit
 - ▶ Very hard to predict stock returns between now and the same time tomorrow
 - ▶ Much easier to predict stock returns between now and the same time 10 years from now.

Structural Breaks: subsamples

D/P predicts returns even better in subsamples

	period	b	R2
k = 10 years	1927-2024	0.49	0.18
k = 10 years	1927-1954	1.64	0.36
k = 10 years	1946-2004	1.27	0.46
k = 10 years	1996-2024	1.60	0.66
k = 5 years	1927-2024	0.23	0.06
k = 5 years	1927-1949	1.82	0.37
k = 5 years	1946-1999	0.61	0.24
k = 5 years	1996-2024	1.06	0.44
k = 1 years	1927-2024	0.08	0.03
k = 1 years	1927-1945	0.55	0.14
k = 1 years	1946-1995	0.23	0.15
k = 1 years	1996-2024	0.44	0.23

Discount vs. Cash Flow

Time-series facts

- ▶ Dividend yields forecast returns with economically large coefficients
- ▶ Cambell-Shiller decomposition:

$$dp_t \approx \sum_{j=1}^k \rho^{j-1} r_{t+j} - \sum_{j=1}^k \rho^{j-1} \Delta d_{t+j} + \rho^k dp_{t+k}$$

- ▶ dividend yields vary because of cash-flow or discount rate?
- ▶ turns out that high prices relative to dividends entirely forecast low returns

Discount vs. Cash Flow

► Regressions

$$\sum_{j=1}^k \rho^{j-1} r_{t+j} = a_r + b_r^k dp_t + \varepsilon_{t+k}^r$$

$$\sum_{j=1}^k \rho^{j-1} \Delta d_{t+j} = a_{\Delta d} + b_{\Delta d}^k dp_t + \varepsilon_{t+k}^{\Delta d}$$

$$\rho^k dp_{t+k} = a_{dp} + b_{dp}^k dp_t + \varepsilon_{t+k}^{dp}$$

► Cambell-Shiller decomposition implies

$$1 \approx b_r^k - b_{\Delta d}^k + b_{dp}^k$$

Regression Results—Table 2 from Cochrane (2010)

Method and Horizon	Coefficient		
	$b_r^{(k)}$	$b_{\Delta d}^{(k)}$	$\rho^k b_{dp}^{(k)}$
Direct regression, $k = 15$	1.01	-0.11	-0.11
Implied by VAR, $k = 15$	1.05	0.27	0.22
VAR, $k = \infty$	1.35	0.35	0.00

- ▶ pd ratio volatility: variation in expected returns!
- ▶ High prices relative to dividends do NOT forecast higher dividend growth
- ▶ High prices relative to dividends forecast lower returns

All price-dividend ratio volatility corresponds to variation in expected returns. None corresponds to variation in expected dividend growth, and none to “rational bubbles.”

Chasing High Yields Works!

- ▶ If you regress future Treasury **bond returns** on the slope of the yield curve, you'll also find evidence of return predictability
- ▶ If you regress future **currency returns** on interest rate differences (=difference between forward rate and spot rate) between countries, you'll also find evidence of return predictability
- ▶ If you regress future **commodity returns** on the difference between the forward price and the spot price, you'll also find evidence of return predictability
- ▶ These strategies are referred to as 'carry' trades

What's carry?

- ▶ Carry is the expected return on an investment when the price does not change:
 1. Currencies: interest rate difference
 2. Commodities: difference between spot price and forward price
 3. Stocks: Dividend yield (roughly)
 4. Bonds: Slope of the yield curve
- ▶ Carry predicts returns (see 'Carry' by Kojien, Moskowitz, Pedersen and Vrugt) in all of these markets

Improving Forecasting Regressions

Campbell & Thompson (2008, RFS)

- ▶ Response to Welch and Goyal (2007)
 - ▶ WG: poor predictability of the equity premium
 - ▶ WG: better to just use average returns as predictor
- ▶ Stock returns are predictable
- ▶ Many of the predictability regressions can serious econometric issues
 - e.g. Stambaugh (1999) bias: if dp ratio follow an AR(1) then we get biased estimations
 - There are several papers trying to fix this and other issues
- ▶ Another issue: out-of-sample performance
 - ▶ We want a model that generate good out-of-sample forecasts

Campbell & Thompson (2008, RFS)

Key contribution:

- ▶ Agrees with WG: unimpressive in-sample results and poor out-of-sample performance
- ▶ Restricted regressions perform considerably better than unrestricted ones
 - ▶ Truncation based on sign:

$$\max\{R_t^{forecast}, 0\}$$

- ▶ Require positive estimated slope

$$\max\{\beta_t^{forecast}, 0\}$$

- ▶ These improve out-of-sample R^2

Campbell & Thompson (2008, RFS)

Data

- ▶ Forecasting variables
 - ▶ valuation ratios: the dividend-price ratio, earnings-price ratio, smoothed earnings-price ratio, and book-to-market ratio
 - ▶ Smoothed measure of accounting real return on equity (ROE). This gives a measure of the total resources available to be divided between real payouts and real growth in book equity.
 - ▶ Nominal interest rates and inflation: the short-term interest rate, the long-term bond yield, the term spread between long- and short-term treasury yields, the default spread between corporate and treasury bond yields, and the lagged rate of inflation.
 - ▶ Equity share of new issues proposed by Baker and Wurgler (2000) and the consumption-wealth ratio of Lettau and Ludvigson (2001).

Campbell & Thompson (2008, RFS)

- ▶ Monthly data and predict simple monthly or annual stock returns on the S&P 500 Index
- ▶ Truncation based on sign:

$$\max\{R_t^{forecast}, 0\}$$

- ▶ Require positive estimated slope

$$\max\{\beta_t^{forecast}, 0\}$$

- ▶ These improve out-of-sample R^2

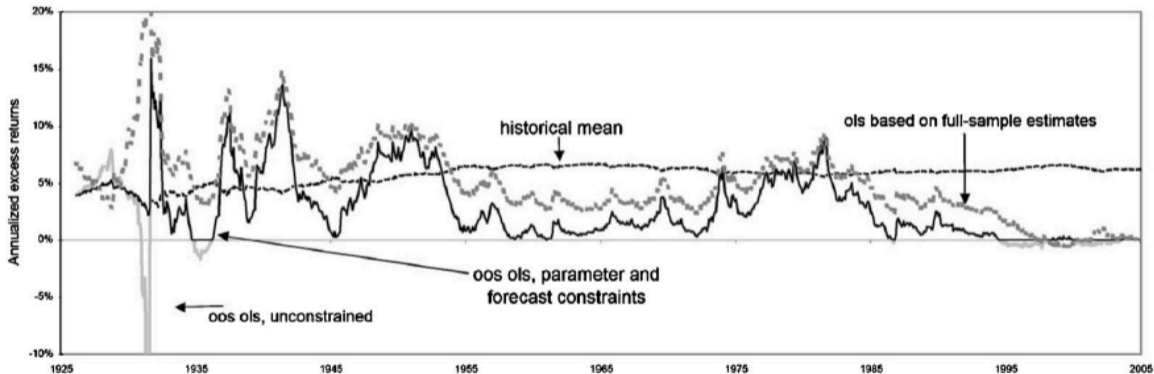
Campbell & Thompson (2008, RFS)

Table 1: Excess return prediction with regression constraints

	Sample Begin	Forecast Begin	In-Sample <i>t</i> -statistic	In-Sample <i>R</i> -squared	Out-of-Sample <i>R</i> -squared with Different Constraints			
					Unconstrained	Positive Slope	Pos. Forecast	Both
A: Monthly Returns								
Dividend-price ratio	1872m2	1927m1	1.25	1.13%	−0.65%	0.05%	0.07%	0.08%
Earnings-price ratio	1872m2	1927m1	2.29	0.71	0.12	0.18	0.14	0.18
Smooth earnings-price ratio	1881m2	1927m1	1.85	1.36	0.33	0.42	0.38	0.43
Book-to-market	1926m6	1946m6	1.96	0.61	−0.43	−0.43	0.00	0.00
ROE	1936m6	1956m6	0.36	0.02	−0.93	−0.06	−0.93	−0.06
T-Bill rate	1920m1	1940m1	2.44	0.86	0.52	0.51	0.57	0.55
Long-term yield	1870m1	1927m1	1.46	0.19	−0.19	−0.19	0.20	0.20
Term spread	1920m1	1940m1	2.16	0.65	0.46	0.47	0.45	0.46
Default spread	1919m1	1939m1	0.74	0.10	−0.19	−0.19	−0.19	−0.19
Inflation	1871m5	1927m1	0.39	0.06	−0.22	−0.21	−0.18	−0.17
Net equity issuance	1927m12	1947m12	1.74	0.48	0.34	0.34	0.50	0.50
Consumption-wealth ratio	1951m12	1971m12	4.57	2.60	−1.36	−1.36	0.27	0.27
B: Annual Returns								
Dividend-price ratio	1872m2	1927m1	2.69	10.8	5.53	5.53	5.63	5.63
Earnings-price ratio	1872m2	1927m1	2.84	6.78	4.93	4.93	4.94	4.94
Smooth earnings-price ratio	1881m2	1927m1	3.01	13.57	7.89	7.89	7.85	7.85
Book-to-market	1926m6	1946m6	1.98	8.26	−3.38	−3.38	1.39	1.39
ROE	1936m6	1956m6	0.35	0.32	−8.60	−0.03	−8.35	−0.03
T-Bill rate	1920m1	1940m1	1.77	4.26	5.54	5.54	7.47	7.47
Long-term yield	1870m1	1927m1	0.91	0.77	−0.15	−0.15	2.26	2.26
Term spread	1920m1	1940m1	1.72	3.10	4.79	4.79	4.74	4.74
Default spread	1919m1	1939m1	0.07	0.01	−3.81	−3.81	−3.81	−3.81
Inflation	1871m5	1927m1	0.17	0.07	−0.71	−0.71	−0.71	−0.71
Net equity issuance	1927m12	1947m12	0.54	0.35	−4.27	−4.27	−2.38	−2.38
Consumption-wealth ratio	1951m12	1971m12	3.76	19.87	−7.75	−7.75	−1.48	−1.48

Campbell & Thompson (2008, RFS)

Panel A: Forecasts Based on Dividend Yield
("oos ols" denotes out-of-sample ordinary least squares)



Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

$$r_{t+1} = \mu + x_t + \varepsilon_{t+1}$$

- ▶ x_t has mean zero and variance σ_x^2
- ▶ ε_t has mean zero and variance σ_ε^2
- ▶ Assume a mean-variance investor with risk aversion γ
- ▶ Unconditional Sharpe ratio:

$$S =$$

- ▶ R^2 for the regression of excess return on predictor variable

$$R^2 =$$

Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

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- ▶ Assume a mean-variance investor with risk aversion γ
- ▶ Unconditional Sharpe ratio:

$$S = \frac{\mu}{\sqrt{\sigma_x^2 + \sigma_\varepsilon^2}}$$

- ▶ R^2 for the regression of excess return on predictor variable

$$R^2 =$$

Campbell & Thompson (2008, RFS)

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- ▶ R^2 for the regression of excess return on predictor variable

$$R^2 = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_\varepsilon^2}$$

Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

- Optimal mean-variance weight on the risky asset:

$$\alpha =$$

and earn an average excess return given by:

$$\mathbb{E}[\alpha r_{t+1}] =$$

Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

- Optimal mean-variance weight on the risky asset:

$$\alpha = \frac{1}{\gamma} \times \frac{\mu}{\sigma_x^2 + \sigma_\varepsilon^2}$$

and earn an average excess return given by:

$$\mathbb{E}[\alpha r_{t+1}] =$$

Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

- Optimal mean-variance weight on the risky asset:

$$\alpha = \frac{1}{\gamma} \times \frac{\mu}{\sigma_x^2 + \sigma_\varepsilon^2}$$

and earn an average excess return given by:

$$\mathbb{E}[\alpha r_{t+1}] = \frac{1}{\gamma} \times \frac{\mu^2}{\sigma_x^2 + \sigma_\varepsilon^2} = \frac{S^2}{\gamma}$$

Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

- Optimal time-varying mean-variance weight:

$$\alpha_t =$$

and earn an average excess return given by:

$$\mathbb{E}[\alpha_t r_{t+1}] =$$

Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

- Optimal time-varying mean-variance weight:

$$\alpha_t = \frac{1}{\gamma} \times \frac{\mu + x_t}{\sigma_\varepsilon^2}$$

and earn an average excess return given by:

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Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

- Optimal time-varying mean-variance weight:

$$\alpha_t = \frac{1}{\gamma} \times \frac{\mu + x_t}{\sigma_\varepsilon^2}$$

and earn an average excess return given by:

$$\mathbb{E}[\alpha_t r_{t+1}] = \frac{1}{\gamma} \times \frac{\mu^2 + \sigma_x^2}{\sigma_\varepsilon^2} = \frac{1}{\gamma} \times \frac{S^2 + R^2}{1 - R^2}$$

Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

- Gain from timing the market:

$$\mathbb{E}[\alpha_t r_{t+1}] - \mathbb{E}[\alpha r_{t+1}] =$$

- Gain from timing the market:

$$\frac{\mathbb{E}[\alpha_t r_{t+1}] - \mathbb{E}[\alpha r_{t+1}]}{\mathbb{E}[\alpha r_{t+1}]} =$$

Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

- Gain from timing the market:

$$\mathbb{E}[\alpha_t r_{t+1}] - \mathbb{E}[\alpha r_{t+1}] = \frac{1}{\gamma} \times \frac{R^2}{1 - R^2} (1 + S^2)$$

- Gain from timing the market:

$$\frac{\mathbb{E}[\alpha_t r_{t+1}] - \mathbb{E}[\alpha r_{t+1}]}{\mathbb{E}[\alpha r_{t+1}]} =$$

Campbell & Thompson (2008, RFS)

Relation between Sharpe Ratio and R^2

- Gain from timing the market:

$$\mathbb{E}[\alpha_t r_{t+1}] - \mathbb{E}[\alpha r_{t+1}] = \frac{1}{\gamma} \times \frac{R^2}{1 - R^2} (1 + S^2)$$

- Gain from timing the market:

$$\frac{\mathbb{E}[\alpha_t r_{t+1}] - \mathbb{E}[\alpha r_{t+1}]}{\mathbb{E}[\alpha r_{t+1}]} = \frac{R^2}{1 - R^2} \times \frac{1 + S^2}{S^2} \approx \frac{R^2}{S^2}$$

Takeaway

- ▶ Stock returns are predictable
- ▶ Unconstrained models perform poorly (Welch and Goyal results)
 - ▶ Follow up paper: [Goyal, Welch, Zafirov \(2024, RFS\)](#)
- ▶ Simple ad-hoc constraint go a long way in terms of improving out-of-sample performance
- ▶ Predictive regressions R^2 look small but it makes sense: benefit from timing the investment is roughly proportional to $\frac{R^2}{S^2}$

Additional Material: Further Improving Forecasting Regressions

Pettenuzo, Timmermann, and Valkanov (2014, JFE)

- ▶ Similar idea to Campbell and Thompson (2008)
- ▶ Economically motivated constraints offer the potential to sharpen forecasts, particularly when the data are noisy and parameter uncertainty is a concern as in return prediction models.

Pettenuzo, Timmermann, and Valkanov (2014, JFE)

- ▶ Estimations constraints:
 - ▶ Non-negative equity premia
 - ▶ Bounds on the conditional Sharpe ratio
- ▶ Bayesian approach: compute the predictive density of the equity premium subject to economic constraints
- ▶ Approach makes efficient use of the entire sequence of observations in computing the predictive density and also accounts for parameter uncertainty
- ▶ What is the difference between this approach and the one developed by Campbell and Thompson?

Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Equity premium constraint

- ▶ Linear model to forecast returns:

$$r_{\tau+1} = \mu + \beta x_{\tau} + \varepsilon_{\tau+1}, \quad \tau = 1, \dots, t-1,$$

$$\varepsilon_{\tau+1} \sim N(0, \sigma_{\varepsilon}^2).$$

- ▶ Equity premium constraint:

$$\mu + \beta x_{\tau} \geq 0 \quad \text{for } \tau = 1, \dots, t.$$

- ▶ Campbell and Thompson approach:

$$\hat{r}_{t+1|t} = \max(0, \hat{\mu}_t + \hat{\beta}_t x_t),$$

- Ignores information about the estimation if negative EP

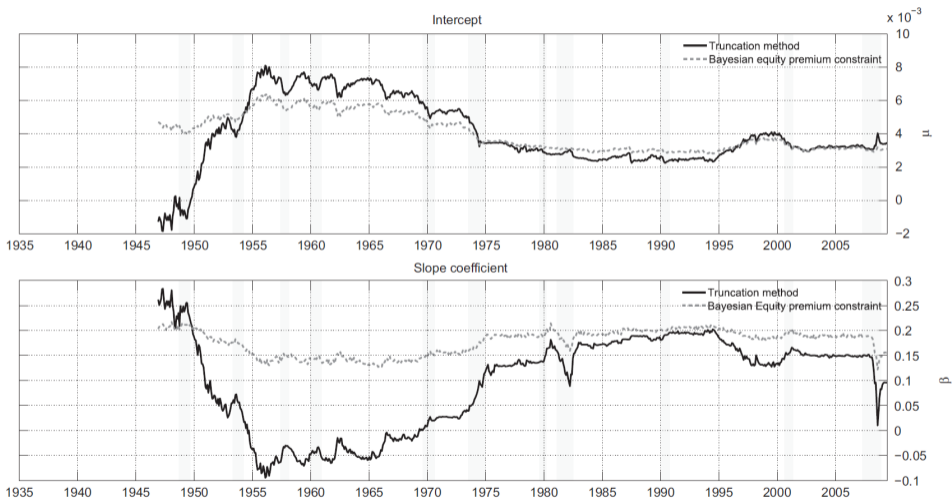
Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Equity premium forecast: one-period ahead



Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Equity premium constraint: using default yield as predictor



Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Sharpe Ratio constraint

- ▶ Linear model with time-varying volatility:

$$r_{\tau+1} = \mu + \beta x_{\tau} + \exp(h_{\tau+1})u_{\tau+1},$$

where the noise term is standard normal

- ▶ The log volatility is a random walk

$$h_{\tau+1} = h_{\tau} + \xi_{\tau+1}$$

where $\xi_{\tau+1} \sim N(0, \sigma_{\xi}^2)$

- ▶ Sharpe ratio is given by

$$SR_{\tau+1|\tau} = \frac{\sqrt{H}(\mu + \beta x_{\tau})}{\exp(h_{\tau} + 0.5\sigma_{\xi}^2)},$$

- How to calculate this expression?

- ▶ Shape ratio bounds

$$SR^l \leq SR_{\tau+1|\tau} \leq SR^u \quad \text{for } \tau = 1, \dots, t$$

Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Sharpe Ratio constraint

- ▶ Shape ratio bounds

$$SR^l \leq SR_{\tau+1|\tau} \leq SR^u \quad \text{for } \tau = 1, \dots, t$$

- ▶ SR should be positive
- ▶ SR around 0.5 are seen as normal
- ▶ Baseline estimation: Sharpe ratios in the unit interval

Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Bayesian estimation

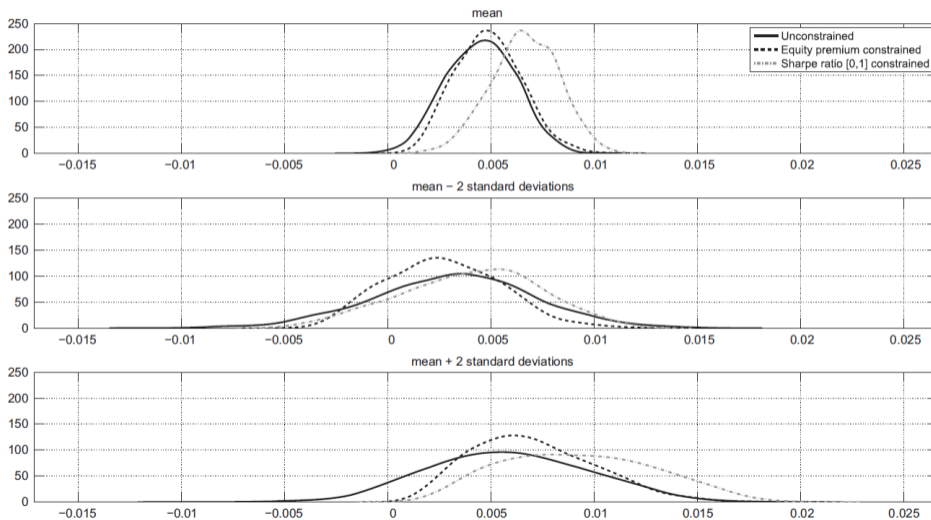
There are three parameters to estimate: μ , β , and σ_ξ

- ▶ Economic constraints are priors
- ▶ Parameters are seen as random variables
- ▶ Start with a prior about the parameters
- ▶ Update your parameters posteriors to get estimates
- ▶ Updating is tricky given the constraints

Solution: simulations

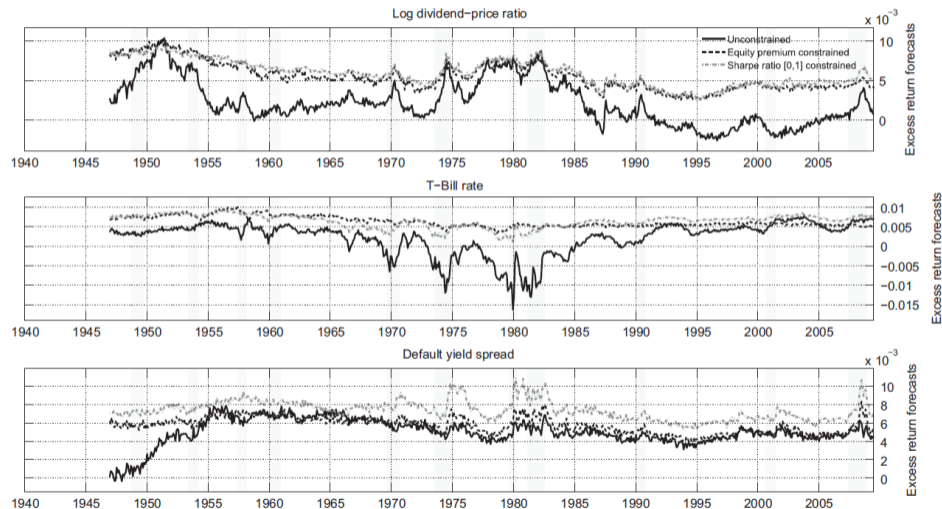
Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Full-sample posterior of the Equity premium: default yield spread



Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Out-of-sample equity premium forecast



Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Out-of-sample R^2

Monthly

Variable	Panel A: full sample (1947–2010)			
	No constraint	CT truncation	EP constraint	SR [0,1] constraint
Log dividend–price ratio	0.10%*	0.25%**	0.64%***	0.49%**

Quarterly

Variable	Panel A: full sample (1947–2010)			
	No constraint	CT truncation	EP constraint	SR [0,1] constraint
Log dividend–price ratio	−0.16%**	1.20%**	2.11%***	1.68%***

Annual

Variable	Panel A: full sample (1947–2010)			
	No constraint	CT truncation	EP constraint	SR [0,1] constraint
Log dividend–price ratio	1.98%*	3.14%*	5.92%***	5.35%***

Takeaway

- ▶ New method to impose constraint that rule out negative equity premia and bounds the Sharpe ratio
- ▶ Constraints affect the estimation procedure
- ▶ More efficient use of information available
- ▶ Smoother predictions: reduced effects of outliers

- ▶ Constraints improve forecasting accuracy

Annex

Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Bayesian estimation: no constraint

Prior of μ and β

$$\begin{bmatrix} \mu \\ \beta \end{bmatrix} \sim N(\underline{b}, \underline{V})$$

where

$$\underline{b} = \begin{bmatrix} \bar{r}_t \\ 0 \end{bmatrix}, \quad \underline{V} = \begin{bmatrix} \psi^2 s_{r,t}^2 & 0 \\ 0 & \psi s_{r,t}^2 / s_{x,t}^2 \end{bmatrix}$$

$$\bar{r}_t = \frac{1}{t-1} \sum_{\tau=1}^{t-1} r_{\tau+1}, \quad s_{r,t}^2 = \frac{1}{t-2} \sum_{\tau=1}^{t-1} (r_{\tau+1} - \bar{r}_t)^2$$

$$\bar{x}_t = \frac{1}{t-1} \sum_{\tau=1}^{t-1} x_{\tau}, \quad s_{x,t}^2 = \frac{1}{t-2} \sum_{\tau=1}^{t-1} (x_{\tau} - \bar{x}_t)^2$$

Ψ controls the tightness of the prior

Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Bayesian estimation: no constraint

Prior for the error precision of the return innovation:

$$\sigma_{\varepsilon}^{-2} \sim G(s_{r,t}^{-2}, \underline{v}_0(t-1))$$

The history of volatility is updated according to

$$p(h^t | \sigma_{\xi}^{-2}) = \prod_{\tau=1}^{t-1} p(h_{\tau+1} | h_{\tau}, \sigma_{\xi}^{-2}) p(h_1)$$

Initial distribution of volatility

$$h_1 \sim N(\ln(s_{r,t}), \underline{k}_h)$$

Prior of σ_{ξ}

$$\sigma_{\xi}^{-2} \sim G(1/\underline{k}_{\xi}, 1)$$

Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Bayesian estimation: imposing constraints

Modified priors

$$\begin{bmatrix} \mu \\ \beta \end{bmatrix} \sim \text{N}(\underline{b}, \underline{V}), \quad \mu, \beta \in A_t$$

EP constraint:

$$A_t = \{\mu + \beta x_\tau \geq 0, \tau = 1, \dots, t\}$$

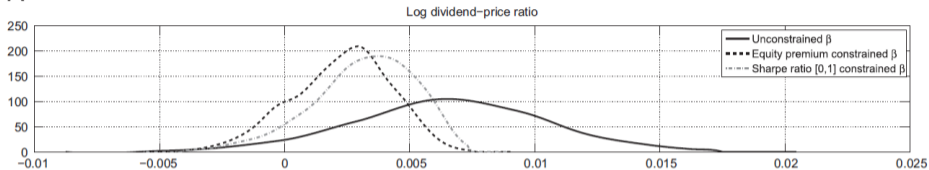
Sharpe ratio constraint

$$\tilde{A}_t = \{SR^l \leq SR_{\tau+1|\tau} \leq SR^u, \tau = 1, \dots, t\}$$

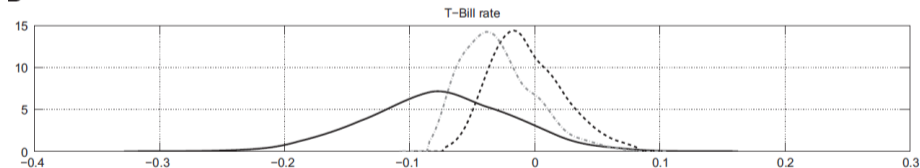
Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Posterior of slope coefficients (using data until 2010)

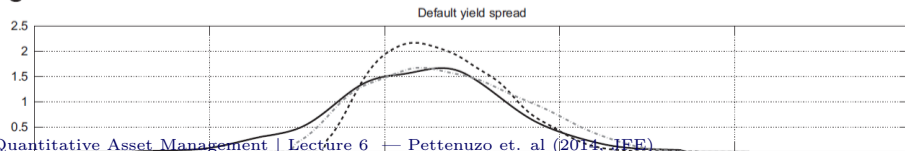
A



B

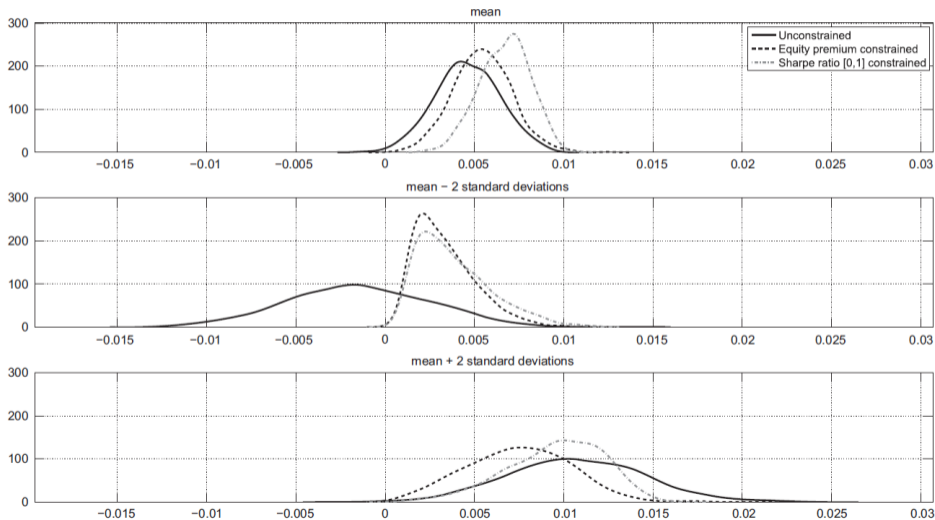


C



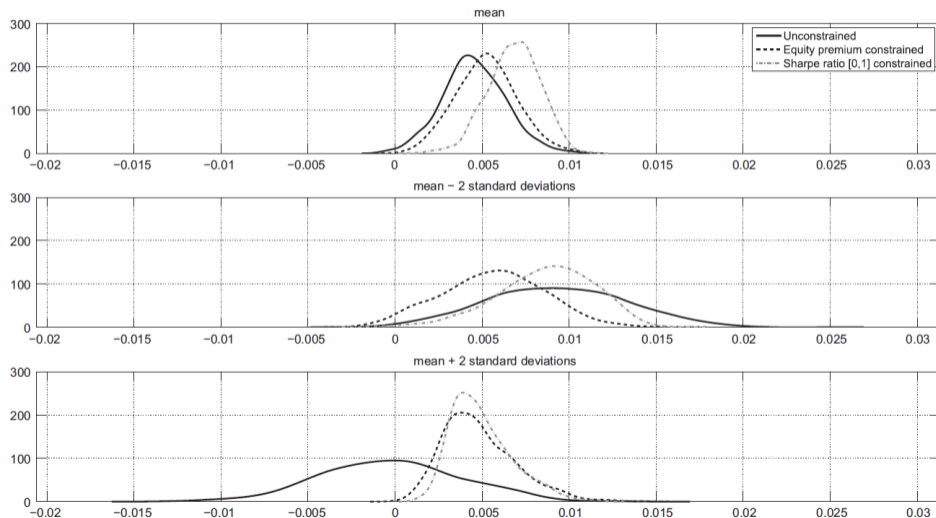
Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Full-sample posterior of the Equity premium: log dividend-price ratio



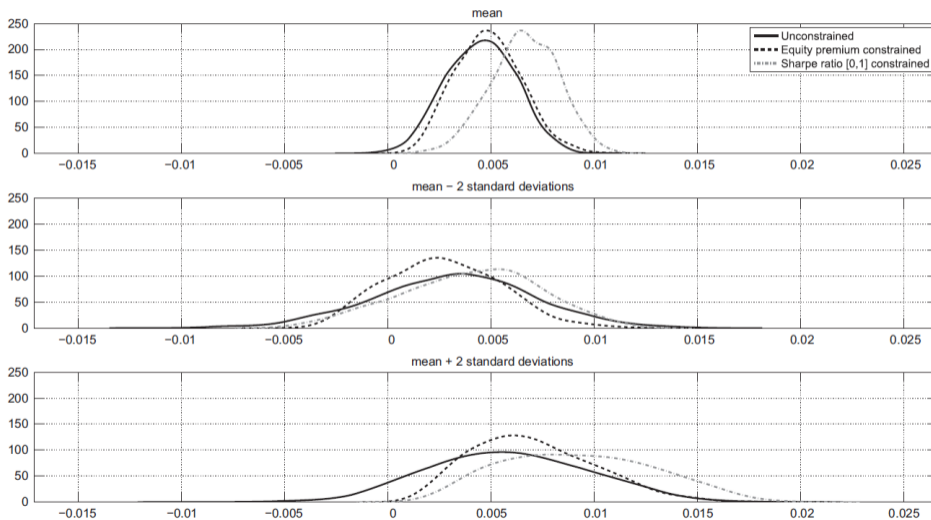
Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Full-sample posterior of the Equity premium: T-bill rate



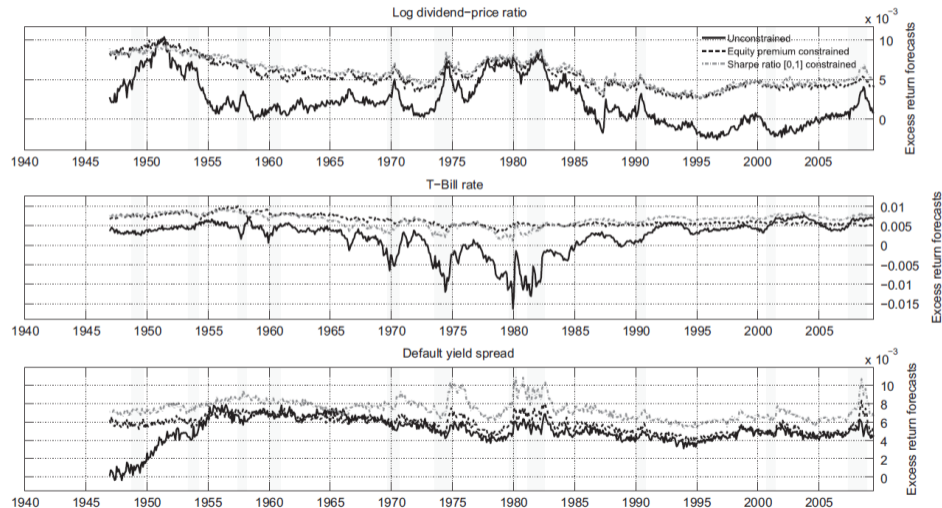
Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Full-sample posterior of the Equity premium: default yield spread



Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Out-of-sample equity premium forecast



Pettenuzo, Timmermann, and Valkanov (2014, JFE)

Conditional Sharpe ratio

