Investments

Topic 1: Basic Valuation

UCLA | Fall 2024

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This version: September 26, 2024

Overview of Topic 1

1. Time Value of Money
2. Present Value Rule
3. Compounding
4. Uncertain Cash Flows
5. Monthly Rates
6. APRs and EARs
7. Perpetuities and Annuities
8. Internal Rate of Return
9. The Principle of No Arbitrage and Present Value
10. Perfect Markets

1. Time Value of Money

Time Value of Money

Example: You have invented a new search algorithm for computer databases. One company offers you \$7,000 for the idea, but first you must develop software that implements the idea. A second company offers \$500 but does not require you to develop the software. A programmer requires \$6,000 payable immediately to program your idea. Programming will take one year. What should you do?

- \$1 today is worth more than \$1 in the future.
- Principle of the **time value of money**:
  - if receiving a fixed sum, want it as soon as possible.
  - if paying a fixed sum, delay payment for as long as possible.

2. Present Value Rule

Present Value Rule

- We need to evaluate a stream of cash flows.
- Basic idea: Convert all cash flows into today's cash flows.
- Cash flows are:

- Suppose that $7,000 in a year is worth x today. Suppose also that the annual interest rate is 5%. Then

$$\begin{aligned} x(1+5\%) &= \$7,000 \\ x &= \$6,667 \end{aligned}$$

- PV in our example:

$$PV_0 = -\$6,000 + \$6,667 = \$667 > \$500 \Rightarrow \text{choose the 1st co}$$

PV Rule (cont'd)

- General formula:

$$\boxed{PV_0 = C_0 + \frac{C_1}{1+r}}$$

First company: $C_0 = -\$6,000,\ \ C_1 = \$7,000$
Second company: $C_0 = \$500$

- What if annual interest rate were 20%?

$$\begin{aligned} x(1+20\%) &= \$7,000 \\ x &= \$5,833 \end{aligned}$$

- PV in our example:

$$PV_0 = -\$6,000 + \$5,833 = -\$167 < \$500 \Rightarrow \text{choose the 2nd co}$$

What Did we Learn?

- Time value of money: \$1 today $\neq$ \$1 in the future.
- Present value (PV) rule:
  - convert future cash flows in today's cash flows.
  - add them up.
  - choose the deal with the highest PV.
- The PV is sensitive to the interest rate.

3. Compounding

Compounding

- Consider investing \$100 at the annual interest rate 10%.
  How much will your investment be worth in 3 years?
  - After one year, you will have: $\$100 + \$100 \times 0.1 = \$110 \qquad (100 \times 1.1)$
  - After two years, you will have: $\$110 + \$110 \times 0.1 = \$121 \qquad (100 \times 1.1^2)$
  - After three years, you will have: $\$121 + \$121 \times 0.1 = \$133.1 \qquad (100 \times 1.1^3)$
  - This is different than $\$100 + \$30 = \$130$.
  - Have to account for the interest on interest.

- More generally, investing C dollars at the annual interest rate r gives

$$C(1 + r)^t$$

  dollars after t years (*future value*).

Multiple Cash Flows

- Consider cash flows C_0, C_1, .., C_T, at years 0, 1, .., T, and suppose that the annual interest rate is r.
- The cash flow C_t is worth
$$\frac{C_t}{(1+r)^t}$$
today.
- The present value of the cash flow stream at year 0 is

$$\boxed{PV_0 = C_0 + \frac{C_1}{1+r} + \ldots + \frac{C_T}{(1+r)^T}}$$

Terminology

- Consider cash flows C_0, C_1, .., C_T, at years 0, 1, .., T, and suppose that the annual interest rate is r.

- The **discount rate** for year t is r, i.e., the interest rate.

- The **discount factor** for year t is

$$\frac{1}{(1+r)^t}.$$

- The **future value** of the cash flow stream at year T is

$$\boxed{FV_T = C_0(1+r)^T + C_1(1+r)^{T-1} + \ldots + C_T}$$

Horizon-Dependent Interest Rate

- So far, we assumed that the interest rate is horizon-independent.
- However, the interest rate generally depends on horizon. There is a **term structure** of interest rates.
- If the discount rate for t years is r_t, the present value of a yearly cash flow stream C_0, C_1, .., C_T, is

$$\boxed{PV_0 = C_0 + \frac{C_1}{1+r_1} + \ldots + \frac{C_T}{(1+r_T)^T}}$$

Time Value of Money is Important..

- You win the \$1 million lottery jackpot which pays \$50,000 per year for next 20 years.
- How much is your prize worth assuming an annual interest rate of 6%?

$$
\begin{aligned}
PV &= \frac{50,000}{1.06} + \frac{50,000}{(1.06)^2} + \cdots + \frac{50,000}{(1.06)^{20}} \\
&= \frac{50,000}{0.06}\left(1 - \frac{1}{(1.06)^{20}}\right) \\
&= 833,333 \times 0.688 \\
&= 573,500
\end{aligned}
$$

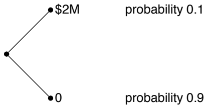
4. Uncertain Cash Flows

Uncertain Cash Flows

- So far, we have considered cash flows that occur with certainty.
- Does the PV rule apply when cash flows are uncertain?
- Example: It is late 2025. A friend of yours operates panzerotti stands near Palaitalia Santa Giulia. Your friend is willing to sell you the rights for operating the stands during the 2026 Olympic Games. The rights cost \$7,500 each. If the weather is gorgeous during the Olympics, you will make \$100,000 per stand. If not, you will make nothing. There is a 10% chance that the weather will be gorgeous. Suppose that you have \$150,000 you can invest in the stands. Should you invest?

An Incorrect Application of the PV Rule

- Compute cash flows.
  - Can buy $150{,}000/7{,}500 = 20$ stands.
  - Cash flows:



- Compute expected cash flow.

$$2M \times 0.1 + 0 \times 0.9 = 0.2M.$$

- Use riskless rate, say 5%.

- PV is

$$-150,000 + \frac{200,000}{1+5\%} = 40,476.$$

Attitudes Towards Risk

By discounting at the riskless rate, we implicitly assume that we will get \$0.2M for sure. However, \$0.2M is only the expected cash flow.

<u>Example:</u>

- Suppose that one offers you a gamble which pays off \$200K or \$0 with equal probabilities. The expected cash flow is \$100K.

- The gamble is not the same as \$100K for sure. The choice between the two depends on your personal attitudes towards risk.
  - **Risk averse:** Prefer \$100K for sure.
  - **Risk loving:** Prefer the gamble.
  - **Risk neutral:** Indifferent.

- A measure of your attitudes towards risk is the amount you would require to walk away from the gamble.

Expected Utility

- In Economics, people are assumed to choose between gambles using **expected utility**.
  - Assign to each payoff x a utility $u(x)$.
  - Compute the expectation over all payoffs.
- Example: Gamble of \$200K or \$0 with equal probabilities vs. \$100K for sure.
  - Expected utility of gamble is

$$u(200) \times 0.5 + u(0) \times 0.5.$$

  - Expected utility of sure payoff is $u(100)$.
- Risk attitudes depend on properties of $u(x)$.
  - Risk averse $\Leftrightarrow$ $u(x)$ concave
  - Risk neutral $\Leftrightarrow$ $u(x)$ linear
  - Risk loving $\Leftrightarrow$ $u(x)$ convex

The PV Rule with Uncertain Cash Flows

- The correct way to apply the PV rule with uncertain cash flows is to use
  - expected cash flows
  - a **risk-adjusted** discount rate.

- We will cover risk-adjusted discount rates in-depth when we talk about Equities.

5. Monthly Rates

Monthly Rates

So far, we have considered yearly cash flows. How does the PV rule apply when cash flows are monthly, quarterly, semiannual, etc? Example: You rent an apartment in Santa Monica for 12 months. The rent is $1,800 per month. Compute the present value of the rent payments, when an annual interest rate is 8%.

- Need monthly rate x
- $1 today is $(1+x)$ in a month and $(1+x)^{12}$ in a year.
- Therefore:

$$\$1 \times (1+x)^{12} = \$1 \times (1+0.08) \Rightarrow 1+x = 1.08^{\frac{1}{12}} \Rightarrow x = 0.643\%$$

- Summarizing:

$$x = (1+r)^{\frac{1}{12}} - 1 \Rightarrow r = (1+x)^{12} - 1.$$

- Present value:

$$PV = 1,800 + \frac{1,800}{1+x} + \cdots + \frac{1,800}{(1+x)^{11}} = 20,868.$$

6. APRs and EARs

APRs and EARs

Citibank quotes you a rate of 7% for your home mortgage. The mortgage involves monthly payments.

- The 7% is not the "true" annual rate. It is only used to derive the monthly rate by:

$$x = \frac{7\%}{12} = 0.583\%.$$

- The "true" annual rate is:

$$(1 + x)^{12} - 1 = 7.23\%.$$

- The 7% is the **Annual Percentage Rate (APR)**. It is the quoted rate.
  The 7.23% is the **Effective Annual Rate (EAR)**. It is the true rate.
- The APR with monthly compounding is sometimes called a "monthly APR".
- "Monthly APR" is not a monthly rate, it is an annual rate!

APRs and EARs: Compounding Frequency

The compounding frequency associated to an APR matters.

Example: Suppose that your bank offers a 1-year CD with a 5% APR. What is the EAR with

- annual compounding? 1.05
- semiannual compounding? $\left(1+\frac{5\%}{2}\right)^{2}=1.0506$
- quarterly compounding? $\left(1+\frac{5\%}{4}\right)^{4}=1.0509$
- monthly compounding? $\left(1+\frac{5\%}{12}\right)^{12}=1.05116$
- daily compounding? $\left(1+\frac{5\%}{365}\right)^{365}=1.051268$
- continuous compounding? $\lim_{T\to\infty}\left(1+\frac{5\%}{T}\right)^{T}=1.05127=e^{0.05}$

General formula: With T compounding periods in a year:

$$\boxed{EAR=\left(1+\frac{APR}{T}\right)^{T}-1}$$

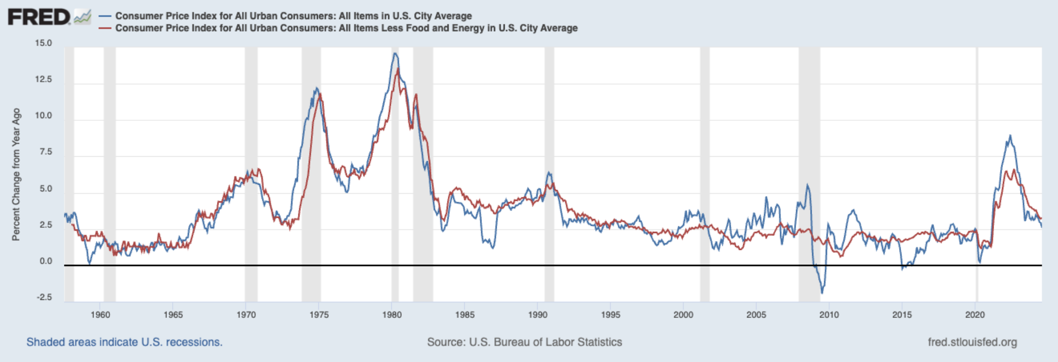
7. Perpetuities and Annuities

Perpetuities: Example

The 1998 tobacco settlement between the US Government and the tobacco industry calls for the tobacco industry to pay $15 billion (inflation-adjusted) each year forever starting in 2023. This is a perpetuity because its cash flows are perpetual. The tax rate for the industry is 40%. The payments are in real dollars, so the appropriate discount rate is the real rate of interest, which we take to be 2.5%. What is the after-tax present value of the perpetuity?

Perpetuity: Stream of constant cash flows, starting one period from today, and lasting forever.

Inflation



FRED
Consumer Price Index for All Urban Consumers: All Items in U.S. City Average
Consumer Price Index for All Urban Consumers: All Items Less Food and Energy in U.S. City Average
Percent Change from Year Ago
15.0
12.5
10.0
7.5
5.0
2.5
0.0
-2.5
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
Shaded areas indicate U.S. recessions.
Source: U.S. Bureau of Labor Statistics
fred.stlouisfed.org

Real vs. Nominal

- Real vs. Nominal Dollars
  - Nominal dollars are “regular” dollars.
  - Real dollars are dollars adjusted for inflation.
  - One nominal dollar received next year is

    $$1/(1+\pi)$$

    real dollars, where π is the inflation rate during the year.

Real vs. Nominal (cont'd)

- Real vs. Nominal Rates
  - Nominal rate (i): 1 dollar now gives $(1+i)$ nominal dollars next year.
  - Real rate (r): 1 dollar now gives $(1+r)$ real dollars next year.
  - Relation between i and r:
    - 1 dollar now gives $(1+i)$ nominal dollars next year, i.e., $(1+i)/(1+\pi)$ real dollars.
    - Therefore,
$$1+r=\frac{1+i}{1+\pi}, \text{ or } r \approx i-\pi.$$

- OK to use either real or nominal dollars. However:
  - If use real dollars, have to use real rate.
  - If use nominal dollars, have to use nominal rate.

PV of the Tobacco Perpetuity

- Let's first compute the PV as of January 2023.

$$PV = \frac{9}{1+2.5\%} + \frac{9}{(1+2.5\%)^2} + \ldots$$

- Generalizing, we need to compute

$$PV = \frac{C_1}{1+r} + \frac{C_1}{(1+r)^2} + \ldots$$

- This is

$$\frac{C_1}{1+r}\left(1 + \frac{1}{1+r} + \ldots\right) = \frac{C_1}{1+r}\frac{1}{1-\frac{1}{1+r}} = \frac{C_1}{r}.$$

- The PV of the tobacco perpetuity as of January 2023 is

$$\frac{9}{2.5\%} = \$360b.$$

- The PV as of January 1998 is

$$\frac{360}{(1+2.5\%)^{25}} = \$194.18b.$$

PV of a Perpetuity

- The PV of a perpetuity is
$$\frac{C_1}{r},$$
where C_1 is the cash flow and r the discount rate.
- The PV of a perpetuity that grows at a constant rate each period (**growing perpetuity**) is
$$\frac{C_1}{r-g},$$
where C_1 is the first cash flow, r the discount rate, and $g < r$ the growth rate of cash flows.
- Points to remember:
  - The first cash flow comes one period from today.
  - If C_1 is an annual (monthly, quarterly, ..) cash flow, use the annual (monthly, quarterly, ..) rate for r.

Annuities: Example

You get a car loan of \$20,000. The rate on the loan is 10% monthly APR, and payment extends over 5 years. What is the monthly payment?

- The monthly rate is

$$\frac{10\%}{12} = 0.83\%.$$

- The present value of the loan repayments is

$$\frac{C}{1+0.83\%} + \frac{C}{(1+0.83\%)^2} + \ldots + \frac{C}{(1+0.83\%)^{60}},$$

  where C is the monthly payment.
- We have to compute

$$\frac{C}{1+r} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^T}.$$

  This is the PV of an annuity.
- **Annuity**: Stream of constant cash flows, starting one period from today, and lasting for T periods.

Annuities: Example (cont'd)

- We can view an annuity as the difference between two perpetuities.
  - First perpetuity: cash flows start one period from today.
  - Second perpetuity: cash flows start $T+1$ periods from today.
- Difference is:

$$\frac{C}{r} - \frac{1}{(1+r)^T}\frac{C}{r} = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right].$$

- This is the PV of an annuity.
- For $r = 0.83\%$ and $T = 60$,

$$PV = C \times 47.07.$$

  Therefore, $C = \$424.90$.
- What is your principal balance after the first payment?
  - The interest part is \$20,000$\times$0.83%=\$166.67
  - The reduction in principal is \$424.90-\$166.67=\$258.23
  - The new balance is \$19,741.77

PV of an Annuity

- The PV of an annuity is

$$C\left[\frac{1}{r}-\frac{1}{r(1+r)^T}\right],$$

  where C is the cash flow, r the discount rate, and T the number of periods.

- Points to remember:
  - The first cash flow comes one period from today.
  - If C is an annual (monthly, quarterly, ..) cash flow, use the annual (monthly, quarterly, ..) rate for r.

8. Internal Rate of Return

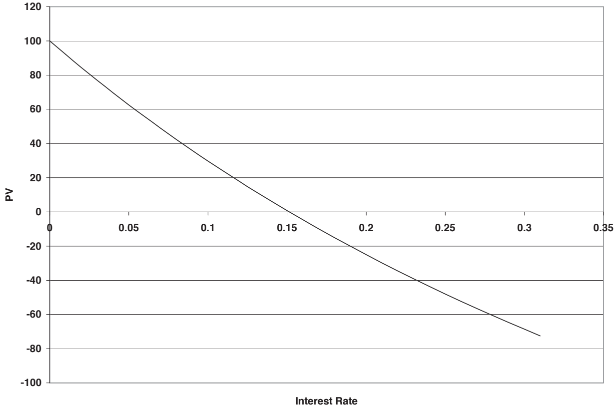
Internal Rate of Return

The **internal rate of return (IRR)** of a project is the discount rate in the PV calculation that makes the PV equal to zero.

Example: What is the IRR of a project that requires an initial investment of \$400 and produces cash flows of \$200 after one year and \$300 after two years?

Computing the IRR

- PV is $-400 + \frac{200}{1+r} + \frac{300}{(1+r)^2}$ decreasing in r!



- IRR is the point at which the PV is 0.
- Solving numerically, we get IRR=15.14%.

Interpretation of the IRR (cont'd)

- IRR is a "break-even" discount rate. If discount rate is smaller than IRR, undertake the project, since positive PV, and vice-versa.

- IRR is a project-specific rate (a rate "internal" to the project).

9. No-Arbitrage

No-Arbitrage

- What is an arbitrage?
  - Getting something for nothing
  - There is no risk
- Pure arbitrages should not exist (and typically don't) and a valid theory of asset prices should not allow arbitrages
- More formally, a collection of possible uncertain payoffs $\tilde{x} = \{\tilde{x}_1, \tilde{x}_2, \ldots\}$ and a pricing function $p(\tilde{x})$ leave **no arbitrage** opportunities if non-negative payoff of a portfolio, $w_1\tilde{x}_1 + w_2\tilde{x}_2 \geq 0$, has a positive price, $p(w_1\tilde{x}_1 + w_2\tilde{x}_2) > 0$
- The **law of one price** is a related concept and says:
  $p(w_1\tilde{x}_1 + w_2\tilde{x}_2) = w_1 p(\tilde{x}_1) + w_2 p(\tilde{x}_2)$
  (price of an ETF is the same as price of its constituents, with appropriate weights)
- If **LOP** is violated, so is **NA**

No-Arbitrage and PV

- Why do we use Present Value? It doesn't allow for arbitrages
- Example: consider an asset that pays \$100 in 1 period
- Suppose you can borrow for 1 period at rate r_1. What should the asset's price, P, be?

	Asset Price	Bank Account	Total Cost
Today	Buy at P ($-P$)	Borrow P at r_1 ($+P$)	0
Next period	$+\$100$	$-P(1+r_1)$	$100 - P(1+r_1)$

No-Arbitrage Price = NPV

- The price has to be

$$P = \frac{100}{1 + r_1}$$

- Why? It precludes arbitrage
- In this case $T = 1, d_1 = 1/(1 + r_1)$, and $C_1 = 100$

Arbitrage and Short-Selling

- What if the PV rule does not hold?

- If the market price is less than PV, you borrow money at r_1 and buy the security.

- What if market price is higher than PV?
  - You need to be able to sell the security, invest the money in the bank and wait until \$100 is paid.
  - How to do this?

- Short selling.

What are Short Sales?

- A short sale consists of selling a stock that we do not own. (Yes, this is possible...)
- Steps:
  - Date 0: Borrow the stock from a broker.
  - Date 0: Sell the stock in the market.
  - Between dates 0 and 1: Compensate the broker for any dividends the stock pays.
  - Date 1: Buy the stock in the market.
  - Date 1: Return the stock to the broker.
- A short sale is profitable if the stock price goes down.
- Naked short positions are rare
- Risks:
  - Unlimited downside
  - Short squeeze
  - Dividend payout
  - Liquidity shortage

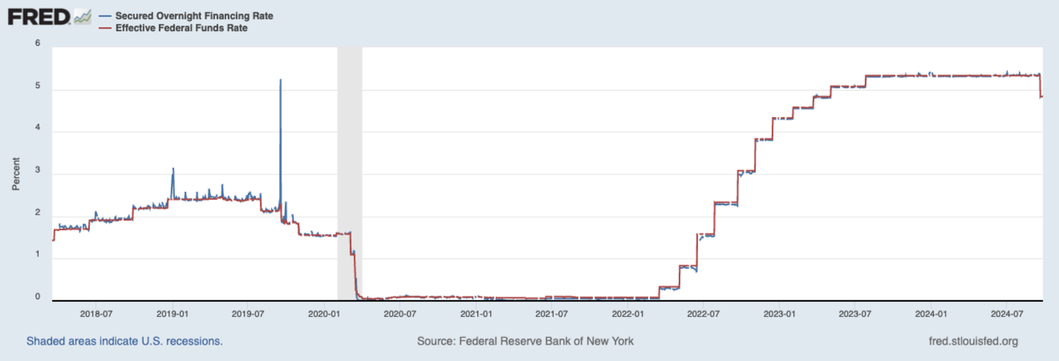
Repo Transactions

- Borrower of a security must leave collateral with the lender, in the form of cash (or other securities).
- **Repo transaction:**
  - Party A borrows a security from Party B, leaving cash as collateral.
  - Party B borrows cash from Party A, leaving the security as collateral.
- Party A (reverse repo):
  - Short sellers.
  - Lenders of cash. (Banks, money market funds, corporations.)
  - When dealers act as lenders of cash, they can re-use collateral (rehypothecate) in other transactions
- Party B (repo):
  - Leverage-constrained security buyers. (Hedge funds, trading desks.)
  - Lenders of securities. (Mutual funds, insurance companies.)
  - When dealers act as lenders of securities, it is usually a 'special' security (as opposed to general collateral, GC) that is needed by cash lenders, e.g., on-the-run Treasuries for hedge funds

Repo Rates and Haircuts

- **Repo rate:** Rate of interest earned by the cash collateral.
  - Equal to the **general collateral rate** for most securities.
  - Lower for securities that are **on special**.
- **Repo haircut:** Price of a security minus cash that can be borrowed with the security as collateral.
  - Increases in the security's riskiness.
- Repos are exempt from bankruptcy stays, which allows these agreements to terminate in the event of bankruptcy, giving the collateral holder the immediate right to use or dispose of the collateral
- We abstract away from issues of collateral

Repo Spiking



Limits to Arbitrage: Palm and 3Com

- In 2000, 3Com basically owned all of Palm.
- On March 2, 2000 3Com sold
  - 4% of its Palm stake in an IPO (at $38/share)
  - 1% to a consortium
  - kept 95% on their own.

- As a result, if you owned 1 share of 3Com, you would indirectly own 1.5 shares of Palm
- APWire: "Shares, which were priced Wednesday night at $38, soared to $140 before dipping to close at **$95**. The 23 million-share offering was originally expected to sell for between $15 and $17 a share, but the company raised its offering price in response to high demand. The stellar IPO gives Palm a preliminary market value of about $54 billion. Meanwhile, shares of 3Com, which owns more than 90 percent of Palm, dipped 21 percent to **$81.81**."

Arbitrage?

- Based on the ownership structure:
    - If you owned 3Com, each share was worth $1.5 \cdot 95 = \$143$ (plus 3Com's ongoing concern value).
    - 3Com was worth only \$81!
- Market capitalization? The slice is worth more than the whole
    - 3Com: \$28 billion
    - Palm: \$54 billion
- What is the arbitrage trade?
- Short 1.5 shares of Palm and buy 1 share of 3Com

Shorting Palm

- From a market participant at the IPO:
  "It may be possible to short Palm stock sometime next week, but there are no guarantees. The brokerage firms and institutional investors that control much of Palm's stock generally agree not to immediately lend the stock to short sellers until sometime after the IPO date."
- If you owned it, would you loan it out?
- Short interest in Palm in March? 147.6% (% of shares shorted)
- What was the cost of shorting Palm?
- Hard to get data, but in July of 2000 (four month later) it was on the order 35% (paid -35%/year on your margin).

10. Perfect Markets

Perfect Markets

- In most cases, we assume perfect markets:
  1. No differences in opinion.
     - Uncertainty is ok, but everyone must agree to exactly what it is. We must not have different information or opinions.
     - (The market efficiency hypothesis addresses the difference in information.)
  2. No taxes.
     - or government interference or regulation [except perfect property rights].
  3. No transaction costs.
     - Neither direct nor indirect.
  4. Competitive markets.
     - or no big sellers/buyers; there must always be more where they came from. No (few) investors or firms are special. If investors differ, there must be infinitely many clones competing.

What do perfect markets mean?

- It is the simplest world, a benchmark
- Analysis is easiest. Life is tough enough even in a perfect market.
- Any logic that fails in this simplest of worlds is surely wrong elsewhere, too. It will be wrong in a more realistic world, too.
- Put differently, as your world gets closer and closer to perfection, your methods need to become closer and closer to what must hold in a perfect market.