

Quantitative Asset Management

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Factor Investing

Factor Investing

- Traded factors are long-short (zero-cost) portfolios:

$$F_t = \sum_i w_{i,t-1} R_{i,t}, \quad \text{where} \quad \sum_i w_{i,t-1} = 0$$

- We can put a superscript j for factor j (e.g., F_t^j)
- For example, HML Factor:

$$\begin{aligned} HML = F_t^{HML} &= \frac{1}{2} \left(R_t^{\text{Small Value port.}} + R_t^{\text{Large Value port.}} \right) \\ &\quad - \frac{1}{2} \left(R_t^{\text{Small Growth port.}} + R_t^{\text{Large Growth port.}} \right) \end{aligned}$$

Factor Investing

- ▶ Factor structure
- ▶ Explain returns using asset-pricing factors
- ▶ E.g. CAPM:

$$R_{i,t}^e = \alpha_i + \beta_i^{MKT} R_t^{e,MKT} + \varepsilon_{i,t}$$

where $R_t^{e,MKT}$ is the market portfolio in excess of the risk free rate of return

- ▶ More generally:

$$R_{i,t}^e = \alpha_i + \sum_{j=1}^K \beta_i^j F_t^j + \varepsilon_{i,t}$$

where $\{F_t^j\}_{j=1}^K$ are asset-pricing factors

Factor Investing

- The HML Factor is a long short portfolio

$$F_t^{HML} = \frac{1}{2} \left(R_t^{\text{Small Value port.}} + R_t^{\text{Large Value port.}} \right)$$

$$\begin{aligned} & - \frac{1}{2} \left(R_t^{\text{Small Growth port.}} + R_t^{\text{Large Growth port.}} \right) \\ &= \frac{1}{2} \left[\sum_{i=1}^n w_{i,t-1}^{SV} R_{i,t} + \sum_{i=1}^n w_{i,t-1}^{LV} R_{i,t} \right] \\ & \quad - \frac{1}{2} \left[\sum_{i=1}^n w_{i,t-1}^{SG} R_{i,t} + \sum_{i=1}^n w_{i,t-1}^{LG} R_{i,t} \right] \end{aligned}$$

- Value weighted weights (lagged market cap) and

$$\sum_{i=1}^n w_{i,t-1}^{SV} = \sum_{i=1}^n w_{i,t-1}^{LV} = \sum_{i=1}^n w_{i,t-1}^{SG} = \sum_{i=1}^n w_{i,t-1}^{LG} = 1$$

Factor Investing

- The HML Factor is a portfolio:

$$\begin{aligned} F_t^{HML} &= \frac{1}{2} \left(R_t^{\text{Small Value port.}} + R_t^{\text{Large Value port.}} \right) \\ &\quad - \frac{1}{2} \left(R_t^{\text{Small Growth port.}} + R_t^{\text{Large Growth port.}} \right) \\ &= \sum_{i=1}^n w_{i,t-1}^{HML} R_{i,t} \end{aligned}$$

where the weight on stock i at period t is

$$w_{i,t-1}^{HML} = \frac{1}{2} [w_{i,t-1}^{SV} + w_{i,t-1}^{LV}] - \frac{1}{2} [w_{i,t-1}^{SG} + w_{i,t-1}^{LG}]$$

Factor Investing

- The HML Factor is a zero-cost portfolio:

$$F_t^{HML} = \sum_{i=1}^n w_{i,t-1}^{HML} R_{i,t}$$

where

$$\sum_{i=1}^n w_{i,t-1}^{HML} = 0$$

- PS 3 and 4: technique to compute weights
 - MOM, SMB and HML

Factor Investing

- ▶ Let us consider the following factor model to explain trading strategy p

$$R_t^{p,e} = \alpha_p + \beta_p^{MKT} R_t^{e,MKT} + \beta_p^{HML} F_t^{HML} + \beta_p^{SMB} F_t^{SMB} \\ + \sum_{j=1}^K \beta_p^j F_t^j + \varepsilon_{p,t}$$

where $\{F_t^j\}_{j=1}^K$ are other asset-pricing factors

- ▶ We won't focus on these other factors today
- ▶ They could include MOM, Investment, Profitability, BAB, ...
- ▶ Alphas and betas relative to the factor model above

Factor Investing

- ▶ Factor investing: choose factor exposures!
- ▶ Construct a portfolio with the exposures you want
- ▶ For example:
 - ▶ I want beta 0.5 on the market and 1.5 on value
 - ▶ I want a market portfolio adjusted towards value:
beta 1 on the market and 0.2 on HML
- ▶ Construct a portfolio (p) with
 - ▶ Market exposure: β_p^{MKT}
 - ▶ HML exposure: β_p^{HML}
 - ▶ No other exposure: $\beta_p^j = 0 \forall j \neq HML, MKT$
- ▶ You can build a function that creates your factor investing strategy

Factor Investing

- ▶ Start with the risk-free asset:

$$R_{p,t} = R_t^f$$

- ▶ Add the factors you want:

$$R_{p,t} = R_t^f + \beta_p^{MKT} R_t^{e,MKT} + \beta_p^{HML} R_t^{HML}$$

$$R_{p,t} = (1 - \beta_p^{MKT}) R_t^f + \beta_p^{MKT} R_t^{MKT} + \beta_p^{HML} R_t^{HML}$$

- ▶ Let $w_{i,t-1}^{MKT}$ be the market weight on asset i at period t
i.e. lagged market cap-based weight
- ▶ Factor investing (FI) strategy
 - ▶ Long $(1 - \beta_p^{MKT})$ the risk-free asset
 - ▶ Factor investing weight on asset i at period t :

$$w_{i,t-1}^{FI} = \beta_p^{MKT} w_{i,t-1}^{MKT} + \beta_p^{HML} w_{i,t-1}^{HML}$$

Factor Investing

- ▶ Factor investing (FI) strategy
 - ▶ Long $(1 - \beta_p^{MKT})$ the risk-free asset
 - ▶ Factor investing weight on asset i at period t :

$$w_{i,t-1}^{FI} = \beta_p^{MKT} w_{i,t-1}^{MKT} + \beta_p^{HML} w_{i,t-1}^{HML}$$

- ▶ If β_p^{HML} is low, you may not even have to short
- ▶ Weights sum to one:

$$(1 - \beta_p^{MKT}) + \sum_{i=1}^n w_{i,t}^{FI} = 1$$

Factor Investing

- ▶ Why is this a factor investing strategy?

- ▶ Portfolio excess returns:

$$R_{p,t} = R_t^f + \beta_p^{MKT} R_t^{e,MKT} + \beta_p^{HML} R_t^{HML}$$

$$R_{p,t}^e = \beta_p^{MKT} R_t^{e,MKT} + \beta_p^{HML} R_t^{HML}$$

- ▶ The factor model estimation will give you:

- ▶ Zero alpha
- ▶ Market exposure: β_p^{MKT}
- ▶ HML exposure: β_p^{HML}
- ▶ No other exposure: $\beta_p^j = 0 \forall j \neq HML, MKT$