Lecture 6
Return Predictability
What drives stock market prices? A present-value decomposition and application of AR models

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Outline

1. Stock Market Predictability
   - Forecasting regressions
   - The Dividend-Yield
   - Cross-equation Restrictions (the Present-Value restriction)
2. References
3. Appendix: Background on Optimal Forecasting

What drives stock price movements?

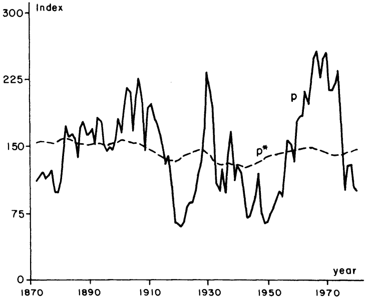


FIGURE 1
Note: Real Standard and Poor's Composite Stock Price Index (solid line p) and *ex post* rational price (dotted line p^*), 1871–1979, both detrended by dividing a long-run exponential growth factor. The variable p^* is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter. Data are from Data Set 1, Appendix.

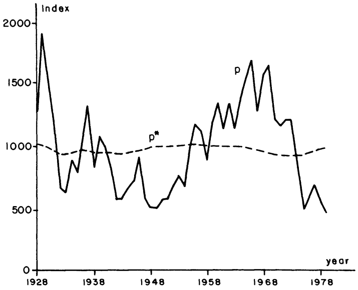


FIGURE 2
Note: Real modified Dow Jones Industrial Average (solid line p) and *ex post* rational price (dotted line p^*), 1928-1979, both detrended by dividing by a long-run exponential growth factor. The variable p^* is the present value of actual subsequent real detrended dividends, subject to an assumption about the present value in 1979 of dividends thereafter. Data are from Data Set 2, Appendix.

Seminal paper by Nobel prize winner Robert Shiller

- Do stock prices move too much to be justified by subsequent dividends?

Stock return predictability

- Let R_{t+1} denote the simple return on the aggregate market, e.g. the CRSP-VW index.
- Let D_t denote aggregate dividends and $d_t = \log(D_t)$.
- The ratio D_t / P_t is called the **dividend yield** while P_t / D_t is called the **price-to-dividend** ratio

Stock return predictability

- A **forecasting regression** is a regression of an outcome at time $t + j$ (with $j > 0$) using an predictor variable known at time t:

$$y_{t+j} = \alpha + \beta x_t + \varepsilon_{t+j}, \quad \text{for } t = 1, ..., T$$

Table: Return Predictability

Regression	slope	t-stat	HAC t-stat	R^2
$R_{t+1} = a + b(D/P)_t + \varepsilon_{t+1}$	3.498	[2.309]	[2.395]	0.062
$R_{t+1} - R_t^f = a + b(D/P)_t + \varepsilon_{t+1}$	3.933	[2.621]	[2.726]	0.078
$r_{t+1} = a_r + b_r(dp)_t + \varepsilon^r_{t+1}$	0.105	[1.989]	[2.075]	0.047
$\Delta d_{t+1} = a_d + b_d(dp)_t + \varepsilon^d_{t+1}$	0.008	0.203	0.185	0.001

Notes: Annual Data. Sample 1927-2009. R_{t+1} is the real return on the CRSP-VW index. r_{t+1} denotes logs of the real return. R^f_{t+1} denotes the return on the real risk-free.

Interpretation

- an increase in the dividend yield of 1 percentage point in deviation from its mean increases the expected real return by 3.49 percentage points (per annum).
- note: when returns are regressed on lagged persistent variables such as the dividend/yield, the disturbances are correlated with the regressor's innovation; this tends to create an upward bias in the case of dividend-yield regressions and is called **Stambaugh bias**; see Stambaugh (1999).
- **Stambaugh bias** implies that OLS coefficients are estimated to be too high.

Relation between Regressions

- note that the log dividend/yield in deviation from its mean is (to a first-order Taylor expansion) given by:

$$dp_t = D_t/P_t \;/\; (D/P)$$

  where D/P is the (unconditional) average dividend/price ratio
- so we can state the return regression :

$$r_{t+1} = a_r + b_r \times dp_t + u_{t+1}$$

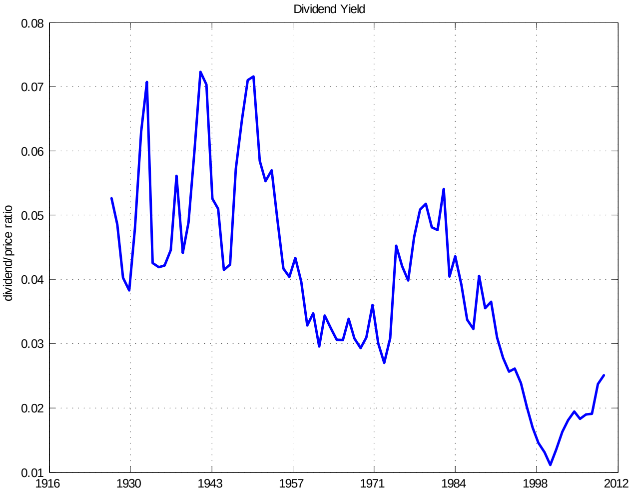
  as follows:

$$r_{t+1} = a_r + b_r \times D_t/P_t \;/\; (D/P) + u_{t+1}$$

- the average dividend yield D/P is .035
- so the implied coefficient for the regression with the dividend yield is

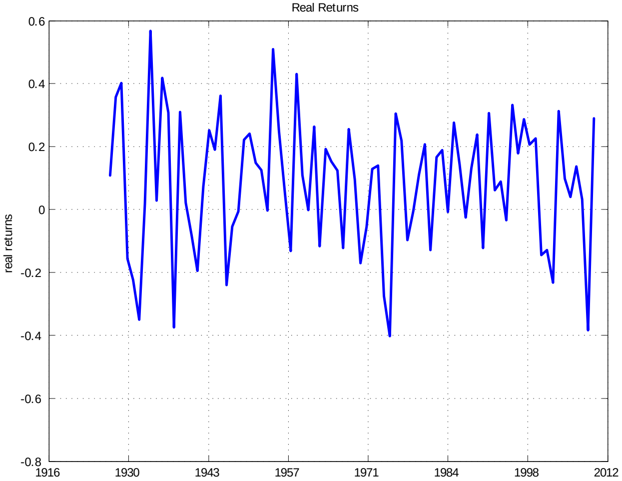
$$\frac{b_r}{D/P} = .105/.035 = 3.00$$

Dividend Yield



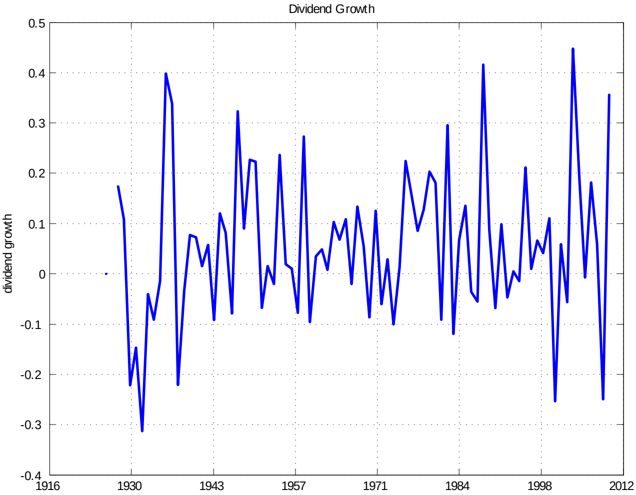
Dividend Yield on CRSP-VW (AMEX-NASDAQ-NYSE). Annual data. 1926-2009.

Real Returns



Real Returns on CRSP-VW (AMEX-NASDAQ-NYSE). Annual data. 1926-2009.

Dividend Growth



Dividend Growth on CRSP-VW (AMEX-NASDAQ-NYSE). Annual data. 1926-2009.

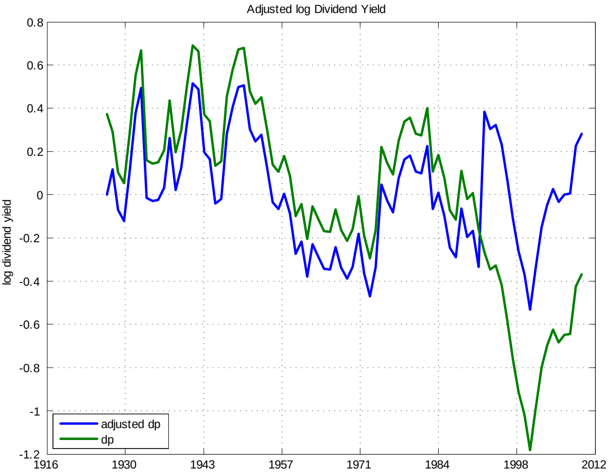
Structural Break in 1991

- Lettau and Van Nieuwerburgh (2007) find a structural break in log dividend yield in 1991.
- defined adjusted dividend yield:

$$
\begin{aligned}
\widetilde{dp}_t &= dp_t - \overline{dp}_1 \\
\widetilde{dp}_t &= dp_t - \overline{dp}_2
\end{aligned}
$$

where $\overline{dp}_1$ denotes the mean in the first sample 1926-1991 and $\overline{dp}_2$ denotes the mean in the second sample 1992-2009.

Log Dividend Yield



Demeaned log Dividend Yield $\widetilde{dp}_t$ on CRSP-VW (AMEX-NASDAQ-NYSE) with break in 1991. Annual data. 1926-2009.

Return Predictability

Table: Return Predictability

Regression	slope	t-stat	HC t-stat	R^2
$r_{t+1} = a_r + b_r(\widetilde{dp})_t + \varepsilon_{t+1}$	0.267	[3.118]	[3.667]	0.107
$\Delta d_{t+1} = a_d + b_d(\widetilde{dp})_t + \varepsilon_{t+1}$	0.039	[0.624]	[0.736]	0.004

Notes: Annual Data. Sample 1927-2009. R_{t+1} is the real return on the CRSP-VW index. R_{t+1} denotes logs of the real return. R^f_{t+1} denotes the return on the real risk-free.

Longer Horizons

- we run the following regression of k period holding returns on the dividend yield:

$$\sum_{i=1}^{k} r_{t+i} = a_r + b_r^k (dp)_t + \varepsilon_{t+k}$$

- as you increase the horizon k, the slope coefficients b_r^k increase and the R^2 increase

- Note: in this case, you should account for autocorrelation of residuals up to and including $k-1$ observations apart mechanically induced by the overlap
  - The next couple of slides shows how to do this using HAC standard errors

HAC robust standard errors

- If OLS residuals exihibit heteroskedasticity and/or autocorrelation (and, potentially, non-normality), OLS is still *consistent*
  - But, not efficient
  - Maximum likelihood is the efficient method in large samples
  - OLS is maximum likelihood only when errors are i.i.d. normally distributed

- If we still choose OLS (as a linear regression is pretty robust and parsimonious), we need to adjust the standard errors
  - HAC (heteroskedasticity and autocorrelation adjusted) standard errors

HAC robust standard errors: theory

- Please refer back to the "Note on Asymptotic Standard Errors" I posted earlier (which have already read)

- Recall, for the case of Asymptotic OLS

$$y_t = x_t \beta + \varepsilon_t, \quad \text{for } t = 1, ..., T$$

$$\hat{\beta}_T - \beta \overset{asymptotically}{\rightarrow} N\left(0, \frac{1}{T} E\left[x_t x_t'\right]^{-1} S E\left[x_t x_t'\right]^{-1}\right)$$

where

$$S = \sum_{j=-\infty}^{\infty} E\left[x_t x_{t-j}' \varepsilon_t \varepsilon_{t-j}\right]$$

and $\hat{\beta}_T$ is the estimate of β in a sample of length T

HAC robust standard errors: theory

- If the residuals are correlated across q leads and lags and zero thereafter

$$corr\left(\varepsilon_t, \varepsilon_{t-j}\right) \begin{cases} \neq 0 & \text{for } |j| \leq q \\ = 0 & \text{for } |j| > q \end{cases}$$

  we have

$$S = \sum_{j=-q}^{q} E\left[x_t x'_{t-j} \varepsilon_t \varepsilon_{t-j}\right]$$

- These are called Hansen-Hodrick standard errors (see next slide)

Hansen-Hodrick standard errors

Define:

$$R_T(v;\beta) = \frac{1}{T}\sum_{t=1+v}^{T} x_t x_{t-v}' \varepsilon_t \varepsilon_{t-v}$$

where the estimate of the spectral density matrix is

$$\hat{S}_T = R_T\left(0;\hat{\beta}_T\right) + \sum_{v=1}^{q}\left[R_T\left(v;\hat{\beta}_T\right) + R_T\left(v;\hat{\beta}_T\right)'\right]$$

The estimate of the covariance matrix is then

$$Est.Asy.Var\left(\hat{\beta}_T\right) = T\left(X_T' X_T\right)^{-1}\hat{S}_T\left(X_T' X_T\right)^{-1}$$

where capital x_t, X_t, is a $T \times K$ matrix with t'th row equal to x_t

Newey-West standard errors

Newey and West (1987) solve an issue for the Hansen-Hodrick standard errors

- The estimated variance covariance matrix of $\hat{\beta}$ can be non-positive definite
- I.e., not invertible, "negative variance"
- To ensure a positive-definite covariance matrix, downweight estimated autocorrelations more the farther from the 0'th lag:

$$\hat{S}_T = R_T\left(0; \hat{\beta}_T\right) + \sum_{v=1}^{q} \frac{q+1-v}{q+1}\left[R_T\left(v; \hat{\beta}_T\right) + R_T\left(v; \hat{\beta}_T\right)'\right]$$

The Newey-West covariance matrix is then

$$Est.Asy.Var\left(\hat{\beta}\right) = T\left(X_T' X_T\right)^{-1} \hat{S}_T \left(X_T' X_T\right)^{-1}$$

- For Newey-West (NW) standard errors, should use $(k-1) \times 1.5$ or so due to the downweighting in the NW procedure
- Note that NW with 0 lags overlap is the same as White standard errors

Long-Horizon Return Predictability with Dividend Yield

Table: Return Predictability

Horizon	1	2	3	4	5
slope	0.105	0.199	0.250	0.282	0.323
OLS	[1.989]	[2.692]	[2.976]	[3.046]	[3.232]
NW	[2.036]	[2.399]	[2.578]	[2.573]	[2.600]
R^2	0.047	0.083	0.101	0.106	0.119

Notes: Annual Data. Sample 1927-2009. Forecasting regression of $\sum_{i=1}^{k} r_{t+i}$ on the log dividend yield. $\sum_{i=1}^{k} r_{t+i}$ denotes the sum of k years of logs of the real return.

Longer Horizons

- we run the following regression of k period holding returns on the dividend yield:

$$\sum_{i=1}^{k} r_{t+i} = a_r + b_r^k (\widetilde{dp})_t + \varepsilon_{t+k}$$

- as you increase the horizon k, the slope coefficients b_r^k increase and the R^2 increase
- Consider the 5 year horizon (next slide) where $\hat{b}_r^5 = 0.826$. An increase in the dividend yield of 1 percentage point in deviation from its mean increases the expected real return by 23.71 percentage points (=.826/.035) or 4.74 percentage points (per annum).

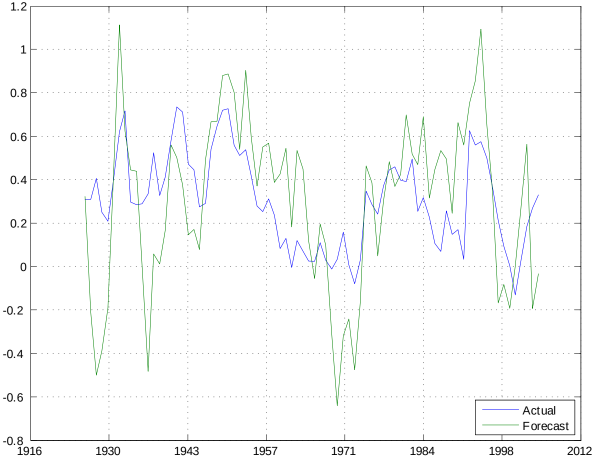
Long-Horizon Return Predictability with Adj. Div. Yield

Table: Return Predictability

Horizon	1	2	3	4	5
slope	0.267	0.478	0.661	0.750	0.826
OLS	[3.137]	[4.075]	[5.179]	[5.578]	[5.892]
NW	[3.480]	[3.960]	[4.977]	[4.559]	[4.157]
R^2	0.107	0.170	0.251	0.283	0.308

Notes: Annual Data. Sample 1927-2009. Forecasting regression of $\sum_{i=1}^{k} r_{t+i}$ on the adjusted log dividend yield. $\sum_{i=1}^{k} r_{t+i}$ denotes the sum of k years of logs of the real return.

5-year return Forecast



5-year log return forecast using Adjusted log Dividend Yield on CRSP-VW (AMEX-NASDAQ-NYSE). Annual data. 1926-2009.

Longer Horizons

- the R^2 in the regression of k period holding returns on the dividend yield is given by:

$$R^2(k) = \frac{V[E_t[r_{t+1}] + \ldots + E_t[r_{t+k}]]}{V[r_{t+1} + r_{t+2} + \ldots + r_{t+k}]}$$

- this grows at rate k initially because
  - realized returns are negatively autocorrelated
  - predicted returns are positively autocorrelated

Linearizing the returns

- consider the return on an asset:

$$R_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\frac{D_{t+1}}{D_t}\left(1 + PD_{t+1}\right)}{PD_t}.$$

- pd_t denotes the log price-dividend ratio

$$pd_t = p_t - d_t = \log\left(\frac{P_t}{D_t}\right),$$

  where price is measured at the end of the period and the dividend flow is over the same period.

- also: note that

$$dp_t = -pd_t$$

Log-Linearizing returns

- Campbell and Shiller (1989) log-linearization of the return equation around the (unconditional) mean log price/dividend ratio delivers the following expression for log returns:

$$r_{t+1} = \Delta d_{t+1} + \rho pd_{t+1} + k - pd_t,$$

  with linearization coefficients ρ and k that depend on the mean of the log price/dividend ratio $\overline{pd}$:

$$\rho = \frac{e^{\overline{pd}}}{e^{\overline{pd}} + 1} < 1 \quad \text{(the } k \text{ coefficient not important)}$$

- this expression is an approximation of an identity. It must hold!

The log of the price/dividend ratio

$$pd_t \;=\; \Delta d_{t+1} + \rho p d_{t+1} + k - r_{t+1}$$

- iterating forward on the linearized return equation
- imposing a no-bubble condition:

$$\lim_{j \to \infty} \rho^j p d_{t+j} = 0$$

- expression for the log price/dividend ratio:

$$pd_t = constant + \overbrace{\sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}}^{\textbf{cash flow}} - \overbrace{\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}}^{\textbf{discount rate}}$$

Price/Dividend Ratios

Price/dividend ratios can only move if they predict returns or cash flows:

$$pd_t = constant + \overbrace{E_t\left[\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j}\right]}^{\textbf{cash flow}} - \overbrace{E_t\left[\sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}\right]}^{\textbf{discount rate}}$$

- a high price-to-dividend ratio pd_t implies that dividends are expected to increase or future returns (discount rates) are expected to decline

Campbell-Shiller Decomposition

The *pd* equation (without expectations) implies that the variance of the price/dividend ratio equals:

$$
\begin{aligned}
V[pd_t] &= V\left[\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j}\right] + V\left[\sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}\right] - 2\text{cov}\left(\sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}, \sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j}\right) \\
&= \text{cov}\left(\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j} - \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}, \sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j}\right) - \text{cov}\left(\sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j} - \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}, \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}\right) \\
&= \text{cov}\left(pd_t, \sum_{j=1}^{\infty}\rho^{j-1}\Delta d_{t+j}\right) - \text{cov}\left(pd_t, \sum_{j=1}^{\infty}\rho^{j-1}r_{t+j}\right)
\end{aligned}
$$

- Campbell and Shiller: the price/dividend ratio has to predict future (long-run) returns and/or dividends if it moves around!
  - the evidence that it predicts returns seems stronger than the evidence that it predicts cash flows

Variance Decomposition

The variance decomposition of the log price/dividend ratio is the difference between two regression slope coefficients:

$$1 = \frac{\text{cov}\left(pd_t, \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}\right)}{V[pd_t]} - \frac{\text{cov}\left(pd_t, \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}\right)}{V[pd_t]}$$

- Is the variance of the *pd*-ratio driven by variation in expected cash flows or expected returns (i.e., discount rates)?

Price/Dividend Ratios

Price/dividend ratios predict future returns.

So do the term spread, the default spread and T-bill rates.

The R^2 increase with the forecasting horizon.

Variance Decomposition

- Recall that $dp_t = -pd_t$. Thus., the dp_t equation (without expectations) implies that the the slope coefficients in a regression of discounted returns and dividend growth on dp_t satisfy the following restriction:

$$
\begin{aligned}
1 &= -\frac{\mathrm{Cov}\left(dp_t, \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}\right)}{V(dp_t)} + \frac{\mathrm{Cov}\left(dp_t, \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}\right)}{V(dp_t)} \\
&= -\beta_d + \beta_r
\end{aligned}
$$

where β_d and β_r are implicitly defined in the above.

Vector autoregressions (VAR)

- consider 1st order restricted VAR:

$$\begin{aligned} r_{t+1} &= a_r + b_r dp_t + \varepsilon^r_{t+1} \\ \Delta d_{t+1} &= a_d + b_d dp_t + \varepsilon^d_{t+1} \\ d_{t+1} - p_{t+1} &= a_{dp} + \phi dp_t + \varepsilon^{dp}_{t+1} \end{aligned}$$

- remember we log-linearized an identity to get:

$$r_{t+1} = \Delta d_{t+1} + \rho pd_{t+1} + \kappa_0 - pd_t.$$

with

$$\rho = \frac{e^{\overline{pd}}}{1 + e^{\overline{pd}}}$$

- this implies that there exists a deterministic relationship between these variables.

Cross-equation restrictions

- take expectations:

$$E_t[r_{t+1}] = E_t[\Delta d_{t+1}] + \rho E_t[pd_{t+1}] + \kappa_0 - pd_t.$$

- go back to the 1st order VAR:

$$\begin{aligned} E_t[r_{t+1}] &= a_r + b_r dp_t \\ E_t[\Delta d_{t+1}] &= a_d + b_d dp_t \\ E_t[d_{t+1} - p_{t+1}] &= a_{dp} + \phi dp_t \end{aligned}$$

- this implies that:

$$a_r + b_r dp_t = a_d + b_d dp_t - \rho(a_{dp} + \phi dp_t) + \kappa_0 - pd_t$$

Cross-equation restrictions

$$
\begin{aligned}
r_{t+1} &= a_r + b_r dp_t + \varepsilon^r_{t+1} \\
\Delta d_{t+1} &= a_d + b_d dp_t + \varepsilon^d_{t+1} \\
dp_{t+1} &= a_{dp} + \phi dp_t + \varepsilon^{dp}_{t+1}
\end{aligned}
$$

- $\Rightarrow$ the coefficients in these three equations must obey:

$$b_r = b_d + 1 - \phi\rho$$

  or equivalently that the following is true:

$$\frac{b_r}{1-\rho\phi} - \frac{b_d}{1-\rho\phi} = 1$$

  - the first term is the slope coefficient in the regression of the discount rate component on the *dp*-ratio
  - the second term is the slope coefficient in the regression of the cash flow component on the *dp*-ratio
  - we show this on the next slide

Slope coefficient background math

Consider two hypothetical regressions:

1. the cash flow component on the *dp*-ratio:

$$\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[\Delta d_{t+j}\right] = \alpha_d + \beta_d dp_t + \varepsilon_t$$

   Substitute in for future dividend growth using the VAR specification (note error term equals zero always):

$$\sum_{j=1}^{\infty} \rho^{j-1} \left(a_d + b_d E_t \left[dp_{t+j-1}\right]\right) = c + \sum_{j=1}^{\infty} \rho^{j-1} b_d \phi^{j-1} dp_t = c + \frac{b_d}{1-\rho\phi} dp_t .$$

   Thus, $\beta_d = \frac{cov\left(dp_t, \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j}\right)}{V\left[dp_t\right]} = \frac{b_d}{1-\rho\phi}$ (and c is a constant term).

2. the discount rate component on the *dp*-ratio:

$$\sum_{j=1}^{\infty} \rho^{j-1} E_t \left[r_{t+j}\right] = \alpha_r + \beta_r dp_t + \varepsilon_t$$

   Similar math as above yields $\beta_r = \frac{cov\left(dp_t, \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}\right)}{V\left[dp_t\right]} = \frac{b_r}{1-\rho\phi}$.

Cross-Equation Restrictions

Table: Cross-Equation Restrictions

	estimate	standard error	*implied*
b_r	0.105	[0.050]	0.114
b_d	0.007	[0.041]	-0.0017
ϕ	0.925	[0.056]	0.935

Notes: Annual Data. Sample 1927-2009. R_{t+1} is the real return on the CRSP-VW index. R_{t+1} denotes logs of the real return. R^f_{t+1} denotes the return on the real risk-free. ρ is .9650. The column calculates each coefficient based on the other two coefficients and the identity $b_r = 1 - \rho\phi + b_d$.

Variance Decomposition

- slope coefficients in predictability regressions represent fractions of variance due to discount rates and cash flows:

$$\frac{b_r}{1-\rho\phi} - \frac{b_d}{1-\rho\phi} = 1$$

  where we have assumed AR(1) for dividend yield with coefficient ϕ, b_r is slope coefficient in return regression, b_d is slope coefficient in dividend growth regression

- plugging in our estimates:

$$\frac{b_r}{1-\rho\phi} = \frac{0.105}{1-.9650\times .925} = .97$$

- discount rates account for 97 % of the variance of the log price/dividend ratio
  - Thus: the market valuation ratio moves around mostly because discount rates (expected returns) vary over time!

Cross-Equation Restrictions with Adjusted Dividend Yield

Table: Cross-Equation Restrictions

	estimate	standard error	*implied*
b_r	0.267	0.072	0.297
b_d	0.039	0.053	0.008
ϕ	0.768	0.768	0.800

Notes: Annual Data. Sample 1927-2008. R_{t+1} is the real return on the CRSP-VW index. R_{t+1} denotes logs of the real return. R^f_{t+1} denotes the return on the real risk-free. ρ is .965. The "implied" column calculates each coefficient based on the other two coefficients and the identity $b_r = 1 - \rho\phi + b_d$. This table uses the adjusted dividend yield $\widetilde{dp}$.

Variance Decomposition

- suppose we use the adjusted dividend yield instead
- go back to variance decomposition:

$$\frac{b_r}{1-\rho\phi} - \frac{b_d}{1-\rho\phi} = 1$$

  where we have assumed AR(1) for dividend yield with coefficient ϕ, b_r is slope coefficient in return regression, b_d is slope coefficient in dividend growth regression
- plugging in our estimates:

$$\frac{b_r}{1-\rho\phi} = \frac{0.267}{1-.965\times.768} = 1.0314$$

- discount rates account for 103 % of the variance of the log price/dividend ratio

The dog that did not bark

- if you believe $b_r = 0$, then you cannot believe $b_d = 0$

$$0 = b_d + 1 - \phi\rho$$

  unless you think $\phi = \rho^{-1} > 1$

- so you need to explain the lack of evidence for dividend growth predictability, see Cochrane (2008)

The dog that did not bark

- Inspector Gregory:

  'Is there any other point to which you would wish to draw my attention?'

- Sherlock Holmes:

  'To the curious incident of the dog in the night-time.'

  'The dog did nothing in the night time.'

  'That was the curious incident.'

  (From "The Adventure of Silver Blaze" by Sir Arthur Conan Doyle.)

The Origins of Return Predictability

- different scenarios
  1. dividend/price ratio is persistent: some predictability
     - ★ $\rho = .9647$
     - ★ $\phi = .90$
     - ★ return regression slope coefficients

       $$b_r = 1 - \rho\phi = .139$$

       close to .097
  2. no persistence in dividend price ratio: too much predictability
     - ★ $\rho = .9647$
     - ★ $\phi = 0$
     - ★ return regression slope coefficients

       $$b_r = 1$$
  3. unit root in dividend price ratio: no predictability

     $$b_r \approx 0$$

Predictability of Returns

Return forecastability follows from the fact that dividends are not forecastable and that the dividend/price ratio is highly but not completely persistent.

Conclusion

- quite some evidence of stock return predictability
- other variables also predict stock returns (like the price/earnings ratio, risk-free rate, the yield spread etc.)
- evidence of predictability is much weaker in out-of-sample; see, Goyal and Welch (2008)

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Appendix

Campbell and Shiller (1989) approximation

Let $p_t = \log(P_t)$ and $d_t = \log(D_t)$.

$$\begin{aligned} r_{t+1} &= \log(1+R_{t+1}) = \log\left(\frac{P_{t+1}+D_{t+1}}{P_t}\right) = \log(P_{t+1}+D_{t+1}) - \log(P_t) \\ &= \log(P_{t+1}+D_{t+1}) - p_{t+1} + p_{t+1} - p_t \\ &= \log\left(\frac{P_{t+1}+D_{t+1}}{P_{t+1}}\right) + p_{t+1} - p_t \\ &= \log(1+\exp(d_{t+1}-p_{t+1})) + p_{t+1} - p_t \end{aligned}$$

- Let $f(x) = \log(1+\exp(x))$ be the function we need to approximate, i.e. $f(d_{t+1}-p_{t+1}) = \log(1+\exp(d_{t+1}-p_{t+1}))$.
- We want to take a first-order Taylor series expansion around the unconditional mean $\bar{dp}$.

Campbell and Shiller (1989) approximation

The first-order Taylor series expansion around $\bar{dp}$ is

$$f(d_{t+1} - p_{t+1}) \approx \log(1 + \exp(\bar{dp})) + \frac{\exp(\bar{dp})}{1 + \exp(\bar{dp})}(d_{t+1} - p_{t+1} - \bar{dp})$$

Plug this expression in to the equation for r_{t+1} above

$$\begin{aligned} r_{t+1} \quad &\approx \quad \log(1 + \exp(\bar{dp})) + \frac{\exp(\bar{dp})}{1 + \exp(\bar{dp})}(d_{t+1} - p_{t+1} - \bar{dp}) + p_{t+1} - p_t \\ &= \quad \log(1 + \exp(\bar{dp})) - \frac{\exp(\bar{dp})}{1 + \exp(\bar{dp})}\bar{dp} + \frac{1}{1 + \exp(\bar{dp})}p_{t+1} \\ &\quad\quad + \frac{\exp(\bar{dp})}{1 + \exp(\bar{dp})}d_{t+1} - p_t \end{aligned}$$

This gives us the expression

$$r_{t+1} \approx \kappa + \rho p_{t+1} + (1 - \rho) d_{t+1} - p_t$$

where $\rho = \frac{1}{1+\exp(\bar{dp})}$ and $\kappa = \log(1 + \exp(\bar{dp})) - \frac{\exp(\bar{dp})}{1+\exp(\bar{dp})}\bar{dp}$.

Appendix: Forecasting

Forecasting

- suppose we want to forecast the value of Y_{t+1} based on a set of predictor variables $\mathbf{X}_t$

Example

If we want to use the last m values of Y_t, then
$\mathbf{X}_t = (Y_t, Y_{t-1}, Y_{t-2}, \ldots, Y_{t-m+1})$

- let $Y^*_{t+1} = g(\mathbf{X}_t)$ denote any forecast function of $\mathbf{X}_t$ consisting of all the variables observed up until time t

- Which function $g(\mathbf{X}_t)$ of the observed data should we choose?

Loss functions

- to evaluate different forecast functions $g(\mathbf{X}_t)$, we need a systematic way to define what 'best' means.
- a loss function $L(Y_{t+1}, Y^*_{t+1})$ describes how we feel about missing our target Y_{t+1} if we choose our forecast to be $Y^*_{t+1} \; = \; g(\mathbf{X}_t)$.

  Examples:

  - squared error loss:

  $$L(Y_{t+1}, Y^*_{t+1}) \; = \; (Y_{t+1} - Y^*_{t+1})^2$$

  - absolute deviation loss:

  $$L(Y_{t+1}, Y^*_{t+1}) \; = \; |Y_{t+1} - Y^*_{t+1}|$$

Optimal forecasts

The **optimal forecast** is the function $Y^*_{t+1} = g(\mathbf{X}_t)$ that minimizes the expected loss

$$\underset{Y^*_{t+1}}{argmin}\; E[L(Y_{t+1}, Y^*_{t+1})|\mathbf{X}_t]$$

- minimizing expected loss is analogous to maximizing expected utility
- the field of **decision theory** studies optimal decision making under uncertainty.

MSE forecasts

Consider the squared error loss function:

$$L(Y_{t+1}, Y^*_{t+1}) \; = \; (Y_{t+1} - Y^*_{t+1})^2$$

The forecast that minimizes the mean squared error

$$\underset{Y^*_{t+1}}{argmin} \; E[(Y_{t+1} - Y^*_{t+1})^2 | \mathbf{X}_t]$$

is the **conditional expectation** of Y_{t+1} given by

$$Y^*_{t+1} \; = \; g(\mathbf{X}_t) \; = \; E[Y_{t+1} | \mathbf{X}_t]$$

- if our loss-function is SE, the **conditional expectation** is optimal

Forecasting

- mean squared error (MSE) is the default choice and is often implicitly assumed.
- There is a large literature in econometrics and statistics on optimal forecasting. Unfortunately, I am only scratching the surface.
- See the (free) book on forecasting: Diebold (2015)

Linear Forecasting

- **suppose we do not commit to a particular process for Y_t**
- suppose we want to forecast the value of Y_{t+1} based on a set of variables $\mathbf{X}_t$ but we only use linear forecasts $\mathbf{X}_t\alpha$

Definition

The **linear projection** of Y_{t+1} on $\mathbf{X}_t$ is defined by finding a vector $\boldsymbol{\alpha}$ such that the forecast error is uncorrelated with $\mathbf{X}_t$:

$$E[\mathbf{X}_t'(Y_{t+1} - \mathbf{X}_t\boldsymbol{\alpha})] = 0$$

Forecasting

The linear forecast $g(\mathbf{X}_t) = \mathbf{X}_t \boldsymbol{\alpha}$ that minimizes the mean squared error

$$\underset{\boldsymbol{\alpha}}{argmin} \;\; E[(Y_{t+1} - \mathbf{X}_t \boldsymbol{\alpha})^2 | \mathbf{X}_t]$$

is the linear projection of Y_t on $\mathbf{X}_t$.

- hence, the best linear forecast satisfies:

$$E[\mathbf{X}_t'(Y_{t+1} - \mathbf{X}_t \boldsymbol{\alpha})] = 0$$

- in other words, the linear projection coefficient satisfies:

$$E(\mathbf{X}_t' Y_{t+1}) = E(\mathbf{X}_t' \mathbf{X}_t) \boldsymbol{\alpha}$$

- hence, the projection coefficient is:

$$\boldsymbol{\alpha} = E(\mathbf{X}_t' \mathbf{X}_t)^{-1} E(\mathbf{X}_t' Y_{t+1})$$

Linear Projection and OLS

- consider the linear regression model:

$$y_{t+1} = \mathbf{x}_t \beta + u_{t+1}$$

- the sample sum of squared residuals:

$$\sum_{t=1}^{T} \left(y_{t+1} - \mathbf{x}_t \hat{\beta} \right)^2$$

- the value of $\hat{\beta}$ that minimizes the SSR is the OLS estimate:

$$\hat{\beta} = \left[\sum_{t=1}^{T} \mathbf{x}_t' \mathbf{x}_t \right]^{-1} \left[\sum_{t=1}^{T} \mathbf{x}_t' y_{t+1} \right]$$

Linear Projection and OLS

- OLS regression coefficients yield consistent estimates of the linear projection coefficients
- sound basis for OLS regressions when forecasting

Comparison with OLS

- recall our OLS estimator of $\boldsymbol{\beta}$:

$$\hat{\beta} = \left[\sum_{t=1}^{T} \mathbf{x}_t' \mathbf{x}_t \right]^{-1} \left[\sum_{t=1}^{T} \mathbf{x}_t y_{t+1} \right]$$

- hence, the OLS estimator uses *sample moments*, while the linear projection uses *population moments*

if $\{\mathbf{X}_t, Y_t\}$ are covariance-stationary, then the sample moments will converge to the population moments and the OLS estimator $\hat{\beta}$ will converge to $\boldsymbol{\alpha}$ as $T \to \infty$

- sound basis for forecasting under very mild conditions!!
- Note that this does not mean OLS is efficient (maximum likelihood is; GLS)