Investments

Topic 2: Fixed Income Intro

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Overview of Topic 2

1. Fixed Income markets
2. Valuation of fixed-income securities
3. Replication and arbitrage
4. Term structure of spot rates
5. Bond returns
6. Yield to Maturity

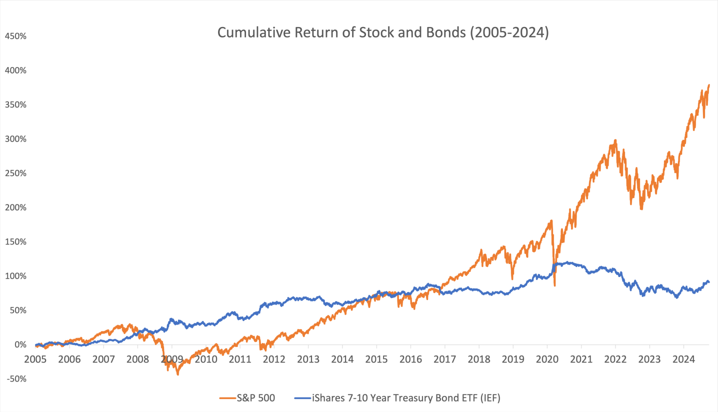
1. Fixed Income Markets

Fixed Income Markets

- Fixed income traditionally refers to securities or agreements that generate fixed (promised) period payments
  - More generally, FI is loosely defined as the markets for borrowing and lending
  - Bank loans and deposits
  - Bonds
  - Derivatives written on these instruments
- Bond markets
  - Governments, companies and institutions borrow and lend
  - Consumers: mortgages and savings
- Derivatives
  - Companies
    - Hedge interest rate risks
    - Change maturity and profile of debt obligations
  - Speculators

Why are fixed income markets interesting?

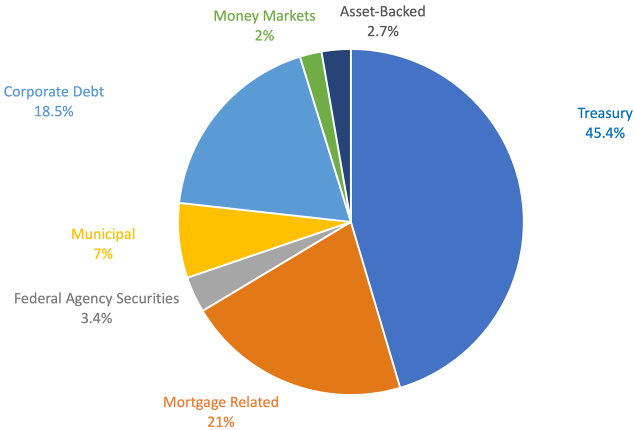
- Sheer size: Q2 of 2024 in $ trillion
  - Equity market cap (Wilshire 5000): 52.88
  - Equity derivatives: 7.8 (notional), 0.58 (gross)
  - U.S. bond market: 58
  - Fixed income derivatives: 529.8 (notional), 12.8 (gross)
- Investment performance



What are we going to learn?

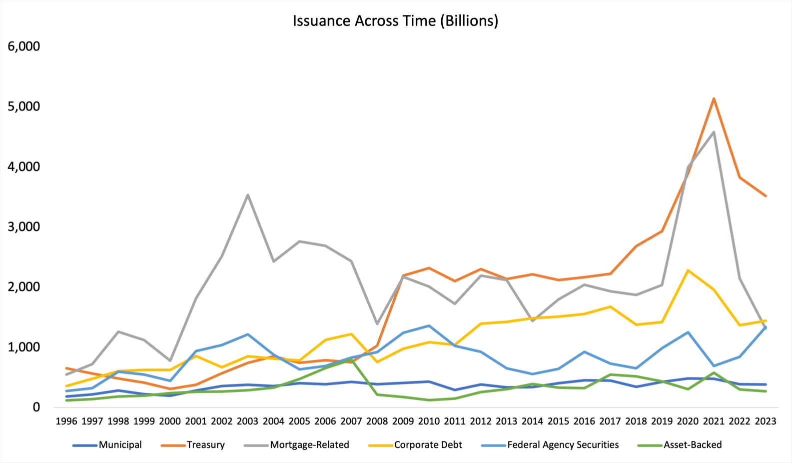
- Institutional features of the markets
- Pricing Theory
  - How to price bonds in a no-arbitrage manner.
  - What are forwards and swaps, how to use them and how to price them.
  - Measures of interest rate return and risk
- Empirics
  - What is going on in the fixed income markets?
  - What do bond price/interest rate changes tell us?

Issuers (2024)

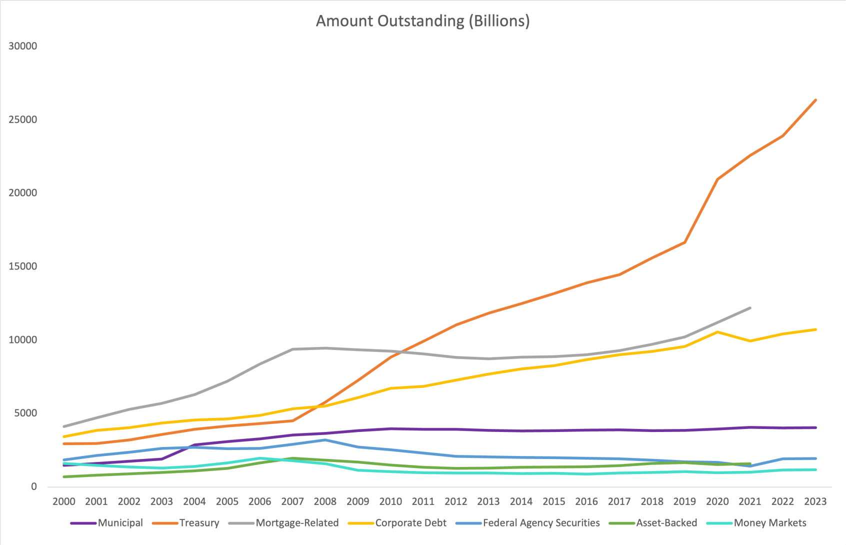


- *Municipal*: States and localities
- *Agencies and GSEs*: Fannie, Freddie, Farm Credit System
- *Treasury*: U.S. Federal Debt
- *Mortgage*: GNMA, FNMA and FHLMC bonds
- *Corporate*: U.S.corporate issuance
- *Money Market*: Commercial paper
- *Asset Backed*: Cars, credit cards, student loans, planes, etc.

Issuance across time



Amounts outstanding across time



Bond characteristics

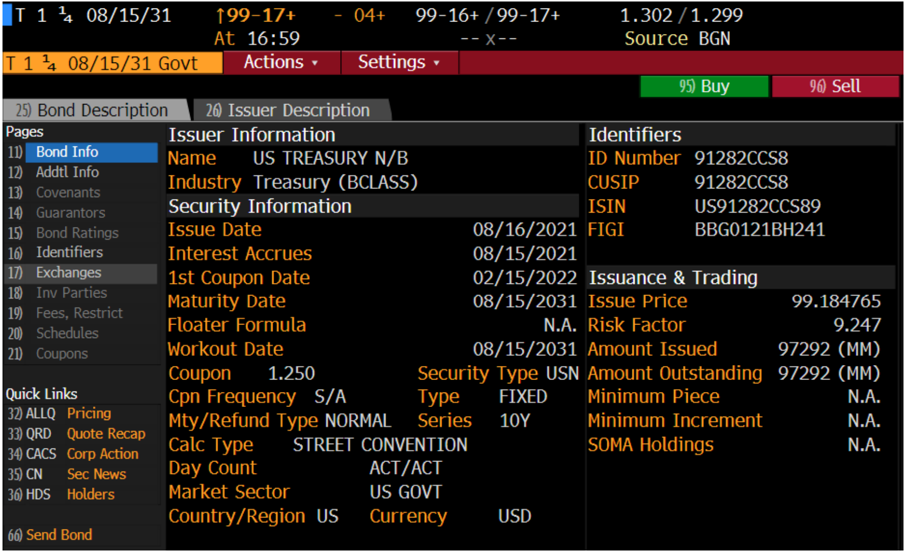
- Issuer
  - Governments, Federal agencies, corporations
- Principal value (or par or face value)
  - Amount received on maturity date: Convention=$100
- Coupon Payments
  - In U.S., semi-annual, a fixed or floating percentage of the principal value
- Maturity
  - Date on which principal (and last coupon) is paid
- Are coupons/par value guaranteed?
  - Default on coupon/principal payments
  - Liquidity: delay in payment
  - Floating rate coupons: interest rate risk
  - Real income: inflation risk
- Recent defaults:
  - US Corporate: Lehman, 2008; WaMu, 2008; AMR, 2011; Toys R US, 2017; Hertz, 2020
  - Sovereign: Argentina, 2001 (restructuring 2005-2015), 2022; Greece, 2012; Puerto Rico, 2016; Russia, 2022; Ukraine, 2022

Lehman Brothers

U.S. Treasury Securities

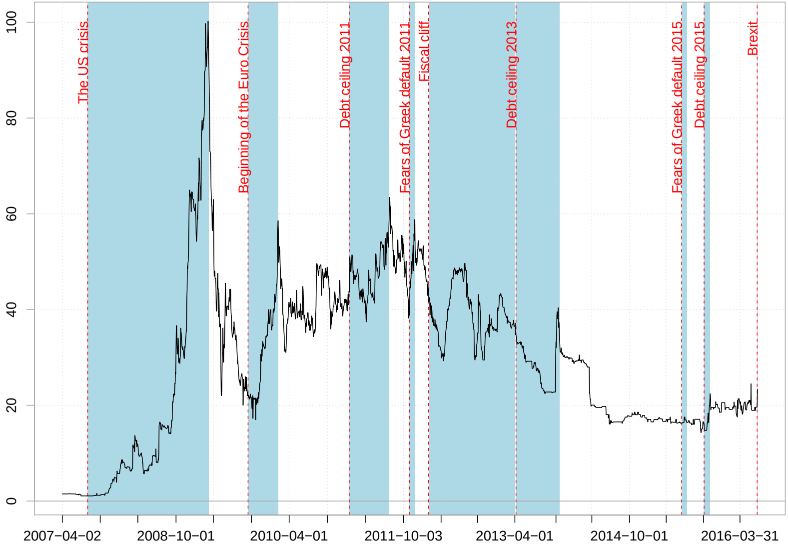
- The only fixed-income securities that involve virtually no default risk are U.S. Treasury securities.
  - only fixed (as opposed to floating) coupons
  - backed by "full faith and credit" of the US government
  - ... but, credit rating of the US government dropped after the crisis
- The US Treasury issues three types of securities:
  - **Treasury Bills** (T-Bills): Maturities up to 1 year. No coupon.
  - **Treasury Notes** (T-Notes): Maturities between 1 and 10 years. Semiannual coupon.
  - **Treasury Bonds** (T-Bonds): Maturities greater than 10 years. Semiannual coupon.
- The US Treasury allows buyers of T-notes and T-bonds to exchange them for the individual coupons and face value. These zero-coupon bonds can be traded and are called **Treasury Strips** (T-Strips).
- We will focus on these securities because they are the simplest to value.

Bond characteristics

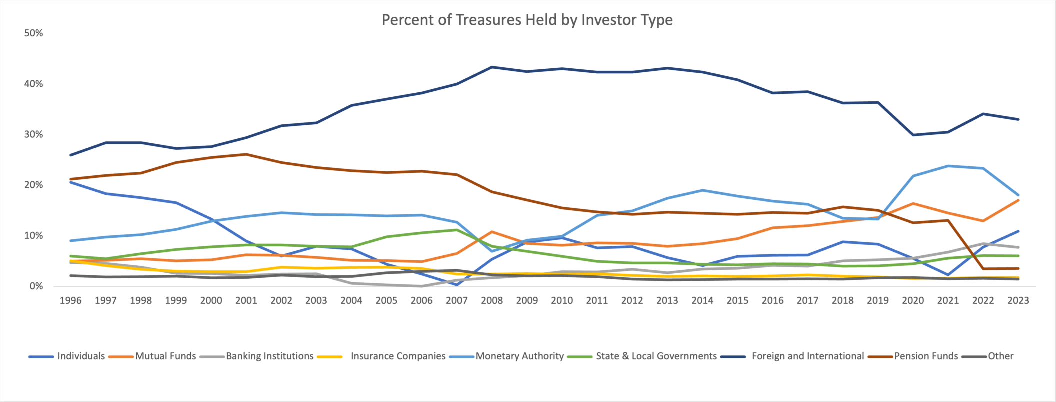


Historical behavior

US CDS premiums



Who holds Treasuries?



Maturity of U.S. Bonds

- What determines long vs short?

Maturity of U.S. Bonds

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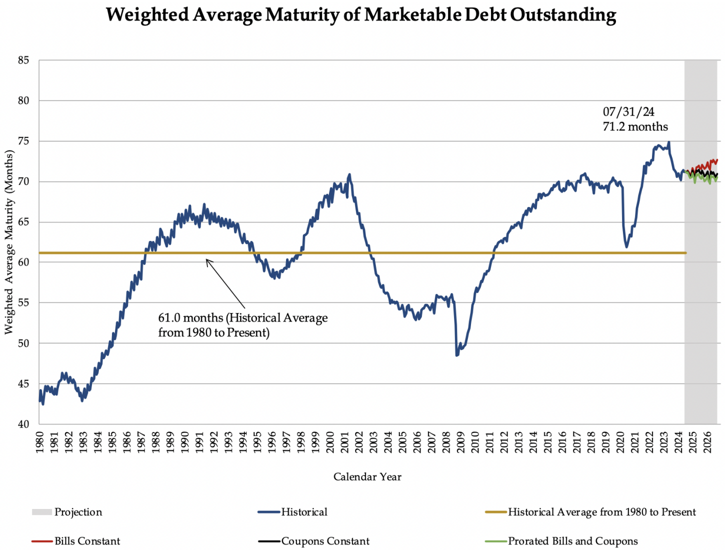


2. Valuation of Fixed-Income Securities

Zero-Coupon Bonds

- Consider a zero-coupon bond with T years to maturity.
- The PV of the bond's cash flows is

$$PV = \frac{100}{(1 + r_T)^T} = 100 \times d_T,$$

  where r_T is the T-year spot rate.
  This is the price of the bond.
- Example: The 4-year spot rate is 5%. Compute the price of a zero-coupon bond with 4 years to maturity.

$$\text{Price} = \frac{100}{(1 + 5\%)^4} = 82.27.$$

- A zero-coupon bond always sells at a "discount" (below face value).

Coupon Bonds

- Consider a bond with annual coupon rate $c\%$ (i.e. annual coupon payments of c), and T years to maturity.
- The PV of the bond's cash flows is

$$\begin{aligned} PV &= \frac{c}{1+r_1} + \frac{c}{(1+r_2)^2} + \cdots + \frac{100+c}{(1+r_T)^T} \\ &= c \times d_1 + c \times d_2 + \cdots + (100+c) \times d_T. \end{aligned}$$

  This is the price of the bond.
- <u>Example:</u> The 1-year spot rate is 6%, the 2-year spot rate is 6.5%, and the 3-year spot rate is 6.7%. Compute the price of a bond with annual coupon rate 8%, and 3 years to maturity.

$$\text{Price} = \frac{8}{1+6\%} + \frac{8}{(1+6.5\%)^2} + \frac{108}{(1+6.7\%)^3} = 103.51.$$

- A coupon bond may sell at a discount (below face value), at par (at face value), or at a premium (above face value).

Semiannual Coupon Payments

- Consider a bond with semiannual coupon rate $c\%$ (i.e. semiannual coupon payments of $c/2$), and T years to maturity.
- Assume that spot rates are quoted as semiannual APRs.
- The PV of the bond's cash flows is

$$PV = \frac{\frac{c}{2}}{1+\frac{r_{0.5}}{2}} + \frac{\frac{c}{2}}{\left(1+\frac{r_1}{2}\right)^2} + \cdots + \frac{100+\frac{c}{2}}{\left(1+\frac{r_T}{2}\right)^{2T}}.$$

  This is the price of the bond, $P_{t,T}$

- Example: Suppose that $r_{0.5} = 8\%$, $r_1 = 8.2\%$, $r_{1.5} = 8.6\%$, and $r_2 = 9\%$. Compute the price of a bond with semiannual coupon rate 8%, and 2 years to maturity.

$$P_{0,2} = \frac{4}{1+4\%} + \frac{4}{(1+4.1\%)^2} + \frac{4}{(1+4.3\%)^3} + \frac{104}{(1+4.5\%)^4} = 98.27.$$

Accrued interest

- What if you are trading a bond in-between coupon dates
- The price discussed above is referred to as "clean" or quoted price
- The price at which the bond trades is the "dirty" price
- Accrued Interest = Dirty - Clean = Coupon rate $\times$ Day count convention
- Day count convention varies with markets
  - Actual/Actual for UST
  - Actual/360

3. Replication and Arbitrage

Replication and Arbitrage

- We determined the price of a bond using the PV rule.
- What ensures that this theoretical rule gives the actual market price?
- We will show that the PV rule gives indeed the market price. This is based on the principle of absence of arbitrage.

A Digression: Obtaining Spot Rates

- So far, we took the spot rates as given. But how can we obtain them?
- Recall that the price of a zero-coupon bond with T years to maturity is

$$\frac{100}{(1+r_T)^T} = 100 \times d_T.$$

- Therefore, if ZCB prices are observable, we can obtain
  - the discount factor d_T, by dividing the price by 100
  - the T-year spot rate, by

$$d_T = \frac{1}{(1+r_T)^T} \Rightarrow r_T = \left(\frac{1}{d_T}\right)^{\frac{1}{T}} - 1.$$

Valuation via Zero-Coupon Bonds

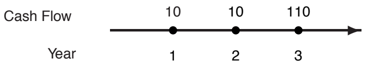
- Recall that the price of a coupon bond is

$$\begin{aligned}\text{Price} &= \frac{c}{1+r_1} + \frac{c}{(1+r_2)^2} + \cdots + \frac{100+c}{(1+r_T)^T} \\ &= c \times d_1 + c \times d_2 + \cdots + (100+c) \times d_T.\end{aligned}$$

  The price depends on the spot rates.
- But we obtained the spot rates from the prices of zero-coupon bonds.
- Therefore, we implicitly obtained the price of a bond from the prices of zero-coupon bonds.
- Knowing the prices of zero-coupon bonds, “nails” the prices of all bonds.

Valuation via Zero-Coupon Bonds: An Example

- Consider a coupon bond with annual coupon rate 10% and 3 years to maturity.
- Cash flows are



- Suppose that the prices of zero-coupon bonds with maturities 1, 2, and 3 years, are 95, 88, and 80, respectively.
- The price of the coupon bond is

$$\begin{aligned}
\text{Price} &= 10 \times d_1 + 10 \times d_2 + 110 \times d_3 \\
&= 10 \times 0.95 + 10 \times 0.88 + 110 \times 0.8 \\
&= 106.3.
\end{aligned}$$

- But what ensures that this "theoretical" price is the actual market price?

Synthetic Replication

- We can construct a portfolio of the three zero-coupon bonds, that has the same cash flow as the coupon bond.

Year	Cash Flow of Coupon Bond	Portfolio of Zeros
1	10	0.1 one-year zeros
2	10	0.1 two-year zeros
3	110	1.1 three-year zeros

- The portfolio of the zero-coupon bonds **synthetically replicates** the coupon bond. It is a **replicating portfolio** for the coupon bond.

Arbitrage

- We have two portfolios with the same cash flows (both the amounts and timing of cash flows is the same):
  - The coupon bond.
  - The replicating portfolio.
- The market value of the replicating portfolio is

$$0.1 \times 95 + 0.1 \times 88 + 1.1 \times 80 = 106.3.$$

- The market value of the coupon bond (the bond's price) has to be the same. Otherwise, there would exist an arbitrage.
- Suppose that a trader offers the coupon bond at 105. Then we can
  - buy the coupon bond at 105.
  - sell the replicating portfolio at 106.3.

  The cash flows in years 1, 2, and 3 cancel, and we are left with a gain of 106.3−105=1.3 today.

What Did we Learn?

- The spot rates can be obtained from the prices of zero-coupon bonds.
- The prices of all bonds can be obtained from the prices of zero-coupon bonds.
- Any bond can be synthetically replicated by a portfolio of zero-coupon bonds.
- Absence of arbitrage implies that a bond must have the same price as its replicating portfolio. This is the price given by the PV rule.

Using Coupon Bonds Instead of Zero-Coupon

- We used zero-coupon bonds to
  - obtain spot rates.
  - obtain prices of other bonds.
  - synthetically replicate other bonds.

- Instead of zero-coupon bonds, we can use coupon bonds.

- The advantage of using coupon bonds is that some coupon bonds are very liquid, and thus their quoted prices are very accurate.

Bootstrapping

- Bootstrapping is a procedure to obtain spot rates from the prices of coupon bonds.

- Example: Consider three bonds with the following characteristics:

Maturity	Coupon Rate (Semiannual)	Price
0.5	0%	98
1.0	0%	95
1.5	8%	102

Determine the 6-month, 1-year, and 18-month spot rates, expressed as semiannual APRs.

Synthetic Replication of a Zero-Coupon Bond

Example: Consider two bonds with the following characteristics available for trading:

Maturity	Coupon Rate (Annual)	Price
2	4%	96.37
2	8%	103.74

I want: \$100 one year from now.

- Questions:
  (i) How do we synthetically create the one-year zero by trading in the two-year bonds?
  (ii) How much will this synthetic one-year zero cost? That is, what is the no-arbitrage price of what I want?

Synthetic Replication of a Zero-Coupon Bond

- Suppose that portfolio consists of x_1 units of the first bond and x_2 units of the second.
- Cash flows are:

Year	Portfolio	Zero-Coupon Bond
1	$4x_1 + 8x_2$	100
2	$104x_1 + 108x_2$	0

- We need to solve the system of equations

$$\begin{aligned} 4x_1 + 8x_2 &= 100 \\ 104x_1 + 108x_2 &= 0. \end{aligned}$$

  The solution is $x_1 = -27$ and $x_2 = 26$.
- We need to sell 27 units of the first bond and buy 26 units of the second.

Synthetic Replication of a Zero-Coupon Bond

- Having synthetically replicated the zero-coupon bond, we can compute its price.

- The market value of the replicating portfolio is

$$-27 \times 96.37 + 26 \times 103.74 = 95.25.$$

- The price of the zero-coupon bond has to be 95.25. Otherwise, there would exist an arbitrage.

The Nelson-Siegel-Svensson model

- Used by the Fed

$$
\begin{aligned}
r_t = \beta_0 + \beta_1 \frac{1 - e^{-t_1}}{t_1} \quad & + \quad \beta_2 \left[\frac{1 - e^{-t_1}}{t_1} - e^{-t_1} \right] \\
& + \quad \beta_3 \left[\frac{1 - e^{-t_2}}{t_2} - e^{-t_2} \right]
\end{aligned}
$$

  with $t_j = t/\tau_j$, and where β_k's and τ_j's are estimated daily to fit observable bond prices

- The "Svensson" part is the second line of the formula

4. Term Structure of Spot Rates

Term Structure of Spot Rates

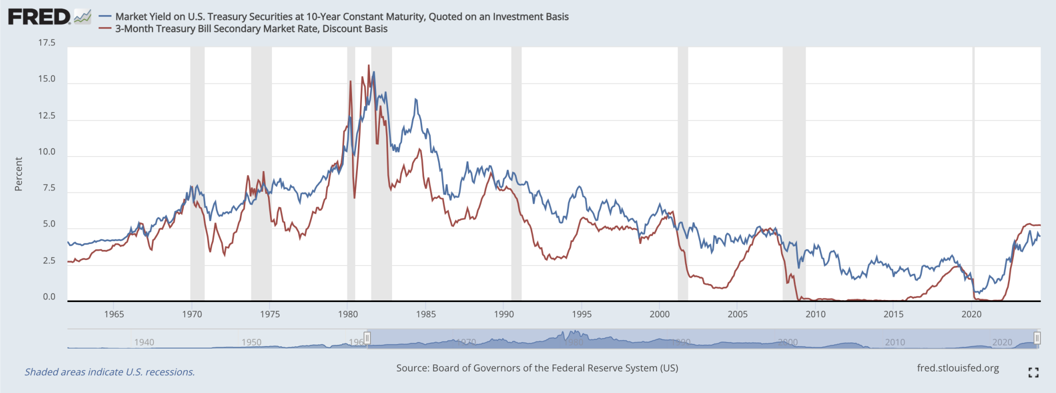
- To value fixed-income securities, it is essential to know the spot rate, r_t, for each maturity t.

- The **term structure of spot rates** (or **yield curve**) represents the spot rates as a function of maturity.

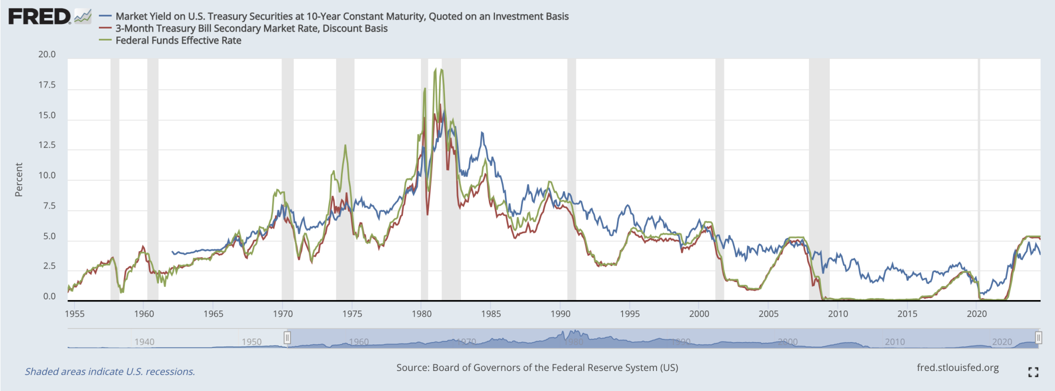
- There are two interesting empirical concepts associated with the yield curve:
  - Time-series of yields and yield spreads
  - Cross-section of bond yields at a given point in time: spot or yield curve (term structure of interest rates)

- Why do the rates move around? What do they tell us?

Movements in the yield curve



Who determines short rates?

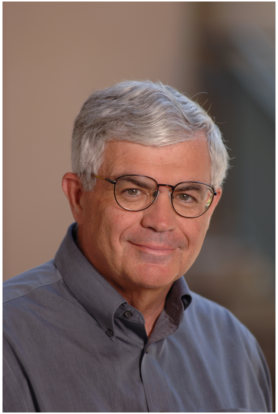


The Taylor rule

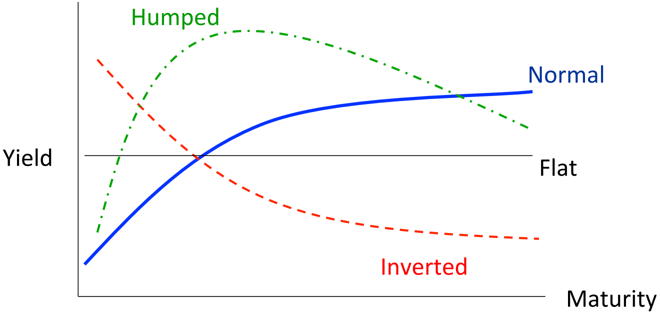
- Fed decisions can be approximated by

  $$\text{FF}_t = r^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5\text{Gap}_t$$

- where
  - FF is the Fed Funds rate
  - r^* is the long-run real interest rate
  - π_t is the current inflation rate
  - π^* is the target inflation rate
  - Gap is the output gap (the difference between real and potential GDP)

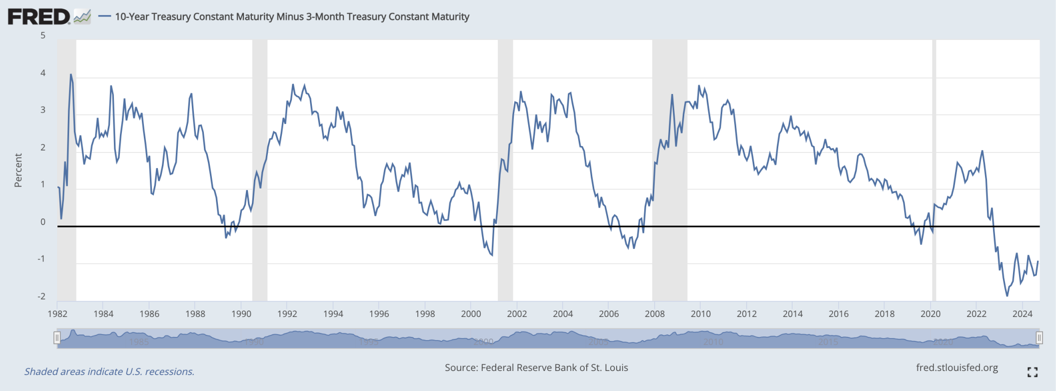


The rest of the yield curve



- Time value of money: $d_t = (1 + r_t)^{-t}$
- Macro determinants
    - Inflation
    - Recession
    - Aggregate risk

Time series of the slope



5. Bond returns

Computing bond returns

- Suppose you buy one of the bonds today and hold to maturity.
- What is the expected return?
- Start with ZCB
  - Buy-and-hold return is $1/P_{t,T} - 1$ is known today
  - One-period return is $P_{t+1,T-1}/P_{t,T} - 1$ is unknown today
- Thus, bonds are exposed to interest rate risk despite being safe assets

Coupon bond returns

- What are the cash flows?
- How to account for intermediate cash flows?
- Re-invest. At what rate?

6. Yield to Maturity

Yield to Maturity

- The **yield to maturity** (YTM) of a bond is the single discount rate that equates the PV of the bond's cash flows to the bond's price.
- Viewing the bond as an "investment project", the YTM is simply the bond's IRR.
- For a bond with annual coupon rate $c\%$ and T years to maturity, the YTM (y) is given by

$$\text{Price} = \frac{c}{1+y} + \frac{c}{(1+y)^2} + \cdots + \frac{100+c}{(1+y)^T}.$$

- Example: If $c = 10$, $T = 3$, and the price is 98, then

$$98 = \frac{10}{1+y} + \frac{10}{(1+y)^2} + \frac{110}{(1+y)^3}.$$

  Solving numerically, we get $y = 10.82\%$.

A Fact About YTM

The relation between a bond's YTM and coupon rate tells us how the bond's price compares to the face value.

- If the YTM is greater than the coupon rate, then the bond sells at a discount (below face value).

- If the YTM is equal to the coupon rate, then the bond sells at par (at face value). The corresponding YTM is known as *par rate*.

- If the YTM is smaller than the coupon rate, then the bond sells at a premium (above face value).

Relation Between YTM and Spot Rates

- Consider a bond with annual coupon rate $c\%$ and T years to maturity. Its price is

$$\text{Price} = \frac{c}{1+r_1} + \frac{c}{(1+r_2)^2} + \cdots + \frac{100+c}{(1+r_T)^T}.$$

- The YTM is given by

$$\text{Price} = \frac{c}{1+y} + \frac{c}{(1+y)^2} + \cdots + \frac{100+c}{(1+y)^T}.$$

- Comparing the two equations:
  - YTM is a complicated average of the spot rates corresponding to the years $1, .., T$.
  - YTM is simple only if the bond is zero-coupon ($c = 0$). It is then equal to the T-year spot rate.

Semiannual Coupon Payments

- When coupon payments are semiannual, the YTM is generally quoted as a semiannual APR.
- For a bond with semiannual coupon rate $c\%$ and T years to maturity, the YTM (y) is given by

$$\text{Price} = \frac{\frac{c}{2}}{1+\frac{y}{2}} + \frac{\frac{c}{2}}{\left(1+\frac{y}{2}\right)^2} + \cdots + \frac{100+\frac{c}{2}}{\left(1+\frac{y}{2}\right)^{2T}}.$$

- For T-strips, T-notes, and T-bonds, the YTM is quoted as a semiannual APR, and is given by the above formula.

Uses and Abuses of YTM

- Main use of YTM: As an alternative way to quote the price of a bond.
- Abuse 1: As a measure of the bond's return.
- Abuse 2: As a tool for choosing between different bonds.

YTM and Return: Zero-Coupon Bonds

- The YTM of a zero-coupon bond is the spot rate corresponding to the bond's time to maturity.
- This is the return from investing in the bond and holding it until maturity.
- But it is not the return for any other investment horizon.

Example: Consider a zero-coupon bond with 3 years to maturity and YTM 6%.

- The return from investing in the bond and holding it for 3 years is 6%.
- However, the return from investing in the bond and selling it after one year is unknown today.
  - The return depends on the bond's price in one year.
  - The price in one year depends on the 2-year spot rate that will prevail in one year. This spot rate is unknown today.

YTM and Return: Coupon Bonds

- The YTM of a coupon bond is the return of investing in the bond, holding it until maturity, and reinvesting the coupons at a rate equal to the YTM.

- Problem 1: The YTM is not the return for any investment horizon other than maturity. This is for the same reason as for zero-coupon bonds.

- Problem 2: The YTM is not the return even for investment horizon equal to maturity. This is because the future spot rates, at which the coupons will be reinvested, may be different than the YTM.

YTM as a Tool for Comparing Bonds

- Using the YTM for comparing bonds is correct only when the bonds have the same coupon and time to maturity. In all other cases it can be very misleading.
- Suppose, for instance, that the bonds have different time to maturity.
  - Recall that the YTM is at best the return of investing until maturity.
  - Therefore, by comparing YTMs we compare returns for different investment horizons.
- The correct way to compare bonds is to think about the risk-return trade-off