Quantitative Asset Management

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Lecture 4: Time Series Momentum and Volatility

1. Time Series Momentum

   Moskowitz, Ooi, and Pedersen (2012, JFE)

2. The cross-section of volatility and expected returns

   Ang, Hodrick, Xing, and Zhang (2006, JF)

Time Series Momentum

Moskowitz, Ooi, and Pedersen (2012, JFE)

Time Series Momentum

- Security's past performance forecast its own future return
  - Large abnormal returns
  - Not crash risk
- Different from cross-sectional momentum
  - Cross-sectional momentum: relative performance
  - Cross-sectional momentum: about cross-section!
- Direct test of whether returns follow a random walk
- Holds globally across different asset classes
  - Equity, currency, commodity and bonds
  - 58 liquid instruments

Data

- 24 commodity futures
- 12 cross-currency forward pairs (9 underlying currencies)
- 9 developed equity index futures
- 13 developed government bond futures

- Dates: January 1965 through December 2009

- Among the most liquid futures contracts in the world

- Source: Datastream, Bloomberg, other exchanges

Data

- Lots of heterogeneity in assets' volatility

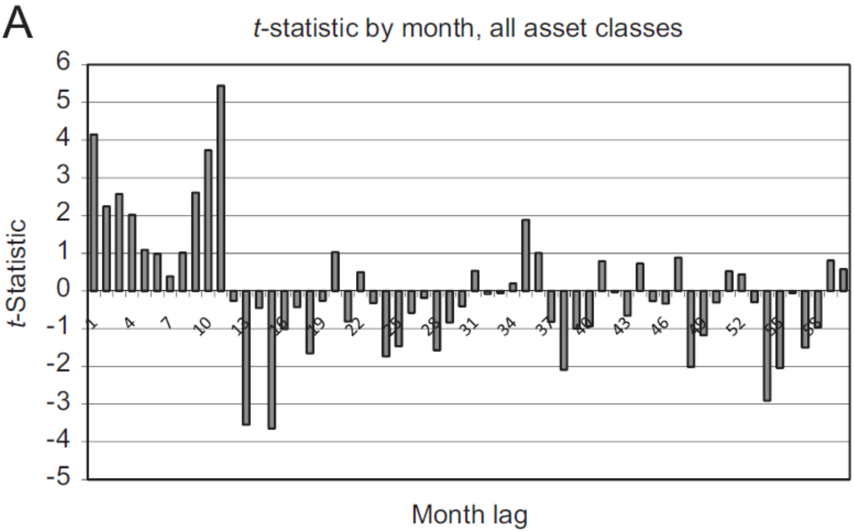
  See Table 1
- Authors scale return by volatility

$$\sigma_t^2 = 261 \sum_{i=0}^{\infty} (1-\delta)\delta^i \left(r_{t-1-i} - \overline{r}_t\right)^2$$

- Change in units
- Use Sharpe ratios instead of return

Regression Evidence

$$r_t^s/\sigma_{t-1}^s = \alpha + \beta_h r_{t-h}^s/\sigma_{t-h-1}^s + \varepsilon_t^s$$

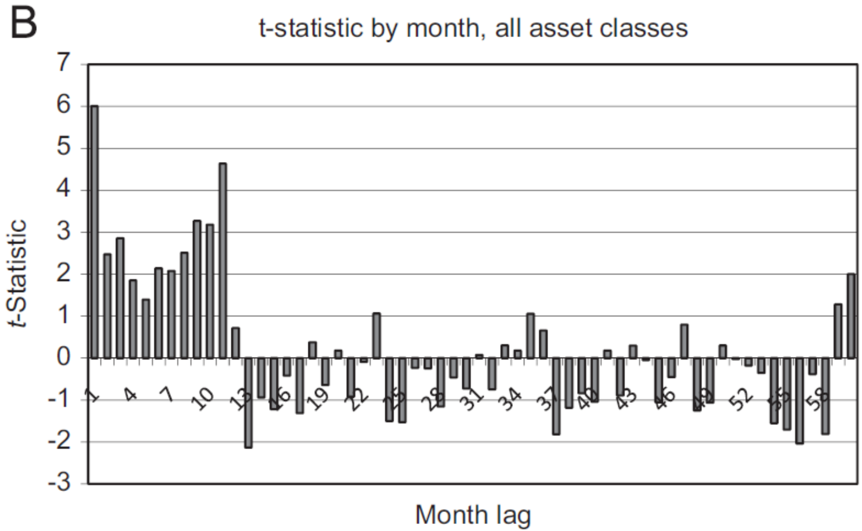


Regression Evidence

- Positive t-stats for first 12 months: suggests significant return continuation
- Negative signs suggest reversals

Regression Evidence

$$r_t^s/\sigma_{t-1}^s = \alpha + \beta_h sign(r_{t-h}^s) + \varepsilon_t^s$$



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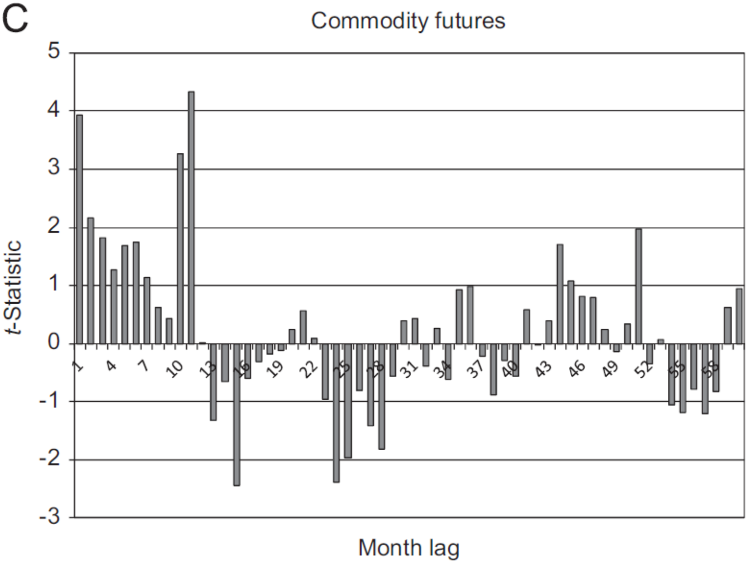

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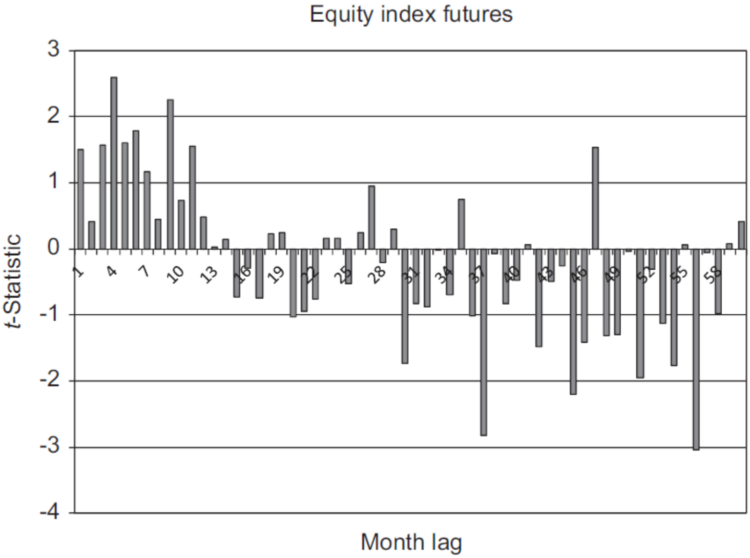

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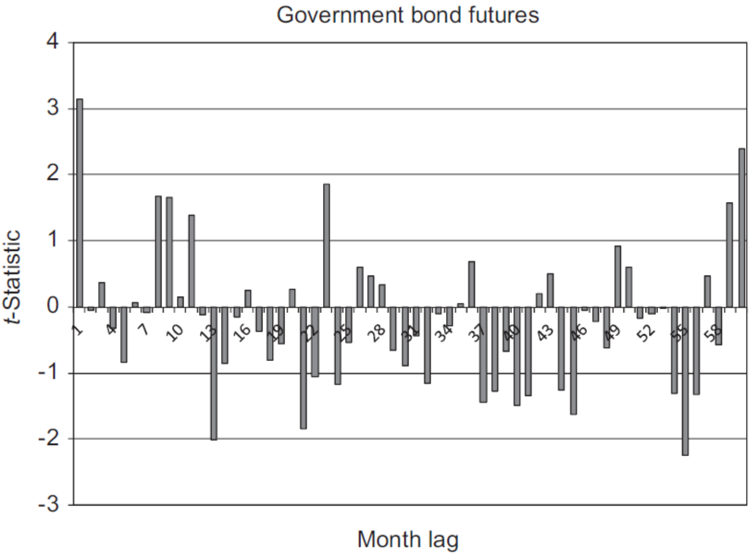

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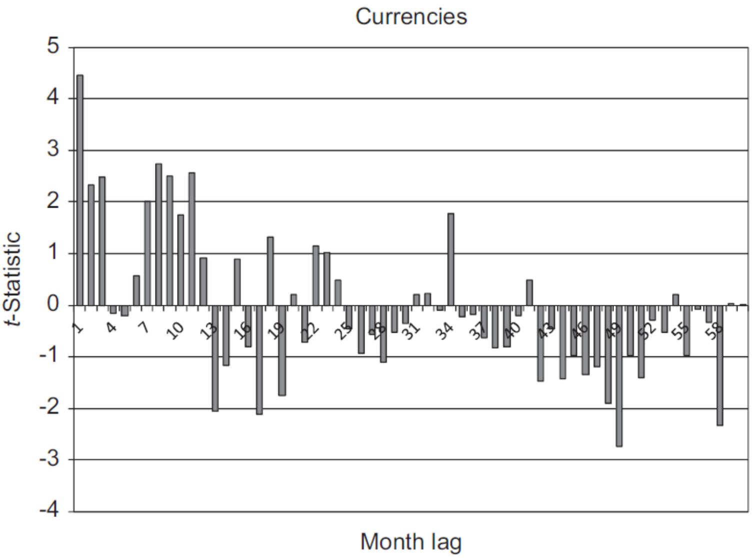


Time Series Momentum Strategy

For each instrument s and month t

- Compute the excess return over the past k months
  - past returns from $t-k-1$ to $t-1$
  - Check whether it is positive or negative
- Long contract with positive excess returns: past k months
- Short contract with negative excess returns: past k months
- Position size: inversely proportional to ex-ante volatility:

$$\frac{1}{\sigma_{t-1}^{s}}$$

Time Series Momentum Strategy

- How to deal with holding periods longer than 1 month?
  - They follow Jegadeesh and Titman (1993)
  - Overlapping holding periods (remember?)
  - "The return at time t represents the average return across all portfolios at that time, namely the return on the portfolio that was constructed last month, the month before that (and still held if the holding period h is greater than two), and so on for all currently 'active' portfolios."

- Does the strategy works?
- Compute alphas relative to the following factor model

$$r_t^{TSMOM(k,h)} = \alpha + \beta_1 MKT_t + \beta_2 BOND_t + \beta_3 GSCI_t + sSMB_t + hHML_t + mUMD_t + \varepsilon_t,$$

Table 2: alphas t-stats

		Holding period (months)							
		1	3	6	9	12	24	36	48
Panel A: All assets									
Lookback period (months)	1	4.34	4.68	3.83	4.29	5.12	3.02	2.74	1.90
	3	5.35	4.42	3.54	4.73	4.50	2.60	1.97	1.52
	6	5.03	4.54	4.93	5.32	4.43	2.79	1.89	1.42
	9	6.06	6.13	5.78	5.07	4.10	2.57	1.45	1.19
	12	6.61	5.60	4.44	3.69	2.85	1.68	0.66	0.46
	24	3.95	3.19	2.44	1.95	1.50	0.20	−0.09	−0.33
	36	2.70	2.20	1.44	0.96	0.62	0.28	0.07	0.20
	48	1.84	1.55	1.16	1.00	0.86	0.38	0.46	0.74

- What is the difference between time series and cross-sectional momentum?

Cross-sectional Momentum

Based on Daniel and Moskowitz (2016, JFE)

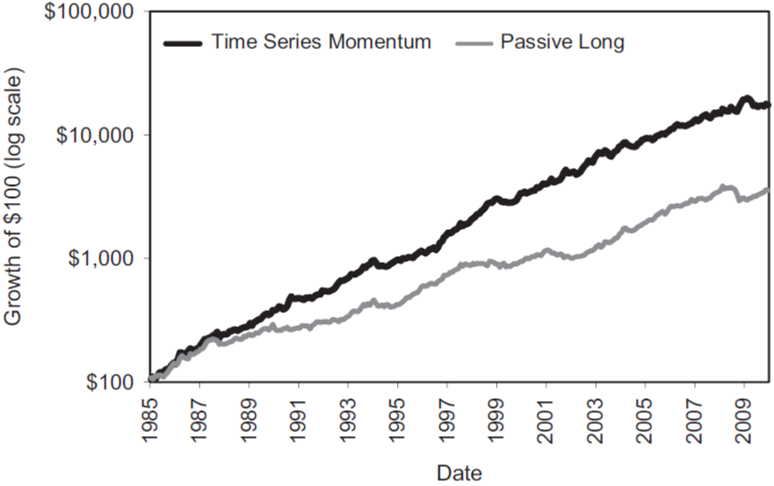
Standard Sample Selection

- CRSP share codes 10 and 11
- NYSE, AMAX, Nasdaq
- Valid share price and number of shares on the formation date
- At least 8 months of return data between $t-12$ and $t-2$ (skip one month)

Momentum Strategy

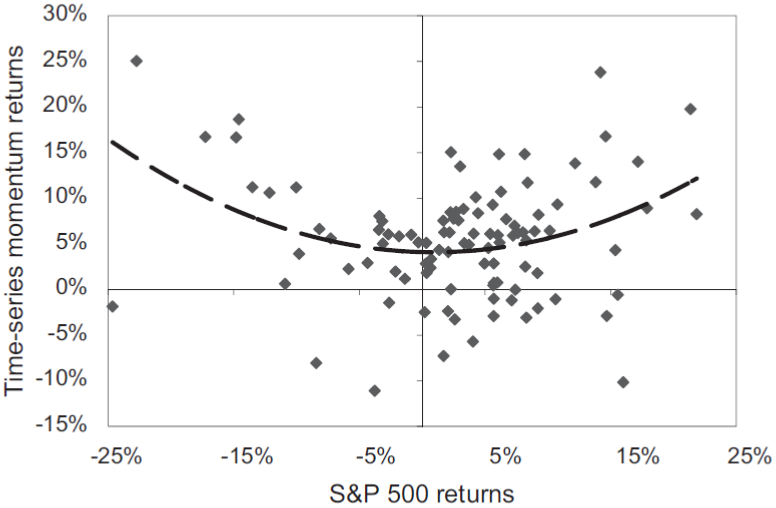
- Rank stock based on their cumulative returns from $t-12$ to $t-2$ (11 months)
- Skip one month as formation period
- Sort stocks into deciles (cross-section)
- Form long-short portfolio

Crash risk?



Cumulative excess return of time series momentum and diversified passive long strategy, January 1985 to December 2009. Plotted are the cumulative excess returns of the diversified TSMOM portfolio and a diversified portfolio of the possible long position in every futures contract we study. The TSMOM portfolio is defined in Eq. (5) and across all futures contracts summed. Sample period is January 1985 to December 2009.

Crash risk?



The time series momentum smile. The non-overlapping quarterly returns on the diversified (equally weighted across all contracts) 12-month time series momentum or trend strategy are plotted against the contemporaneous returns on the S&P 500

Takeaway

- Time series momentum in equity indexes, commodity, bonds, and currency
- Persistence in returns in liquid contracts
- Time series momentum is different from cross-sectional momentum
- It seems that speculators profit from time series momentum
- Time series momentum not driven by:
  - transaction costs (liquidity)
  - Crash risk
  - Standard risk factors (3FF+MOM, bond, commodity, etc)

The cross-section of volatility and expected returns

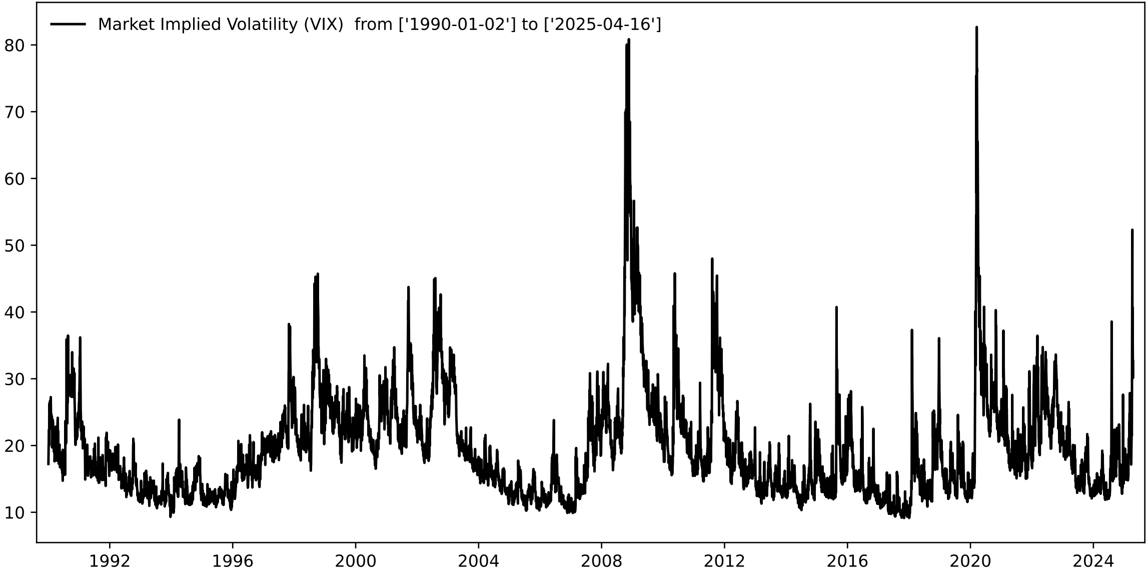
Ang, Hodrick, Xing, and Zhang (2006, JF)

Is volatility priced in cross-section of stocks returns?

The cross-section of volatility and expected returns

- Verify whether volatility is priced
- *exposure* to vol $\Rightarrow$ lower average returns
- Puzzle: stocks with *high* idiosyncratic volatility have low returns

Daily VIX



Market Implied Volatility (VIX) from ['1990-01-02'] to ['2025-04-16']
80
70
60
50
40
30
20
10
1992
1996
2000
2004
2008
2012
2016
2020
2024

The cross-section of volatility and expected returns

Portfolio Sorting

- Pre-formation betas

$$r_t^i = \beta_0 + \beta_{MKT}^i MKT_t + \beta_{\Delta VIX}^i \Delta VIX_t + \varepsilon_t^i$$

- Use daily data
- Get monthly beta estimates
- Sort stocks into Vol-beta ($\beta_{\Delta VIX}^i$) quintiles

The cross-section of volatility and expected returns

Factor Mimicking Portfolio

- Use volatility-beta sorted portfolios as test assets: X_t
- Regress changes in VIX on test assets:

$$\Delta VIX_t = c + b' X_t + u_t$$

- Regression at daily frequency every month
- $FVIX_t \equiv b' X_t$
- $FVIX_t$ is traded and it is a zero-cost portfolio

Portfolios Sorted by Exposure to Changes in VIX

We form value-weighted quintile portfolios every month by regressing excess individual stock returns on ΔVIX, controlling for the MKT factor as in equation (3), using daily data over the previous month. Stocks are sorted into quintiles based on the coefficient $\beta_{\Delta VIX}$ from lowest (quintile 1) to highest (quintile 5). The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "5-1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report Jensen's alpha with respect to the CAPM or the Fama–French (1993) three-factor model. The pre-formation betas refer to the value-weighted $\beta_{\Delta VIX}$ or β_{FVIX} within each quintile portfolio at the start of the month. We report the pre-formation $\beta_{\Delta VIX}$ and β_{FVIX} averaged across the whole sample. The second to last column reports the $\beta_{\Delta VIX}$ loading computed over the next month with daily data. The column reports the next month $\beta_{\Delta VIX}$ loadings averaged across months. The last column reports ex post β_{FVIX} factor loadings over the whole sample, where $FVIX$ is the factor mimicking aggregate volatility risk. To correspond with the Fama–French alphas, we compute the ex post betas by running a four-factor regression with the three Fama–French factors together with the $FVIX$ factor that mimics aggregate volatility risk, following the regression in equation (6). The row labeled "Joint test p-value" reports a Gibbons, Ross and Shanken (1989) test for the alphas equal to zero, and a robust joint test that the factor loadings are equal to zero. Robust Newey–West (1987) t-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

								Factor Loadings			
Rank	Mean	Std. Dev.	% Mkt Share	Size	B/M	CAPM Alpha	FF-3 Alpha	Pre-Formation $\beta_{\Delta VIX}$	Pre-Formation β_{FVIX}	Next Month Post-Formation $\beta_{\Delta VIX}$	Full Sample Post-Formation β_{FVIX}
1	1.64	5.53	9.4%	3.70	0.89	0.27	0.30	−2.09	−2.00	−0.033	−5.06
						[1.66]	[1.77]				[−4.06]
2	1.39	4.43	28.7%	4.77	0.73	0.18	0.09	−0.46	−0.42	−0.014	−2.72
						[1.82]	[1.18]				[−2.64]
3	1.36	4.40	30.4%	4.77	0.76	0.13	0.08	0.03	0.08	0.005	−1.55
						[1.32]	[1.00]				[−2.86]
4	1.21	4.79	24.0%	4.76	0.73	−0.08	−0.06	0.54	0.62	0.015	3.62
						[−0.87]	[−0.65]				[4.53]
5	0.60	6.55	7.4%	3.73	0.89	−0.88	−0.53	2.18	2.31	0.018	8.07
						[−3.42]	[−2.88]				[5.32]
5-1	−1.04					−1.15	−0.83				
	[−3.90]					[−3.54]	[−2.93]				
Joint test p-value						0.01	0.03				0.00

Factor Correlation

The table reports correlations of first differences in *VIX*, *FVIX*, and *STR* with various factors. The variable ΔVIX ($\Delta_m VIX$) represents the daily (monthly) change in the *VIX* index, and *FVIX* is the mimicking aggregate volatility risk factor. The factor *STR* is constructed by Coval and Shumway (2001) from the returns of zero-beta straddle positions. The factors *MKT*, *SMB*, *HML* are the Fama and French (1993) factors, the momentum factor *UMD* is constructed by Kenneth French, and *LIQ* is the Pástor and Stambaugh (2003) liquidity factor. The sample period is January 1986 to December 2000, except for correlations involving *STR*, which are computed over the sample period January 1986 to December 1995.

Panel A: Daily Correlation

	ΔVIX
FVIX	0.91

Panel B: Monthly Correlations

	FVIX	$\Delta_m VIX$	*MKT*	*SMB*	*HML*	*UMD*	*LIQ*
$\Delta_m VIX$	0.70	1.00	−0.58	−0.18	0.22	−0.11	−0.33
FVIX	1.00	0.70	−0.66	−0.14	0.26	−0.25	−0.40
STR	0.75	0.83	−0.39	−0.39	0.08	−0.26	−0.59

Volatility-beta sorted portfolios

Robustness

- VIX innovations calculations
- Portfolio formation window
- Characteristics controls: BM and size

  "Every month, each stock is matched with one of the Fama–French 25 size and book-to-market portfolios according to its size and book-to-market characteristics. The table reports value-weighted simple returns in excess of the characteristic-matched returns."
- Liquidity effect
  - One-way sort controlling for liquidity: first sort on liquidity, then sort each quintile bucket by $\beta_{\Delta VIX}$ (sequential double sort). Finally, for each volatility group (low to high), we average across all liquidity buckets.
- Also control for volume and momentum

Prices of Risk

Fama and MacBeth

$$r_t^i = c + \beta_{MKT}^i \lambda_{MKT} + \beta_{FVIX}^i \lambda_{FVIX} + \beta_{SMB}^i \lambda_{SMB}$$
$$+\beta_{HML}^i \lambda_{HML} + \beta_{UMD}^i \lambda_{UMD} + \beta_{LIQ}^i \lambda_{LIQ} + \varepsilon_t^i$$

- Test assets: 25 portfolio double sorted on market- and volatility-betas

Panel A reports the Fama–MacBeth (1973) factor premiums on 25 portfolios sorted first on β_{MKT} and then on $\beta_{\Delta VIX}$. *MKT* is the excess return on the market portfolio, *FVIX* is the mimicking factor for aggregate volatility innovations, *STR* is Coval and Shumway's (2001) zero-beta straddle return, *SMB* and *HML* are the Fama–French (1993) size and value factors, *UMD* is the momentum factor constructed by Kenneth French, and *LIQ* is the aggregate liquidity measure from Pástor and Stambaugh (2003). In Panel B, we report ex post factor loadings on *FVIX*, from the regression specification I (Fama–French model plus *FVIX*). Robust t-statistics that account for the errors-in-variables for the first-stage estimation in the factor loadings are reported in square brackets. The sample period is from January 1986 to December 2000, except for the Fama–MacBeth regressions with *STR*, which are from January 1986 to December 1995.

Panel A: Fama–MacBeth (1973) Factor Premiums

	I	II	III	IV
Constant	−0.145	−0.527	−0.202	−0.247
	[−0.23]	[−0.88]	[−0.31]	[−0.36]
MKT	0.977	1.276	1.034	1.042
	[1.11]	[1.47]	[1.13]	[1.13]
FVIX	−0.080		−0.082	−0.071
	[−2.49]		[−2.39]	[−2.02]
STR		−0.194		
		[−2.32]		
SMB	−0.638	−0.246	−0.608	−0.699
	[−1.24]	[−0.59]	[−1.13]	[−1.25]
HML	−0.590	−0.247	−0.533	−0.232
	[−0.95]	[−0.40]	[−0.82]	[−0.34]
UMD			0.827	0.612
			[0.83]	[0.59]
LIQ				−0.021
				[−1.00]
Adj R^2	0.67	0.56	0.65	0.79

Idiosyncratic Volatility

- Idiosyncratic Volatility: vol of factor model residuals
$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \varepsilon_t^i$$
- Asset-specific volatility measure

- Strategy: construct idiosyncratic volatility sorted portfolios
- Estimate vol using 1 month of data
- Hold portfolio for 1 month

Idiosyncratic volatility sorted portfolios

Rank	Mean	Std. Dev.	% Mkt Share	Size	B/M	CAPM Alpha	FF-3 Alpha
Panel A: Portfolios Sorted by Total Volatility							
1	1.06	3.71	41.7%	4.66	0.88	0.14	0.03
						[1.84]	[0.53]
2	1.15	4.48	33.7%	4.70	0.81	0.13	0.08
						[2.14]	[1.41]
3	1.22	5.63	15.5%	4.10	0.82	0.07	0.12
						[0.72]	[1.55]
4	0.99	7.15	6.7%	3.47	0.86	−0.28	−0.17
						[−1.73]	[−1.42]
5	0.09	8.30	2.4%	2.57	1.08	−1.21	−1.16
						[−5.07]	[−6.85]
5-1	−0.97					−1.35	−1.19
	[−2.86]					[−4.62]	[−5.92]
Panel B: Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3							
1	1.04	3.83	53.5%	4.86	0.85	0.11	0.04
						[1.57]	[0.99]
2	1.16	4.74	27.4%	4.72	0.80	0.11	0.09
						[1.98]	[1.51]
3	1.20	5.85	11.9%	4.07	0.82	0.04	0.08
						[0.37]	[1.04]
4	0.87	7.13	5.2%	3.42	0.87	−0.38	−0.32
						[−2.32]	[−3.15]
5	−0.02	8.16	1.9%	2.52	1.10	−1.27	−1.27
						[−5.09]	[−7.68]
5-1	−1.06					−1.38	−1.31
	[−3.10]					[−4.56]	[−7.00]

Takeaways

- Volatility is priced in the cross-section
- *exposure* to vol $\Rightarrow$ lower average returns
- Puzzle: stocks with *high* idiosyncratic volatility have low returns
  - Relationship is flat, except for the 5th quintile
- From my research: Idiosyncratic volatility obeys a strong factor structure and the common factor in idiosyncratic volatility carries a negative price of risk (Herskovic, Kelly, Lustig, and Van Nieuwerburgh, 2016 JFE).