MACHINE LEARNING LAB MANUAL

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1. Scheme and Syllabus:

MACHINE LEARNING LABORATORY

[As per Choice Based Credit System (CBCS) scheme]
(Effective from the academic year 2016 -2017)

SEMESTER – VII

Subject Code	15CSL76	IA Marks	20
Number of Lecture Hours/Week	01I + 02P	Exam Marks	80
Total Number of Lecture Hours	40	Exam Hours	3

CREDITS - 02

Course objectives: This course will enable students to

- 1. Make use of Data sets in implementing the machine learning algorithms
- 2. Implement the machine learning concepts and algorithms in any suitable language of choice.

Description (If any):

- 1. The programs can be implemented in either JAVA or Python.
- 2. For Problems 1 to 6 and 10, programs are to be developed without using the built-in classes or APIs of Java/Python.
- 3. Data sets can be taken from standard repositories (https://archive.ics.uci.edu/ml/datasets.html) or constructed by the students.

Lab Experiments:

- Implement and demonstrate the **FIND-Salgorithm** for finding the most specific hypothesis based on a given set of training data samples. Read the training data from a .CSV file. (**Page NO: 4**)
- For a given set of training data examples stored in a .CSV file, implement and demonstrate the **Candidate-Elimination algorithm**to output a description of the set of all hypotheses consistent with the training examples. (**Page NO:8**)
- Write a program to demonstrate the working of the decision tree based **ID3 algorithm**. Use an appropriate data set for building the decision tree and apply this knowledge toclassify a new sample. (**Page NO18:**)
- Build an Artificial Neural Network by implementing the **Backpropagation algorithm** and test the same using appropriate data sets. (**Page NO:38**)
- Write a program to implement the **naïve Bayesian classifier** for a sample training data set stored as a .CSV file. Compute the accuracy of the classifier, considering few test data sets. (**Page NO:44**)
- Assuming a set of documents that need to be classified, use the **naïve Bayesian**Classifier model to perform this task. Built-in Java classes/API can be used to write the program. Calculate the accuracy, precision, and recall for your data set. (Page NO:52)
- Write a program to construct a**Bayesian network** considering medical data. Use this model to demonstrate the diagnosis of heart patients using standard Heart Disease Data Set. You can use Java/Python ML library classes/API. (**Page NO: 61**)
- Apply **EM algorithm** to cluster a set of data stored in a .CSV file. Use the same data set for clustering using *k*-Means algorithm. Compare the results of these two algorithms and comment on the quality of clustering. You can add Java/Python ML library classes/API in the program. (**Page NO: 67**)
- 9 Write a program to implement *k*-Nearest Neighbour algorithm to classify the iris data set. Print both correct and wrong predictions. Java/Python ML library classes can

be used for this problem. (Page NO: 77)

Implement the non-parametric **Locally Weighted Regressionalgorithm** in order to fit data points. Select appropriate data set for your experiment and draw graphs. (**Page NO: 82**)

Course outcomes: The students should be able to:

- 1. Understand the implementation procedures for the machine learning algorithms.
- 2. Design Java/Python programs for various Learning algorithms.
- 3. Applyappropriate data sets to the Machine Learning algorithms.
- 4. Identify and apply Machine Learning algorithms to solve real world problems.

Conduction of Practical Examination:

- All laboratory experiments are to be included for practical examination.
- Students are allowed to pick one experiment from the lot.
- Strictly follow the instructions as printed on the cover page of answer script
- Marks distribution: Procedure + Conduction + Viva: 20 + 50 + 10 (80)

Change of experiment is allowed only once and marks allotted to the procedure part to be made zero.

Problem1: Implement and demonstrate the **FIND-S** algorithm for finding the most specific hypothesis based on a given set of training data samples. Read the training data from a **.CSV file.**

Algorithm:

- 1. Initialize **h** to the most specific hypothesis in **H**
- 2. **For** each positive training instance **x**
 - For each attribute constraint a_i in h
 If the constraint a_i in h is satisfied by x then do nothing
 else replace a_i in h by the next more general constraint that is satisfied by x
- 3. Output hypothesis h

Illustration:

Step1: Find S

Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

Initialize h to the most specific hypothesis in H

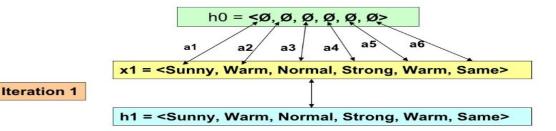
Step2: Find S

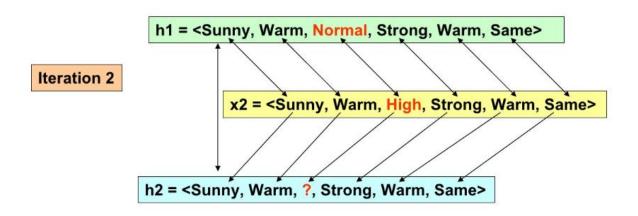
2. For each positive training instance x

For each attribute constraint a_i in h
 If the constraint a_i is satisfied by x

Then do nothing

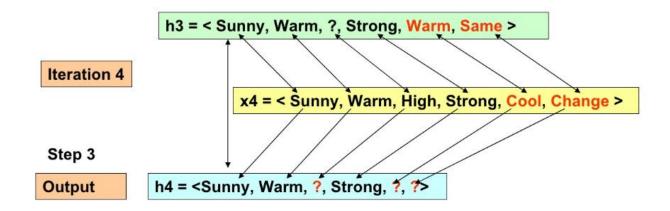
Else replace a_i in h by the next more general constraint that is satisfied by x





Iteration 3 Ignore h3 = <Sunny, Warm, ?, Strong, Warm, Same>

Iteration 4 and Step 3: Find S



Source Code of the Program:

```
import random
import csv
attributes = [['Sunny', 'Rainy'],
              ['Warm','Cold'],
              ['Normal', 'High'],
              ['Strong','Weak'],
              ['Warm','Cool'],
              ['Same','Change']]
num attributes = len(attributes)
print (" \n The most general hypothesis : ['?','?','?','?','?','
?']\n")
print ("\n The most specific hypothesis : ['0', '0', '0', '0', '0', '0']
0']\n")
a = []
print("\n The Given Training Data Set \n")
with open('C:\\Users\\thyagaragu\\Desktop\\Data\\ws.csv', 'r') a
s csvFile:
    reader = csv.reader(csvFile)
    for row in reader:
        a.append (row)
        print(row)
print("\n The initial value of hypothesis: ")
hypothesis = ['0'] * num attributes
print(hypothesis)
# Comparing with First Training Example
for j in range(0, num attributes):
        hypothesis[j] = a[0][j];
# Comparing with Remaining Training Examples of Given Data Set
print("\n Find S: Finding a Maximally Specific Hypothesis\n")
for i in range(0,len(a)):
    if a[i][num attributes] == 'Yes':
```

```
for j in range(0, num_attributes):
    if a[i][j]!=hypothesis[j]:
        hypothesis[j]='?'
    else:
        hypothesis[j]= a[i][j]
    print(" For Training Example No :{0} the hypothesis is ".for mat(i), hypothesis)
print("\n The Maximally Specific Hypothesis for a given Training Examples :\n")
print(hypothesis)
```

Output:

```
The most general hypothesis : ['?','?','?','?','?','?']
The most specific hypothesis : ['0','0','0','0','0','0']
The Given Training Data Set
['sunny', 'warm', 'normal', 'strong', 'warm', 'same', 'Yes']
['sunny', 'warm', 'high', 'strong', 'warm', 'same', 'Yes']
['rainy', 'cold', 'high', 'strong', 'warm', 'change', 'No']
['sunny', 'warm', 'high', 'strong', 'cool', 'change', 'Yes']
The initial value of hypothesis:
['0', '0', '0', '0', '0', '0']
Find S: Finding a Maximally Specific Hypothesis
For Training Example No :0 the hypothesis is ['sunny', 'warm', 'normal',
'strong', 'warm', 'same']
For Training Example No :1 the hypothesis is ['sunny', 'warm', '?', 'str
ong', 'warm', 'same']
For Training Example No :2 the hypothesis is ['sunny', 'warm', '?', 'str
ong', 'warm', 'same']
For Training Example No :3 the hypothesis is ['sunny', 'warm', '?', 'str
ong', '?', '?']
The Maximally Specific Hypothesis for a given Training Examples:
['sunny', 'warm', '?', 'strong', '?', '?']
```

Program2: For a given set of training data examples stored in a .CSV file, implement and demonstrate the Candidate - elimination algorithm to output a description of the set of all hypotheses consistent with the training examples.

Algorithm:

 $G \leftarrow$ maximally general hypotheses in H

S ← maximally specific hypotheses in H

For each training example d=<x,c(x)>

Case 1: If d is a positive example

Remove from G any hypothesis that is inconsistent with d For each hypothesis s in S that is not consistent with d

- Remove s from S.
- Add to S all minimal generalizations h of s such that
 - h consistent with d
 - Some member of G is more general than h
- Remove from S any hypothesis that is more general than another hypothesis in S

Case 2: If d is a negative example

Remove from S any hypothesis that is inconsistent with d For each hypothesis g in G that is not consistent with d

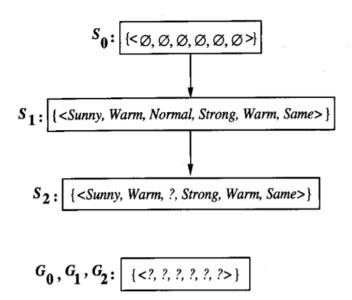
- *Remove g from G.*
- Add to G all minimal specializations h of g such that
 - o h consistent with d
 - Some member of S is more specific than h
- Remove from G any hypothesis that is less general than another hypothesis in G

Illustration:

						-	
Example	Sky	AirTemp	Humidity	Wind	Water	Forecast	EnjoySport
1	Sunny	Warm	Normal	Strong	Warm	Same	Yes
2	Sunny	Warm	High	Strong	Warm	Same	Yes
3	Rainy	Cold	High	Strong	Warm	Change	No
4	Sunny	Warm	High	Strong	Cool	Change	Yes

$$\begin{split} &S_0 = \{<\varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing > \} \\ &G_0 = \{, ?, ?, ?, ?, ? \} \\ &S_1 = \{<\text{Sunny, Warm, Normal, Strong, Warm, Same} > \} \\ &G_1 = \{, ?, ?, ?, ?, ? \} \\ &S_2 = \{<\text{Sunny, Warm, ?, Strong, Warm, Same} > \} \\ &G_2 = \{, ?, ?, ?, ?, ? \} \end{split}$$

Trace1:

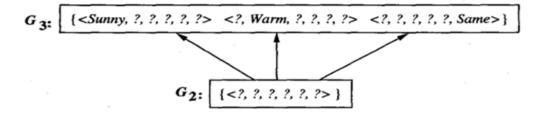


Training examples:

- 1. <Sunny, Warm, Normal, Strong, Warm, Same>, Enjoy Sport = Yes
- 2. <Sunny, Warm, High, Strong, Warm, Same>, Enjoy Sport = Yes

Candidate-Elimination Trace 1. S_0 and G_0 are the initial boundary sets corresponding to the most specific and most general hypotheses. Training examples 1 and 2 force the S boundary to become more general, as in the Find-S algorithm. They have no effect on the G boundary.

Trace 2:

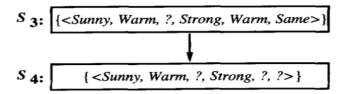


Training Example:

3. <Rainy, Cold, High, Strong, Warm, Change>, EnjoySport=No

CANDIDATE-ELIMINATION Trace 2. Training example 3 is a negative example that forces the G_2 boundary to be specialized to G_3 . Note several alternative maximally general hypotheses are included in G_3 .

Trace3:

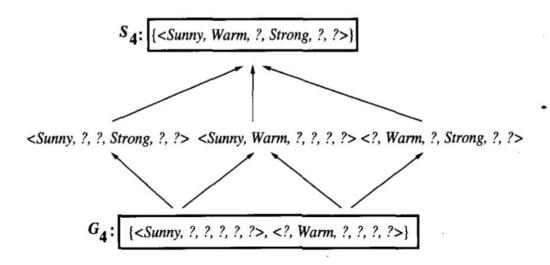


Training Example:

4. <Sunny, Warm, High, Strong, Cool, Change>, EnjoySport = Yes

CANDIDATE-ELIMINATION Trace 3. The positive training example generalizes the S boundary, from S_3 to S_4 . One member of G_3 must also be deleted, because it is no longer more general than the S_4 boundary.

Final Version Space:



The final version space for the *EnjoySport* concept learning problem and training examples described earlier.

Source Code:

```
import random
import csv
class Factors:

    factors={}
    attributes = ()

    def __init__ (self,attr):
        self.attributes = attr
        for i in attr:
            self.factors[i]=[]

    def add_values(self,factor,values):
        self.factors[factor]=values
```

```
class Candidate_elimination:
    Positive={}
    Negative={}

# Constructor
def __init__(self,data,fact):
        self.num_factors = len(data[0][0])
        self.factors = fact.factors
        self.attr = fact.attributes
```

```
self.dataset = data
   # Main Algorithm Method
    def run algo(self):
        G = self.initializeG()
        S = self.initializeS()
        i=1
        for example in self.dataset: # For Each Training Example Data ,d
            if self.is positive(example):# Positive Training Examples
                #Remove from G any hypothesis which is inconsistent with d
                G = self.remove inconsistent G(G,example[0])
                S new = S[:]
                for s in S:
                     if not self.consistent(s,example[0]):
                         S new.remove(s)
                         generalization = self.generalize inconsistent S(s,
example[0])
                         generalization = self.get general(generalization, G
                         if generalization:
                             S new.append(generalization)
                    S = S \text{ new}[:]
                    S = self.remove more general(S)
                    # print("S+:\n",S)
                    #print("G+:\n",G)
            else: # Negative Training Examples
                     S = self.remove inconsistent S(S,example[0])
                    G \text{ new} = G[:]
                     for g in G:
                             if self.consistent(g,example[0]):
                                 G new.remove(g)
                                 specializations = self.specialize inconsis
tent G(q,example[0])
                                 specializationss = self.get specific(speci
alizations, S)
                                 if specializations != []:
                                     G new += specializations
                    G = G \text{ new}[:]
                    G = self.remove more specific(G)
                    #print("S-:\n",S)
                    #print("G-:\n",G)
            print("S[%d]:" %i,S,"\n")
            print("G[%d]:" %i,G,"\n")
            i=i+1
        #print ("Final S:",S)
        #print ("Final G:",G)
```

```
def initializeS(self):
    ''' Initialize the specific boundary '''
    S = tuple(['0' for factor in range(self.num factors)])
    return [S]
def initializeG(self):
    ''' Initialize the general boundary '''
    G = tuple(['?' for factor in range(self.num factors)])
    return [G]
def is positive(self, example):
    ''' Check if a given training example is positive '''
    if example[1] == 'Y':
        return True
    elif example[1] == 'N':
        return False
    else:
        raise TypeError("invalid target value")
def is negative(self, example):
    ''' Check if a given training example is negative '''
    if example[1] == 'N':
        return False
    elif example[1] == 'Y':
        return True
    else:
        raise TypeError("invalid target value")
def match factor(self, value1, value2):
    ''' Check for the factors values match,
        necessary while checking the consistency of
        training example with the hypothesis '''
    if value1 == '?' or value2 == '?':
        return True
    elif value1 == value2 :
        return True
    return False
def consistent(self, hypothesis, instance):
    ''' Check whether the instance is part of the hypothesis '''
    for i, factor in enumerate(hypothesis):
        if not self.match factor(factor,instance[i]):
            return False
    return True
def remove inconsistent G(self, hypotheses, instance):
    ''' For a positive example, the hypotheses in G
```

```
inconsistent with it should be removed '''
       G new = hypotheses[:]
        for g in hypotheses:
            if not self.consistent(q,instance):
               G new.remove(g)
        return G new
   def remove inconsistent S(self, hypotheses, instance):
        ''' For a negative example, the hypotheses in S
            inconsistent with it should be removed '''
        S new = hypotheses[:]
        for s in hypotheses:
            if self.consistent(s,instance):
               S new.remove(s)
        return S new
   def remove more general(self, hypotheses):
        ''' After generalizing S for a positive example,
        the hypothesis in S general than others in S should
        be removed '''
        S new = hypotheses[:]
        for old in hypotheses:
            for new in S new:
                if old!=new and self.more general(new,old):
                    S new.remove[new]
        return S new
   def remove more specific(self, hypotheses):
        ''' After specializing G for a negative example,
        the hypothesis in G
        specific than others in G should be removed '''
        G new = hypotheses[:]
        for old in hypotheses:
            for new in G new:
                if old!=new and self.more specific(new,old):
                    G new.remove[new]
        return G new
   def generalize inconsistent S(self, hypothesis, instance):
        ''' When a inconsistent hypothesis for positive example
        is seen in the specific boundary S, it should be generalized
        to be consistent with the example ... we will get one hypothesis''
       hypo = list(hypothesis) # convert tuple to list for mutability
        for i, factor in enumerate(hypo):
            if factor == '0':
                hypo[i] = instance[i]
            elif not self.match factor(factor,instance[i]):
                hypo[i] = '?'
       generalization = tuple(hypo) # convert list back to tuple for immu
tability
       return generalization
```

```
def specialize inconsistent G(self, hypothesis, instance):
        "" When a inconsistent hypothesis for negative example is
        seen in the general boundary G should be
        specialized to be consistent with the example.. we will get a set
of hypotheses '''
        specializations = []
        hypo = list(hypothesis) # convert tuple to list for mutability
        for i, factor in enumerate(hypo):
            if factor == '?':
                values = self.factors[self.attr[i]]
                for j in values:
                    if instance[i] != j:
                        hyp=hypo[:]
                        hyp[i]=j
                        hyp=tuple(hyp) # convert list back to tuple for im
mutability
                        specializations.append(hyp)
        return specializations
    def get general(self,generalization,G):
        ''' Checks if there is more general hypothesis in G
            for a generalization of inconsistent hypothesis in S
            in case of positive example and returns valid generalization '
, ,
        for g in G:
            if self.more general(g, generalization):
                return generalization
        return None
    def get specific(self, specializations, S):
        ''' Checks if there is more specific hypothesis in S
            for each of hypothesis in specializations of an
            inconsistent hypothesis in G in case of negative example
            and return the valid specializations'''
        valid specializations = []
        for hypo in specializations:
            for s in S:
                if self.more specific(s,hypo) or s==self.initializeS()[0]:
                    valid specializations.append(hypo)
        return valid specializations
    def exists general(self,hypothesis,G):
        '''Used to check if there exists a more general hypothesis in
            general boundary for version space'''
        for g in G:
            if self.more general(q,hypothesis):
                return True
```

```
return False
    def exists specific(self, hypothesis, S):
        '''Used to check if there exists a more specific hypothesis in
            general boundary for version space'''
        for s in S:
            if self.more specific(s, hypothesis):
                return True
        return False
    def get version space(self, specific, general):
        ''' Given the specific and the general boundary of the
            version space, evaluate the version space in between '''
        while get order(VS):
            for hypothesis in VS[:]:
                hypo = list(hypothesis) # convert tuple to list for mutabi
lity
                for i, factor in enumerate(hypo):
                    if factor != '?':
                        hyp=hypo[:]
                        hyp[i]='?'
                        if self.exists general(hyp,general)and self.exists
specific(hyp, specific):
                            VS.append(tuple(hyp))
        return VS
    def get order(self, hypothesis):
        pass
    def more general(self,hyp1,hyp2):
        ''' Check whether hyp1 is more general than hyp2 '''
        hyp = zip(hyp1, hyp2)
        for i, j in hyp:
            if i == '?':
                continue
            elif j == '?':
                if i != '?':
                    return False
            elif i != j:
                return False
                                             # i==i
            else:
                continue
        return True
    def more specific(self, hyp1, hyp2):
        ''' hyp1 more specific than hyp2 is
            equivalent to hyp2 being more general than hyp1 '''
        return self.more general(hyp2,hyp1)
```

```
1.1.1
dataset=[(('Sunny','Warm','Normal','Strong','Warm','Same'),'Y'),
         (('Sunny','Warm','High','Strong','Warm','Same'),'Y'),
         (('Rainy', 'Cold', 'High', 'Strong', 'Warm', 'Change'), 'N'),
         (('Sunny','Warm','High','Strong','Cool','Change'),'Y')]
with open('C:\\Users\\Dr.Thyagaraju\\Desktop\\Data\\wsce.csv', 'r') as csv
File:
        dataset = [tuple([tuple(line[:-1]),''.join(line[-1:])]) for line i
n csv.reader(csvFile)]
attributes = ('Sky','Temp','Humidity','Wind','Water','Forecast')
f = Factors(attributes)
f.add values('Sky',('Sunny','Rainy'))
f.add values('Temp',('Warm','Cold'))
f.add_values('Humidity',('Normal','High'))
f.add values('Wind',('Strong','Weak'))
f.add values('Water',('Warm','Cool'))
f.add values('Forecast', ('Same', 'Change'))
a = Candidate elimination(dataset, f)
a.run algo()
```

Output:

Program3: Write a program to demonstrate the working of the decision tree based ID3 algorithm. Use an appropriate data set for building the decision tree and apply this knowledge to classify a new sample.

Algorithm:

ID3 - Algorithm

ID3(Examples, TargetAttribute, Attributes)

- Create a Root node for the tree
- If all Examples are positive, Return the single-node tree Root, with label = +
- If all Examples are negative, Return the single-node tree Root, with label = -
- If Attributes is empty, Return the single-node tree Root, with label = most common value of TargetAttribute in Examples
- Otherwise Begin
 - $-A \leftarrow$ the attribute from *Attributes* that best classifies *Examples*
 - The decision attribute for Root \leftarrow A
 - For each possible value, vi, of A,
 - Add a new tree branch below Root, corresponding to the test A = vi
 - Let Examples_{vi} be the subset of Examples that have value vi for A
 - If $Examples_{vi}$ is empty
 - Then below this new branch add a leaf node with label = most common value of *TargetAttribute* in *Examples*
 - Else below this new branch add the subtree
 ID3(Examples_{vi}, TargetAttribute, Attributes {A})
- End
- Return Root

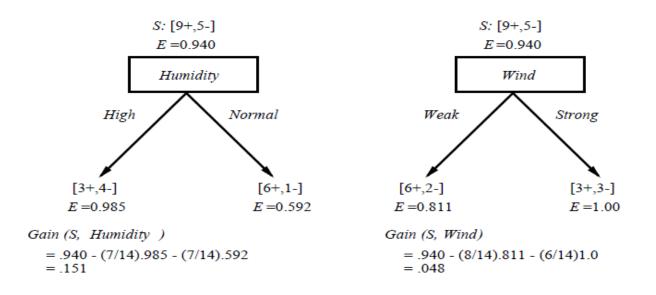
Illustration:

To illustrate the operation of ID3, let's consider the learning task represented by the below examples

Dorr	Outlook	Tomporatura	Unmiditar	Wind	DlawTannia
Day	Outlook	Temperature	Humidity		PlayTennis
D1	Sunny	Hot	High	Weak	\mathbf{No}
D2	Sunny	Hot	High	Strong	\mathbf{No}
D3	Overcast	Hot	High	Weak	\mathbf{Yes}
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	\mathbf{No}
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Compute the Gain and identify which attribute is the best as illustrated below

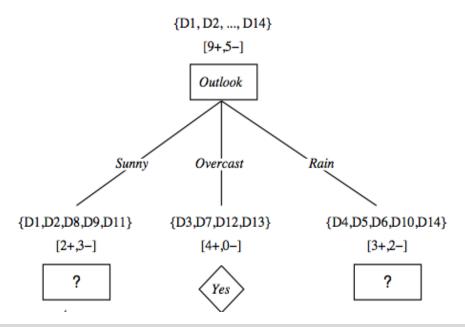
Which attribute is the best classifier?



Which attribute to test at the root?

- Which attribute should be tested at the root?
 - *Gain(S, Outlook)* = 0.246
 - Gain(S, Humidity) = 0.151
 - Gain(S, Wind) = 0.048
 - Gain(S, Temperature) = 0.029
- Outlook provides the best prediction for the target
- Lets grow the tree:
 - add to the tree a successor for each possible value of Outlook
 - partition the training samples according to the value of Outlook

After first step



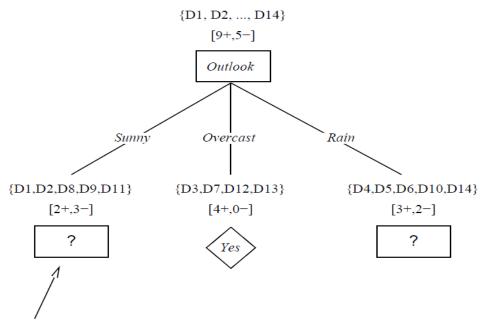
Second step

■ Working on *Outlook=Sunny* node:

Gain(
$$S_{Sunny}$$
, Humidity) = $0.970 - 3/5 \times 0.0 - 2/5 \times 0.0 = 0.970$
Gain(S_{Sunny} , Wind) = $0.970 - 2/5 \times 1.0 - 3.5 \times 0.918 = 0.019$
Gain(S_{Sunny} , Temp.) = $0.970 - 2/5 \times 0.0 - 2/5 \times 1.0 - 1/5 \times 0.0 = 0.570$

- Humidity provides the best prediction for the target
- Lets grow the tree:
 - add to the tree a successor for each possible value of Humidity
 - partition the training samples according to the value of *Humidity*

Second and third steps



Which attribute should be tested here?

$$S_{sunny} = \{D1,D2,D8,D9,D11\}$$

 $Gain (S_{sunny}, Humidity) = .970 - (3/5) 0.0 - (2/5) 0.0 = .970$
 $Gain (S_{sunny}, Temperature) = .970 - (2/5) 0.0 - (2/5) 1.0 - (1/5) 0.0 = .570$
 $Gain (S_{sunny}, Wind) = .970 - (2/5) 1.0 - (3/5) .918 = .019$

Source Code:

Import Play Tennis Data

```
import pandas as pd
from pandas import DataFrame
df_tennis = DataFrame.from_csv('C:\\Users\\Dr.Thyagaraju\\Deskto
p\\Data\\PlayTennis.csv')
df_tennis
```

Output:

	PlayTennis	Outlook	Temperature	Humidity	Wind
0	No	Sunny	Hot	High	Weak
1	No	Sunny	Hot	High	Strong
2	Yes	Overcast	Hot	High	Weak
3	Yes	Rain	Mild	High	Weak
4	Yes	Rain	Cool	Normal	Weak
5	No	Rain	Cool	Normal	Strong
6	Yes	Overcast	Cool	Normal	Strong
7	No	Sunny	Mild	High	Weak
8	Yes	Sunny	Cool	Normal	Weak
9	Yes	Rain	Mild	Normal	Weak
10	Yes	Sunny	Mild	Normal	Strong
11	Yes	Overcast	Mild	High	Strong
12	Yes	Overcast	Hot	Normal	Weak
13	No	Rain	Mild	High	Strong

Entropy of the Training Data Set

```
def entropy(probs): # Calulate the Entropy of given probability
   import math
   return sum([-prob*math.log(prob, 2) for prob in probs])

def entropy_of_list(a_list): # Entropy calculation of list of discrete val
   ues (YES/NO)
    from collections import Counter
   cnt = Counter(x for x in a list)
```

```
print("No and Yes Classes:",a_list.name,cnt)
num_instances = len(a_list)*1.0
probs = [x / num_instances for x in cnt.values()]
return entropy(probs) # Call Entropy:

# The initial entropy of the YES/NO attribute for our dataset.
#print(df_tennis['PlayTennis'])
total_entropy = entropy_of_list(df_tennis['PlayTennis'])
print("Entropy of given PlayTennis Data Set:",total_entropy)
```

Output:

```
No and Yes Classes: PlayTennis Counter({'Yes': 9, 'No': 5}) Entropy of given PlayTennis Data Set: 0.9402859586706309
```

Information Gain of Attributes

```
def information gain (df, split attribute name, target attribute name, trac
    print("Information Gain Calculation of ", split attribute name)
    , , ,
    Takes a DataFrame of attributes, and quantifies the entropy of a target
attribute after performing a split along the values of another attribute.
    # Split Data by Possible Vals of Attribute:
    df split = df.groupby(split attribute name)
    #print(df split.groups)
    for name, group in df split:
       print(name)
       print(group)
    # Calculate Entropy for Target Attribute, as well as
    # Proportion of Obs in Each Data-Split
    nobs = len(df.index) * 1.0
    #print("NOBS", nobs)
    df agg ent = df split.agg({target attribute name : [entropy of list, 1
ambda x: len(x)/nobs] })[target attribute name]
    #print("DFAGGENT",df agg ent)
    df_agg_ent.columns = ['Entropy', 'PropObservations']
    #if trace: # helps understand what fxn is doing:
    # print(df agg ent)
    # Calculate Information Gain:
   new entropy = sum( df agg ent['Entropy'] * df agg ent['PropObservation
   old entropy = entropy of list(df[target attribute name])
    return old_entropy - new_entropy
```

```
print('Info-gain for Outlook is :'+str( information_gain(df_tennis, 'Outlo
ok', 'PlayTennis')),"\n")
print('\n Info-gain for Humidity is: ' + str( information_gain(df_tennis,
    'Humidity', 'PlayTennis')),"\n")
print('\n Info-gain for Wind is:' + str( information_gain(df_tennis, 'Wind
    ', 'PlayTennis')),"\n")
print('\n Info-gain for Temperature is:' + str( information_gain(df_tennis, 'Temperature', 'PlayTennis')),"\n")
```

Output:

Information	Gain Calc	ulation of Ou	tlook			
Overcast						
PlayTenn	is Outlo	ok Temperature	Humidity	Wind		
2 Y	es Overca	st Hot	High	Weak		
6 Ye	es Overca	st Cool	Normal	Strong		
11 Y	es Overca	st Mild	High	Strong		
12 Y	es Overca	st Hot	Normal	Weak		
Rain						
PlayTenn	is Outlook	Temperature H	umidity	Wind		
3 Y	es Rain	Mild	High	Weak		
	es Rain	Cool	Normal	Weak		
5	No Rain	Cool	Normal	Strong		
9 Y	es Rain	Mild	Normal	Weak		
13	No Rain	Mild	High	Strong		
Sunny						
PlayTenn	is Outlook	Temperature H	umidity	Wind		
0	No Sunny	Hot	High	Weak		
1	No Sunny	Hot	High	Strong		
7	No Sunny	Mild	High	Weak		
8 Ye	es Sunny	Cool	Normal	Weak		
10 Y	es Sunny	Mild	Normal	Strong		
No and Yes	Classes: P	layTennis Coun	ter({'Yes	': 4 _})		
No and Yes	Classes: P	layTennis Coun	ter({'Yes	': 3, 'No'	: 2})	
No and Yes	Classes: P	layTennis Coun	ter({'No'	: 3, 'Yes'	: 2})	
No and Yes	Classes: P	layTennis Coun	ter({'Yes	': 9, 'No'	: 5})	

Info-gain for Outlook is :0.246749819774

In	formation	Gain (Calculatio	n of	Humidity
Hi	gh				
	PlayTennis	Outlook	Temperature	Humidity	y Wind
0	No	Sunny	Hot	High	n Weak

1	No	Sunny	Hot	High	Strong	
2	Yes	Overcast	Hot	High	Weak	
3	Yes	Rain	Mild	High	Weak	
7	No	Sunny	Mild	High	Weak	
11	Yes	Overcast	Mild	High	Strong	
13	No	Rain	Mild	High	Strong	
No	rmal					
	PlayTennis	Outlook	Temperature	Humidity	Wind	
4	Yes	Rain	Cool	Normal	Weak	
5	No	Rain	Cool	Normal	Strong	
6	Yes	Overcast	Cool	Normal	Strong	
8	Yes	Sunny	Cool	Normal	Weak	
9	Yes	Rain	Mild	Normal	Weak	
10	Yes	Sunny	Mild	Normal	Strong	
12	Yes	Overcast	Hot	Normal	Weak	
No	and Yes Cl	asses: Play	Tennis Count	ter({'No':	4, 'Yes': 3})
No	and Yes Cl	asses: Play	Tennis Count	ter({'Yes'	: 6, 'No': 1})
No	and Yes Cl	asses: Play	Tennis Count	ter({'Yes'	: 9, 'No': 5})
I	nfo-gain	for Hum	idity is:	0.1518	35501362	
T		O-: C			7.º3	
		n Gain C	Calculation	on of W	ind	
	rong					
St	rong PlayTennis	Outlook	Temperature	Humidity	Wind	
St:	rong PlayTennis No	Outlook Sunny	Temperature Hot	Humidity High	Wind Strong	
1 5	rong PlayTennis No No	Outlook Sunny Rain	Temperature Hot Cool	Humidity High Normal	Wind Strong Strong	
1 5 6	rong PlayTennis No No Yes	Outlook Sunny Rain Overcast	Temperature Hot Cool Cool	Humidity High Normal	Wind Strong Strong Strong	
1 5 6	rong PlayTennis No No Yes Yes	Outlook Sunny Rain Overcast Sunny	Temperature Hot Cool Cool Mild	Humidity High Normal Normal	Wind Strong Strong Strong Strong	
1 5 6 10	rong PlayTennis No No Yes Yes	Outlook Sunny Rain Overcast Sunny Overcast	Temperature Hot Cool Cool Mild Mild	Humidity High Normal Normal Normal	Wind Strong Strong Strong Strong	
1 5 6	rong PlayTennis No No Yes Yes	Outlook Sunny Rain Overcast Sunny	Temperature Hot Cool Cool Mild	Humidity High Normal Normal	Wind Strong Strong Strong Strong	
1 5 6 10	rong PlayTennis No No Yes Yes Yes No	Outlook Sunny Rain Overcast Sunny Overcast Rain	Temperature Hot Cool Cool Mild Mild Mild	Humidity High Normal Normal Normal High High	Wind Strong Strong Strong Strong Strong Strong	
1 5 6 10 11 13 We	rong PlayTennis No No Yes Yes Yes No ak PlayTennis	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook	Temperature Hot Cool Cool Mild Mild	Humidity High Normal Normal High High High	Wind Strong Strong Strong Strong Strong Strong Wind	
1 5 6 10 11 13 We	rong PlayTennis No No Yes Yes Yes No ak PlayTennis	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny	Temperature Hot Cool Cool Mild Mild Mild Temperature Hot	Humidity High Normal Normal High High High	Wind Strong Strong Strong Strong Strong Strong Wind Weak	
1 5 6 10 11 13 We.	rong PlayTennis No No Yes Yes Yes No ak PlayTennis	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast	Temperature Hot Cool Cool Mild Mild Mild Temperature Hot	Humidity High Normal Normal High High High Humidity High High	Wind Strong Strong Strong Strong Strong Wind Weak Weak	
1 5 6 10 11 13 We 2 3	rong PlayTennis No No Yes Yes Yes No ak PlayTennis	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast Rain	Temperature Hot Cool Cool Mild Mild Mild Temperature Hot Hot	Humidity High Normal Normal High High High Humidity High High High	Wind Strong Strong Strong Strong Strong Wind Weak Weak Weak	
1 5 6 10 11 13 We- 0 2 3 4	rong PlayTennis No No Yes Yes Yes No ak PlayTennis No Yes	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast Rain	Temperature Hot Cool Cool Mild Mild Mild Mild Hot Hot Hot	Humidity High Normal Normal High High High High Humidity High High High Normal	Wind Strong Strong Strong Strong Strong Strong Wind Weak Weak Weak Weak	
1 5 6 10 11 13 We 2 3 4 7	rong PlayTennis No No Yes Yes No ak PlayTennis No Yes	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast Rain Rain	Temperature Hot Cool Cool Mild Mild Mild Temperature Hot Hot	Humidity High Normal Normal High High High Humidity High High High High High High	Wind Strong Strong Strong Strong Strong Wind Weak Weak Weak	
1 5 6 10 11 13 We- 0 2 3 4	rong PlayTennis No No Yes Yes No ak PlayTennis No Yes Yes	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast Rain Rain Rain Sunny	Temperature Hot Cool Cool Mild Mild Mild Mild Hot Hot Hot	Humidity High Normal Normal High High High High Humidity High High High Normal	Wind Strong Strong Strong Strong Strong Strong Wind Weak Weak Weak Weak	
1 5 6 10 11 13 We 2 3 4 7	rong PlayTennis No No Yes Yes Yes No ak PlayTennis No Yes Yes Yes	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast Rain Sunny	Temperature Hot Cool Cool Mild Mild Mild Temperature Hot Hot Mild Cool	Humidity High Normal Normal High High High Humidity High High High High High High	Wind Strong Strong Strong Strong Strong Strong Wind Weak Weak Weak Weak Weak Weak	
1 5 6 10 11 13 We 2 3 4 7	rong PlayTennis No No Yes Yes Yes No ak PlayTennis No Yes Yes Yes Yes	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast Rain Rain Sunny Sunny Sunny	Temperature Hot Cool Cool Mild Mild Mild Mild Mild Cool Mild Mild	Humidity High Normal Normal High High High High High High High High	Wind Strong Strong Strong Strong Strong Strong Wind Weak Weak Weak Weak Weak Weak Weak	
1 5 6 10 11 13 Wed 2 3 4 7 8 9	rong PlayTennis No No Yes Yes No ak PlayTennis No Yes Yes Yes Yes Yes Yes Yes Yes Yes	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast Rain Sunny Sunny Sunny Rain Overcast	Temperature Hot Cool Cool Mild Mild Mild Mild Temperature Hot Hot Mild Cool Mild	Humidity High Normal Normal High High High High High High High High	Wind Strong Strong Strong Strong Strong Strong Wind Weak Weak Weak Weak Weak Weak Weak Weak)
1 5 6 10 11 13 We. 0 2 3 4 7 8 9 12 No	rong PlayTennis No No Yes Yes Yes No ak PlayTennis No Yes Yes Yes Yes Yes Yes And Yes	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast Rain Rain Sunny Sunny Sunny Overcast	Temperature Hot Cool Cool Mild Mild Mild Mild Temperature Hot Hot Hot Mild Cool Mild	Humidity High Normal Normal Normal High High High High High Normal Normal Hormal High	Wind Strong Strong Strong Strong Strong Strong Wind Weak Weak Weak Weak Weak Weak Weak Weak	
1 5 6 10 11 13 We 2 3 4 7 8 9 12 No No	PlayTennis No No Yes Yes Yes No ak PlayTennis No Yes Yes Yes Yes Yes Yes And Yes And Yes And Yes Cl	Outlook Sunny Rain Overcast Sunny Overcast Rain Outlook Sunny Overcast Rain Rain Sunny Sunny Sunny Rain Overcast asses: Play	Temperature Hot Cool Cool Mild Mild Mild Mild Temperature Hot Mot Mild Cool Mild Cool	Humidity High Normal Normal Normal High High High High High Normal Normal High Vormal High Normal Vormal Normal Normal ter({'No':	Wind Strong Strong Strong Strong Strong Strong Wind Weak Weak Weak Weak Weak Weak Weak Weak)

Inf	fo-gain	for Win	d is:0.04	8127030)4083	
Info	rmation Ga	ain Calcula	ation of Ter	mperature		
Cool				-		
Pla	ayTennis	Outlook T	Cemperature B	Humidity	Wind	
4	Yes	Rain	Cool	Normal	Weak	
5	No	Rain	Cool	Normal	Strong	
6	Yes	Overcast	Cool	Normal	Strong	
8	Yes	Sunny	Cool	Normal	Weak	
Hot						
Pl	layTennis	Outlook	Temperature	Humidity	Wind	
0	No	Sunny	Hot	High	Weak	
1	No	Sunny	Hot	High	Strong	
2	Yes	Overcast	Hot	High	Weak	
12	Yes	Overcast	Hot	Normal	Weak	
Mild						
	layTennis		Temperature	Humidity	Wind	
3	Yes	Rain	Mild	High	Weak	
7	No	Sunny	Mild	High		
9	Yes	Rain	Mild	Normal	Weak	
10	Yes	Sunny	Mild	Normal	Strong	
11	Yes	Overcast	Mild	High	Strong	
13	No	Rain	Mild	High	Strong	
					': 3, 'No': 1}	
					: 2, 'Yes': 2}	
					': 4, 'No': 2}	
No ar	nd Yes Cla	asses: Play	Tennis Count	ter({'Yes	': 9, 'No': 5})
Inf	fo-gain	for Tem	perature	is:0.02	29222565659	

ID3 Algorithm

```
def id3(df, target_attribute_name, attribute_names, default_class=None):
    ## Tally target attribute:
    from collections import Counter
    cnt = Counter(x for x in df[target_attribute_name])# class of YES /NO

## First check: Is this split of the dataset homogeneous?
if len(cnt) == 1:
    return next(iter(cnt))

## Second check: Is this split of the dataset empty?
# if yes, return a default value
elif df.empty or (not attribute_names):
    return default_class
```

```
## Otherwise: This dataset is ready to be divvied up!
   else:
       # Get Default Value for next recursive call of this function:
       default class = max(cnt.keys()) #[index of max] # most common valu
e of target attribute in dataset
        # Choose Best Attribute to split on:
       gainz = [information gain(df, attr, target attribute name) for att
r in attribute names]
       index of max = gainz.index(max(gainz))
       best attr = attribute names[index of max]
        # Create an empty tree, to be populated in a moment
       tree = {best attr:{}}
       remaining attribute names = [i for i in attribute names if i != be
st attr]
        # Split dataset
        # On each split, recursively call this algorithm.
        # populate the empty tree with subtrees, which
        # are the result of the recursive call
        for attr val, data subset in df.groupby(best attr):
            subtree = id3(data subset,
                        target attribute name,
                        remaining attribute names,
                        default class)
            tree[best_attr][attr_val] = subtree
        return tree
```

Predicting Attributes

```
# Get Predictor Names (all but 'class')
attribute_names = list(df_tennis.columns)
print("List of Attributes:", attribute_names)
attribute_names.remove('PlayTennis') #Remove the class attribute
print("Predicting Attributes:", attribute_names)
Output:
```

```
List of Attributes: ['PlayTennis', 'Outlook', 'Temperature', 'Humidity', 'Wind']

Predicting Attributes: ['Outlook', 'Temperature', 'Humidity', 'Wind']
```

Tree Construction

```
# Run Algorithm:
from pprint import pprint
tree = id3(df_tennis,'PlayTennis',attribute_names)
print("\n\nThe Resultant Decision Tree is :\n")
pprint(tree)
```

Output

In:	formation Ga	ain Calcula	ation of Ou	tlook					
	ercast								
	PlayTennis	Outlook	Temperature	Humidity	Wind				
2	Yes	Overcast	Hot	High	Weak				
6	Yes	Overcast	Cool	Normal	Strong				
11	Yes	Overcast	Mild	High	Strong				
12	Yes	Overcast	Hot	Normal	Weak				
Ra	in								
	PlayTennis	Outlook Te	emperature H	umidity	Wind				
3	Yes	Rain	Mild	High	Weak				
4	Yes	Rain	Cool	Normal	Weak				
5	No	Rain	Cool	Normal	Strong				
9	Yes	Rain	Mild	Normal	Weak				
13	No	Rain	Mild	High	Strong				
Sur	nny								
	PlayTennis	Outlook Te	emperature H	umidity	Wind				
0	No	Sunny	Hot	High	Weak				
1	No	Sunny	Hot	High	Strong				
7	No	Sunny	Mild	High	Weak				
8	Yes	Sunny	Cool	Normal	Weak				
10	Yes	Sunny	Mild		Strong				
No and Yes Classes: PlayTennis Counter({'Yes': 4})									
			Tennis Coun						
			Tennis Coun						
			Tennis Coun			5})			
	formation Ga	ain Calcula	ation of Ter	mperature					
Cod									
	PlayTennis		Temperature 1		Wind				
4	Yes	Rain	Cool	Normal	Weak				
5	No	Rain	Cool	Normal	Strong				
6	Yes	Overcast		Normal	Strong				
8	Yes	Sunny	Cool	Normal	Weak				
Hot									
	PlayTennis		Temperature						
0	No	Sunny	Hot	High					
1	No	Sunny	Hot	High					
2	Yes	Overcast	Hot	High					
12	Yes	Overcast	Hot	Normal	Weak				
Mil									
	PlayTennis		Temperature	_					
3	Yes	Rain	Mild	High					
7	No	Sunny	Mild	High					
9	Yes	Rain	Mild	Normal	Weak				

1.0		7	7	0-			M-1-1	N	1	C+		
10			/es		unny		Mild	Norma		Strong		
11			Yes				Mild	Hig		Strong		
13			No		Rain		Mild	Hig		Strong		
										3, 'No':		
-										2, 'Yes':		
-						<u> </u>				4, 'No':		
									es ':	9, 'No':	5})	
In	forma	atior	n G	ain Cai	lcula	ation of	f Hun	nidity				
Hi	gh											
	Play	_Z Tenr	nis	Out	look	Tempera	ature	Humidit	У	Wind		
0			No	Sı	unny		Hot	Hig	jh	Weak		
1			No	Sı	unny		Hot	Hig	jh	Strong		
2		7	Yes	Over	cast		Hot	Hig	jh_	Weak		
3		7	Yes		Rain		Mild	Hiç	γh	Weak		
7			No	Sı	unny		Mild	Hig	jh	Weak		
11		7	Yes	Over	cast		Mild	Hig	jh .	Strong		
13			No]	Rain		Mild	Hiç	јh	Strong		
No	rmal											
	Play	/Tenr	nis	Out	look	Tempera	ature	Humidit	У	Wind		
4		7	Yes]	Rain		Cool	Norma	1	Weak		
5			No]	Rain		Cool	Norma	ıl	Strong		
6			res	Over	cast		Cool	Norma	ıl	Strong		
8			Yes	Sı	unny		Cool	Norma	ıl	Weak		
9		7	Yes]	Rain		Mild	Norma	1	Weak		
10			Yes		unny		Mild	Norma	1	Strong		
12			res	Over	cast		Hot	Norma	1	Weak		
No	and	Yes	Cl	asses:	Play	Tennis	Count	cer({'No	·':	4, 'Yes':	3})	
_										6, 'No':		
-										9, 'No':		
						ation of					- , ,	
	rong											
		/Tenr	nis	011†	look	Tempera	ature	Humidit	. V	Wind		
1			No		unny	T O T- 0	Hot	Hic		Strong		
5			No		Rain		Cool	Norma		Strong		
6		7	res	Over			Cool	Norma		Strong		
10			les		unny		Mild	Norma		Strong		
11			les				Mild	Hic		Strong		
13			No		Rain		Mild	Hic		Strong		
Wea	a k		110					1119	,	~ 01 0119		
****		/Tenr	nie	O11+	100k	Tempera	aturo	Humidit	. 7.7	Wind		
0	ттау	, T C 111	No		unny	TOWNET	Hot	Hic		Weak		
2		7	res	Over			Hot	Hig		Weak		
3			les les		Rain		Mild			Weak		
4								Hiç Norma		Weak		
			Yes No		Rain		Cool					
7			No	Si	unny		Mild	Hig	111	Weak		

```
Sunny
                               Cool
                                      Normal Weak
          Yes
9
          Yes
                   Rain
                               Mild
                                      Normal Weak
12
          Yes
               Overcast
                                Hot
                                      Normal Weak
No and Yes Classes: PlayTennis Counter({'No': 3, 'Yes': 3})
No and Yes Classes: PlayTennis Counter({'Yes': 6,
No and Yes Classes: PlayTennis Counter({'Yes': 9,
                                                   'No': 5})
Information Gain Calculation of Temperature
  PlayTennis Outlook Temperature Humidity
                                              Wind
                                              Weak
         Yes
                Rain
                            Cool
                                   Normal
5
                Rain
                            Cool
                                   Normal
                                           Strong
          No
Mild
  PlayTennis Outlook Temperature Humidity
                                               Wind
          Yes
                 Rain
                             Mild
                                       High
                                               Weak
9
          Yes
                 Rain
                             Mild
                                     Normal
                                               Weak
13
           No
                 Rain
                             Mild
                                      High Strong
No and Yes Classes: PlayTennis Counter({ 'Yes': 1, 'No': 1})
No and Yes Classes: PlayTennis Counter({'Yes': 2, 'No': 1})
No and Yes Classes: PlayTennis Counter({'Yes': 3, 'No': 2})
Information Gain Calculation of Humidity
High
   PlayTennis Outlook Temperature Humidity
                                               Wind
          Yes
                 Rain
                             Mild
                                       High
                                               Weak
13
                             Mild
           No
                 Rain
                                       High
                                             Strong
Normal
  PlayTennis Outlook Temperature Humidity
                                              Wind
                                              Weak
         Yes
                Rain
                            Cool
                                   Normal
5
                            Cool
          No
                Rain
                                   Normal Strong
         Yes
                Rain
                            Mild
                                   Normal
                                              Weak
No and Yes Classes: PlayTennis Counter({'Yes': 1, 'No': 1})
No and Yes Classes: PlayTennis Counter({'Yes': 2,
                                                   'No': 1})
No and Yes Classes: PlayTennis Counter({'Yes': 3,
Information Gain Calculation of
                                 Wind
Strong
   PlayTennis Outlook Temperature Humidity
                                               Wind
5
                 Rain
                             Cool
                                     Normal
                                             Strong
13
                             Mild
           Nο
                 Rain
                                       High
                                             Strong
 PlayTennis Outlook Temperature Humidity
                                            Wind
         Yes
                Rain
                            Mild
                                      High
                                            Weak
         Yes
                Rain
                            Cool
                                   Normal
                                           Weak
                            Mild
         Yes
                Rain
                                   Normal
No and Yes Classes: PlayTennis Counter({'No': 2})
No and Yes Classes: PlayTennis Counter({'Yes': 3})
No and Yes Classes: PlayTennis Counter({'Yes': 3, 'No': 2})
```

```
Information Gain Calculation of Temperature
Cool
 PlayTennis Outlook Temperature Humidity
         Yes
                            Cool
                                    Normal
               Sunny
                                            Weak
Hot
 PlayTennis Outlook Temperature Humidity
                                              Wind
                                              Weak
          No
               Sunny
                             Hot
                                      High
1
          No
               Sunny
                             Hot
                                      High Strong
Mild
  PlayTennis Outlook Temperature Humidity
                                               Wind
                             Mild
                                               Weak
           No
                Sunny
                                       High
                             Mild
10
          Yes
                Sunny
                                     Normal
                                             Strong
No and Yes Classes: PlayTennis Counter({'Yes': 1})
No and Yes Classes: PlayTennis Counter({'No': 2})
No and Yes Classes: PlayTennis Counter({'No': 1, 'Yes': 1})
No and Yes Classes: PlayTennis Counter({'No': 3, 'Yes': 2})
Information Gain Calculation of Humidity
High
 PlayTennis Outlook Temperature Humidity
                                              Wind
                             Hot
                                      High
                                              Weak
          No
               Sunny
1
          No
               Sunny
                             Hot
                                      High Strong
                            Mild
          No
               Sunny
                                      High
                                              Weak
Normal
                                               Wind
   PlayTennis Outlook Temperature Humidity
          Yes
                Sunny
                             Cool
                                     Normal
                                               Weak
10
          Yes
                Sunny
                             Mild
                                     Normal
                                             Strong
No and Yes Classes: PlayTennis Counter({'No': 3})
No and Yes Classes: PlayTennis Counter({'Yes': 2})
No and Yes Classes: PlayTennis Counter({'No': 3, 'Yes': 2})
Information Gain Calculation of Wind
Strong
   PlayTennis Outlook Temperature Humidity
                                               Wind
                                       High
           No
                Sunny
                              Hot
                                             Strong
10
                             Mild
          Yes
                Sunny
                                     Normal
                                             Strong
  PlayTennis Outlook Temperature Humidity
                                            Wind
               Sunny
                             Hot
                                      High
                                            Weak
          No
                            Mild
          No
                                      High
               Sunny
                                            Weak
8
         Yes
               Sunny
                            Cool
                                    Normal
                                            Weak
No and Yes Classes: PlayTennis Counter({'No': 1, 'Yes': 1})
No and Yes Classes: PlayTennis Counter({'No': 2, 'Yes': 1})
No and Yes Classes: PlayTennis Counter({'No': 3, 'Yes': 2})
The Resultant Decision Tree is:
```

Classification Accuracy

```
def classify(instance, tree, default=None):
    attribute = next(iter(tree)) #tree.keys()[0]
    if instance[attribute] in tree[attribute].keys():
        result = tree[attribute][instance[attribute]]
        if isinstance(result, dict): # this is a tree, delve deeper
            return classify(instance, result)
        else:
            return result # this is a label
    else:
        return default
```

```
df_tennis['predicted'] = df_tennis.apply(classify, axis=1, args=(tree,'No'))
    # classify func allows for a default arg: when tree doesn't have answe
r for a particular
    # combitation of attribute-values, we can use 'no' as the default gues
s

print('Accuracy is:' + str( sum(df_tennis['PlayTennis']==df_tennis['predicted']) / (1.0*len(df_tennis.index)) ))

df_tennis[['PlayTennis', 'predicted']]
```

Output :

Accuracy is:1.0

	Play Tennis	predicted
0	No	No
1	No	No
2	Yes	Yes
3	Yes	Yes
4	Yes	Yes
5	No	No
6	Yes	Yes
7	No	No
8	Yes	Yes
9	Yes	Yes
10	Yes	Yes
11	Yes	Yes
12	Yes	Yes
13	No	No

Classification Accuracy: Training/Testing Set

Output:

```
Information Gain Calculation of Outlook
```

	PlayTennis	011+1001	Temperature	Humidit.	Wind	predicted
2	Yes		Hot			Yes
<u>.</u> 5	Yes	Overcast	Cool			Yes
	in	Overcast	0001	ΝΟΙΜαΙ	berong	163
_		Outlook Te	emperature H	umidity	Wind n	redicted
3	Yes	Rain	Mild	High	Weak	Yes
4	Yes	Rain	Cool	Normal	Weak	Yes
5	No	Rain	Cool		Strong	No
9	Yes	Rain	Mild	Normal	Weak	Yes
	nny					
		Outlook Te	emperature H	umiditv	Wind pi	redicted
L	No	Sunny	Hot	_	Strong	No
7	No	Sunny	Mild	High	Weak	No
3	Yes	Sunny	Cool	Normal	Weak	Yes
0	and Yes Cl	Lasses: Pla	ayTennis Cou	nter({'Ye		
			ayTennis Cou			No': 1})
0	and Yes Cl	lasses: Pla	ayTennis Cou	nter({'No	': 2, 'Ye	es': 1})
10	and Yes Cl	Lasses: Pla	ayTennis Cou	nter({'Ye	s': 6, 'N	To': 3})
n	formation (Gain Calcul	lation of Te	emperature	e	
0	ol					
	PlayTennis	Outlook	Temperature	Humidity	Wind	predicted
1	Yes	Rain	Cool	Normal	Weak	Yes
)	No	Rain	Cool	Normal	Strong	No
ĵ	Yes	Overcast	Cool	Normal	Strong	Yes
	Yes	Sunny	Cool	Normal	Weak	Yes
0	t					
	PlayTennis	Outlook	Temperature	Humidity	Wind	predicted
1	No	Sunny	Hot	High	Strong	No
2	Yes	Overcast	Hot	High	Weak	Yes
	ld					
	_		emperature H		Wind pred	
3	Yes	Rain	Mild		Weak	Yes
7	No	Sunny	Mild		Weak	No
9	Yes	Rain	Mild		Weak	Yes
			ayTennis Cou			
			ayTennis Cou			es': 1})
_			ayTennis Cou			No': 1})
			ayTennis Cou		s': 6, 'N	1o': 3})
	formation (Gain Calcui	lation of H	umidity		
	gh 					
	PlayTennis		Temperature			predicted
1	No	Sunny	Hot			No
2 3	Yes	Overcast	Hot			Yes
	Yes	Rain	Mild	High	Weak	Yes

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7	No	Sunny	Mild	High	Weak	No	
	rmal			1			
	PlayTennis		Temperature	-		predicted	
4	Yes	Rain	Cool	Normal	Weak	Yes	
5	No	Rain	Cool	Normal	Strong	No	
6	Yes	Overcast	Cool	Normal		Yes	
8	Yes	Sunny	Cool	Normal	Weak	Yes	
9	Yes	Rain	Mild	Normal	Weak	Yes	
No	and Yes Cl	asses: Pla	yTennis Cour	nter({'No'	: 2, 'Ye	es': 2})	
No	and Yes Cl	asses: Pla	yTennis Cour	nter({'Yes	s': 4, 'I	No': 1})	
No	and Yes Cl	asses: Pla	yTennis Cour	nter({'Yes	s': 6, '1	No': 3})	
In	formation G	ain Calcul	ation of Wi	ind			
St	rong						
]	PlayTennis	Outlook	Temperature	Humidity	Wind	predicted	
1	No	Sunny	Hot	High	Strong	No	
5	No	Rain	Cool	Normal	Strong	No	
6	Yes	Overcast	Cool	Normal	Strong	Yes	
Wea	ak						
]	PlayTennis	Outlook	Temperature	Humidity	Wind p	redicted	
2	Yes	Overcast	Hot	High	Weak	Yes	
3	Yes	Rain	Mild	High	Weak	Yes	
4	Yes	Rain	Cool	Normal		Yes	
7	No	Sunny	Mild	High		No	
8	Yes	Sunny	Cool	Normal	Weak	Yes	
9	Yes	Rain	Mild	Normal	Weak	Yes	
			yTennis Cour				
			yTennis Cour				
			yTennis Cour				
	formation G			emperature		. 3),	
Cod		rain Caicui	acton of 16	emperacure	-		
			mperature Hu	ımidit.	Wind n	redicted	
	_						
4 5	Yes	Rain	Cool	Normal	Weak	Yes	
	No	Rain	Cool	Normal S	Strong	No	
Mi		O-+11 =			المسادة	14	
			mperature Hu		lind pred		
3	Yes	Rain	Mild		leak	Yes	
9	Yes	Rain	Mild		leak	Yes	
			yTennis Cour			No': 1})	
			yTennis Cour				
			yTennis Cour	nter({'Yes	s': 3, 'I	No': 1})	
In	formation G	ain Calcul	ation of Hu	umidity			
Hi	gh						
1	PlayTennis	Outlook Te	mperature Hu	umidity W	lind pred	dicted	
		Rain	Mild	High W	leak	Yes	

```
PlayTennis Outlook Temperature Humidity
                                            Wind predicted
        Yes
               Rain
                          Cool
                                  Normal
                                            Weak
                                                        Yes
5
         No
               Rain
                           Cool
                                  Normal Strong
                                                        No
9
         Yes
               Rain
                           Mild
                                  Normal
                                                        Yes
                                             Weak
No and Yes Classes: PlayTennis Counter({'Yes': 1})
No and Yes Classes: PlayTennis Counter({'Yes': 2, 'No': 1})
No and Yes Classes: PlayTennis Counter({'Yes': 3, 'No': 1})
Information Gain Calculation of Wind
Strong
 PlayTennis Outlook Temperature Humidity
                                            Wind predicted
               Rain
                            Cool
         No
                                  Normal
                                          Strong
Weak
 PlayTennis Outlook Temperature Humidity
                                          Wind predicted
               Rain
                           Mild
                                    High
                                          Weak
4
         Yes
               Rain
                            Cool
                                  Normal Weak
                                                      Yes
                                 Normal
         Yes
               Rain
                           Mild
                                          Weak
                                                      Yes
No and Yes Classes: PlayTennis Counter({'No': 1})
No and Yes Classes: PlayTennis Counter({ 'Yes': 3})
No and Yes Classes: PlayTennis Counter({'Yes': 3, 'No': 1})
Information Gain Calculation of Temperature
Cool
 PlayTennis Outlook Temperature Humidity Wind predicted
         Yes
               Sunny
                            Cool
                                  Normal
                                          Weak
Hot
 PlayTennis Outlook Temperature Humidity
                                           Wind predicted
          No
               Sunny
                            Hot
                                     High Strong
Mild
PlayTennis Outlook Temperature Humidity Wind predicted
         No
              Sunny
                           Mild
                                    High Weak
No and Yes Classes: PlayTennis Counter({ 'Yes': 1})
No and Yes Classes: PlayTennis Counter({'No': 1})
No and Yes Classes: PlayTennis Counter({'No': 1})
No and Yes Classes: PlayTennis Counter({'No': 2, 'Yes': 1})
Information Gain Calculation of Humidity
High
  PlayTennis Outlook Temperature Humidity
                                           Wind predicted
              Sunny
                            Hot
                                    High Strong
         No
                                                        No
                           Mild
          No
                                    High
                                            Weak
              Sunny
                                                        No
Normal
  PlayTennis Outlook Temperature Humidity Wind predicted
         Yes
              Sunny
                           Cool
                                  Normal Weak
                                                      Yes
No and Yes Classes: PlayTennis Counter({'No': 2})
No and Yes Classes: PlayTennis Counter({'Yes': 1})
No and Yes Classes: PlayTennis Counter({'No': 2, 'Yes': 1})
Information Gain Calculation of Wind
```

St	rong								
	Play	Геnni	S	Outlook	Temperature	Humidity	Wir	nd predicted	
1		N	Io	Sunny	Hot	High	Stror	ng No	
₩e	ak								
	Play	Геппі	.S	Outlook	Temperature	Humidity	Wind	predicted	
7		N	10	Sunny	Mild	High	Weak	No	
8		Υe	s	Sunny	Cool	Normal	Weak	Yes	
No	and	Yes	Cl	asses:	PlayTennis Co	ounter({'N	fo ': 1}	+)	
Nc	and	Yes	Cl	asses:	PlayTennis Co	ounter({'N	o': 1,	'Yes': 1})	
No	and	Yes	Cl	asses:	PlayTennis Co	ounter({'N	lo': 2,	'Yes': 1})	

Accuracy is : 0.75

Lab Exercise: Apply above Program to clasify the new sample /new data set.

Program4: Build an Artificial Neural Network by implementing Backpropagation algorithm and test the same appropriate data sets

Algorithm:

function BackProp (D, η , n_{in} , n_{hidden} , n_{out})

- D is the training set consists of m pairs: $\{(x_i, y_i)^m\}$
- η is the learning rate as an example (0.1)
- $-n_{\rm in}$, $n_{\rm hidden}$ e $n_{\rm out}$ are the number of imput hidden and output unit of neural network

Make a feed-forward network with $n_{\rm in}$, $n_{\rm hidden}$ e $n_{\rm out}$ units

Initialize all the weight to short randomly number (es. [-0.05 0.05])

Repeat until termination condition are verifyed:

For any sample in D:

Forward propagate the network computing the output o_u of every unit u of the network

Back propagate the errors onto the network: – For every output unit k, compute the error δ_{L} :

- $\delta_k = o_k (1 o_k)(t_k o_k)$
- For every hidden unit h compute the error δ_h : $\delta_h = o_h (1 o_h) \sum_{k \in outputs} w_{kh} \delta_k$
- Update the network weight wii:
- $w_{ji} = w_{ji} + \Delta w_{ji}$, where $\Delta w_{ji} = \eta \delta_j x_{ji}$

 $(x_{ii} \text{ is the input of unit } j \text{ from coming from unit } i)$

The Backpropagation Algorithm for a feed-forward 2-layer network of sigmoid units, the stochastic version

Idea: Gradient descent over the entire vector of network weights.

Initialize all weights to small random numbers.

Until satisfied, // stopping criterion to be (later) defined for each training example,

- 1. input the training example to the network, and compute the network outputs
- 2. for each output unit k:

$$\delta_k \leftarrow o_k (1 - o_k) (t_k - o_k)$$

3. for each hidden unit h:

$$\delta_h \leftarrow o_h(1 - o_h) \sum_{k \in outputs} w_{kh} \delta_k$$

4. update each network weight w_{ii} : $w_{ji} \leftarrow w_{ji} + \Delta w_{ji} \text{ where } \Delta w_{ji} = \eta \delta_j x_{ji},$ and x_{ii} is the *i*th input to unit *j*.

Source Code:

Below is a small contrived dataset that we can use to test out training our neural network.

X1	X2	Υ
2.7810836	2.550537003	0
1.465489372	2.362125076	0
3.396561688	4.400293529	0
1.38807019	1.850220317	0
3.06407232	3.005305973	0
7.627531214	2.759262235	1
5.332441248	2.088626775	1
6.922596716	1.77106367	1
8.675418651	-0.242068655	1
7.673756466	3.508563011	1

Below is the complete example. We will use 2 neurons in the hidden layer. It is a binary classification problem (2 classes) so there will be two neurons in the output layer. The network will be trained for 20 epochs with a learning rate of 0.5, which is high because we are training for so few iterations.

```
import random
from math import exp
from random import seed
# Initialize a network
def initialize network(n inputs, n hidden, n outputs):
   network = list()
   hidden layer = [{'weights':[random.uniform(-0.5,0.5) for i in range(n
inputs + 1)]} for i in range(n hidden)]
   network.append(hidden layer)
   output layer = [{'weights': [random.uniform(-0.5,0.5) for i in range(n
hidden + 1)]} for i in range(n outputs)]
   network.append(output layer)
   return network
# Calculate neuron activation for an input
def activate(weights, inputs):
   activation = weights[-1]
   for i in range(len(weights)-1):
       activation += weights[i] * inputs[i]
   return activation
```

```
# Transfer neuron activation
def transfer(activation):
    return 1.0 / (1.0 + exp(-activation))
# Forward propagate input to a network output
def forward propagate(network, row):
    inputs = row
    for layer in network:
        new inputs = []
        for neuron in layer:
            activation = activate(neuron['weights'], inputs)
            neuron['output'] = transfer(activation)
            new inputs.append(neuron['output'])
        inputs = new inputs
    return inputs
# Calculate the derivative of an neuron output
def transfer derivative(output):
    return output * (1.0 - output)
# Backpropagate error and store in neurons
def backward propagate error(network, expected):
    for i in reversed(range(len(network))):
        layer = network[i]
        errors = list()
        if i != len(network)-1:
            for j in range(len(layer)):
                error = 0.0
                for neuron in network[i + 1]:
                    error += (neuron['weights'][j] * neuron['delta'])
                errors.append(error)
        else:
            for j in range(len(layer)):
                neuron = layer[i]
                errors.append(expected[j] - neuron['output'])
        for j in range(len(layer)):
            neuron = layer[j]
            neuron['delta'] = errors[j] * transfer derivative(neuron['outp
ut'1)
# Update network weights with error
def update weights(network, row, l rate):
    for i in range(len(network)):
        inputs = row[:-1]
        if i != 0:
            inputs = [neuron['output'] for neuron in network[i - 1]]
        for neuron in network[i]:
            for j in range(len(inputs)):
                neuron['weights'][j] += l rate * neuron['delta'] * inputs[
j]
            neuron['weights'][-1] += 1 rate * neuron['delta']
# Train a network for a fixed number of epochs
def train network(network, train, 1 rate, n epoch, n outputs):
```

```
for epoch in range(n epoch):
        sum error = 0
        for row in train:
            outputs = forward propagate (network, row)
            expected = [0 for i in range(n outputs)]
            expected[row[-1]] = 1
            sum error += sum([(expected[i]-outputs[i])**2 for i in range(1
en(expected))])
            backward propagate error(network, expected)
            update weights(network, row, l rate)
        print('>epoch=%d, lrate=%.3f, error=%.3f' % (epoch, l rate, sum er
ror))
#Test training backprop algorithm
seed(1)
dataset = [[2.7810836, 2.550537003, 0],
    [1.465489372, 2.362125076, 0],
    [3.396561688, 4.400293529, 0],
    [1.38807019,1.850220317,0],
    [3.06407232,3.005305973,0],
    [7.627531214,2.759262235,1],
    [5.332441248, 2.088626775, 1],
    [6.922596716,1.77106367,1],
    [8.675418651, -0.242068655, 1],
    [7.673756466,3.508563011,1]]
n inputs = len(dataset[0]) - 1
n outputs = len(set([row[-1] for row in dataset]))
network = initialize network(n inputs, 2, n outputs)
train network(network, dataset, 0.5, 20, n outputs)
#for layer in network:
# print(layer)
i = 1
for layer in network:
    j=1
    for sub in layer:
        print("\n Layer[%d] Node[%d]:\n" %(i,j), sub)
        j=j+1
    i=i+1
```

Output:

```
>epoch=0, lrate=0.500, error=4.763

>epoch=1, lrate=0.500, error=4.558

>epoch=2, lrate=0.500, error=4.316

>epoch=3, lrate=0.500, error=3.733

>epoch=4, lrate=0.500, error=3.733

>epoch=5, lrate=0.500, error=3.428

>epoch=6, lrate=0.500, error=3.132

>epoch=7, lrate=0.500, error=2.850

>epoch=8, lrate=0.500, error=2.588

>epoch=9, lrate=0.500, error=2.348
```

```
>epoch=10, lrate=0.500, error=2.128
>epoch=11, lrate=0.500, error=1.931
>epoch=12, lrate=0.500, error=1.753
>epoch=13, lrate=0.500, error=1.595
>epoch=14, lrate=0.500, error=1.454
>epoch=15, lrate=0.500, error=1.329
>epoch=16, lrate=0.500, error=1.218
>epoch=17, lrate=0.500, error=1.120
>epoch=18, lrate=0.500, error=1.033
>epoch=19, lrate=0.500, error=0.956
Layer[1] Node[1]:
 {'weights': [-1.435239043819221, 1.8587338175173547, 0.7917644224148094],
'output': 0.029795197360175857, 'delta': -0.006018730117768358}
Layer[1] Node[2]:
 \{\text{'weights': } [-0.7704959899742789, 0.8257894037467045, 0.21154103288579731 \}
], 'output': 0.06771641538441577, 'delta': -0.005025585510232048}
Layer[2] Node[1]:
 {'weights': [2.223584933362892, 1.2428928053374768, -1.3519548925527454],
'output': 0.23499833662766154, 'delta': -0.042246618795029306}
Layer[2] Node[2]:
 {'weights': [-2.509732251870173, -0.5925943219491905, 1.259965727484093],
'output': 0.7543931062537561, 'delta': 0.04550706392557862}
```

Predict

Making predictions with a trained neural network is easy enough. We have already seen how to forward-propagate an input pattern to get an output. This is all we need to do to make a prediction. We can use the output values themselves directly as the probability of a pattern belonging to each output class. It may be more useful to turn this output back into a crisp class prediction. We can do this by selecting the class value with the larger probability. This is also called the arg max function. Below is a function named predict() that implements this procedure. It returns the index in the network output that has the largest probability. It assumes that class values have been converted to integers starting at 0.

```
# Calculate neuron activation for an input
def activate(weights, inputs):
    activation = weights[-1]
    for i in range(len(weights)-1):
        activation += weights[i] * inputs[i]
    return activation
```

```
# Transfer neuron activation
def transfer(activation):
    return 1.0 / (1.0 + exp(-activation))
# Forward propagate input to a network output
def forward propagate(network, row):
    inputs = row
    for layer in network:
        new inputs = []
        for neuron in layer:
            activation = activate(neuron['weights'], inputs)
            neuron['output'] = transfer(activation)
            new inputs.append(neuron['output'])
        inputs = new inputs
    return inputs
# Make a prediction with a network
def predict(network, row):
    outputs = forward propagate(network, row)
    return outputs.index(max(outputs))
# Test making predictions with the network
dataset = [[2.7810836, 2.550537003, 0],
    [1.465489372, 2.362125076, 0],
    [3.396561688, 4.400293529, 0],
    [1.38807019, 1.850220317, 0],
    [3.06407232,3.005305973,0],
    [7.627531214,2.759262235,1],
    [5.332441248, 2.088626775, 1],
    [6.922596716,1.77106367,1],
    [8.675418651,-0.242068655,1],
    [7.673756466,3.508563011,1]]
network = [[{\daggerights': [-1.482313569067226, 1.8308790073202204, 1.0783819
22048799]}, {'weights': [0.23244990332399884, 0.3621998343835864, 0.402898
21191094327]}],
    [{'weights': [2.5001872433501404, 0.7887233511355132, -1.1026649757805
829]}, {'weights': [-2.429350576245497, 0.8357651039198697, 1.069921718128
06561}11
for row in dataset:
    prediction = predict(network, row)
   print('Expected=%d, Got=%d' % (row[-1], prediction))
Expected=0, Got=0
Expected=0, Got=0
Expected=0, Got=0
Expected=0, Got=0
Expected=0, Got=0
Expected=1, Got=1
Expected=1, Got=1
Expected=1, Got=1
Expected=1, Got=1
Expected=1, Got=1
```

Program5: Write a program to implement the **naïve Bayesian classifier** for a sample training data set stored as a .CSV file. Compute the accuracy of the classifier, considering few test data sets.

Bayesian Theorem:

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)}$$

- P(h) = prior probability of hypothesis h
- P(D) = prior probability of training data D
- P(h|D) = probability of h given D
- P(D|h) = probability of D given h

Naive Bayes: For the Bayesian Rule above, we have to extend it so that we have

$$P(C|X_1, X_2, ..., X_n) = \frac{P(X_1, X_2, ..., X_n|C) P(C)}{P(X_1, X_2, ..., X_n)}$$

Bayes' rule:

Given a set of variables, $X = \{x1,x2,x...,xd\}$, we want to construct the posterior probability for the event Cj among a set of possible outcomes $C = \{c1,c2,c...,cd\}$, the Bayes Rule is

$$p(C_j | x_1, x_2, ..., x_d) \propto p(x_1, x_2, ..., x_d | C_j) p(C_j)$$

Since Naive Bayes assumes that the conditional probabilities of the independent variables are statistically independent we can decompose the likelihood to a product of terms:

$$p(X \mid C_j) \propto \prod_{k=1}^{d} p(x_k \mid C_j)$$

and rewrite the posterior as:

$$p(C_j \mid X) \propto p(C_j) \prod_{k=1}^{d} p(x_k \mid C_j)$$

Using Bayes' rule above, we label a new case X with a class level Cj that achieves the highest posterior probability.

Naive Bayes can be modeled in several different ways including normal, lognormal, gamma and Poisson density functions:

$$\begin{cases} \frac{1}{\sigma_{ij}\sqrt{2\pi}}\exp\left(\frac{-\left(\mathbf{x}-\mu_{ij}\right)^{2}}{2\sigma_{ij}}\right), & -\infty < x < \infty, -\infty < \mu_{ij} <, \sigma_{ij} > 0 \end{cases} & \text{Normal} \\ \mu_{ij}: \text{mean, } \sigma_{ij}: \text{standard deviation} \\ \frac{1}{x\sigma_{ij}(2\pi)^{1/2}}\exp\left\{\frac{-\left[\log\left(x/m_{ij}\right)\right]^{2}}{2\sigma_{ij}^{2}}\right\}, & 0 < x < \infty, m_{ij} > 0, \sigma_{ij} > 0 \end{cases} & \text{Lognormal} \\ m_{ij}: \text{scale parameter, } \sigma_{ij}: \text{shape parameter} \\ \left\{\frac{x}{b_{ij}}\right\}_{b_{ij}\Gamma\left(\mathbf{c}_{ij}\right)}^{c_{ij}1}\exp\left(\frac{-x}{b_{ij}}\right), & 0 \le x < \infty, b_{ij} > 0, c_{ij} > 0 \end{cases} & \text{Gamma} \\ b_{ij}: \text{scale parameter, } c_{ij}: \text{shape parameter} \\ \frac{\lambda_{ij}\exp\left(-\lambda_{ij}\right)}{x!}, & 0 \le x < \infty, \lambda_{ij} > 0, x = 0, 1, 2, \dots \end{cases} & \text{Poisson} \\ \lambda_{ij}: \text{mean} \end{cases}$$

Types

Gaussian: It is used in classification and it assumes that features follow a normal distribution.
 Gaussian Naive Bayes is used in cases when all our features are continuous. For example in Iris dataset features are sepal width, petal width, sepal length, petal length.

$$P(x_i \mid y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left(-\frac{(x_i - \mu_y)^2}{2\sigma_y^2}\right)$$

Multinomial Naive Bayes: Its is used when we have discrete data (e.g. movie ratings ranging 1
and 5 as each rating will have certain frequency to represent). In text learning we have the
count of each word to predict the class or label

$$p(\mathbf{x} \mid C_k) = rac{(\sum_i x_i)!}{\prod_i x_i!} \prod_i p_{ki}{}^{x_i}$$

$$\hat{P}(x_i \mid \omega_j) = rac{\sum t f(x_i, d \in \omega_j) + lpha}{\sum N_{d \in \omega_j} + lpha \cdot V}$$

• **Bernoulli Naive Bayes:** It assumes that all our features are binary such that they take only two values. Means 0s can represent "word does not occur in the document" and 1s as "word occurs in the document"

$$P(x_i \mid y) = P(i \mid y)x_i + (1 - P(i \mid y))(1 - x_i)$$

Source Code:

```
# Example of Naive Bayes implemented from Scratch in Python
#http://machinelearningmastery.com/naive-bayes-classifier-scratch-python/
import csv
import random
import math
# 1.Data Handling
# 1.1 Loading the Data from csv file of Pima indians diabetes dataset.
def loadcsv(filename):
    lines = csv.reader(open(filename, "r"))
    dataset = list(lines)
    for i in range(len(dataset)):
        # converting the attributes from string to floating point numbers
        dataset[i] = [float(x) for x in dataset[i]]
    return dataset
#1.2 Splitting the Data set into Training Set
def splitDataset(dataset, splitRatio):
    trainSize = int(len(dataset) * splitRatio)
    trainSet = []
    copy = list(dataset)
    while len(trainSet) < trainSize:</pre>
        index = random.randrange(len(copy)) # random index
        trainSet.append(copy.pop(index))
    return [trainSet, copy]
#2.Summarize Data
#The naive bayes model is comprised of a
#summary of the data in the training dataset.
#This summary is then used when making predictions.
#involves the mean and the standard deviation for each attribute, by class
value
#2.1: Separate Data By Class
#Function to categorize the dataset in terms of classes
#The function assumes that the last attribute (-1) is the class value.
#The function returns a map of class values to lists of data instances.
def separateByClass(dataset):
    separated = {}
    for i in range(len(dataset)):
        vector = dataset[i]
        if (vector[-1] not in separated):
            separated[vector[-1]] = []
        separated[vector[-1]].append(vector)
    return separated
#The mean is the central middle or central tendency of the data,
# and we will use it as the middle of our gaussian distribution
# when calculating probabilities
#2.2 : Calculate Mean
```

```
def mean(numbers):
    return sum(numbers)/float(len(numbers))
#The standard deviation describes the variation of spread of the data,
#and we will use it to characterize the expected spread of each attribute
#in our Gaussian distribution when calculating probabilities.
#2.3 : Calculate Standard Deviation
def stdev(numbers):
   avg = mean(numbers)
   variance = sum([pow(x-avg,2) for x in numbers])/float(len(numbers)-1)
   return math.sqrt(variance)
#2.4 : Summarize Dataset
#Summarize Data Set for a list of instances (for a class value)
#The zip function groups the values for each attribute across our data ins
#into their own lists so that we can compute the mean and standard deviati
on values
#for the attribute.
def summarize(dataset):
   summaries = [(mean(attribute), stdev(attribute)) for attribute in zip(
*dataset)]
   del summaries[-1]
   return summaries
#2.5 : Summarize Attributes By Class
#We can pull it all together by first separating our training dataset into
#instances grouped by class. Then calculate the summaries for each attribut
e.
def summarizeByClass(dataset):
    separated = separateByClass(dataset)
   summaries = {}
   for classValue, instances in separated.items():
        summaries[classValue] = summarize(instances)
   return summaries
#3. Make Prediction
#3.1 Calculate Probaility Density Function
def calculateProbability(x, mean, stdev):
   exponent = math.exp(-(math.pow(x-mean,2)/(2*math.pow(stdev,2))))
   return (1 / (math.sqrt(2*math.pi) * stdev)) * exponent
#3.2 Calculate Class Probabilities
def calculateClassProbabilities(summaries, inputVector):
   probabilities = {}
    for classValue, classSummaries in summaries.items():
        probabilities[classValue] = 1
        for i in range(len(classSummaries)):
           mean, stdev = classSummaries[i]
            x = inputVector[i]
```

```
probabilities[classValue] *= calculateProbability(x, mean, std
ev)
    return probabilities
#3.3 Prediction : look for the largest probability and return the associat
ed class
def predict(summaries, inputVector):
    probabilities = calculateClassProbabilities(summaries, inputVector)
    bestLabel, bestProb = None, -1
    for classValue, probability in probabilities.items():
        if bestLabel is None or probability > bestProb:
            bestProb = probability
            bestLabel = classValue
    return bestLabel
#4.Make Predictions
# Function which return predictions for list of predictions
# For each instance
def getPredictions(summaries, testSet):
    predictions = []
    for i in range(len(testSet)):
        result = predict(summaries, testSet[i])
        predictions.append(result)
    return predictions
#5. Computing Accuracy
def getAccuracy(testSet, predictions):
    correct = 0
    for i in range(len(testSet)):
        if testSet[i][-1] == predictions[i]:
            correct += 1
    return (correct/float(len(testSet))) * 100.0
#Main Function
def main():
    filename = 'C:\\Users\\Dr.Thyagaraju\\Desktop\\Data\\pima-indians-diab
etes.csv'
    splitRatio = 0.67
    dataset = loadcsv(filename)
    #print("\n The Data Set :\n",dataset)
    print("\n The length of the Data Set : ",len(dataset))
    print("\n The Data Set Splitting into Training and Testing \n")
    trainingSet, testSet = splitDataset(dataset, splitRatio)
    print('\n Number of Rows in Training Set:{0} rows'.format(len(training
Set)))
    print('\n Number of Rows in Testing Set:{0} rows'.format(len(testSet))
    print("\n First Five Rows of Training Set:\n")
    for i in range (0,5):
```

```
print(trainingSet[i],"\n")

print("\n First Five Rows of Testing Set:\n")
for i in range(0,5):
    print(testSet[i],"\n")

# prepare model
summaries = summarizeByClass(trainingSet)
print("\n Model Summaries:\n",summaries)

# test model
predictions = getPredictions(summaries, testSet)
print("\nPredictions:\n",predictions)

accuracy = getAccuracy(testSet, predictions)
print('\n Accuracy: {0}%'.format(accuracy))
main()
```

Output:

```
The length of the Data Set: 768

The Data Set Splitting into Training and Testing

Number of Rows in Training Set:514 rows

Number of Rows in Testing Set:254 rows

First Five Rows of Training Set:

[4.0, 116.0, 72.0, 12.0, 87.0, 22.1, 0.463, 37.0, 0.0]

[0.0, 84.0, 64.0, 22.0, 66.0, 35.8, 0.545, 21.0, 0.0]

[0.0, 162.0, 76.0, 36.0, 0.0, 49.6, 0.364, 26.0, 1.0]

[10.0, 101.0, 86.0, 37.0, 0.0, 45.6, 1.136, 38.0, 1.0]

[5.0, 78.0, 48.0, 0.0, 0.0, 33.7, 0.654, 25.0, 0.0]
```

First Five Rows of Testing Set:

```
[1.0, 85.0, 66.0, 29.0, 0.0, 26.6, 0.351, 31.0, 0.0]

[8.0, 183.0, 64.0, 0.0, 0.0, 23.3, 0.672, 32.0, 1.0]

[4.0, 110.0, 92.0, 0.0, 0.0, 37.6, 0.191, 30.0, 0.0]

[10.0, 139.0, 80.0, 0.0, 0.0, 27.1, 1.441, 57.0, 0.0]

[7.0, 100.0, 0.0, 0.0, 0.0, 30.0, 0.484, 32.0, 1.0]
```

Model Summaries:

{0.0: [(3.3474320241691844, 3.045635385378286), (111.54380664652568, 26.0 40069054720693), (68.45921450151057, 18.15540652389224), (19.9456193353474 3, 14.709615608767137), (71.50151057401813, 101.04863439385403), (30.86314 1993957708, 7.207208162103949), (0.4341842900302116, 0.2960911906946818), (31.613293051359516, 12.100651311117689)], 1.0: [(4.469945355191257, 3.736 9440851983082), (139.3879781420765, 33.733070931373234), (71.1475409836065 6, 20.694403393963842), (22.92896174863388, 18.151995092528765), (107.9781

4207650273, 146.92526156736633), (35.28633879781422, 7.783342260348583), (0.5569726775956286, 0.3942245334398509), (36.78688524590164, 11.174610282702282)]}

Predictions:

Accuracy: 80.31496062992126%

Program6: Assuming a set of documents that need to be classified, use the naïve Bayesian Classifier model to perform this task. Built-in Java classes/API can be used to write the program. Calculate the accuracy, precision, and recall for your data set.

Algorithm:

Learning to Classify Text: Preliminaries

Target concept Interesting? : *Document* \rightarrow {+, -}

- 1. Represent each document by vector of words
 - one attribute per word position in document
- 2. Learning: Use training examples to estimate
 - P(+) P(-)
 - P(doc|+) P(doc|-)

Naive Bayes conditional independence assumption

$$P(doc|v_j) = \prod_{i=1}^{length(doc)} P(a_i = w_k|v_j)$$

where $P(a_i = w_k \mid v_j)$ is probability that word in position i is w_k , given v_j one more assumption:

 $P(a_i = w_k | v_i) = P(a_m = w_k | v_i), \forall i, m$

Learning to Classify Text: Algorithm

\$1: LEARN_NAIVE_BAYES_TEXT (*Examples*, V)

S2: CLASSIFY_NAIVE_BAYES_TEXT (*Doc*)

• Examples is a set of text documents along with their target values. V is the set of all possible target values. This function learns the probability terms P(wk Iv,), describing the probability that a randomly drawn word from a document in class vj will be the English word wk. It also learns the class prior probabilities P(vj).

S1: LEARN_NAIVE_BAYES_TEXT (Examples, V)

- 1. collect all words and other tokens that occur in Examples
 - Vocabulary ← all distinct words and other tokens in Examples
- **2.** calculate the required $P(v_i)$ and $P(w_k \mid v_i)$ probability terms
 - For each target value v_i in V do

$$P(v_j) \leftarrow \frac{|docs_j|}{|Examples|}$$

- o $docs_i \leftarrow$ subset of *Examples* for which the target value is v_i
- o $Text_i \leftarrow$ a single document created by concatenating all members of $docs_i$
- $n \leftarrow \text{total number of words in } Text_i \text{ (counting duplicate words multiple times)}$
- for each word w_k in *Vocabulary*
 - * $n_k \leftarrow$ number of times word w_k occurs in $Text_j$

$$P(w_k|v_j) \leftarrow \frac{n_k+1}{n+|Vocabulary|}$$

S2: CLASSIFY_NAIVE_BAYES_TEXT (Doc)

- positions ← all word positions in Doc that contain tokens found in Vocabulary
- Return *v_{NB}* where

$$v_{NB} = \underset{v_j \in V}{\operatorname{argmax}} P(v_j) \prod_{i \in positions} P(a_i | v_j)$$

Twenty News Groups

• Given 1000 training documents from each group Learn to classify new documents according to which newsgroup it came from

comp.graphics	misc.forsale	alt.atheism	sci.space
comp.os.ms-windows.misc	rec.autos	soc.religion.christian	sci.crypt
comp.sys.ibm.pc.hardware	rec.motorcycles	talk.religion.misc	sci.electronics
comp.sys.mac.hardware	rec.sport.baseball	talk.politics.mideast	sci.med
comp.windows.x	rec.sport.hockey	talk.politics.misc	
		talk.politics.guns	

Naive Bayes: 89% classification accuracy

Learning Curve for 20 Newsgroups

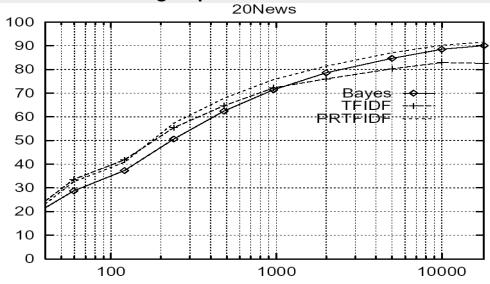


Fig: Accuracy vs. Training set size (1/3 withheld for test

Example:

- In the example, we are given a sentence "A very close game", a training set of five sentences (as shown below), and their corresponding category (Sports or Not Sports).
- The goal is to build a Naive Bayes classifier that will tell us which category the sentence "A very close game" belongs to. applying a Naive Bayes classifier, thus the strategy would be calculating the probability of both "A very close game is Sports", as well as it's *Not Sports*. The one with the higher probability will be the result.
- to calculate P(*Sports* | *A very close game*), i.e. the probability that the category of the sentence is *Sports*given that the sentence is "A very close game".

Text	Category
"A great game"	Sports
"The election was over"	Not sports
"Very clean match"	Sports
"A clean but forgettable game"	Sports
"It was a close election"	Not sports
<u> </u>	·

Step 1: Feature Engineering

= 0.0000143

- word frequencies, i.e., counting the occurrence of every word in the document.
- P(a very close game) = P(a)XP(very)XP(close)XP(game)
- P(a very close game | Sports) = P(a|Sports) X P(Very|Sports) X P(close|Sports) X P(game|Sports)
- P(a very close game | Not Sports) = P(a | Not Sports) x P(very | Not Sports) x P(close | Not Sports) x P(game | Not Sports)

Step 2: Calculating the probabilities

- Here, the word "close" does not exist in the category Sports, thus P(close | Sports) = 0, leading to P(a very close game | Sports)=0.
- Given an observation x = (x1, ..., xd) from a multinomial distribution with N trials and parameter vector $\theta = (\theta 1, ..., \theta d)$, a "smoothed" version of the data gives the estimator.

$$\hat{ heta}_i = rac{x_i + lpha}{N + lpha d} \qquad (i = 1, \ldots, d),$$

• where the pseudo count $\alpha > 0$ is the smoothing parameter ($\alpha = 0$ corresponds to no smoothing)

Word	P(word Sports)	P(word Not Sports)
а	$\frac{2+1}{11+14}$	$\frac{1+1}{9+14}$
very	$\frac{1+1}{11+14}$	$\frac{0+1}{9+14}$
close	$\frac{0+1}{11+14}$	$\frac{1+1}{9+14}$
game	$\frac{2+1}{11+14}$	$\frac{0+1}{9+14}$

$$\begin{split} &P(a|Sports)\times P(very|Sports)\times P(close|Sports)\times P(game|Sports)\times \\ &P(Sports)\\ &=4.61\times 10^{-5}\\ &=0.0000461 \end{split}$$

$$\begin{aligned} &P(a-\text{Not Sports})\times P(very|Not\,Sports)\times P(close|Not\,Sports)\times P(game|Not\,Sports)\times \\ &P(Not\,Sports)\\ &=1.43\times 10^{-5} \end{aligned}$$

As seen from the results shown below, P(a very close game | Sports) gives a higher probability, suggesting that the sentence belongs to the Sports category.

Multinomial Naive Bayes

Term Frequency

A alternative approach to characterize text documents — rather than binary values — is the *term* frequency (tf(t, d)). The term frequency is typically defined as the number of times a given term t (i.e., word or token) appears in a document d (this approach is sometimes also called *raw frequency*). In practice, the term frequency is often normalized by dividing the raw term frequency by the document length.

normalized term frequency =
$$\frac{tf(t,d)}{n_d}$$

where

- tf(t,d): Raw term frequency (the count of term t in document d).
- n_d : The total number of terms in document d.

The term frequencies can then be used to compute the maximum-likelihood estimate based on the training data to estimate the class-conditional probabilities in the multinomial model:

$$\hat{P}(x_i \mid \omega_j) = rac{\sum t f(x_i, d \in \omega_j) + lpha}{\sum N_{d \in \omega_j} + lpha \cdot V}$$

where

- x_i : A word from the feature vector \mathbf{x} of a particular sample.
- $\sum t f(x_i, d \in \omega_j)$: The sum of raw term frequencies of word x_i from all documents in the training sample that belong to class ω_i .
- $\sum N_{d\in\omega j}$: The sum of all term frequencies in the training dataset for class ω_j .
- lpha: An additive smoothing parameter (lpha=1 for Laplace smoothing).
- V: The size of the vocabulary (number of different words in the training set).

The class-conditional probability of encountering the text \mathbf{x} can be calculated as the product from the likelihoods of the individual words (under the *naive* assumption of conditional independence).

$$P(\mathbf{x} \mid \omega_j) = P(x_1 \mid \omega_j) \cdot P(x_2 \mid \omega_j) \cdot \ldots \cdot P(x_n \mid \omega_j) = \prod_{i=1}^m P(x_i \mid \omega_j)$$

Source Code:

Loading the 20 newsgroups dataset: The dataset is called "Twenty Newsgroups". Here is the official description, quoted from the website: http://qwone.com/~jason/20Newsgroups/

The 20 Newsgroups data set is a collection of approximately 20,000 newsgroup documents, partitioned (nearly) evenly across 20 different newsgroups. To the best of our knowledge, it was originally collected by Ken Lang, probably for his paper "Newsweeder: Learning to filter netnews," though he does not explicitly mention this collection. The 20 newsgroups collection has become a popular data set for experiments in text applications of machine learning techniques, such as text classification and text clustering.

```
from sklearn.datasets import fetch_20newsgroups
twenty_train = fetch_20newsgroups(subset='train', shuffle=True)
x = len(twenty_train.target_names)
print("\n The number of categories:",x)
print("\n The %d Different Categories of 20Newsgroups\n" %x)
i=1
for cat in twenty_train.target_names:
    print("Category[%d]:" %i,cat)
    i=i+1
print("\n Length of training data is",len(twenty_train.data))
print("\n Length of file names is ",len(twenty_train.filenames))

print("\n The Content/Data of First File is :\n")

print(twenty_train.data[0])

print("\n The Contents/Data of First 10 Files is in Training Data :\n")
for i in range(0,10):
```

Considering only four Categories

print("\n FILE NO:%d \n"%(i+1))

print(twenty train.data[i])

```
categories = ['alt.atheism', 'soc.religion.christian','comp.graphics', 'sc
i.med']
twenty_train = fetch_20newsgroups(subset='train', categories=categorie
s, shuffle=True, random_state=42)
print("\n Reduced Target Names:\n", twenty_train.target_names)
print("\n Reduced Target Length:\n", len(twenty_train.data))
print("\nFirst Document : ",twenty train.data[0])
```

Extracting features from text files

Word Occurrences

```
from sklearn.feature_extraction.text import CountVectorizer
count_vect = CountVectorizer()
X_train_counts = count_vect.fit_transform(twenty_train.data)
print("\n(Target Length , Distinct Words):",X_train_counts.shape)
```

```
print("\n Frequency of the word algorithm:", count_vect.vocabulary_.get('a lgorithm'))
```

From occurrences to frequencies

```
(Target Length , Distinct Words): (2257, 35788)

Frequency of the word algorithm: 4690
```

From occurrences to frequencies

Term Frequencies: Divide the number of occurrences of each word in a document by the total number of words in the document: these new features are called tf for Term Frequencies. Another refinement on top of tf is to downscale weights for words that occur in many documents in the corpus and are therefore less informative than those that occur only in a smaller portion of the corpus. This downscaling is called tf–idf for "Term Frequency times Inverse Document Frequency". Both tf and tf–idf can be computed as follows:

```
from sklearn.feature_extraction.text import TfidfTransformer

tf_transformer = TfidfTransformer(use_idf=False).fit(X_train_counts)

X_train_tf = tf_transformer.transform(X_train_counts)

X_train_tf.shape
(2257, 35788)
```

In the above example-code, we firstly use the fit(..) method to fit our estimator to the data and secondly the transform(..) method to transform our count-matrix to a tf-idf representation. These two steps can be combined to achieve the same end result faster by skipping redundant processing. This is done through using the fit_transform(..) method as shown below, and as mentioned in the note in the previous section:

```
tfidf_transformer = TfidfTransformer()
X_train_tfidf = tfidf_transformer.fit_transform(X_train_counts)
X_train_tfidf.shape
(2257, 35788)
```

Now that we have our features, we can train a classifier to try to predict the category of a post. Let's start with a naïve Bayes classifier, which provides a nice baseline for this task. scikit-learn includes several variants of this classifier; the one most suitable for word counts is the multinomial variant:

Training a classifier

```
from sklearn.naive_bayes import MultinomialNB
clf = MultinomialNB().fit(X train tfidf, twenty train.target)
```

Predicting the Outcome

To try to predict the outcome on a new document we need to extract the features using almost the same feature extracting chain as before. The difference is that we call transform instead of fit_transform on the transformers, since they have already been fit to the training set:

```
docs_new = ['God is love', 'OpenGL on the GPU is fast']
X_new_counts = count_vect.transform(docs_new)
X_new_tfidf = tfidf_transformer.transform(X_new_counts)
predicted = clf.predict(X_new_tfidf)
```

```
for doc, category in zip(docs_new, predicted):
    print('%r => %s' % (doc, twenty_train.target_names[category]))
'God is love' => soc.religion.christian
'OpenGL on the GPU is fast' => comp.graphics
```

Building a pipeline

In order to make the vectorizer => transformer => classifier easier to work with, scikit-learn provides a Pipeline class that behaves like a compound classifier:

The names vect, tfidf and clf (classifier) are arbitrary. We shall see their use in the section on grid search, below. We can now train the model with a single command:

```
text_clf.fit(twenty_train.data, twenty_train.target)
Pipeline(memory=None,
    steps=[('vect', CountVectorizer(analyzer='word', binary=False, decode_er
ror='strict',
    dtype=<class 'numpy.int64'>, encoding='utf-8', input='content',
    lowercase=True, max_df=1.0, max_features=None, min_df=1,
    ngram_range=(1, 1), preprocessor=None, stop_words=None,
    strip...inear_tf=False, use_idf=True)), ('clf', MultinomialNB(alpha=1.0, class_prior=None, fit_prior=True))])
```

Evaluation of the performance on the test set

```
0.97 0.60 0.74
                                         319
       alt.atheism
                           0.89
                                  0.92
      comp.graphics
                   0.96
                                         389
          sci.med
                   0.97
                          0.81
                                 0.88
                                         396
soc.religion.christian
                          0.99
                                 0.78
                                         398
                   0.65
       avg / total 0.88 0.83
                                  0.84
                                         1502
```

As expected the confusion matrix shows that posts from the newsgroups on atheism and christian are more often confused for one another than with computer graphics.

Reference : http://scikit-learn.org/stable/tutorial/text_analytics/working_with_text_data.html

Program7: Write a program to construct a **Bayesian network** considering medical data. Use this model to demonstrate the diagnosis of heart patients using standard Heart Disease Data Set. You can use Java/Python ML library classes/API.

Algorithm:

Bayesian Network (BAYESIAN BELIEF NETWORKS

Bayesian Belief networks describe conditional independence among subsets of variables

 → allows combining prior knowledge about (in)dependencies among variables with observed
 training data (also called Bayes Nets)

Conditional Independence

Definition: X is conditionally independent of Y given Z if the probability distribution governing
 X is independent of the value of Y given the value of Z; that is, if

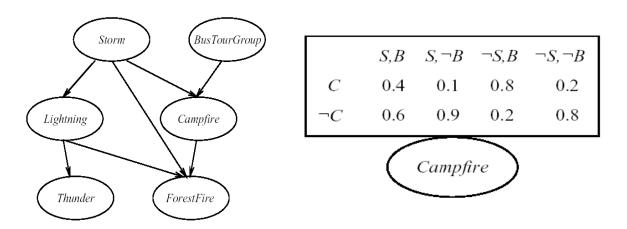
$$(\forall x_i, y_j, z_k) P(X=x_i | Y=y_j, Z=z_k) = P(X=x_i | Z=z_k)$$

more compactly, we write
 $P(X|Y,Z) = P(X|Z)$

- Example: Thunder is conditionally independent of Rain, given Lightning
 P(Thunder | Rain, Lightning) = P(Thunder | Lightning)
- · Naive Bayes uses cond. indep. to justify

$$P(X, Y|Z) = P(X|Y, Z) P(Y|Z) = P(X|Z) P(Y|Z)$$

Bayesian Belief Network



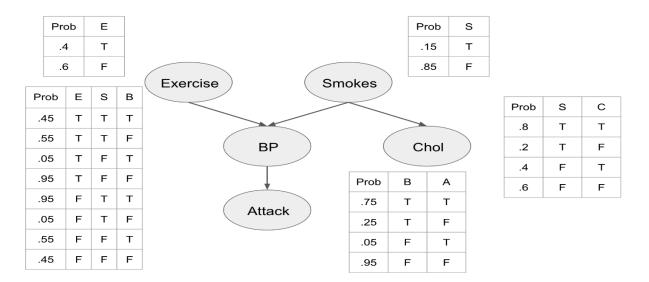
- Represents a set of conditional independence assertions:
 - Each node is asserted to be conditionally independent of its non descendants, given its immediate predecessors.
 - · Directed acyclic graph
- Represents joint probability distribution over all variables
 - e.g., P(Storm, BusTourGroup, . . . , ForestFire)
 - · in general,

$$P(y_1, \dots, y_n) = \prod_{i=1}^n P(y_i|Parents(Y_i))$$

where $Parents(Y_i)$ denotes immediate predecessors of Y_i in graph

• so, joint distribution is fully defined by graph, plus the $P(y_i | Parents(Y_i))$

Example 1:



Example2:

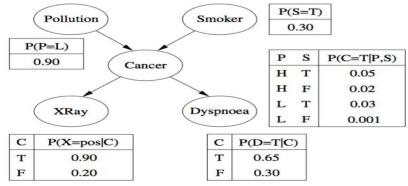


FIGURE 2.1

A BN for the lung cancer problem.

Source Code:

7.1. Constructing a Bayesian Network considering Medical Data 7.1.1 Defining a Structure with nodes and edges

7.1.2 Creation of Conditional Probability Table

```
# Now defining the parameters.
from pgmpy.factors.discrete import TabularCPD
cpd poll = TabularCPD(variable='Pollution', variable card=2,
                      values=[[0.9], [0.1]])
cpd smoke = TabularCPD(variable='Smoker', variable card=2,
                      values=[[0.3], [0.7]])
cpd cancer = TabularCPD(variable='Cancer', variable card=2,
                        values=[[0.03, 0.05, 0.001, 0.02],
                                [0.97, 0.95, 0.999, 0.98]],
                        evidence=['Smoker', 'Pollution'],
                        evidence card=[2, 2])
cpd xray = TabularCPD(variable='Xray', variable_card=2,
                      values=[[0.9, 0.2], [0.1, 0.8]],
                      evidence=['Cancer'], evidence card=[2])
cpd dysp = TabularCPD(variable='Dyspnoea', variable card=2,
                      values=[[0.65, 0.3], [0.35, 0.7]],
                      evidence=['Cancer'], evidence card=[2])
```

7.1.3 Associating Conditional probabilities with the Bayesian Structure

```
# Associating the parameters with the model structure.
cancer_model.add_cpds(cpd_poll, cpd_smoke, cpd_cancer, cpd_xray, cpd_dysp)
# Checking if the cpds are valid for the model.
cancer_model.check_model()

# Doing some simple queries on the network
cancer_model.is_active_trail('Pollution', 'Smoker')
cancer_model.is_active_trail('Pollution', 'Smoker', observed=['Cancer'])
cancer_model.get_cpds()
```

```
print(cancer_model.get_cpds('Pollution'))
print(cancer_model.get_cpds('Smoker'))

print(cancer_model.get_cpds('Xray'))
print(cancer_model.get_cpds('Dyspnoea'))
print(cancer_model.get_cpds('Cancer'))
```

7.1.4 Determining the Local independencies

```
cancer_model.local_independencies('Xray')
cancer_model.local_independencies('Pollution')
cancer_model.local_independencies('Smoker')
cancer_model.local_independencies('Dyspnoea')
cancer_model.local_independencies('Cancer')
cancer_model.get_independencies()
```

7.1.5.Inferencing with Bayesian Network

```
# Doing exact inference using Variable Elimination
from pgmpy.inference import VariableElimination
cancer_infer = VariableElimination(cancer_model)

# Computing the probability of bronc given smoke.
q = cancer_infer.query(variables=['Cancer'], evidence={'Smoker': 1})
print(q['Cancer'])
# Computing the probability of bronc given smoke.
q = cancer_infer.query(variables=['Cancer'], evidence={'Smoker': 1})
print(q['Cancer'])
# Computing the probability of bronc given smoke.
q = cancer_infer.query(variables=['Cancer'], evidence={'Smoker': 1, 'Pollution': 1})
print(q['Cancer'])
```

7.2 Diagnosis of heart patients using standard Heart Disease Data Set

```
import numpy as np
from urllib.request import urlopen
import urllib
import matplotlib.pyplot as plt # Visuals
import seaborn as sns
import sklearn as skl
import pandas as pd
```

7.2.1 Importing Heart Disease Data Set and Customizing

```
Cleveland data URL = 'http://archive.ics.uci.edu/ml/machine-learning-datab
ases/heart-disease/processed.hungarian.data'
#Hungarian data URL = 'http://archive.ics.uci.edu/ml/machine-learning-data
bases/heart-disease/processed.hungarian.data'
#Switzerland data URL = 'http://archive.ics.uci.edu/ml/machine-learning-da
tabases/heart-disease/processed.switzerland.data'
np.set printoptions(threshold=np.nan) #see a whole array when we output it
names = ['age', 'sex', 'cp', 'trestbps', 'chol', 'fbs', 'restecg', 'thalac
h', 'exang', 'oldpeak', 'slope', 'ca', 'thal', 'heartdisease']
heartDisease = pd.read csv(urlopen(Cleveland data_URL), names = names) #ge
ts Cleveland data
#HungarianHeartDisease = pd.read csv(urlopen(Hungarian data URL), names =
names) #gets Hungary data
#SwitzerlandHeartDisease = pd.read csv(urlopen(Switzerland data URL), name
s = names) #gets Switzerland data
#datatemp = [ClevelandHeartDisease, HungarianHeartDisease, SwitzerlandHear
tDisease] #combines all arrays into a list
#heartDisease = pd.concat(datatemp) #combines list into one array
heartDisease.head()
del heartDisease['ca']
del heartDisease['slope']
del heartDisease['thal']
del heartDisease['oldpeak']
heartDisease = heartDisease.replace('?', np.nan)
heartDisease.dtypes
heartDisease.columns
```

7.2.2 Modeling Heart Disease Data

```
from pgmpy.models import BayesianModel
from pgmpy.estimators import MaximumLikelihoodEstimator, BayesianEstimator
model = BayesianModel([('age', 'trestbps'), ('age', 'fbs'), ('sex', 'trest
bps'), ('sex', 'trestbps'),
                        ('exang', 'trestbps'), ('trestbps', 'heartdisease'), (
'fbs', 'heartdisease'),
                       ('heartdisease', 'restecg'), ('heartdisease', 'thalach'
),('heartdisease','chol')])
# Learing CPDs using Maximum Likelihood Estimators
model.fit(heartDisease, estimator=MaximumLikelihoodEstimator)
#for cpd in model.get cpds():
# print("CPD of {variable}:".format(variable=cpd.variable))
  # print(cpd)
print(model.get cpds('age'))
print(model.get cpds('chol'))
print(model.get cpds('sex'))
model.get independencies()
```

7.2.3.Inferencing with Bayesian Network

0.6333

0.3667

	In	[3!	5]:
<pre>q = HeartDisease_infer.query(variables=['heartdisease'], evidence={'c</pre>	hol'	:	10
0})			
<pre>print(q['heartdisease'])</pre>			

heartdisease	phi(heartdisease)
heartdisease_0	1.0000
heartdisease_1	0.0000

heartdisease_0

heartdisease 1

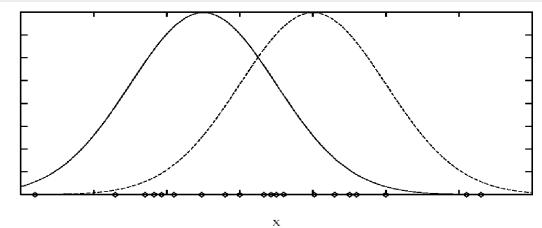
Program 8: Apply **EM algorithm** to cluster a set of data stored in a .CSV file. Use the same data set for clustering using **k-Means algorithm**. Compare the results of these two algorithms and comment on the quality of clustering. You can add Java/Python ML library classes/API in the program.

Algorithm:

Expectation Maximization (EM) Algorithm

- When to use:
 - Data is only partially observable
 - Unsupervised clustering (target value unobservable)
 - Supervised learning (some instance attributes unobservable)
- Some uses:
 - Train Bayesian Belief Networks
 - Unsupervised clustering (AUTOCLASS)
 - Learning Hidden Markov Models

Generating Data from Mixture of *k* Gaussians



(x)

Each instance x generated by

- 1. Choosing one of the k Gaussians with uniform probability
- 2. Generating an instance at random according to that Gaussian

EM for Estimating k Means

- Given:
 - Instances from X generated by mixture of k Gaussian distributions
 - Unknown means $<\mu_1,...,\mu_k>$ of the k Gaussians
 - Don't know which instance x_i was generated by which Gaussian
- Determine:
 - Maximum likelihood estimates of $\langle \mu_1,...,\mu_k \rangle$
- Think of full description of each instance as

 $y_i = \langle x_i, z_{i1}, z_{i2} \rangle$ where

- z_{ij} is 1 if x_i generated by jth Gaussian
- *x_i* observable
- z_{ij} unobservable

• EM Algorithm: Pick random initial $h = \langle \mu_1, \mu_2 \rangle$ then iterate

E step: Calculate the expected value $E[z_{ij}]$ of each hidden variable z_{ij} , assuming the current hypothesis

 $h = <\mu_1, \ \mu_2> \text{ holds.}$

$$E[z_{ij}] = \frac{p(x = x_i | \mu = \mu_j)}{\sum_{n=1}^{2} p(x = x_i | \mu = \mu_n)}$$
$$= \frac{e^{-\frac{1}{2\sigma^2}(x_i - \mu_j)^2}}{\sum_{n=1}^{2} e^{-\frac{1}{2\sigma^2}(x_i - \mu_n)^2}}$$

M step: Calculate a new maximum likelihood hypothesis $h' = \langle \mu'_1, \mu'_2 \rangle$, assuming the value taken on by each hidden variable z_{ij} is its expected value $E[z_{ij}]$ calculated above. Replace $h = \langle \mu_1, \mu_2 \rangle$ by $h' = \langle \mu'_1, \mu'_2 \rangle$.

$$\mu_j \leftarrow \frac{\sum_{i=1}^m E[z_{ij}] \ x_i}{\sum_{i=1}^m E[z_{ij}]}$$

K Means Algorithm

- 1. The sample space is initially partitioned into K clusters and the observations are randomly assigned to the clusters.
- 2. For each sample:
 - Calculate the distance from the observation to the centroid of the cluster.
 - IF the sample is closest to its own cluster THEN leave it ELSE select another cluster.
- 3. Repeat steps 1 and 2 untill no observations are moved from one cluster to another

Distance functions

$$\sqrt{\sum_{i=1}^{k} (x_i - y_i)^2}$$

$$\sum_{i=1}^{k} \left| x_i - y_i \right|$$

$$\left(\sum_{i=1}^{k} \left(\left|x_{i}-y_{i}\right|\right)^{q}\right)^{1/q}$$

Basic Algorithm of K-means

Algorithm 1 Basic K-means Algorithm.

- Select K points as the initial centroids.
- 2: repeat
- 3: Form K clusters by assigning all points to the closest centroid.
- 4: Recompute the centroid of each cluster.
- 5: until The centroids don't change

Details of K-means

- 1. Initial centroids are often chosen randomly.
 - Clusters produced vary from one run to another
- 2. The centroid is (typically) the mean of the points in the cluster.
- 3. 'Closeness' is measured by **Euclidean distance**, cosine similarity, correlation, etc.
- 4. K-means will converge for common similarity measures mentioned above.
- 5. Most of the convergence happens in the first few iterations.
 - Often the stopping condition is changed to 'Until relatively few points change clusters'

Euclidean Distance

$$d(i,j) = \sqrt{|x_{i_1} - x_{j_1}|^2 + |x_{i_2} - x_{j_2}|^2 + \dots + |x_{i_p} - x_{j_p}|^2}$$

A simple example: Find the distance between two points, the original and the point (3,4)

$$d_{E}(O, A) = \sqrt{3^2 + 4^2} = 5$$

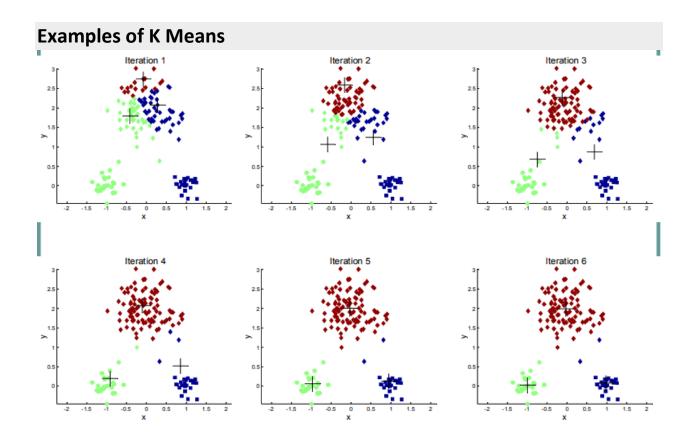
Update Centroid

We use the following equation to calculate the n dimensional centroid point amid k n-dimensional points

$$CP(x_{1},x_{2},...,x_{k}) = (\frac{\sum_{i=1}^{k} x1st_{i}}{k}, \frac{\sum_{i=1}^{k} x2nd_{i}}{k}, ..., \frac{\sum_{i=1}^{k} xnth_{i}}{k})$$

Example: Find the centroid of 3 2D points, (2,4), (5,2) and (8,9)

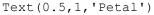
$$CP = (\frac{2+5+8}{3}, \frac{4+2+9}{3}) = (5,5)$$

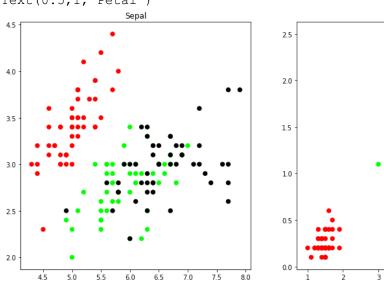


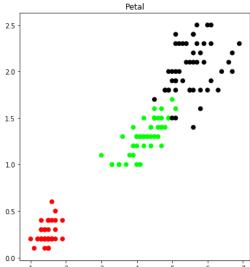
Source Code:

```
import matplotlib.pyplot as plt
from sklearn import datasets
from sklearn.cluster import KMeans
import sklearn.metrics as sm
import pandas as pd
import numpy as np
%matplotlib inline
# import some data to play with
iris = datasets.load iris()
#print("\n IRIS DATA :",iris.data);
#print("\n IRIS FEATURES :\n",iris.feature names)
#print("\n IRIS TARGET :\n",iris.target)
#print("\n IRIS TARGET NAMES:\n",iris.target names)
# Store the inputs as a Pandas Dataframe and set the column names
X = pd.DataFrame(iris.data)
#print(X)
X.columns = ['Sepal Length', 'Sepal Width', 'Petal Length', 'Petal Width']
```

```
#print(X.columns)
#print("X:",x)
#print("Y:",y)
y = pd.DataFrame(iris.target)
y.columns = ['Targets']
# Set the size of the plot
plt.figure(figsize=(14,7))
# Create a colormap
colormap = np.array(['red', 'lime', 'black'])
# Plot Sepal
plt.subplot(1, 2, 1)
plt.scatter(X.Sepal Length, X.Sepal Width, c=colormap[y.Targets], s=40)
plt.title('Sepal')
plt.subplot(1, 2, 2)
plt.scatter(X.Petal Length, X.Petal Width, c=colormap[y.Targets], s=40)
plt.title('Petal')
```







Build the K Means Model

```
# K Means Cluster
model = KMeans(n_clusters=3)
model.fit(X)
# This is what KMeans thought
model.labels_
```

Visualise the classifier results

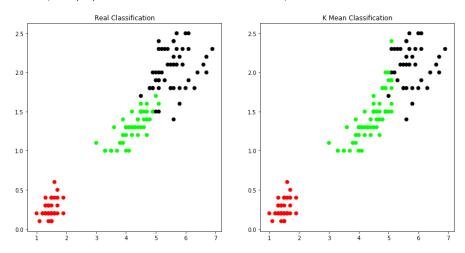
```
# View the results
# Set the size of the plot
plt.figure(figsize=(14,7))

# Create a colormap
colormap = np.array(['red', 'lime', 'black'])

# Plot the Original Classifications
plt.subplot(1, 2, 1)
plt.scatter(X.Petal_Length, X.Petal_Width, c=colormap[y.Targets], s=40)
plt.title('Real Classification')

# Plot the Models Classifications
plt.subplot(1, 2, 2)
plt.scatter(X.Petal_Length, X.Petal_Width, c=colormap[model.labels_], s=40)
)
plt.title('K Mean Classification')
```

Text(0.5,1,'K Mean Classification')



The Fix

```
# The fix, we convert all the 1s to 0s and 0s to 1s.
predY = np.choose(model.labels_, [0, 1, 2]).astype(np.int64)
print (predY)
```

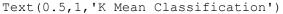
Re-plot

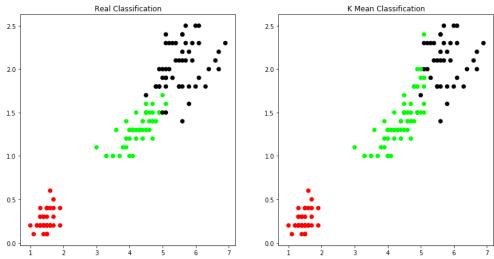
```
# View the results
# Set the size of the plot
plt.figure(figsize=(14,7))

# Create a colormap
colormap = np.array(['red', 'lime', 'black'])

# Plot Orginal
plt.subplot(1, 2, 1)
plt.scatter(X.Petal_Length, X.Petal_Width, c=colormap[y.Targets], s=40)
plt.title('Real Classification')

# Plot Predicted with corrected values
plt.subplot(1, 2, 2)
plt.scatter(X.Petal_Length, X.Petal_Width, c=colormap[predY], s=40)
plt.title('K Mean Classification')
```





Performance Measures

Accuracy

```
sm.accuracy_score(y, model.labels_)
```

Confusion Matrix

GMM

```
from sklearn import preprocessing

scaler = preprocessing.StandardScaler()

scaler.fit(X)

xsa = scaler.transform(X)

xs = pd.DataFrame(xsa, columns = X.columns)

xs.sample(5)
```

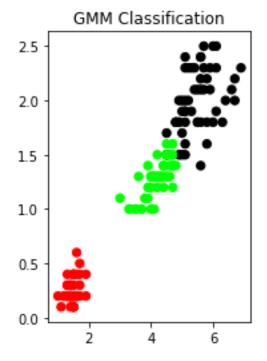
	Sepal_Length	Sepal_Width	Petal_Length	Petal_Width	
132	0.674501	-0.587764	1.047087	1.316483	
110	0.795669	0.337848	0.762759	1.053537	
93	-1.021849	-1.744778	-0.260824 -0.261193		
24	-1.264185	0.800654	-1.056944	-1.312977	
111	0.674501	-0.819166	0.876490	0.922064	

from sklearn.mixture import GaussianMixture

```
gmm = GaussianMixture(n_components=3)
gmm.fit(xs)

GaussianMixture(covariance_type='full', init_params='kmeans', max_iter=100
,
    means_init=None, n_components=3, n_init=1, precisions_init=None,
    random_state=None, reg_covar=1e-06, tol=0.001, verbose=0,
    verbose interval=10, warm start=False, weights init=None)
```

ext(0.5,1,'GMM Classification')



sm.accuracy score(y, y cluster gmm)

0.96666666666666667

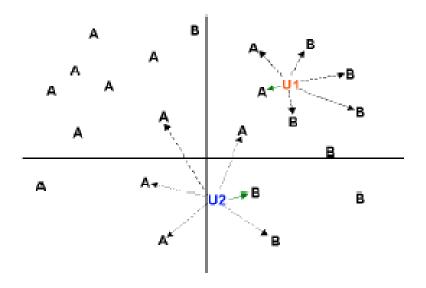
so the GMM clustering matched the true labels more closely than the Kmea ns, # as expected from the plots.

Program9: Write a program to implement k-Nearest Neighbour algorithm to classify the iris data set. Print both correct and wrong predictions. Java/Python ML library classes can be used for this problem.

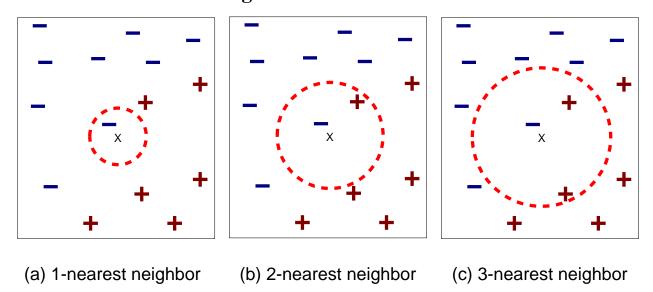
Algorithm:

K-Nearest-Neighbor Algorithm

• Principle: points (documents) that are close in the space belong to the same class



Definition of Nearest Neighbor



Distance Metrics

Minkowsky:

Manhattan / city-block:

$$D(x,y) = \left(\sum_{i=1}^{m} |x_i - y_i|^r\right)^{1/r} \qquad D(x,y) = \sqrt{\sum_{i=1}^{m} (x_i - y_i)^2} \qquad D(x,y) = \sum_{i=1}^{m} |x_i - y_i|$$

$$D(x,y) = \sqrt{\sum_{i=1}^{m} (x_i - y_i)^2}$$

$$D(\boldsymbol{x}, \boldsymbol{y}) = \sum_{i=1}^{m} |x_i - y_i|$$

Camberra:

$$D(x,y) = \sum_{i=1}^{m} \frac{|x_i - y_i|}{|x_i + y_i|}$$

Chebychev:
$$D(x,y) = \max_{i=1}^{m} |x_i - y_i|$$

Quadratic:
$$D(x,y) = (x - y)^T Q(x - y) = \sum_{j=1}^m \left(\sum_{i=1}^m (x_i - y_i)q_{ji}\right)(x_j - y_j)$$

definite $m \times m$ weight matrix

Mahalanobis:

$$D(x, y) = [\det V]^{1/m} (x - y)^{\mathrm{T}} V^{-1} (x - y)$$

V is the covariance matrix of $A_1.A_m$, and A_i is the vector of values for attribute *i* occuring in the training set instances 1..n.

Correlation: $D(x,y) = \frac{\sum_{i=1}^{m} (x_i - \overline{x_i})(y_i - \overline{y_i})}{\sqrt{\sum_{i=1}^{m} (x_i - \overline{x_i})^2 \sum_{i=1}^{m} (y_i - \overline{y_i})^2}}$

 $\overline{x_i} = \overline{y_i}$ and is the average value for attribute i occuring in the training set.

Chi-square: $D(x,y) = \sum_{sum_i}^{m} \left(\frac{x_i}{size} - \frac{y_i}{size} \right)^2$

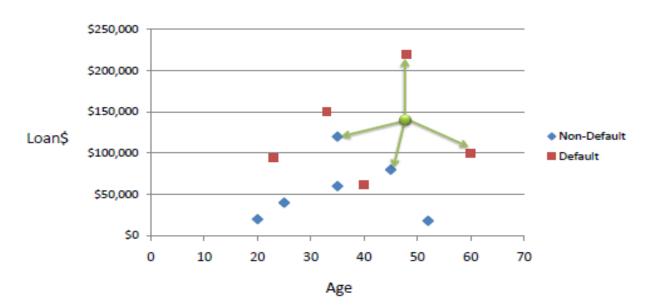
 sum_i is the sum of all values for attribute i occurring in the training set, and $size_x$ is the sum of all values in the vector x.

Kendall's Rank Correlation: sign(x)=-1, 0 or 1 if x < 0,x = 0, or x > 0, respectively.

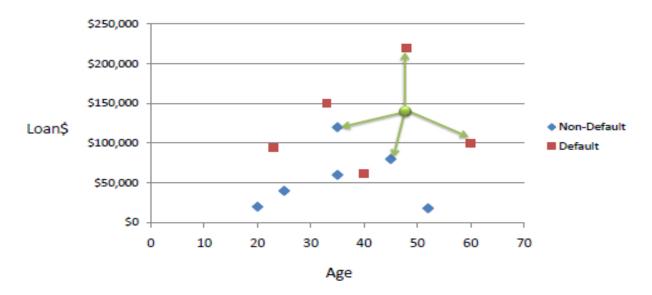
$$D(x,y) = 1 - \frac{2}{n(n-1)} \sum_{i=1}^{m} \sum_{j=1}^{i-1} \text{sign}(x_i - x_j) \text{sign}(y_i - y_j)$$

Figure 1. Equations of selected distance functions. (x and y are vectors of m attribute values).

Example: Consider the following data concerning credit default. Age and Loan are two numerical variables (predictors) and Default is the target.



We can now use the training set to classify an unknown case (Age=48 and Loan=\$142,000) using Euclidean distance. If K=1 then the nearest neighbor is the last case in the training set with Default=Y.



 $D = Sqrt[(48-33)^2 + (142000-150000)^2] = 8000.01 >> Default=Y$

	Age	Loan	Default	Distance					
	25	\$40,000	N	102000					
	35	\$60,000	N	82000					
	45	\$80,000	N	62000					
	20	\$20,000	N	122000					
	35	\$120,000	N	22000	2				
	52	\$18,000	N	124000					
	23	\$95,000	Υ	47000					
	40	\$62,000	Υ	80000					
	60	\$100,000	Υ	42000	3				
	48	\$220,000	Υ	78000					
	33	\$150,000	Υ <table-cell-columns></table-cell-columns>	8000	1				
			Ţ						
	48	\$142,000	?						
$D = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$ Euclidean Distance									

With K=3, there are two Default=Y and one Default=N out of three closest neighbors. The prediction for the unknown case is again Default=Y.

Source Code:

```
# Python program to demonstrate
# KNN classification algorithm
# on IRIS dataset

from sklearn.datasets import load_iris
from sklearn.neighbors import KNeighborsClassifier
import numpy as np
from sklearn.model_selection import train_test_split
iris_dataset=load_iris()

print("\n IRIS FEATURES \ TARGET NAMES: \n ", iris_dataset.target_names)
for i in range(len(iris_dataset.target_names)):
    print("\n[{0}]:[{1}]".format(i,iris_dataset.target_names[i]))
```

```
print("\n IRIS DATA :\n", iris dataset["data"])
X train, X test, y train, y test = train test split(iris dataset["data"],
iris dataset["target"], random state=0)
print("\n Target :\n", iris dataset["target"])
print("\n X TRAIN \n", X train)
print("\n X TEST \n", X test)
print("\n Y TRAIN \n", y train)
print("\n Y TEST \n", y test)
kn = KNeighborsClassifier(n neighbors=1)
kn.fit(X train, y train)
x \text{ new} = \text{np.array}([[5, 2.9, 1, 0.2]])
print("\n XNEW \n", x new)
prediction = kn.predict(x new)
print("\n Predicted target value: {}\n".format(prediction))
print("\n Predicted feature name: {}\n".format
    (iris dataset["target names"][prediction]))
i=1
x= X test[i]
x new = np.array([x])
print("\n XNEW \n", x new)
for i in range(len(X test)):
    x = X test[i]
    x new = np.array([x])
    prediction = kn.predict(x new)
    print("\n Actual : {0} {1}, Predicted :{2}{3}".format(y test[i], iris d
ataset["target names"][y test[i]],prediction,iris dataset["target names"][
prediction]))
print("\n TEST SCORE[ACCURACY]: {:..2f}\n".format(kn.score(X test, y test))
```

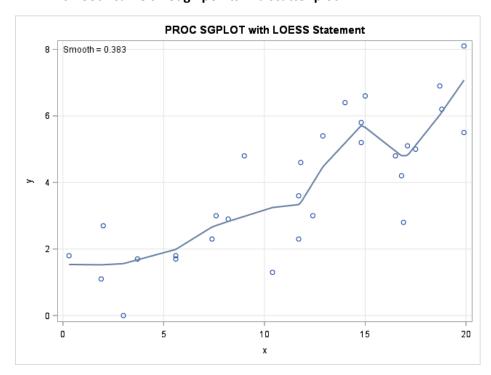
Output:

```
Actual: 2 virginica, Predicted: [2]['virginica']
Actual: 1 versicolor, Predicted: [1]['versicolor']
Actual: 0 setosa, Predicted: [0]['setosa']
Actual: 2 virginica, Predicted: [2]['virginica']
Actual: 0 setosa, Predicted: [0]['setosa']
-----
Actual: 1 versicolor, Predicted: [2]['virginica']
TEST SCORE[ACCURACY]: 0.97
```

Program10: Implement the non-parametric Locally Weighted Regression algorithm in order to fit data points. Select appropriate data set for your experiment and draw graphs.

Algorithm: Loess/Lowess Regression

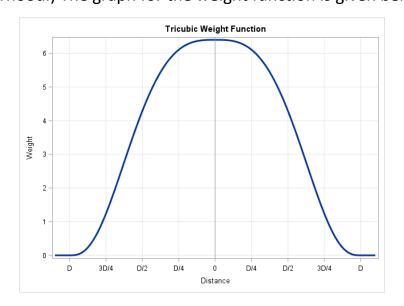
• Loess regression is a nonparametric technique that uses local weighted regression to fit a smooth curve through points in a scatter plot.



Overview of the loess regression algorithm

- 1. Choose a smoothing parameter: The smoothing parameter, s, is a value in (0,1] that represents the proportion of observations to use for local regression. If there are n observations, then the k = floor(n*s) points closest to x_0 (in the X direction) form a *local neighborhood* near x_0 .
- 2. Find the k nearest neighbors to x_0 :

3. Assign weights to the nearest neighbors: The loess algorithm uses a tricubic weight function to weight each point in the local neighborhood of x_0 . The weight for the i_th point in the neighborhood is $w_i = (32/5) (1 - (d_i / D)^3)^3$ where D is the largest distance in the neighborhood and d_i is the distance to the i_th point. (The weight function is zero outside of the local neighborhood.) The graph for the weight function is given below:



4. Perform local weighted regression: The points in the local neighborhood of x_0 are used to <u>fit and score a local weighted regression model</u> at x_0 .

Source Code:

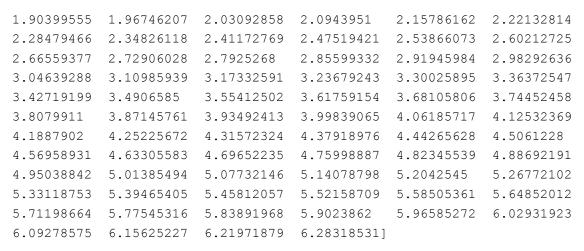
```
# Author: Dr Thyagaraju GS , Context Innovations Lab

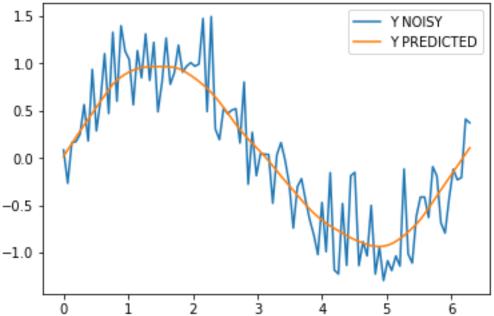
from math import ceil
import numpy as np
from scipy import linalg

def lowess(x, y, f= 2. / 3., iter=3):

    n = len(x) # Number of x points
    r = int(ceil(f * n)) # Computing the residual of smoothing functions
    h = [np.sort(np.abs(x - x[i]))[r] for i in range(n)] #
    w = np.clip(np.abs((x[:, None] - x[None, :]) / h), 0.0, 1.0) # Weight
Function
    w = (1 - w ** 3) ** 3 # Tricube Weight Function
```

```
ypred = np.zeros(n) # Initialisation of predictor
    delta = np.ones(n) # Initialisation of delta
    for iteration in range(iter):
        for i in range(n):
            weights = delta * w[:, i] # Cumulative Weights
            b = np.array([np.sum(weights * y), np.sum(weights * y * x)]) #
Matrix B
            A = np.array([[np.sum(weights), np.sum(weights * x)],
                          [np.sum(weights * x), np.sum(weights * x * x)]])
# Matrix A
            beta = linalg.solve(A, b) # Beta, Solution of AX= B equation
            ypred[i] = beta[0] + beta[1] * x[i]
        residuals = y - ypred # Finding Residuals
        s = np.median(np.abs(residuals)) # Median of Residuals
        delta = np.clip(residuals / (6.0 * s), -1, 1) # Delta
        delta = (1 - delta ** 2) ** 2  # Delta
    return ypred
if name == ' main ': # Main Function
    import math
    n = 100 # Number of data points
    #Case1: Sinusoidal Fitting
    x = np.linspace(0, 2 * math.pi, n)
    y = np.sin(x) + 0.3 * np.random.randn(n)
    #Case2 : Straight Line Fitting
    \#x=np.linspace(0,2.5,n) # For Linear
    \#y=1 + 0.25*np.random.randn(n) \# For Linear
    f = 0.25
    ypred = lowess(x, y, f=f, iter=3)
    import pylab as pl
    pl.clf()
    pl.plot(x, y, label='Y NOISY')
    pl.plot(x, ypred, label='Y PREDICTED')
    pl.legend()
  pl.show()
[ 0.
            0.06346652 0.12693304 0.19039955 0.25386607 0.31733259
 0.38079911 \quad 0.44426563 \quad 0.50773215 \quad 0.57119866 \quad 0.63466518 \quad 0.6981317
 0.76159822 0.82506474 0.88853126 0.95199777 1.01546429 1.07893081
 1.14239733 1.20586385 1.26933037 1.33279688 1.3962634 1.45972992
 1.52319644 1.58666296 1.65012947 1.71359599 1.77706251 1.84052903
```





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