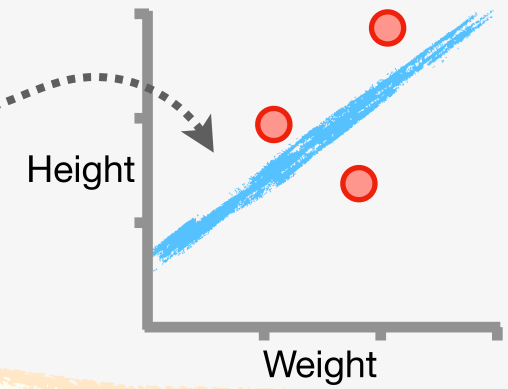
The Sum of the Squared Residuals: Main Ideas Part 1

1 **The Problem:** We have a model that makes predictions. In this case, we're using Weight to predict Height. However, we need to quantify the quality of the model and its predictions.

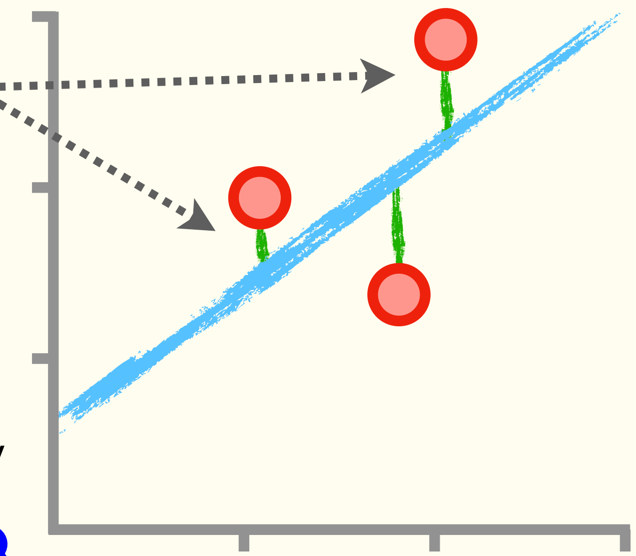


2 **A Solution:** One way quantify the quality of a model and its predictions is to calculate the **Sum of the Squared Residuals.**

As the name implies, we start by calculating **Residuals**, the differences between the **Observed** values and the values **Predicted** by the model.

Residual = Observed - Predicted

Visually, we can draw **Residuals** with these **green lines**.



Since, in general, the smaller the **Residuals**, the better the model fits the data, it's tempting to compare models by comparing the sum of their **Residuals**, but the **Residuals** below the **blue line** would cancel out the ones above it!!!

So, instead of calculating the sum of the **Residuals**, we square the **Residuals** first and calculate the **Sum of the Squared Residuals** (**SSR**).

The Sum of Squared Residuals (SSR) is usually defined with fancy **Sigma** notation and the right-hand side reads: "The sum of all observations of the squared difference between the observed and predicted values."

$$\text{SSR} = \sum_{i=1}^{n} (\text{Observed}_i - \text{Predicted}_i)^2$$

n = the number of **Observations.**

i = the index for each **Observation**. For example, ***i* = 1** refers to the first **Observation**.

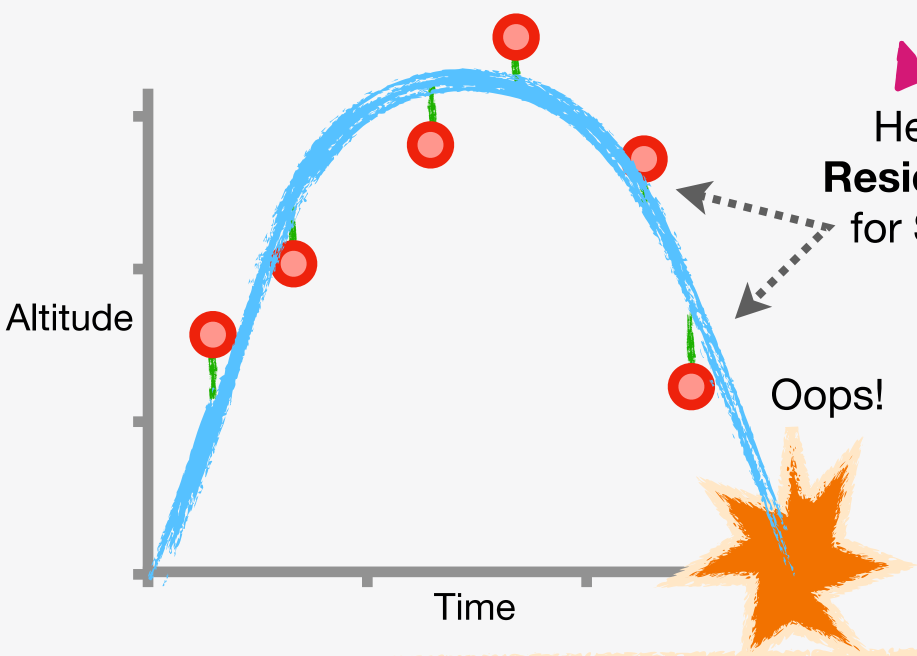
The **Sigma** symbol, **Σ**, tells us to do a **summation.**

NOTE: ***Squaring***, as opposed to taking the ***absolute value***, makes it easy to take the derivative, which will come in handy when we do **Gradient Descent** in **Chapter 5**.

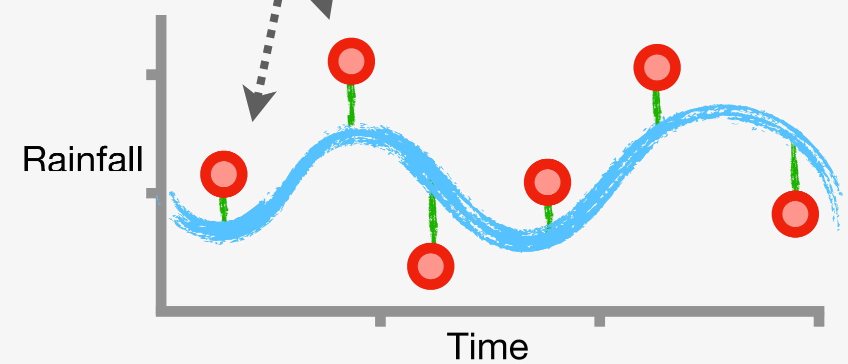
The Sum of the Squared Residuals: Main Ideas Part 2

3 So far, we've looked at the **SSR** only in terms of a simple straight line model, but we can calculate it for all kinds of models.

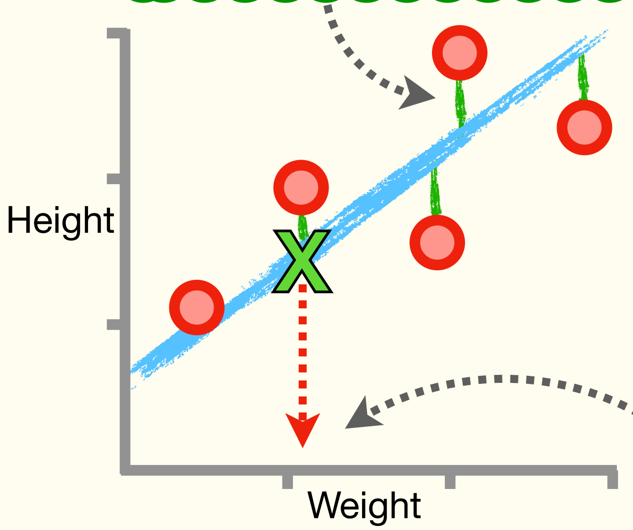
Here's an example of the **Residuals** for altitude vs. time for **SpaceX's SN9** rocket…



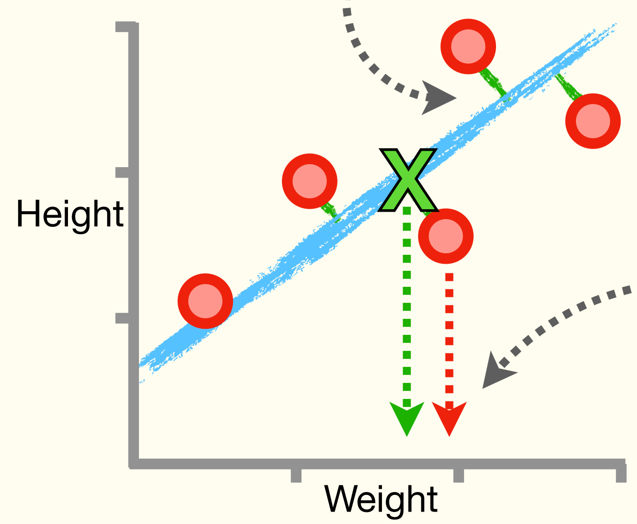
…and here's an example of the **Residuals** for a sinusoidal model of rainfall. Some months are more rainy than others, and the pattern is cyclical over time. If you can calculate the **Residuals**, you can square them and add them up!



4 **NOTE:** When we calculate the **Residuals**, we use the **vertical distance** to the **model**…



…instead of the shortest distance, the **perpendicular distance**…



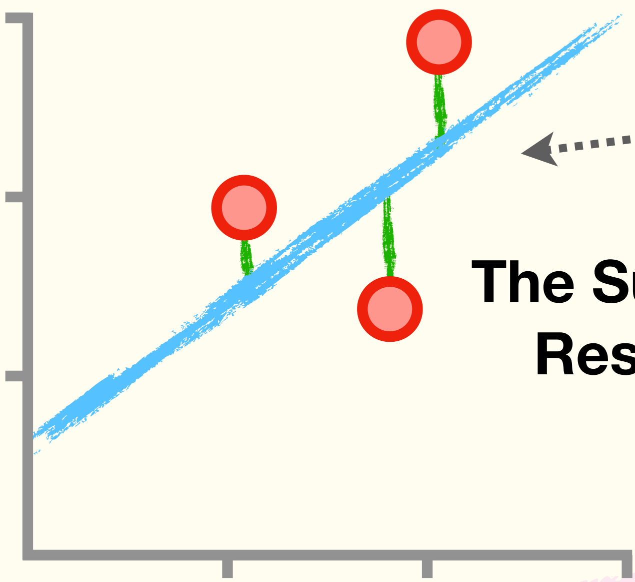
…because, in this example, perpendicular lines result in different Weights for the Observed and Predicted Heights.

In contrast, the vertical distance allows both the Observed and Predicted Heights to correspond to the same Weight.

5 Now that we understand the main ideas of the **SSR**, let's walk through an example of how it's calculated, step-by-step.

SSR: Step-by-Step

1 In this example, we have **3** Observations, so ***n* = 3**, and we expand the summation into **3** terms.

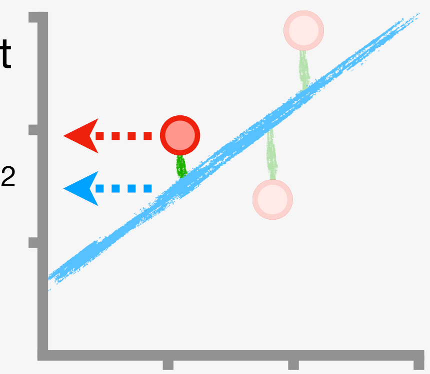


The Sum of Squared Residuals (SSR) $= \sum_{i=1}^{n} (\text{Observed}_i - \text{Predicted}_i)^2$

Observed =
Predicted =
Residual =

For ***i* = 1**, the term for the first Observation…

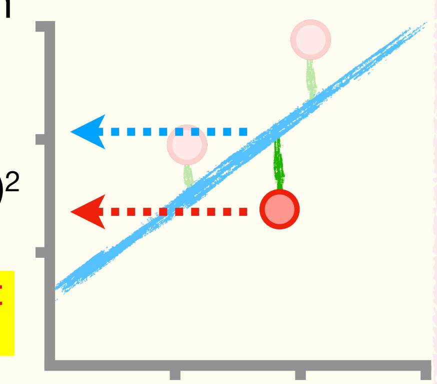
$(1.9 - 1.7)^2$



For ***i* = 2**, the term for the second Observation…

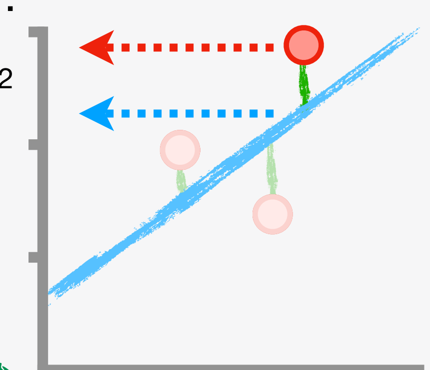
$(1.6 - 2.0)^2$

otherwise we get negative value



For ***i* = 3**, the term for the third Observation…

$(2.9 - 2.2)^2$



2 Once we expand the summation, we plug in the **Residuals** for each Observation.

$$\textbf{SSR} = (\text{Observed}_1 - \text{Predicted}_1)^2 + (\text{Observed}_2 - \text{Predicted}_2)^2 + (\text{Observed}_3 - \text{Predicted}_3)^2$$

3 Now, we just do the math, and the final **Sum of Squared Residuals (SSR)** is **0.69**.

$$\textbf{SSR} = (1.9 - 1.7)^2 + (1.6 - 2.0)^2 + (2.9 - 2.2)^2 = 0.69$$

BAM!!!

Don't get me wrong, the **SSR** is awesome, but it has a pretty big problem that we'll talk about on the next page.