vectors

1. Find the value of $\vec{p}$ if

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}.$$

2. If $\vec{\mathbf{p}}$ is a unit vector and $(\vec{\mathbf{x}} - \vec{\mathbf{p}}) \cdot (\vec{\mathbf{x}} + \vec{\mathbf{p}}) = 80$, then find $|\vec{\mathbf{x}}|$.

3. Find the shortest distance between the following two lines:

$$\vec{r} = (1 + \lambda)\hat{i} + (2 - \lambda)\hat{j} + (\lambda + 1)\hat{k};$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}).$$

4. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.