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Amrita School of Engineering, Coimbatore
B.Tech Mid-term – September 03, 2024
5th Semester
Cybersecurity
20CYS301 Digital Communications

Duration: Two hours Maximum: 50 Marks

Course Outcomes (COs):

CO	Course Outcomes
CO01	Understand the fundamental principles of digital modulation and demodulation methods
CO02	Identify and list various issues present in the design of a communication system
CO03	Apply the time domain and frequency domain concepts of signals in data communication
CO04	Design suitable error detection and error correction algorithms to achieve error free data communication
CO05	
CO06	

NOTE to exam invigilator

- Please do NOT permit anyone to leave the desk until the last 30 minute of the exam.

NOTE to students

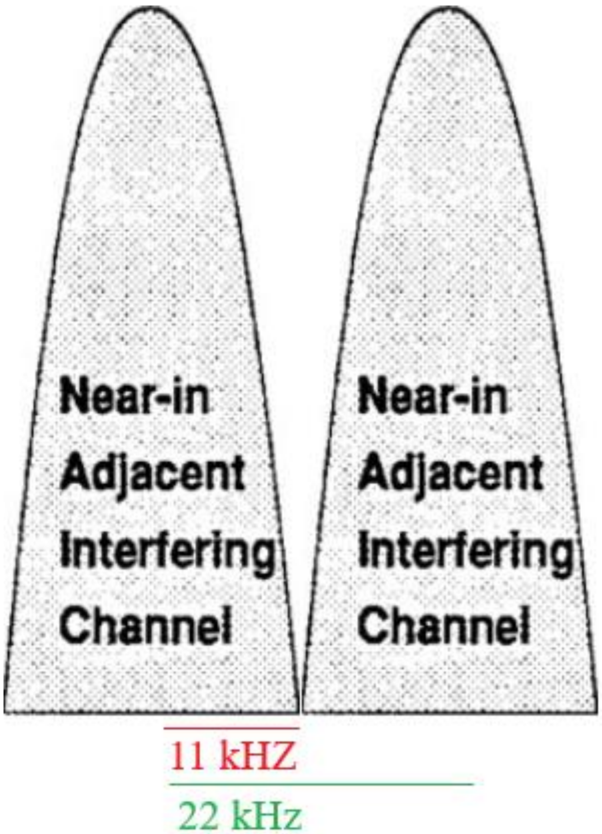
- Answer ALL questions
- When answering non-problem/proof-based questions, write in bullet points.
- Rough work should be struck out to ensure an evaluator is able to distinguish rough work from solutions.
- Use any colored pen or pencil except red

Formulae

1. Message bandwidth is 11 KHz. It is used to AM-modulate a carrier at 1 MHz frequency.

What is the minimum distance between sub-carrier frequencies in case of DSB SC AM? Justify.

[5.0] [C01] [BTL2]



Justification?

2. Light speed in vacuum is 3 lac km per second. A 2G phone is set for operation at 900 MHz band (in Bharata, it can be 900 or 1800 MHz). Max operating frequency of 5G phones in Bharata is 5 GHz. For effective coupling between generated and transmitted energy, antenna length must be $> \frac{1}{10}\lambda$.

The antenna length is larger in the case of 2G or 5G? State the reason.

Compute the minimum size of the antenna needed for 2G and for 5G.

[5.0] [C02] [BTL3]

Larger antenna is needed for larger wavelength. Wavelength is inversely proportional to frequency. As frequency for 2G is smaller, its antenna length is larger.

For effectively coupling, antenna length must be $> \frac{1}{10}\lambda$. Thus, the antenna that works with 900 MHz will also work with 1,800 MHz.

$\lambda_{900} = \frac{c}{f} = \frac{3*10^8}{900*10^6} = 0.33m.$

Thus, min antenna length must be > $\frac{1}{10}\lambda$, i.e., > $\frac{1}{10}33.33\ cm$, i.e., > 3.33 cm.

This is the min size of the antenna needed.

3. An antenna is mounted at a height of 240 meters. It transmits a signal 100 GHz at 20 dB. Signals at 100 GHz travel line-of-sight (LOS). Find the max distance over which signal can be received by a ground-based receptor Modulation scheme is DSB SC AM

when there is: a) *No rain* b) *Light rain* c) *Heavy rain*

Minimum signal strength needed at receiver is 12 dB

You will find these useful:

line-of-sight radius is $d = \sqrt{15h}\ km$ where h is the height at which the antenna is placed above the local ground

Note that attenuation in air: $\frac{0.02dB}{km}$ attenuation in light rain: $\frac{0.1dB}{km}$ attenuation in heavy rain: $\frac{6dB}{km}$

[5.0] [C02] [BTL3]

LOS distance is $d = \sqrt{15h} = \sqrt{15 * 240} = 60km$

Without attention directed towards LOS limitation, signal transmission distance is $\frac{(Strength_{Tx} - Min_{strength_{Rx}})}{Attenuation}$ which

in case a) is $\frac{(20-12)}{0.02} = 400km$

in case b) is $\frac{(20-12)}{0.1} = 80km$

in case c) is $\frac{(20-12)}{6} = 1.33km$

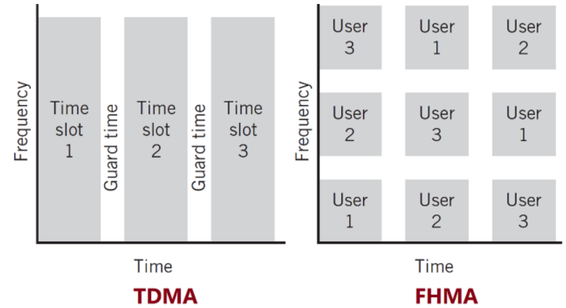
As max distance is $MIN(LOS, distance\ over\ which\ signal\ is\ strong\ enough\ to\ be\ received)$,

in case a) it is $MIN(60, 400) = 60km$

in case b) it is $MIN(60, 80) = 60km$

in case c) it is $MIN(60, 1.33) = 1.33km$

4. The figure shows usage of spectrum across time and frequency for TDMA and FHMA.



Spatial division multiple access (SDMA) is common in satellite-ground communications.

Does SDMA have guard bands across time axis?

Does SDMA have guard bands across frequency axis?

Which of the three, TDMA, FHMA or SDMA supports the max number of users? Justify.

[5.0] [C02] [BTL2]

Without guard bands, there will be local area interference. Thus, SDMA uses guard bands across both frequency and time axes.

SDMA, in addition to multiplexing in time and frequency domain, it also multiplexes in the spatial domain. Thus, both time slots, and frequency slots are reused in areas that do not overlap (transmitted power being too small or LOS limitation coming in beyond a certain distance from the transmitter). Thus, SDMA supports the max number of users as

5. Message signal is $m(t) = acos(2\pi f_m t)$. It *a*) frequency modulates and *b*) phase modulates a carrier $c(t) = A_c cos(2\pi f_c t)$. What is the modulated signal?

[5] [C03] [BTL2]

For PM: $\varphi(t) = k_p m(t) = k_p acos(2\pi f_m t)$.

Thus, $u_{PM}(t) = A_c cos\left[2\pi f_c t + k_p acos(2\pi f_m t)\right] = A_c cos\left[2\pi f_c t + \beta_p \cdot cos(2\pi f_m t)\right]$

For FM: $\varphi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau = 2\pi k_f \int_{-\infty}^{t} acos(2\pi f_m \tau)d\tau = \frac{a \cdot 2\pi k_f}{2\pi f_m} \cdot sin(2\pi f_m \tau)|_{-\infty}^{t}$

Thus, $\varphi(t) = \frac{k_f a}{f_m} \cdot sin(2\pi f_m \tau)$

known as modulation indices of PM & FM systems

Thus, $u_{FM}(t) = A_c cos\left[2\pi f_c t + \frac{k_f a}{f_m} \cdot sin(2\pi f_m t)\right] = A_c cos\left[2\pi f_c t + \beta_f \cdot sin(2\pi f_m \tau)\right]$

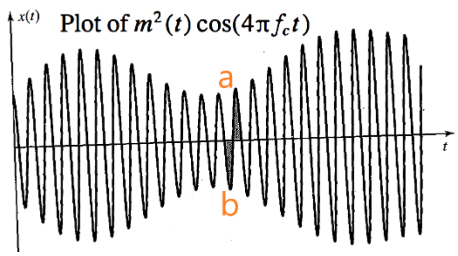
6. Derive the expression for power of DSB-SC AM message signal $A_c \cdot m(t) \cdot cos(2 \cdot \pi \cdot f_c \cdot t)$. Signal power is $P[u(t)] = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u^2(t)dt$

[7.5] [C01] [BTL3]

$$P[u(t)] = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u^2(t)dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} {A_c}^2 m^2(t) C^2(2\pi f_c t)dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{{A_c}^2}{2} m^2(t)[1 + cos(4\pi f_c t)]dt \ldots \because C^2(t) = \frac{[1+C(2T)]}{2}$$

$$= \frac{{A_c}^2}{2}\left[\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t)dt\right] + \frac{{A_c}^2}{2}\left[\cancel{\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t)cos(4\pi f_c t)dt}\right] \approx \frac{{A_c}^2}{2} P_m$$



$\frac{{A_c}^2}{2}\left[\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t)cos(4\pi f_c t)dt\right] \to 0$ because:

a. *The two areas 'a' and 'b' are nearly equal as because $f_m \ll f_c$, the two nearly cancel each other over the period and,*

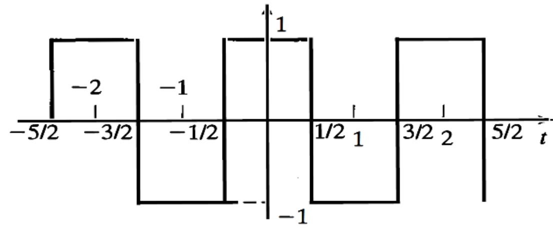
b. *The term is divided by $T \to \infty$.*

7. Derive the expression for fourier transform for the waveform shown. Plug in the values of n=0 and n=6 into the final equation and compute.

The period of the waveform is 2.

sin$x$ = $x - x^3/3! + x^5/5! \pm O(x^7)$ and cos$x$ = $1 - x^2/2! + x^4/4! \pm O(x^6)$

FT for continuous periodic waveform is $X(n) = \frac{1}{T}\int_T x(t)e^{-jn\pi t}d\tau$



[7.5] [C03] [BTL4]

$x(t)$ is a periodic signal with $T_0 = 2$.

Thus, $X(n) = \frac{1}{T}\int_{-\frac{1}{2}}^{\frac{3}{2}} x(t)e^{-jn\pi t}d\tau = \frac{1}{2}\int_{-1/2}^{1/2}(1)e^{-jn\pi t}d\tau + \frac{1}{2}\int_{1/2}^{3/2}(-1)e^{-jn\pi t}d\tau$

$= \frac{1}{-2jn\pi}\left[e^{-jn\pi/2} - e^{jn\pi/2}\right] - \frac{1}{-2jn\pi}\left[e^{-jn3\pi/2} - e^{-jn\pi/2}\right]$

$e^{-jn\pi/2} = C(-n\pi/2) + jS(-n\pi/2)$

$\therefore$ **1st term** is $\frac{1}{-2jn\pi}[\cancel{C(\theta)} - jS(\theta) - \cancel{C(\theta)} - jS(\theta)] = \frac{S(n\pi/2)}{n\pi}$

2nd term is $-\frac{1}{-2jn\pi}\left[e^{-jn3\pi/2} - e^{-jn\pi/2}\right] = \frac{1}{2jn\pi}\left[e^{-jn\pi}e^{-jn\pi/2} - e^{-jn\pi/2}\right]$

$= \frac{e^{-jn\pi}}{2jn\pi}\left[e^{-jn\pi/2} - e^{+jn\pi/2}\right] = \frac{e^{-jn\pi}}{2jn\pi}[-2jS(n\pi/2)] = \frac{-e^{-jn\pi}}{n\pi}[S(n\pi/2)]$

Thus, total term is $\frac{S(n\pi/2)}{n\pi} - \frac{e^{-jn\pi}[S(n\pi/2)]}{n\pi} = \frac{S(n\pi/2)}{n\pi}\left[1 - e^{-jn\pi}\right]$

$= \frac{S(n\pi/2)}{n\pi}\left[1 - \left(C(-n\pi) + jS(-n\pi)\right)\right] = \frac{S(n\pi/2)}{n\pi}[1 - C(n\pi)]$

If n is even then, 0

If n is 1 then, $\frac{2}{\pi}$ If n is 3 then, $\frac{-2}{2\cdot\pi}$, etc.

If n is 0 then, in the limits, $\frac{(n\pi/2-(n\pi/2)^2/2!+\cdots-\cdots)}{n\pi}[1 - 1 + (n\pi)^3/3! - \cdots + \cdots]$

$= [1 - n\pi/2/2! + \cdots - \cdots]\cdot[(n\pi)^2/3! - \cdots + \cdots]$ which, in the limits, is 0 because of 2nd term.

Thus, the value is 0 in both cases where n= 0 and n= 6.

8. Derive the mathematical expressions associated with demodulation of a Quadrature Carrier Multiplexed (QCM) signal. $u(t) = A \cdot m_1(t) \cdot cos(2 \cdot \pi \cdot f_c \cdot t) + A \cdot m_2(t) \cdot sin(2 \cdot \pi \cdot f_c \cdot t)$

[10] [C01] [BTL4]

Multiply $u(t) = A_c m_1(t)cos(2\pi f_c t) + A_c m_2(t)sin(2\pi f_c t)$ by $cos(2\pi f_c t)$.

$\therefore u(t)cos(2\pi f_c t) = A_c m_1(t)cos^2(2\pi f_c t) + A_c m_2(t)sin(2\pi f_c t)cos(2\pi f_c t)$

$= \frac{A_c}{2} m_1(t) \cdot [1 + cos(4\pi f_c t)] + \frac{A_c m_2(t)sin(4\pi f_c t)}{2}$

$= \frac{A_c}{2} m_1(t) + \frac{A_c}{2} m_1(t)cos(4\pi f_c t) + \frac{A_c}{2} m_2(t)\, sin(4\pi f_c t)$

Low pass filter will remove the highlighted term, leaving only the envelope of $m_1(t)$.

To recover $m_2(t)$, multiply $u(t) = A_c m_1(t)cos(2\pi f_c t) + A_c m_2(t)sin(2\pi f_c t)$, with $sin(2\pi f_c t)$.

Thus, $u(t)cos(2\pi f_c t) = A_c m_1(t)cos(2\pi f_c t)sin(2\pi f_c t) + A_c m_2(t)sin^2(2\pi f_c t)$

$= \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + A_c m_2(t)sin^2(2\pi f_c t)$

$= \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + A_c m_2(t)[1 - cos^2(2\pi f_c t)]$

$= \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + A_c m_2(t)\left[1 - \frac{1+cos(4\pi f_c t)}{2}\right] = \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + A_c m_2(t)\frac{1-cos(4\pi f_c t)}{2}$

$= \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + \frac{A_c}{2} m_2(t) - \frac{A_c}{2} m_2(t)\, cos(4\pi f_c t)$

A low pass filter eliminates the highlighted terms, leaving just the envelop of m_2.

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Course Outcome /Bloom's Taxonomy Level (BTL) Mark Distribution Table

CO	**Marks**	**BTL**	**Marks**
CO01	**22.5**	BTL 1	**0.0**
CO02	**15.0**	BTL 2	**15.0**
CO03	**12.5**	BTL 3	**17.5**
CO04	**0.0**	BTL 4	**17.5**
CO05		BTL 5	
CO06		BTL 6	

Bloom's Taxonomy Levels (attached for reference)

Level 1 – Remember | Level 2 – Understand | Level 3 – Apply | Level 4 – Analyze |

Level 5 – Evaluate | Level 6- Create