20CYS301

Digital Communications

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Amplitude Modulation [AM]

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I. DSB-SC AM

II. DSB-SC AM Demodulation

III. Conventional AM

IV. Conventional AM Demodulation

V. Quadrature Carrier Multiplexing

DSB-SC AM [1/7]

Amplitude Modulation is modulation of an analog carrier using analog data. Examples of such data include speech, music, images or videos.

We discuss:

1. how amplitude modulation can be constructed mathematically (*the analog message signals change the amplitude of an analog carrier*)
2. methods for demodulation of the carrier-modulated signal to recover the analog information

Treatment of system performance under various noise conditions & power efficiency will be considered after we discuss angle modulation.

$m(t)$ is the analog signal to be transmitted. It is assumed to be a lowpass signal with bandwidth W. Thus, if $|f| > W$, $M(f) = 0$.

Power of the signal is $\left\{P_m = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |m(t)|^2 dt\right\}$ x

Carrier is $c(t) = A_c cos(2\pi f_c t + \varphi_c)$

Modulation changes $m(t)$ from lowpass to bandpass, in the neighborhood of carrier frequency.

DSB-SC AM [2/7]

double side band - suppressed carrier amplitude modification

Types of AMs:

1. Double-sideband-suppressed-carrier DSB-SC AM
2. Conventional double-sideband AM
3. Single-sideband SSB AM
4. Vestigial-sideband VSB AM

types 4

multiplying message signal with carrier signal

DSB-SC AM: message signal is $m(t)$. $c(t) = m(t)A_c cos(2\pi f_c t + \varphi_c)$ is carrier signal. Ignore Tx-Rx synchronization issues. Thus, φ_c can be set to 0. Thus, Tx signal is: $u(t) = A_c m(t) cos(2\pi f_c t)$.

the efficiency of the modulated message will be 50%

Note: multiplication in time-domain is convolution in frequency domain.

Frequency Spectrum of DSB-SC AM Waveform:

Problem: Prove that $\Im[u(t)] = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$

Solution: $\Im[u(t)] = \Im[A_c m(t) cos(2\pi f_c t)] = \frac{A_c}{2}\Im\left[m(t)\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)\right]$

learn expansions

$= \frac{A_c}{2}\Im\left[m(t)e^{j2\pi f_c t}\right] + \frac{A_c}{2}\Im\left[m(t)e^{-j2\pi f_c t}\right]$

DSB-SC AM [3/7]

images (x)

$$= \frac{A_c}{2}\mathfrak{I}\left[m(t)e^{j2\pi f_c t}\right] + \frac{A_c}{2}\mathfrak{I}\left[m(t)e^{-j2\pi f_c t}\right]$$

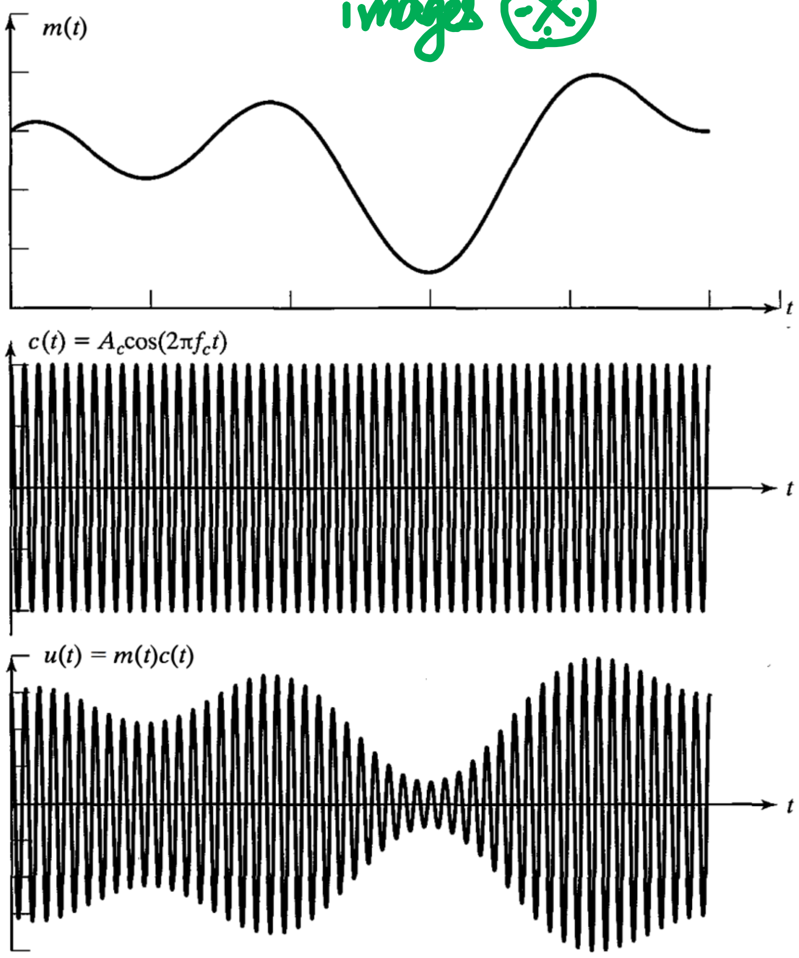
$$= \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{j2\pi f_c t}e^{-j2\pi f t}dt + \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{-j2\pi f_c t}e^{-j2\pi f t}dt$$

$$= \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{j2\pi f_c t}e^{-j2\pi f t}dt + \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{-j2\pi f_c t}e^{-j2\pi f t}dt$$

this is the fourier transformation of f-fc

$$= \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{-j2\pi (f-f_c) t}dt + \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{-j2\pi (f+f_c) t}dt$$

$$= \frac{A_c}{2}M(f-f_c) + \frac{A_c}{2}M(f+f_c) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$



The spectrum of the message signal is shifted by f_c. If the message signal width was W then it occupies $2W$ after amplitude modulation (AM). Thus, channel bandwidth needed is $B_c = 2W$.

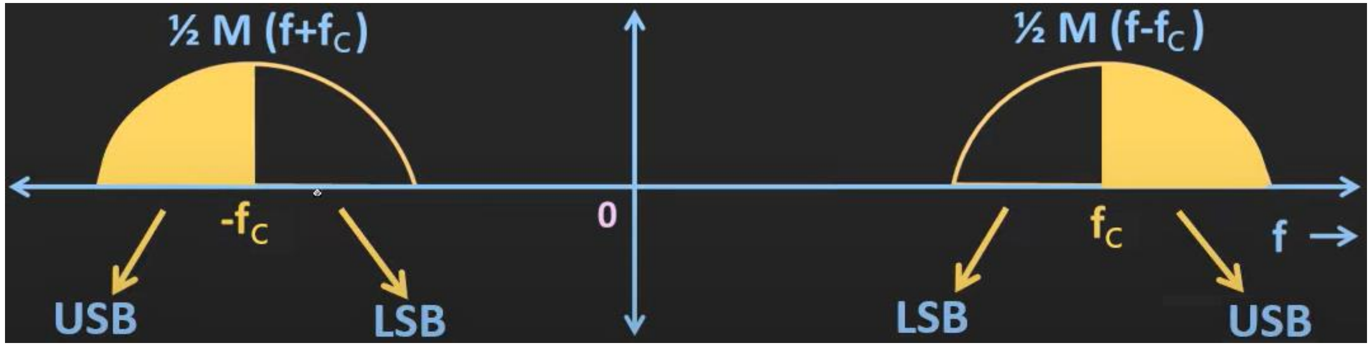
The frequency content of the modulated signal $u(t)$ in the frequency band $|f| > f_c$ is upper sideband of $U(f)$ and that in $|f| < f_c$ is lower sideband of $U(f)$. As $U(f)$ comprises both USB and LSB, it is called a double-sideband (DSB) AM signal.

fourier[A.f(t)] = A.(1/D).

this is wrong. switch lower and upper

DSB-SC AM [4/7]

$U_f = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$ upper side band (USB) & lower side band (LSB) of U_f.



Either of the two bands have all frequencies in $M(f)$. Message is shifted by f_c and channel bandwidth is $B_c = 2W$.

Note, the carrier is not transmitted. Thus, that bit of power is conserved.

the message has multiple frequencies ranging from 0 to f

Problem: How do we know from U_f or the chart that the carrier was not transmitted?

Solution: Had the carrier been transmitted, you will see $\delta(f \pm f_c)$ terms also in U_f.

Thus, the transmission is double side band (DSB) suppressed carrier (SC) amplitude modulated (AM) signal or, DSB SC AM signal, or, simply, DSBSC signal.

Problem: Express the product of 2 signals $u_t = A cos(2\pi f_m t) cos(2\pi f_c t)$ as sum of two signals.

Solution: As $A \cdot C[2\pi(f_m + f_c)t] = A \cdot C(2\pi f_m t)C(2\pi f_c t) - A \cdot S(2\pi f_m t)S(2\pi f_c t)$& $A \cdot C[2\pi(f_m - f_c)t] = A \cdot C(2\pi f_m t)C(2\pi f_c t) + A \cdot S(2\pi f_m t)S(2\pi f_c t)$,

$\frac{1}{2}A \cdot C[2\pi(f_m + f_c)t] + \frac{1}{2}A \cdot C[2\pi(f_m - f_c)t] = A \cdot cos(2\pi f_m t)cos(2\pi f_c t)$

DSB-SC AM [5/7]

here they are different in phases as well

Problem: In the previous problem, what if the signal is $A \cdot cos(2\pi f_m t + \theta)cos(2\pi f_c t + \varphi)$?

Solution: DIY!

this is in the form A.C(alpha).C(beta) = 1/2(C[a+b]+c[a-b]... its same as prev ques

Problem: Find fourier transform of $u_t = A \cdot cos(2\pi f_c t)$.

Solution: $\Im[u_t] = \Im[A \cdot C(2\pi f_c t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} A \cdot C(2\pi f_c t) e^{-j2\pi f_m t} dt$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A \cdot \frac{C(2\pi f_c t)+jS(2\pi f_c t)+C(2\pi f_c t)-jS(2\pi f_c t)}{2} e^{-j2\pi f_m t} dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A \cdot \left[\frac{C(2\pi f_c t)+jS(2\pi f_c t)}{2} + \frac{C(2\pi f_c t)-jS(2\pi f_c t)}{2}\right] e^{-j2\pi f_m t} dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A \cdot \left[\frac{e^{j(2\pi f_c)t}+e^{-j(2\pi f_c)t}}{2}\right] e^{-j2\pi f_m t} dt = \frac{A}{4\pi}\int_{-\infty}^{\infty} e^{j2\pi(f_c - f_m)t} + e^{-j2\pi(f_c+f_m)t} dt$$

$$= \frac{A}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j2\pi(f_m - f_c)t} dt\right] + \frac{A}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j2\pi(f_c + f_m)t} dt\right]$$

$$= \frac{A}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} 1 \cdot e^{-j2\pi(f_m - f_c)t} dt\right] + \frac{A}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} 1 \cdot e^{-j2\pi(f_c + f_m)t} dt\right]$$

$$= \frac{A}{2}\delta(f - (f_m - f_c)) + \frac{A}{2}\delta(f - (f_m + f_c))$$

X

Thus, the Fourier Transformation of $A \cdot cos(2\pi f_c t)$ are two delta functions of half the amplitude.

DSB-SC AM [6/7]

Problem: Message signal is $m(t) = sinc(10^4 t)$. Find DSB-SC-AM signal and its bandwidth if the carrier is a sinusoid of frequency 1 MHz, i.e., $c(t) = cos(2\pi 10^6 t)$.

Solution: $u(t) = m(t)c(t)$ If we can get the Fourier transform of $m(t)$ then, as we know the signal is DSB-SC-AM signal, bandwidth of the signal will be 2 times the bandwidth of $m(t)$.

$\therefore \Im[m(t)] = \Im[sinc(10^4 t)] = \frac{1}{|10^4|}\Pi\left[\frac{f}{10^4}\right] \because \Im[at] = \frac{1}{|a|}F\left[\frac{f}{a}\right]$ and $F[f]$ is rectangle function Π.

The rectangle function has a bandwidth of 10^4 Hz, centered at 0. Thus, DSB-SC-AM signal bandwidth is $10KHz$.

What about "Find DSB-SC-AM signal"?

Problem: What is the power of DSB-SC-AM signal?

Solution: $\mathrm{P}[u(t)] = \lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} u^2(t)dt = \lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} {A_c}^2 m^2(t) C^2(2\pi f_c t)dt$

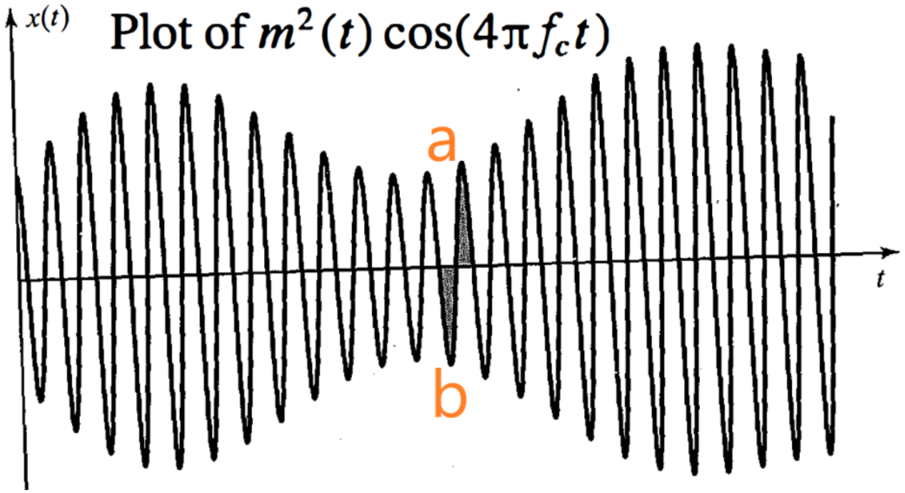
$$= \lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{{A_c}^2}{2} m^2(t)[1 + cos(4\pi f_c t)]dt \ldots \because C^2(t) = \frac{[1+C(2T)]}{2}$$

cutting it as it tends to 0

$$= \frac{{A_c}^2}{2}\left[\lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} m^2(t)dt\right] + \frac{{A_c}^2}{2}\left[\cancel{\lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} m^2(t)cos(4\pi f_c t)dt}\right] \approx \frac{{A_c}^2}{2} P_m$$

DSB-SC AM [7/7]

$$\frac{A_c{}^2}{2}\left[\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} m^2(t)cos(4\pi f_c t)dt\right] \to 0 \text{ because:}$$



when the message or m(t) is a sinusoid wave, this is the illustration

a. *The two areas 'a' and 'b' are nearly equal as because $f_m \ll f_c$, the two nearly cancel each other over the period and,*

b. *The term is divided by $T \to \infty$.*

Problem: What is the power of USB-SC-AM and LSB-SC-AM signal?

Solution: Do it yourself!

since its symmetric, it will be 50% of dsb sc am

When the message freq is 2% or super small, only then the a and b part will be almost similar... so the adjacent ones ie; a and b keep getting equalised with each other as the frequency decreases... and since t tends to infinity, then a and b keeps cancelling out each other, and the whole thing tends to 0

what happens when the carrier is not supressed? ie its USB AM and LSB AM signal?

When carrier is transmitted, the delta function is added to the wave and the final power is just +1

((Ac^2)/2) + 1 --> DSB AM final power
((Ac^2)/4) + 1 --> USB AM or LSB AM final power

((Ac^2)/2) --> DSB SC AM final power
((Ac^2)/4) --> USB SC AM or LSB SC AM final power

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DSB-SC AM Demodulation [1/2]

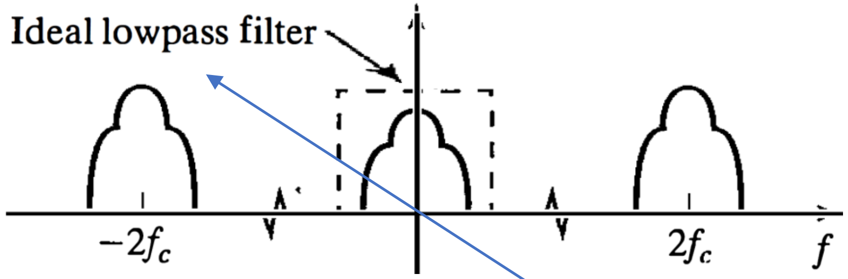
If a DSB-SC AM signal $u(t)$ is transmitted through an ideal channel then the received signal, $r(t) = u(t) = A_c m(t)\cos(2\pi f_c t)$.

Ideal channel: *it may be defined as an impulse function with a sampling frequency at least twice of the maximum frequency in* $m(t)$*.*

ie; if there's 10 MHz bandwidth, we have to sample at 20 MHz

To demodulate, we multiply $r(t)$ with a locally generated sinusoid.

Assume the receiver crystal operates at the same frequency as that of the carrier, f_c, but is out of sync in phase with the transmitter by φ.



Thus, $r(t)\cos(2\pi f_c t + \varphi) = [A_c m(t)\cos(2\pi f_c t)] \cdot \cos(2\pi f_c t + \varphi)$.

As $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$, we get:

$\frac{1}{2}A_c m(t)\cos(\cancel{4\pi f_c t + \varphi}) + \frac{1}{2}A_c m(t)\cos(\varphi)$ … the 1st term can be filtered out by a low pass filter.

this is being cut because the filter removes it

This is possible in a cost-effective manner only because signal bandwidth $W \ll f_c$.

The output of an ideal DSB-SC AM lowpass filter is $\frac{1}{2}A_c m(t)\cos(\varphi)$.

DSB-SC AM Demodulation [2/2]

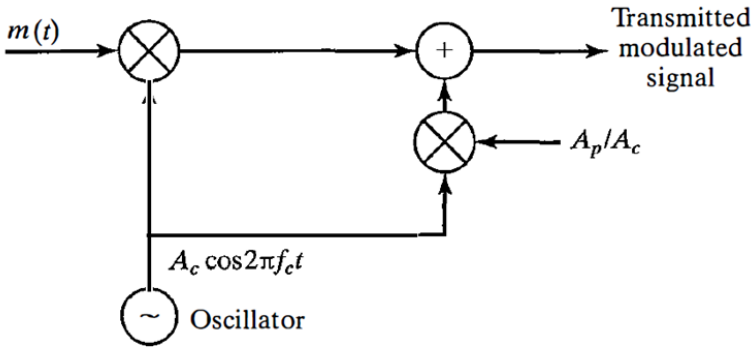
Note that the demodulated signal is weaker. Its power decreases by a factor of $cos^2(\varphi)$.

If Rx is $45°$ out of phase with the Tx then, the power of demodulated signal is $cos^2(45°) = (1/\sqrt{2})^2 = 50\%$ of the received signal.

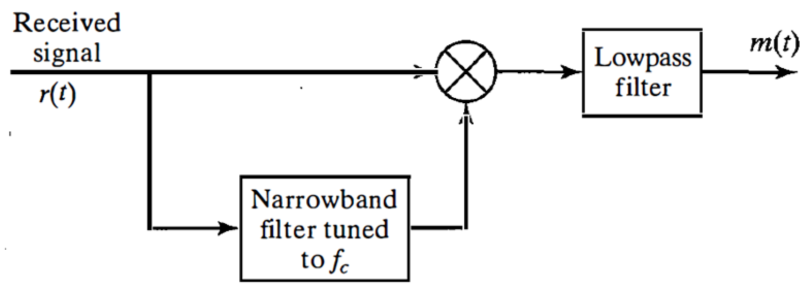
This can be addressed in two ways:

this is for synchronisation

1. *Adding a carrier component, called a pilot tone, to the carrier component whose magnitude, A_p (power is ${A_p}^2/2$), is much smaller than that of the modulated signal. This introduces a DC component and the signal is no longer a DSB-SC AM signal.*
2. *Generating a phase-locked sinusoidal carrier from the received signal $r(t)$ without the need of a pilot signal by using a phase-locked loop* (TBD *later*).



Addition of pilot tone to a DSB-AM signal



Use of a pilot tone to demodulate a DSB-AM signal

I. *DSB-SC AM*
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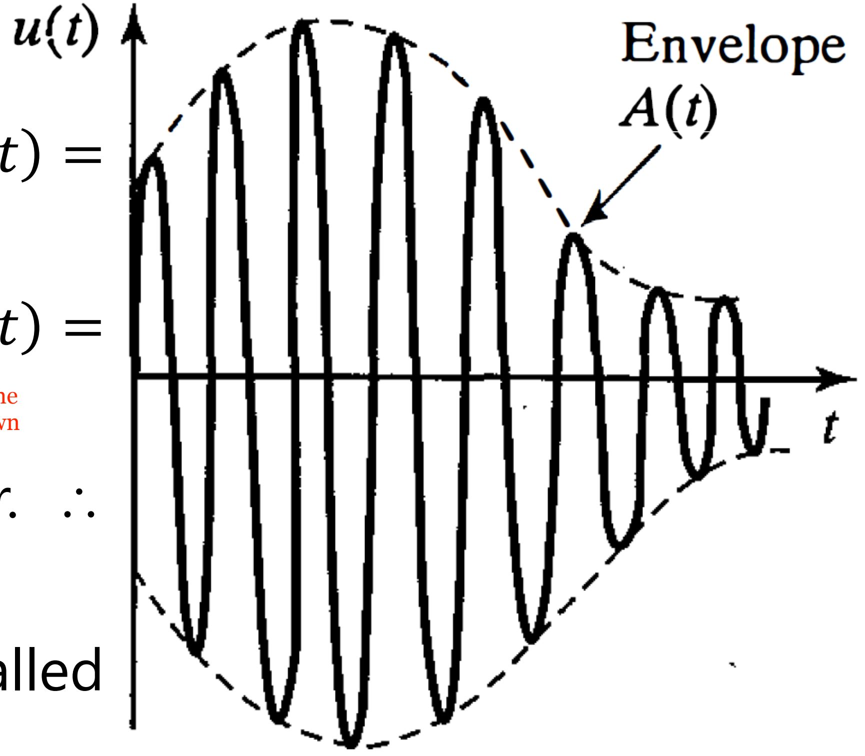
check formula

Conventional AM [1/4]



Recall that the received signal for DSB-SC AM was $r(t) = u(t) = A_c m(t)\cos(2\pi f_c t)$.

we cant generally express message as an equation

here there is no SC or surpressed carrier

In case of conventional AM, i.e., just AM, it is $r(t) = u(t) = A_c[1 + m(t)]\cos(2\pi f_c t)$ where $|m(t)| \le 1$.

if m(t) is more than one, then the downside of the graph will go positive and the upside will go down and be negative

If $|m(t)| \le 1$, then $A_c[1 + m(t)]$ is +ve & demodulation is easier. $\therefore$ commercial broadcasters use this kind of modulation.

If $|m(t)| > 1$, then $A_c[1 + m(t)]$ is –ve. This signal is called overmodulated & its demodulation is complex.

its overmodulated when the A(t) crosses the x axis

$m(t) = a m_n(t)$; $-1 \le m_n(t) \le 1$ and $0 \le a \le 1$. Thus, $r(t) = u(t) = A_c[1 + a m_n(t)]\cos(2\pi f_c t)$.

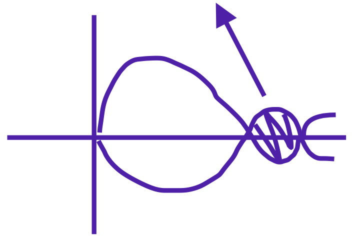
Spectrum of AM Signal: $\mathfrak{I}[u(t)] = U(f) = \mathfrak{I}[A_c[1 + a m_n(t)]\cos(2\pi f_c t)]$

$$= \mathfrak{I}[A_c a m_n(t)\cos(2\pi f_c t)] + \mathfrak{I}[A_c \cos(2\pi f_c t)]$$

this was computed before

$$= \frac{A_c a}{2}[M_n(f - f_c) + M_n(f + f_c)] + \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

this area is hard to recover if m(t) > 1



Note: Conventional AM is same as DSB-SC where $m(t)$ is replaced by $[1 + a m_n(t)]$

bandwidth of DSB-SC will be twice the message bandwidth
q) whats the bandwidth of usb sc/usb/dsb/lsb-sc/lsb

Conventional AM [2/4]

Like in the case of DSB-SC AM, conventional AM will occupy twice the bandwidth of the message signal $m(t)$.

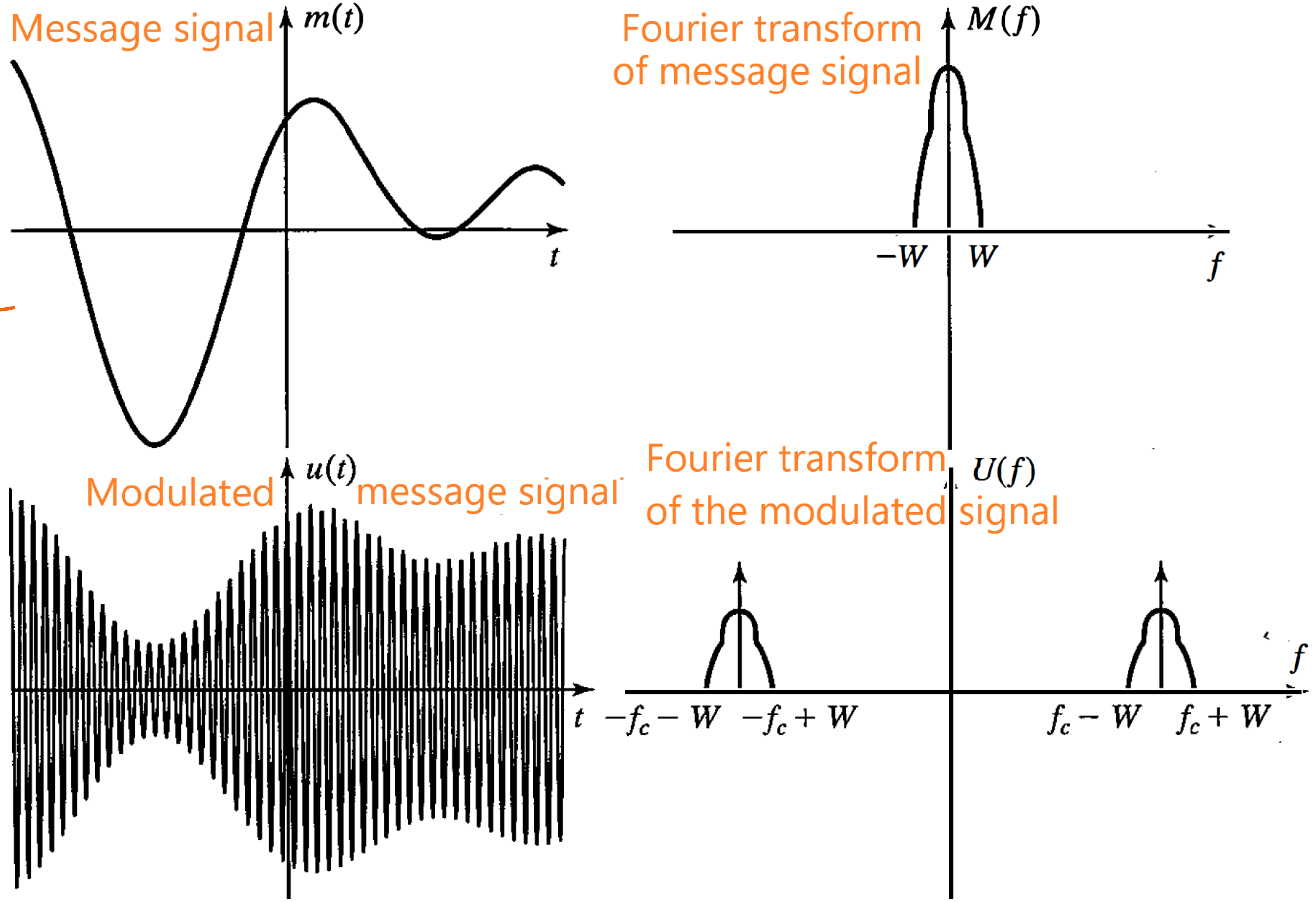
If message signal $m(t) = cos(2\pi f_m t)$; $\cancel{f_m \ll f_c}$
then, $u(t) = A_c[1 + a\,cos(2\pi f_m t)]\,cos(2\pi f_c t)$

$$= A_c cos(2\pi f_c t) + A_c a\,cos(2\pi f_m t)\,cos(2\pi f_c t)$$

$$u(t) = A_c cos(2\pi f_c t) + \frac{A_c a}{2} cos[2\pi(f_c - f_m)t] + \frac{A_c a}{2} cos[2\pi(f_c + f_m)t]$$

LSB component: $u_l(t) = \frac{A_c a}{2} cos[2\pi(f_c - f_m)t]$

USB component: $u_l(t) = \frac{A_c a}{2} cos[2\pi(f_c + f_m)t]$

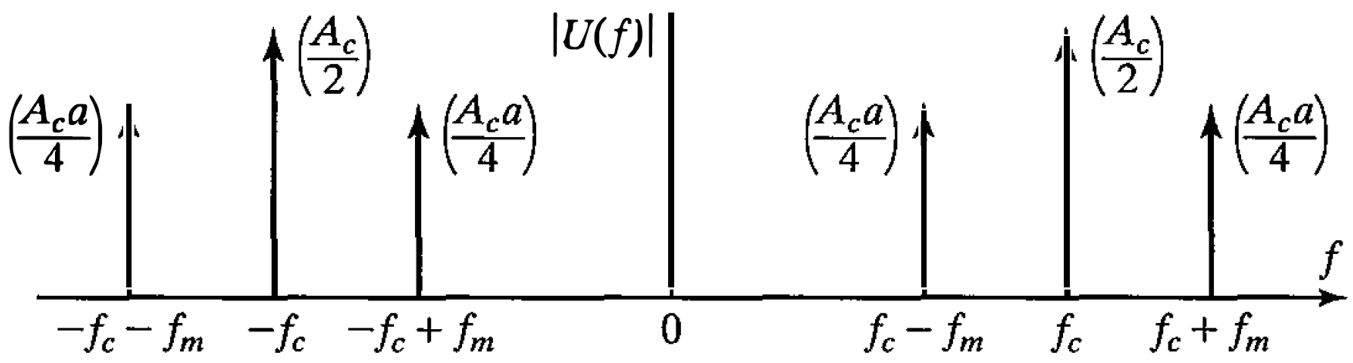


Conventional AM [3/4]

the message will have multiple frequencies

Problem: Compute power of conventional AM signal using $u(t)$ derived on last slide. Is it the same if $U(f)$ is used? **Solution:** DIY.

The answer is $A_c{}^2/2 + a^2 A_c{}^2/4$. Note that carrier power $\geq 2x$ of message signal.



Depicted in the figure is the spectrum of conventional AM signal, i.e., DSB AM signal.

Recall that received power of DSB-SC AM is $\frac{A_c{}^2}{2} P_m$. Compare this with received power of DSB AM of $\frac{A_c{}^2}{2} + \frac{a^2 A_c{}^2}{4}$ if the message signal is $cos(2\pi f_m t)$ or, $\frac{A_c{}^2}{2} + \frac{a^2 A_c{}^2}{2} P_m$ for a general case.

here we have the message signal and the disturbance coming from the carrier

$$\frac{Efficiency_{DSB\ AM}}{Efficiency_{DSB-SC\ AM}} = \frac{\cancel{\frac{A_c{}^2}{2}} + \frac{a^2 A_c{}^2}{2} P_m}{\frac{A_c{}^2}{2} P_m} = \cancel{\frac{1}{P_m}} \cancel{+} a^2 \dots \dots$$

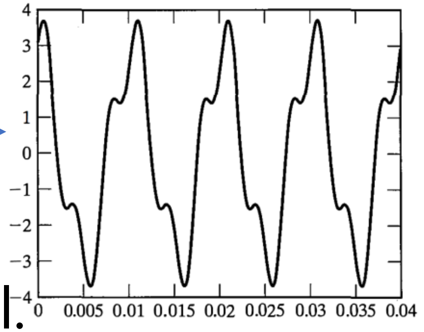
why did we cancel out the terms?

a has to be less than 1 or the signal will be overmodulated and Pm will be much larger than 1
so the whole thing will be super small

in SC there is no small a like numerator

As a is typically quite small, power efficiency of conventional demodulators is much smaller than that of DSB-SC AM transmissions. Still, the former is preferred due to ease in demodulation, which in this case, makes receiver demodulators materially cheaper.

Conventional AM [4/4]



Problem: Message signal $m(t) = 3\,cos(200\pi t) + sin(600\pi t)$.

this is in the form 2pi.f

Carrier signal $c(t) = cos(2 \times 10^5 t)$. Modulation index $a = 0.85$.

Determine the power of sidebands & carrier components of the modulated signal.

Solution: Step 1: find extremum of the signal. To do this, take derivative and set it to 0.

min and maximum

Thus, $\frac{d(m(t))}{dt} = 3 \cdot 200\pi\,[-sin(200\pi t)] + 600\pi \cdot cos(600\pi t) = 0.\ \ \therefore\ cos(600\pi t) = sin(200\pi t).$

As $sin(\theta) = cos\left(\frac{\pi}{2} - \theta\right)$, $cos(600\pi t) = cos\left(\frac{\pi}{2} - 200\pi t\right)\ \therefore\ cos^{-1}[cos(600\pi t)] = cos^{-1}\left[cos\left(\frac{\pi}{2} - 200\pi t\right)\right]$, i.e., $600\pi t = \frac{\pi}{2} - 200\pi t$, i.e., $t = \frac{1}{1600}$.

$$m(t)_{|t=\frac{1}{1600}} = 3\,cos\left(200\pi \cdot \frac{1}{1600}\right) + sin\left(600\pi \cdot \frac{1}{1600}\right) = 3.696.$$

Thus, maximum value of normalized message signal is $m_n(t) = \frac{3\,cos(200\pi t) + sin(600\pi t)}{3.696} = 0.82\,cos(200\pi t) + 0.27\,sin(600\pi t)$.

Power in sum of 2 sinusoids is the sum of powers of each sinusoid, i.e., $P_{m_n} = \frac{1}{2}[0.82^2 + 0.27^2] = 0.37$. Carrier power: $\frac{{A_c}^2}{2} = 0.5$ & sideband power: $\frac{{A_c}^2 \cdot a^2 \cdot P_{m_n}}{2} = \frac{1}{2} \times 0.85^2 \times 0.37 = 0.133$.

Conventional AM [explanatory note for last slide]

In these slides on slide "Conventional AM [4/4]", the numerical is indeed correct.

The integration of the cross term, 0.44 cos(200πt)*sin(600πt) indeed goes to zero when you integrate it.

∴ the statement "Power in sum of 2 sinusoids is the sum of powers of each sinusoid" is correct.

One can convince oneself of this using 3 methods:

- do it via actual integration or
- graphically assess it as on slide "DSB-SC AM [7/7]" or,
- by looking at the figure on slide "Conventional AM [3/4]"

In the exam, you do not need to show how the integration of the cross terms go to zero.

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Conventional AM Demodulation [1/1]

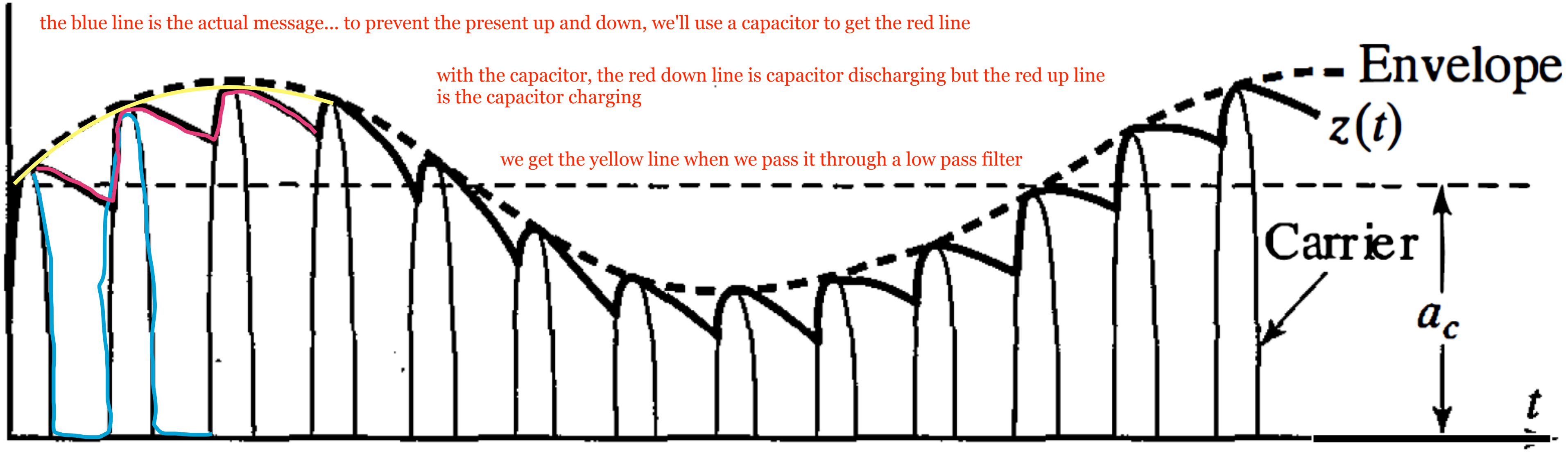
Demodulation of Conventional DSB-AM Signal: The demodulation is easier than that of DSB-SC AM as there is no need to synchronize demodulator.

As long as $|m(t)| < 1$, the envelope $1 + m(t) > 0$.

If we rectify the signal then the negative values are eliminated.

The rectified signal is then passed through a low-pass filter whose bandwidth matches that of the message signal.

Rectifier followed by lowpass filter is called *envelop detector.* The envelope then must be passed through a DC blocker to get the message signal.



usb

we use halfwave rectifier here

the dc blocker will be a high pass filter

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Quadrature Carrier Multiplexing [1/3]

Signal Multiplexing: If we wish to send ≥ 2 messages simultaneously then you could modulate each with a different carrier frequency.

Problem: Is this an example of time division or frequency division multiplexing? Justify.

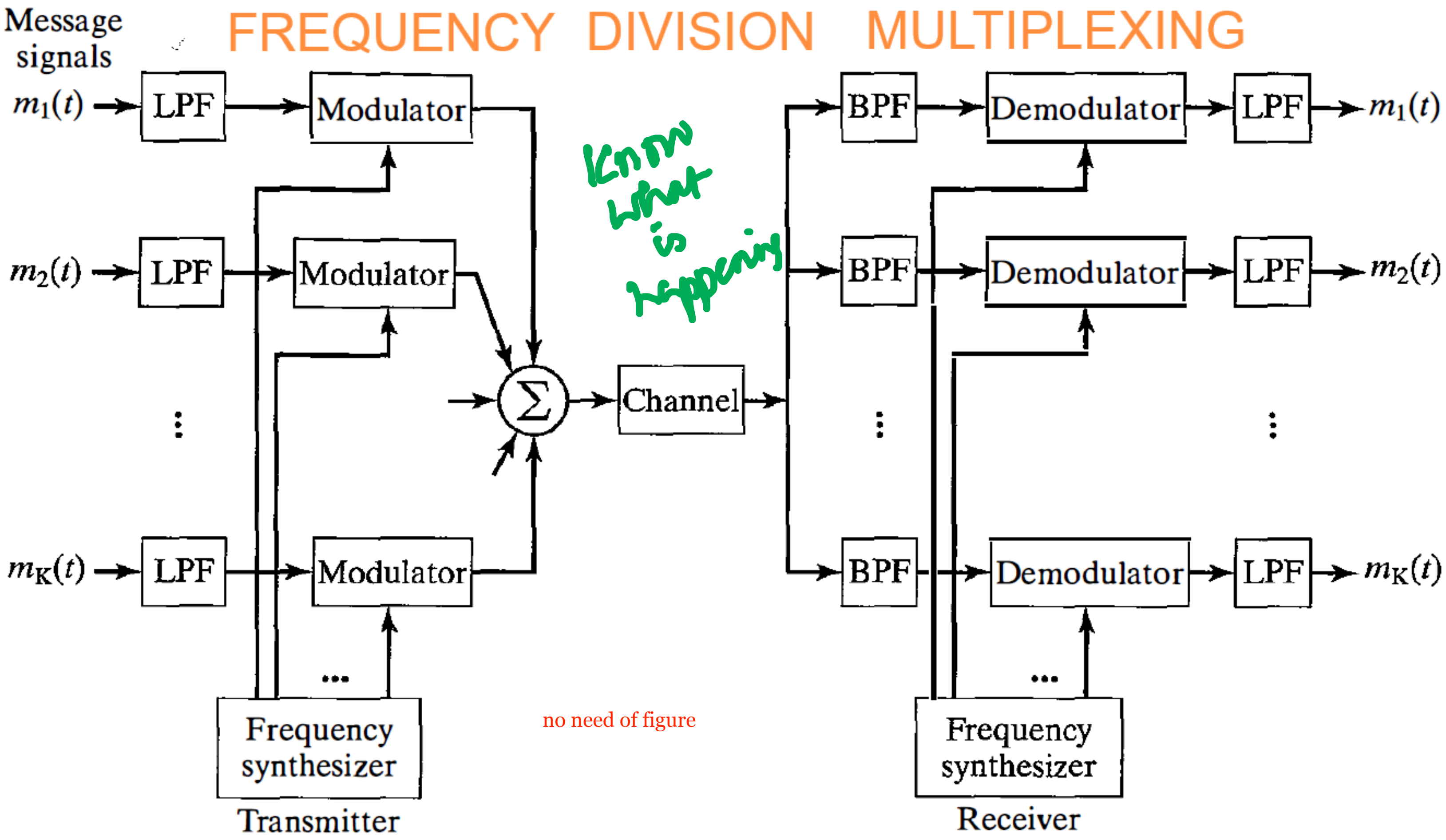
Solution: DIY.

FREQUENCY DIVISION MULTIPLEXING

Problem: What is the min distance between carrier frequencies for SSB? for DSB? Justify.

Solution: DIY.

Time division multiplexing is typically not used for transmitting analog information.



no need of figure

Quadrature Carrier Multiplexing [2/3]

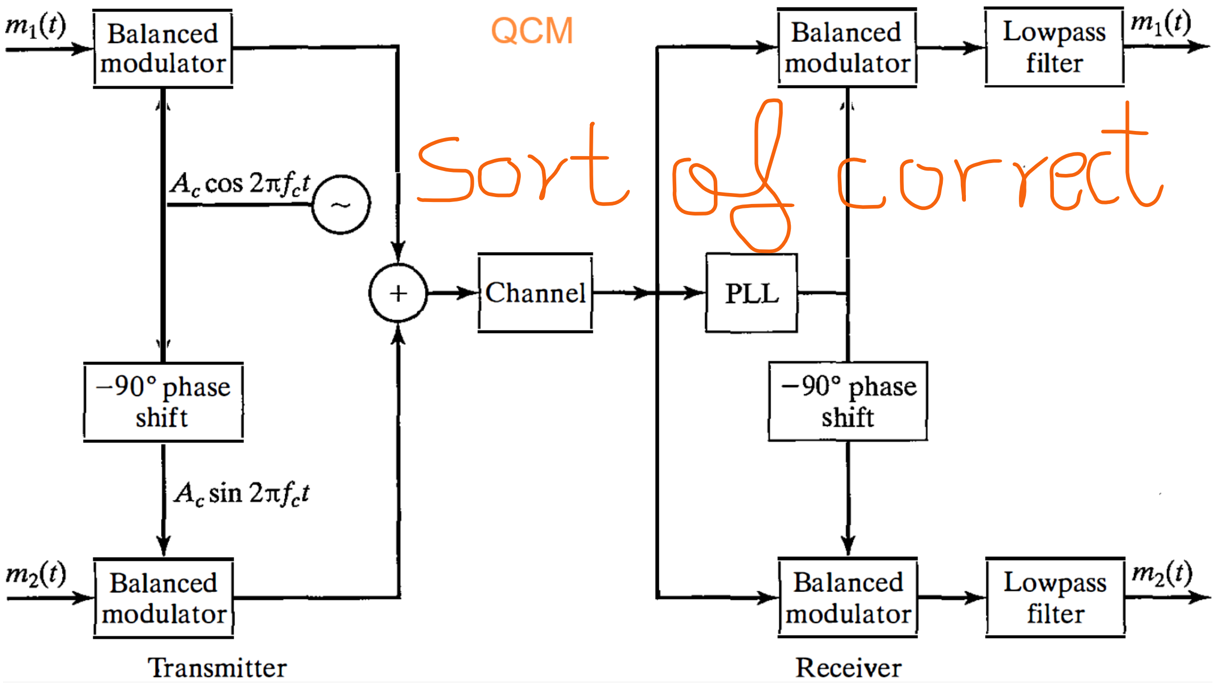
Quadrature Carrier Multiplexing (QCM): We can instead, send the data on two carriers: $A_c cos(2\pi f_c t)$ and $A_c sin(2\pi f_c t)$. Thus, $u(t) = A_c m_1(t) cos(2\pi f_c t) + A_c m_2(t) sin(2\pi f_c t)$.

Problem: Bandwidth efficiency of QCM is higher or lower or the same as that of SSB?

Solution: DIY

A Balanced Modulator is a (de)modulator that implements DSB SC modulation.

PLL is phase-locked loop.



Demodulation: multiply by $cos(2\pi f_c t)$.

$\therefore\ u(t)cos(2\pi f_c t) = A_c m_1(t) cos^2(2\pi f_c t) + A_c m_2(t) sin(2\pi f_c t) cos(2\pi f_c t)$

$$= \frac{A_c}{2} m_1(t) \cdot [1 + cos(4\pi f_c t)] + \frac{A_c m_2(t) sin(4\pi f_c t)}{2}$$

$$= \frac{A_c}{2} m_1(t) + \frac{A_c}{2} m_1(t) cos(4\pi f_c t) + \frac{A_c}{2} m_2(t)\, sin(4\pi f_c t)$$

Quadrature Carrier Multiplexing [3/3]

Low pass filter will remove the highlighted term, leaving only the envelope of $m_1(t)$.

Problem: How do we recover $m_2(t)$?

Solution: Multiply $u(t) = A_c m_1(t) cos(2\pi f_c t) + A_c m_2(t) sin(2\pi f_c t)$ with $sin(2\pi f_c t)$.

Thus, $u(t) cos(2\pi f_c t) = A_c m_1(t) cos(2\pi f_c t) sin(2\pi f_c t) + A_c m_2(t) sin^2(2\pi f_c t)$

$$= \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + A_c m_2(t) sin^2(2\pi f_c t)$$

$$= \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + A_c m_2(t)[1 - cos^2(2\pi f_c t)]$$

$$= \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + A_c m_2(t) \left[1 - \frac{1+cos(4\pi f_c t)}{2}\right] = \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + A_c m_2(t) \frac{1-cos(4\pi f_c t)}{2}$$

$$= \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + \frac{A_c}{2} m_2(t) - \frac{A_c}{2} m_2(t)\, cos(4\pi f_c t)$$

A low pass filter eliminates the highlighted terms, leaving just the envelop of m_2.