20CYS301

Digital Communications

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Angle Modulation, i.e., Phase and Frequency Modulation

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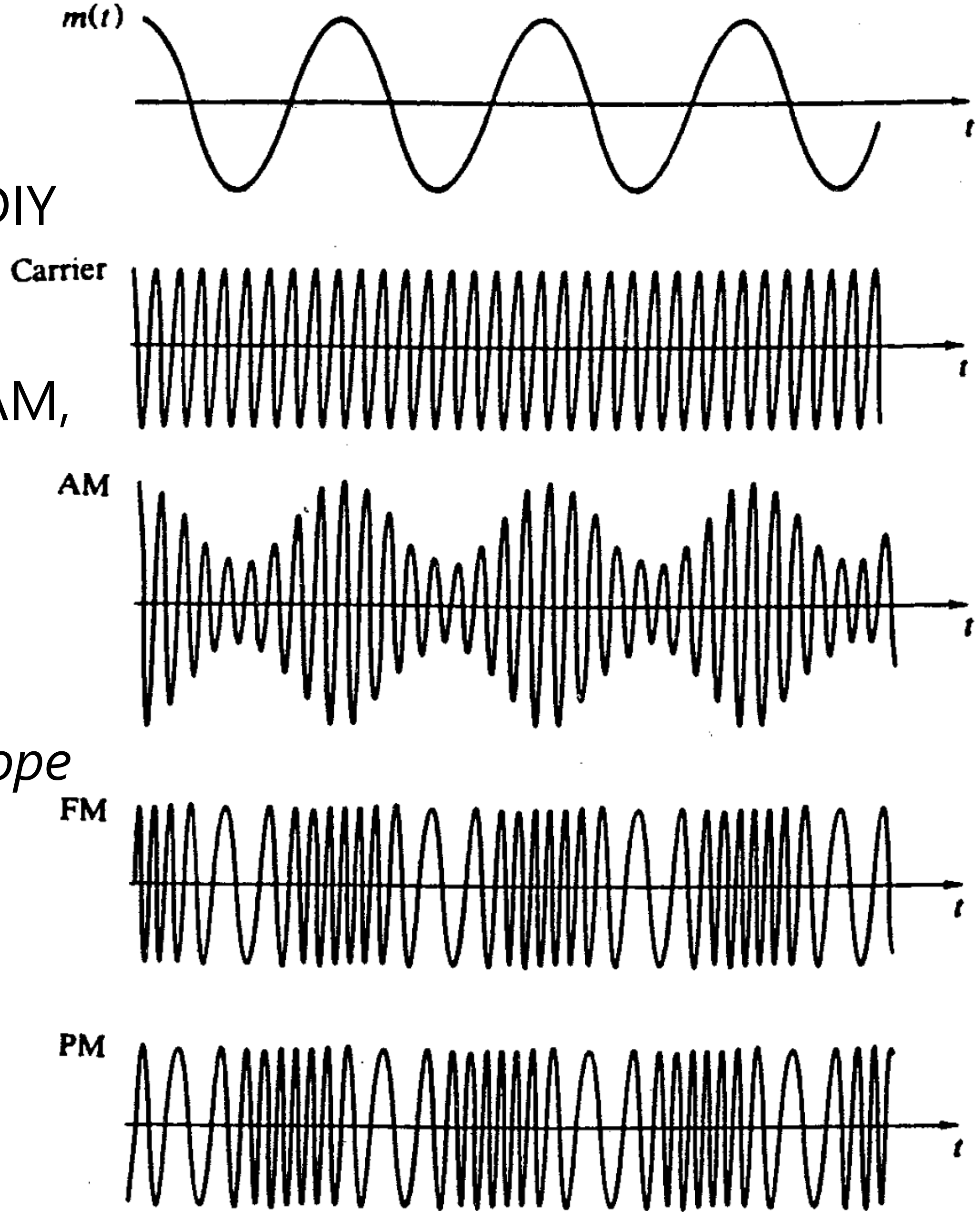
I. Similarities between FM & PM
II. *Angle Modulation*
III. *Narrowband Angle Modulation*
IV. *Spectral Characteristics of Angle Modulated Signals*
V. *Angle Modulation by an Arbitrary Message Signal*
VI. *Angle Modulator Implementation*
VII. *Angle Demodulator Implementation*

Similarities between FM & PM [1/1]

Problem: Which one is AM, FM, PM? Justify. **Solution:** DIY



FM and PM are computationally more complex than AM, but they have:

- *more bandwidth efficiency over AM*
- *greater noise immunity*
- *less impacted by amplifier non-linearities as their envelope is constant*
- *greater power efficiency*

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Angle Modulation [1/3]

General representation of an angle-modulated signal: $u(t) = A_c \cos[\underbrace{2\pi f_c t + \varphi(t)}_{\theta(t) = \omega_c t + \varphi(t)}]$ ... (1)

$\therefore \frac{d\theta(t)}{dt} = \omega_c + \frac{d\varphi(t)}{dt}$. $\therefore$ rate of deviation of $\angle$ from base is $\left[\frac{d\theta(t)}{dt} - \omega_c\right] = [\omega_i - \omega_c] = \frac{d\varphi(t)}{dt}$ ... (2)

Phase deviation is simply $\varphi(t)$ ... (3)

For phase modulation (PM): phase is proportional to message signal $\therefore \varphi(t) = k_p m(t)$ ... (4)

From (2) & (4), $\omega_i = \omega_c + \frac{d\varphi(t)}{dt} \therefore \omega_i = \omega_c + \frac{d[k_p m(t)]}{dt}$ ... (5)

$\therefore \omega_i = \omega_c + k_p \frac{dm(t)}{dt}$ ... (6)

Also, we note that by putting (4) in (1), $u_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$ ... (7)

$\therefore$ We say that in PM, we vary the phase $\angle$ with the message.

Angle Modulation [2/3]

For FM: $u(t) = A_c \cos\left(2\pi \int_0^t f(\tau)\, d\tau\right) = A_c \cos\left(2\pi \int_0^t [f_c + k_f m(\tau)]\, d\tau\right)$

Thus, $u_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau\right]$ … (8)

$\therefore$ We say that in FM, we vary the phase $\angle$ with the respect to the integral of the message.

Problem: Message signal is $m(t) = a\cos(2\pi f_m t)$. It *a*) frequency modulates and *b*) phase modulates a carrier $c(t) = A_c \cos(2\pi f_c t)$. What is the modulated signal?

Solution: For PM: $\varphi(t) = k_p m(t) = k_p a\cos(2\pi f_m t)$.

Thus, $u_{PM}(t) = A_c \cos[2\pi f_c t + k_p a \cos(2\pi f_m t)] = A_c \cos[2\pi f_c t + \beta_p \cdot \cos(2\pi f_m t)]$

For FM: $\varphi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau = 2\pi k_f \int_{-\infty}^{t} a\cos(2\pi f_m \tau) d\tau = \frac{a \cdot 2\pi k_f}{2\pi f_m} \cdot \sin(2\pi f_m \tau)\big|_{-\infty}^{t}$

Thus, $\varphi(t) = \frac{k_f a}{f_m} \cdot \sin(2\pi f_m \tau)$

known as modulation indices of PM & FM systems

Thus, $u_{FM}(t) = A_c \cos\left[2\pi f_c t + \frac{k_f a}{f_m} \cdot \sin(2\pi f_m t)\right] = A_c \cos[2\pi f_c t + \beta_f \cdot \sin(2\pi f_m \tau)]$

Angle Modulation [3/3]

For sinusoidal message signals: $\beta_p = k_p a$ and $\beta_f = \frac{k_f a}{f_m}$

For non-sinusoidal message signals:

$\beta_p = k_p \cdot \max\{|m(t)|\}$ and

$\beta_f = \frac{k_f \cdot \max\{|m(t)|\}}{W}$ *where W is the bandwidth of the message signal*

β_p is max possible phase deviation &

β_f is [max possible frequency deviation ÷ message bandwidth]

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Narrowband Angle Modulation [1/2]

Recall angle-modulated signal is: $u(t) = A_c \cos[2\pi f_c t + \varphi(t)]$ … (1)

$= A_c\, cos(2\pi f_c t) cos\varphi(t) - A_c\, sin(2\pi f_c t) sin\varphi(t)$ … (2)

$u(t) \approx A_c\, cos(2\pi f_c t) - A_c \varphi(t)\, sin(2\pi f_c t)$ *if modulation index is small* … (3)

Recall that for Convention AM, $u(t) = A_c[1 + m(t)]\, cos(2\pi f_c t)$

i.e., $u(t) = A_c\, cos(2\pi f_c t) + A_c m(t)\, cos(2\pi f_c t)$ … (4)

Thus, comparing (3) & (4), if modulation index is small, i.e., for a narrow-band angle modulation, the angle-modulated signal is similar to Conventional AM signal but with message signal modulated on a sine instead of a cosine carrier.

Problem: Is bandwidth of message signal for narrow band the same, or much larger or much smaller than that of Conventional AM signal?

Solution: We know for PM: $\varphi(t) = k_p a cos(2\pi f_m t)$ … (5)

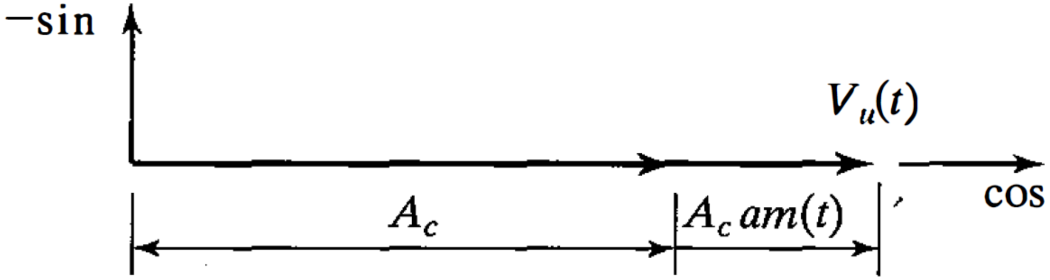
some const [not a fn of fm]

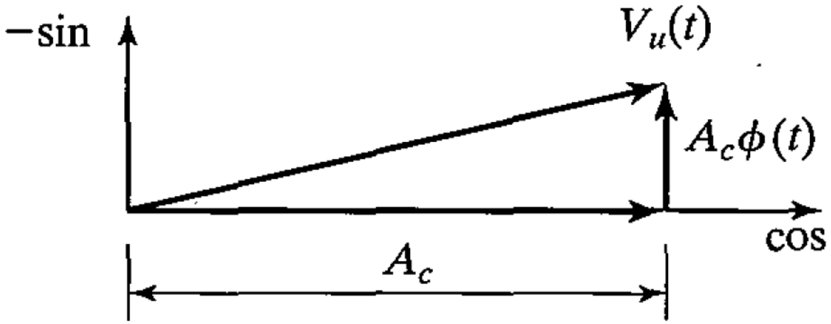
$m(t)$ for sinusoidal signal is $acos(2\pi f_m t)$. $\therefore$ for PM, the bandwidths are identical.

for FM: $\varphi(t) = \frac{k_f a}{f_m} \cdot sin(2\pi f_m \tau)$ … (6) Thus, from (5) & (6), the two bandwidths are the same.

Narrowband Angle Modulation [2/2]

1. Phasor diagrams for these 2 signals are shown.
2. Narrowband angle-modulation scheme has << amplitude variations than Conventional AM. Ideally an angle-modulation system has constant amplitude and, hence, there should be no amplitude variations in the phasor-diagram representation of the system. These slight variations are due to the first-order approximation.
3. Narrowband angle-modulation method does not provide better noise immunity than a conventional AM system (*to be seen later*). ∴ narrowband angle modulation is seldom used in practice for communication purposes.
4. However, these systems can be used as an intermediate stage for the generation of wideband angle-modulated signals (*to be seen later*).





Phasor Diagram for **A)** Conventional AM **B)** Narrowband Angle Modulation

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Spectral Characteristics of $\angle$-Modulated Signals [1/9]

To study spectral characteristics of $\angle$-modulated] signals, we study those for:

1. *Simple modulating signals*
2. *Use approximations*
3. *Use 1. & 2. to generalize for complicated message signals*

Thus, we will first study spectral characteristics of $\angle$-modulated signals where the modulating signal is a sinusoidal signal.

As seen on slide 6, : $u(t) = \begin{cases} A_c \cos\left[2\pi f_c t + \beta_p cos(2\pi f_m t)\right] \text{ ... for } PM \\ A_c \cos\left[2\pi f_c t + \beta_f sin(2\pi f_m t)\right] \text{ ... for } FM \end{cases}$ ... (1)

For FM, $u(t) = Re\left(A_c \cdot e^{\left[2\pi f_c t + \beta_f sin(2\pi f_m t)\right]}\right) = Re\left[A_c \cdot e^{j2\pi f_c t} \cdot e^{j\beta_f sin(2\pi f_m t)}\right]$ ... (2)

$sin(2\pi f_m t)$ is a periodic signal with period $T_m = \frac{1}{f_m}$. The same is true for $e^{j\beta_f sin(2\pi f_m t)}$.

$\therefore$ We can use a Fourier series representation.

Spectral Characteristics of ∠-Modulated Signals [2/9]

$$\mathfrak{F}\left[e^{j\beta_f sin(2\pi f_m t)}\right] = \frac{1}{T_m}\int_0^{T_m} e^{j\beta_f sin(2\pi f_m t)} e^{-jn2\pi f_m t} dt$$

$= f_m \int_0^{\frac{1}{f_m}} e^{j\beta_f sin(2\pi f_m t)} e^{-jn2\pi f_m t} dt$ If $\boldsymbol{2\pi f_m t = u}$ then

$$\mathfrak{F}\left[e^{j\beta_f sin(u)}\right] = \frac{1}{T_m}\int_0^{T_m} e^{j\beta_f sinu} e^{-jnu}\left(\frac{du}{2\pi f_m}\right) = \frac{1}{\cancel{T_m}}\int_0^{T_m} e^{j\beta_f sinu} e^{-jnu}\left(\frac{\cancel{T_m} du}{2\pi}\right)$$

$= \frac{1}{2\pi}\int_0^{T_m} e^{j\beta_f sinu} e^{-jnu} du$ … (3) Thus, $\mathfrak{F}\left[e^{j\beta_f sin(u)}\right] = \frac{1}{2\pi}\int_0^{2\pi} e^{j(\beta_f sinu - nu)} du$

This is Bessel function of the 1st kind of order n, $J_n(\beta_f)$ … (4)

Thus, the signal, $e^{j\beta_f sin(u)} = e^{j\beta_f sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta_f) e^{jn2\pi f_m t}$ … (5)

Thus, from (2) & (5), $u(t) = Re\left[A_c \cdot e^{j2\pi f_c t} \cdot e^{j\beta_f sin(2\pi f_m t)}\right]$

Thus, $u(t) = Re\left[A_c \cdot e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} J_n(\beta_f) e^{jn2\pi f_m t}\right]$ … (6)

And $\because$ $e^{j2\pi f_c t}$ is constant with respect to the summation term:

$$u(t) = Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta_f) e^{jn2\pi f_m t} e^{j2\pi f_c t}\right] = Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta_f) e^{j2\pi(nf_m + f_c)t}\right]$$

Spectral Characteristics of $\angle$-Modulated Signals [3/9]

Thus, $u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta_f) \cos[2\pi(f_c + nf_m)t]$ … (7)

Problem: ~~Derive the Bessel function equation for PM just as it has been done for FM above.~~

Solution: ~~DYI~~.

(7) shows that for $\angle$-modulated signals, even if the modulating signal is a single sinusoid of frequency, f_m, the $\angle$-modulated signal has ∞ frequencies: $f_c \pm f_m, f_c \pm 2f_m, \dots, f_c \pm nf_m \pm \cdots$.

Thus, the bandwidth of $\angle$-modulated signals is ∞.

Effective bandwidth of modulated signal: As discussed earlier, $J_n(\beta_f) = \mathfrak{F}[e^{j\beta_f sin(u)}] = \frac{1}{2\pi}\int_0^{2\pi} e^{j(\beta_f sinu - nu)} du$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[1 + j(\beta_f sinu - nu) - \frac{(\beta_f sinu - nu)^2}{2!} - j\frac{(\beta_f sinu - nu)^3}{3!} + \frac{(\beta_f sinu - nu)^4}{4!} - \cdots\right] du$$

Here, for small values of β_f, i.e., small values of $\frac{k_f a}{f_m}$ for sinusoid modulating signals or for small values of $\frac{k_f \cdot \max\{|m(t)|\}}{W}$ for general modulating signals, $J_n(\beta_f) = \frac{{\beta_f}^n}{2^n n!}$ … (8) … *stated without proof*

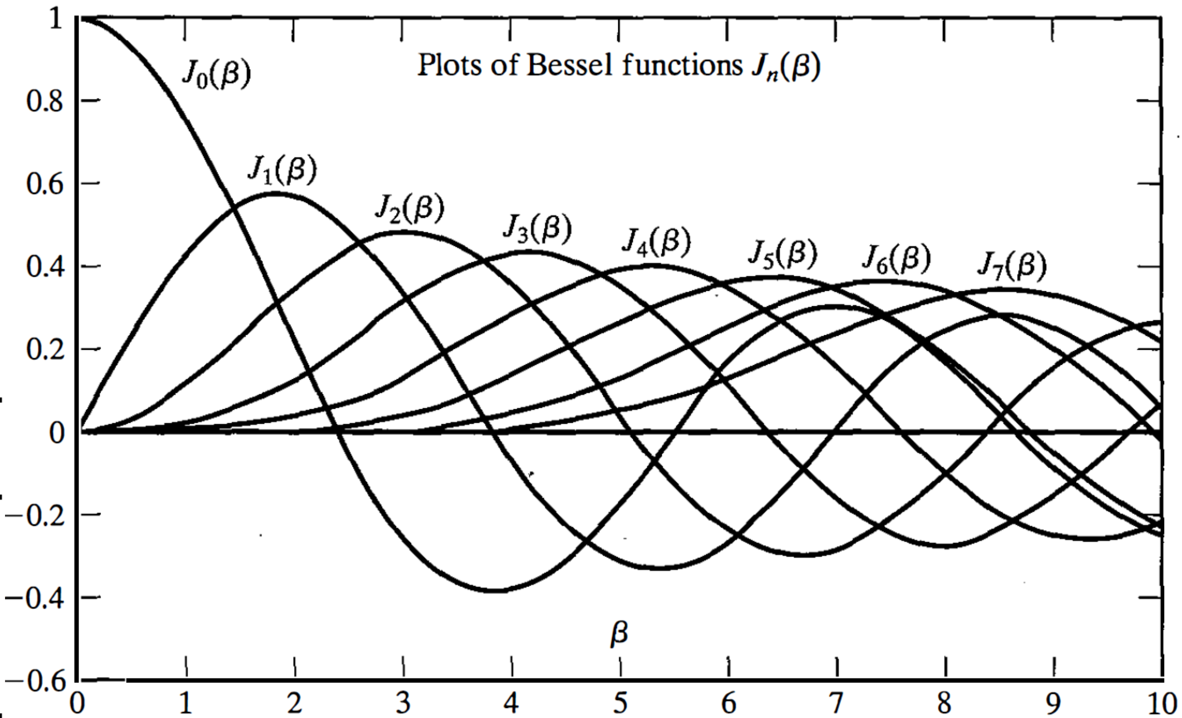
Spectral Characteristics of ∠-Modulated Signals [4/9]

If β_f is small, ${\beta_f}^2 \ll \beta_f$. Thus, only the first sideband corresponding to $n = 1$ is important.

A property of Bessel function: $J_{-n}(\beta_f) = \begin{cases} J_n(\beta_f) & ; for\ even\ n \\ -J_n(\beta_f) & ; for\ odd\ n \end{cases}$ … (9) … *stated without proof*

REQUIRED NUMBER OF HARMONICS IN FM

Power (%)	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 5$	$\beta = 8$	$\beta = 10$	$\beta = 15$
80	—	1	2	4	7	9	14
90	1	1	2	5	8	10	15
98	1	2	3	6	9	11	16



Spectral Characteristics of $\angle$-Modulated Signals [5/9]

Problem: $c(t) = 10\cos(2\pi f_c t)$ is carrier signal. $m(t) = \cos(20\pi t)$ is message signal. $k_f = 50$. *a*) Write expression for the modulated signal, $u(t)$. *b*) Write it as Bessel function. *c*) How many harmonics should be selected to contain 98% of the modulated signal power?

Solution: Carrier power is $P_c = \frac{{A_c}^2}{2} = \frac{10^2}{2} = 50$. As the signal is frequency-modulated:

$$u(t) = 10\,cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{\tau} m(\tau)\,dt\right], = 10\,cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} cos(20\pi\tau)\,d\tau\right]$$

$$= 10cos\left[2\pi f_c t + \frac{2\pi k_f}{20\pi} \cdot sin(20\pi\tau)\Big|_{-\infty}^{t}\right] = 10cos\left[2\pi f_c t + \frac{2\pi \cdot 50}{20\pi} \cdot sin(20\pi t)\right]$$

Thus, $u(t) = 10cos[2\pi f_c t + 5sin(20\pi t)]$ *where* $\beta_f = \frac{k_f \cdot \max\{|m(t)|\}}{W} = 5.$ … (1)

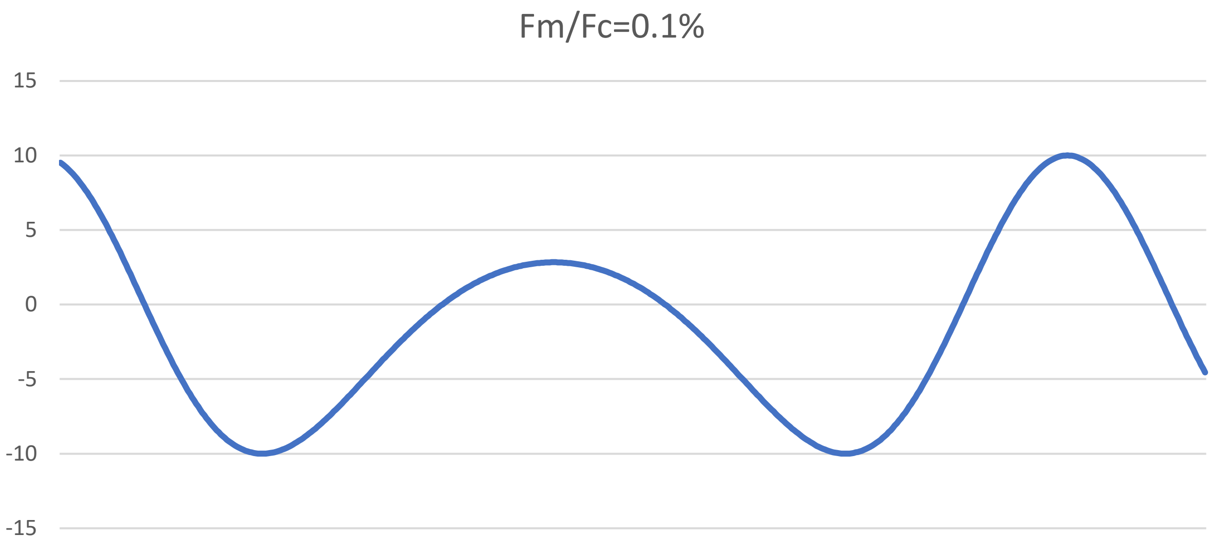
As $m(t) = \cos(20\pi t) = \cos(2\pi f_m t)$, $f_m = 10$ … (2) As $c(t) = 10\cos(2\pi f_c t)$, $A_c = 10$ … (3)

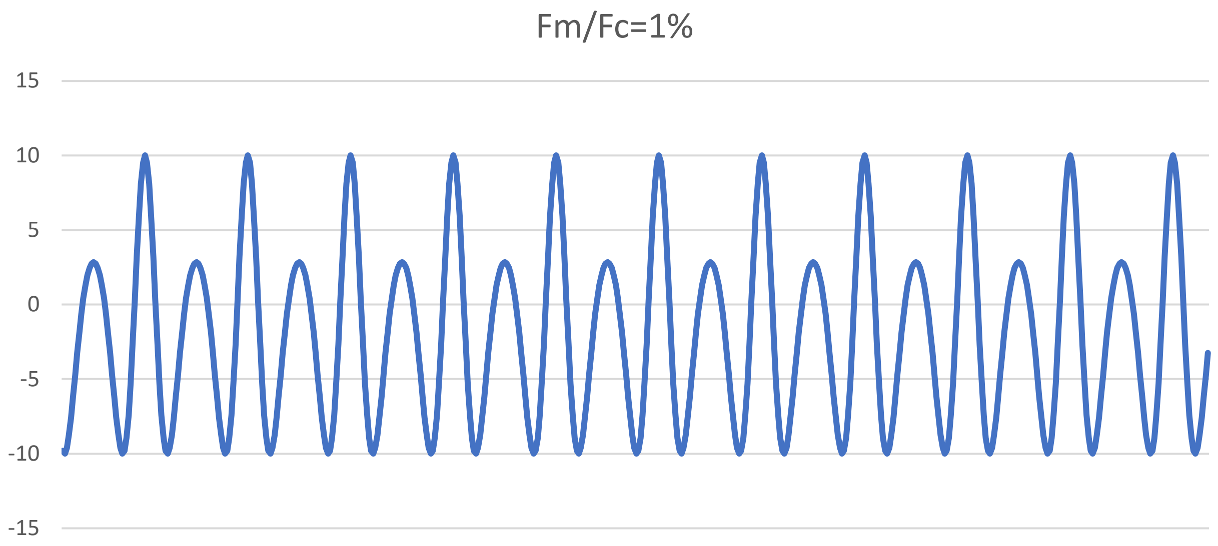
To find harmonics, rewrite $u(t)$ as a Bessel function. Thus:

$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta_f)\cos[2\pi(f_c + nf_m)t]$ … (4)

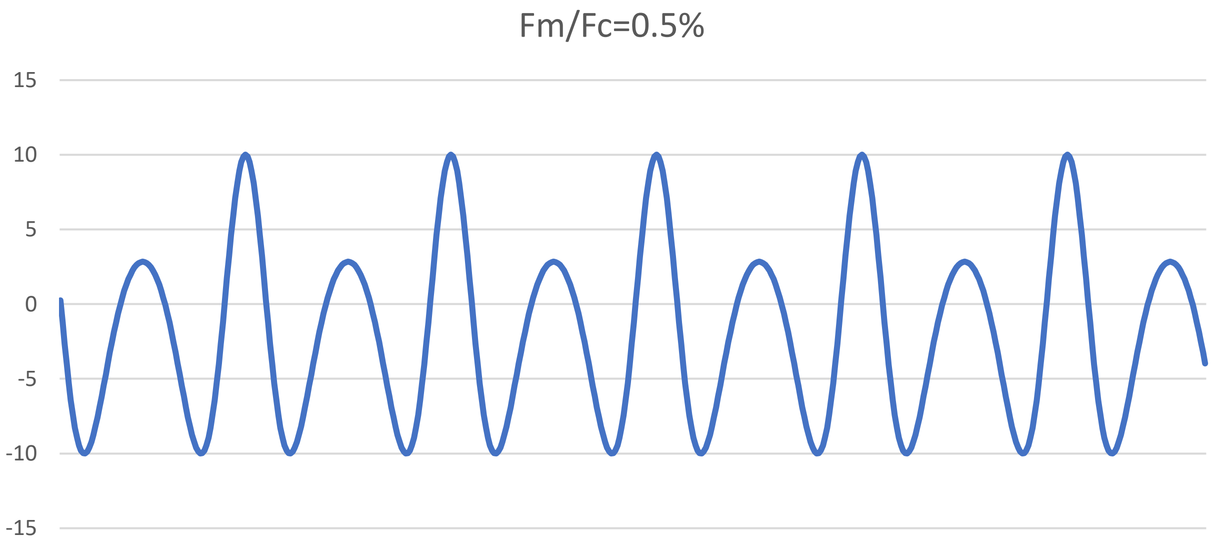
Put (1) to (3) in (4) $\therefore$ $u(t) = \sum_{n=-\infty}^{\infty} 10 J_n(5)\cos[2\pi(f_c + 10n)t]$ … (5)

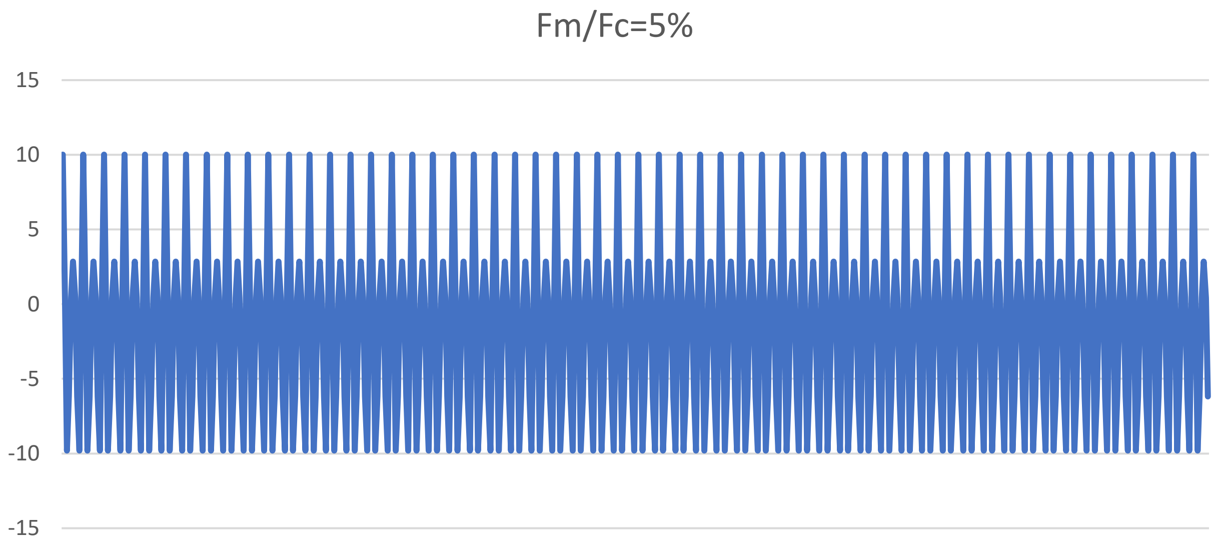
Spectral Characteristics of ∠-Modulated Signals [6/9]





shows results from previous slide for various modulation ratios





Spectral Characteristics of $\angle$-Modulated Signals [7/9]

Thus, the FM-modulated signal is concentrated at $f_c + 10n$ (*just a spectral property of modulated signal*).

Modulated signal power is $P_u = \frac{1}{T}\int_{-T/2}^{T/2} u^2(t)\, dt = \sum_{n=-\infty}^{\infty}(10J_n(5)\, cos[2\pi(f_c + 10n)t])^2\, dt$

This long-winded method is in the textbook. It is correct but not needed, once you get β_f, just use the lookup table

$= \sum_{n=-\infty}^{\infty} \frac{1}{T} \cdot \int_{-T/2}^{T/2} 100{J_n}^2(5) cos^2[2\pi(f_c + 10n)t]\, dt$

$= \sum_{n=-\infty}^{\infty} 100{J_n}^2(5) \int_{-T/2}^{T/2} \frac{1}{T} \cdot cos^2[2\pi(f_c + 10n)t]\, dt = \sum_{n=-\infty}^{\infty} 100{J_n}^2(5) \cdot \frac{1}{2} = \sum_{n=-\infty}^{\infty} 50{J_n}^2(5)$

This must be greater than 98% of the total power, i.e., $\geq 98\% P_c$, i.e., $\geq 98\% \cdot 50$, i.e., $\sum_{n=-k}^{k} 50{J_n}^2(5) \geq 49.5$, i.e., $\sum_{n=-k}^{k} {J_n}^2(5) \geq 0.98$.

As $J_{-n}(\beta_f) = \begin{cases} J_n(\beta_f) & ; for\ even\ n \\ -J_n(\beta_f) & ; for\ odd\ n \end{cases}$, ${J_n}^2(\beta_f)$ is an even function. Thus, $\sum_{n=-k}^{k} {J_n}^2(5) = {J_0}^2(5) + 2 \cdot \sum_{n=1}^{k} {J_n}^2(5)$. Using approximation methods (*not covered here*) the value of k for which ${J_0}^2(5) + 2 \cdot \sum_{n=1}^{k} {J_n}^2(5) \geq 0.98$ is 6. Note, we get this from the table on slide 15.

Spectral Characteristics of $\angle$-Modulated Signals [8/9]

Thus, if the modulated signal is passed through an ideal bandpass filter centred at f_c with bandwidth $\geq 2 \cdot 10 \cdot 60\ Hz$, i.e., $\geq 120\ Hz$, 98% of the signal power is captured.

Formula for effective bandwidth that captures 98% of the transmitted signal power for a sinusoidal message signal is $B_c = 2\left(1 + \beta_f\right) f_m$.

Next up, using ${J_0}^2(5) + 2 \cdot \sum_{n=1}^{k} {J_n}^2(5) \geq 0.98$ & the equation above, we get:

Spectral Characteristics of $\angle$-Modulated Signals [9/9]

$$B_{modulated\ signal} = 2\left(1+\beta_f\right) f_m = \begin{cases} 2\left(1+k_p a\right) f_m = 2\left(f_m + k_p a f_m\right); & for\ PM \quad \text{multiplicative} \\ 2\left(1+\frac{k_f a}{f_m}\right) f_m = 2\left(f_m + k_f a\right)\ ; & for\ FM \quad \text{additive} \end{cases}$$

Thus, if amplitude or frequency of message signal ↑ then the effective bandwidth ↑.

Number of harmonics in the bandwidth (including the carrier) is:

$$N_{harmonics-modulated-signal} = 2\left(1+\lfloor\beta_f\rfloor\right) + 1 = 2\lfloor\beta_f\rfloor + 3 = \begin{cases} 2\lfloor k_p a\rfloor + 3\ ; & for\ PM \\ 2\left\lfloor\frac{k_f a}{f_m}\right\rfloor + 3\ ; & for\ FM \end{cases}$$

$\therefore$ if message signal amplitude increases, number of harmonics increases for both PM & FM.
The number of harmonics for PM is independent of f_m but falls almost linearly wrt f_m for FM.

Net-on-net, for FM modulation, the bandwidth increases only slightly with increase in message bandwidth due to the compensatory effect of reduction in number of harmonics.

In PM, however, the number of harmonics remains constant and only the spacing between them increases. Therefore, the net effect is a linear increase in bandwidth.

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Spectral Characteristics of $\angle$-Modulated Signals [1/2]

$\angle$-modulation by an arbitrary message: The spectral characteristic analysis is quite involved but an approximation exists in the name of Carson's Rule: $B_c = 2(1+\beta)W$ *where*

$$\beta = \begin{cases} k_p \max\{|m(t)|\} & ; \; for\ PM \\ \dfrac{k_f \max\{|m(t)|\}}{W} & ; \; for\ FM \end{cases}$$

where W *is signal bandwidth and* $\max\{|m(t)|\}$ *is maximum of the absolute amplitudes of the message signal.*

As β is typically > 5, BW of an $\angle$-modulated signals is $\gg$ BW of amplitude-modulated signal.

Problem: Message signal is $m(t) = 10 sinc(10^4 t)$. Find the transmission BW of the FM-modulated signal with $k_f = 4000$. What is the minimum separation between channels?

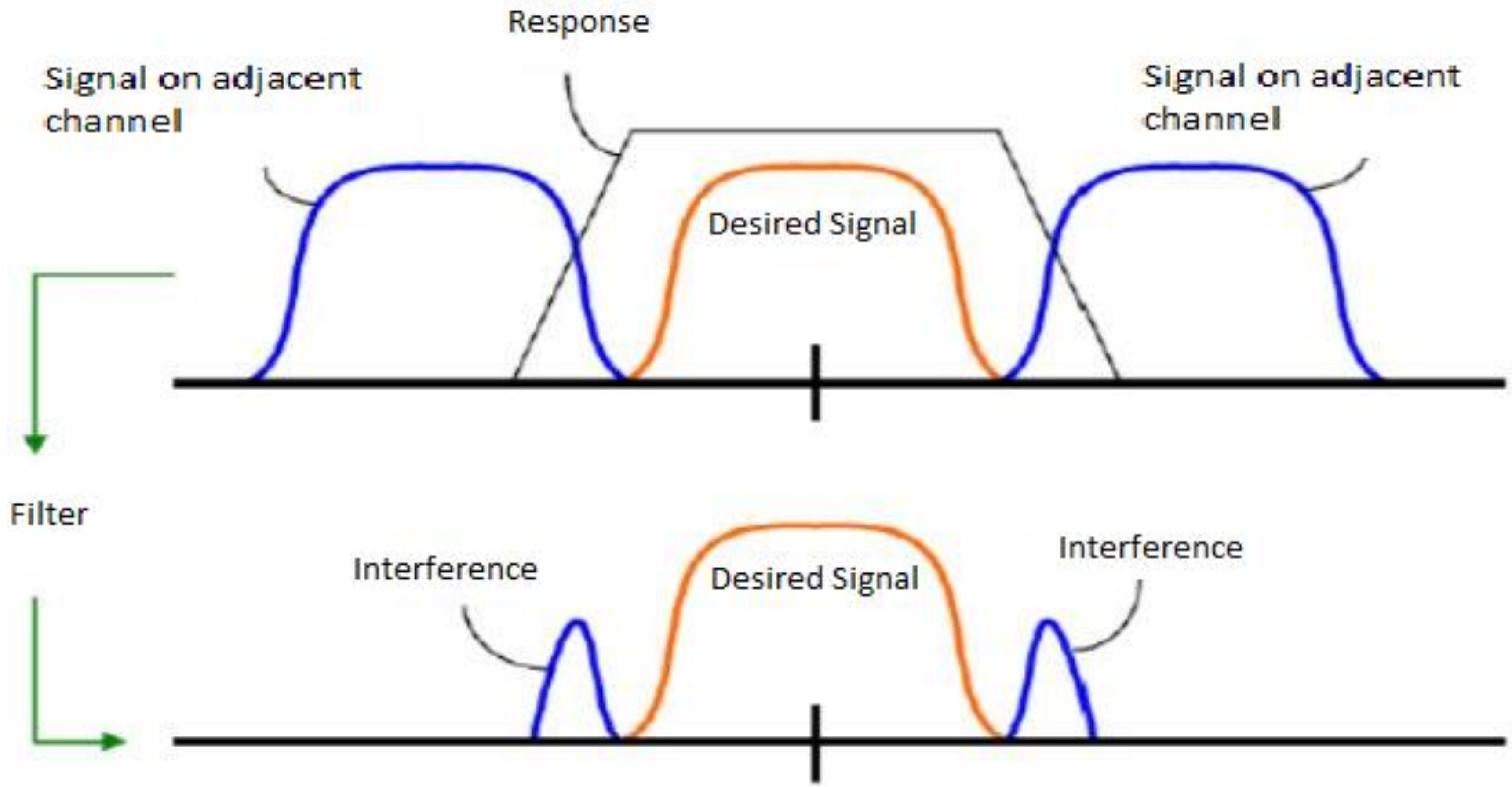
Solution: $B_{modulated-signal} = 2(1+\beta)W = 2\left(1 + \frac{k_f \max\{|m(t)|\}}{W}\right)W = 2W + 2k_f \max\{|m(t)|\}$

Now: $\max\{|m(t)|\} = 10$ which occurs at t = 0, $k_f = 4000$ & in frequency space, $m(t)$ is a box function with box-width 10^4, i.e., $W = 10^4/2 = 5000Hz$.

$\therefore B_c = 2 \cdot 5000 + 2 \cdot 4000 \cdot 10 = 90\ KHz.$

Spectral Characteristics of $\angle$-Modulated Signals [2/2]

$90\ KHz$ is also the minimum separation between adjacent channels.



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Angle Modulator Implementation [1/4]

diode whose capacitance varies with reverse voltage

As we saw, $\angle$-modulation generates additional frequencies that were not present in the message signal. This is also true for amplitude modulation due to noise.

An LTI system cannot generate frequencies in the output which were not in the input. Thus, modulator/demodulator cannot be modeled as an LTI system. We will study two modulators.

I. Varactor Diode-based $\angle$-modulators: To generate an FM signal, design an oscillator whose frequency varies as voltage; the voltage varies as message signal, i.e., $f_O \propto V_{input} \propto m(t)$.

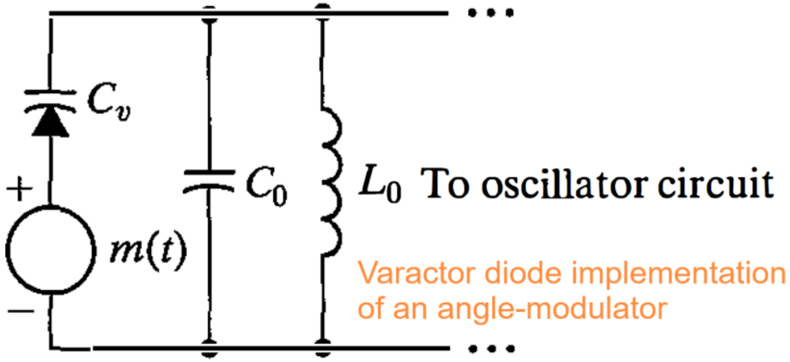
This oscillator is called voltage-controlled oscillator (VCO).

A VCO can be implemented using a varactor diode. It has capacitor whose capacitance varies with the applied voltage, i.e., $c_O \propto V_{input}$.

In the figure: $C(t) = C_0 + k_0 m(t)$ … (1)

If $m(t) = 0$ then tuned circuit frequency is $f_C = \frac{1}{2\pi\sqrt{L_0 C_0}}$ … (2)

So, $f_C(t) = \frac{1}{2\pi\sqrt{L_0\left(C_0 + k_0 m(t)\right)}} = \frac{1}{2\pi\sqrt{L_0 C_0}} \frac{1}{\sqrt{1+\frac{k_0 m(t)}{C_0}}} = f_c \frac{1}{\sqrt{1+\frac{k_0 m(t)}{C_0}}}$ … (3)



Varactor diode implementation of an angle-modulator

Angle Modulator Implementation [2/4]

So, $f_C(t) = f_c \frac{1}{\sqrt{1+\frac{k_0}{C_0}m(t)}} = f_c \frac{1}{\sqrt{1+\epsilon(t)}}$ … (3)

Taylor series expansion of $\frac{1}{\sqrt{1+\epsilon(t)}}$ around $\epsilon(t) = 0$ is $1 - \frac{\epsilon(t)}{2} + 3\frac{\epsilon(t)^2}{8} - 5\frac{\epsilon(t)^3}{16} + O(\epsilon(t)^4)$ … (4)

If $\epsilon(t) \ll 1$ then, $\frac{1}{\sqrt{1+\epsilon(t)}} \approx 1 - \frac{\epsilon(t)}{2}$ … (5)

Thus, $f_C(t) \approx f_c\left[1 - \frac{\epsilon(t)}{2}\right] = f_c\left[1 - \frac{k_0}{2C_0}m(t)\right]$ … (6)

Taylor series of function f(x) at a is defined as:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

$$= 1 + \frac{\frac{d}{dx}\left(\frac{1}{\sqrt{1+x}}\right)(0)}{1!}x + \frac{\frac{d^2}{dx^2}\left(\frac{1}{\sqrt{1+x}}\right)(0)}{2!}x^2 + \frac{\frac{d^3}{dx^3}\left(\frac{1}{\sqrt{1+x}}\right)(0)}{3!}x^3 + \ldots$$

II. Reactance Tube-based ∠-modulators: To generate an FM signal, design an oscillator whose inductor's inductance varies as voltage i.e., $f_O \propto L_{input} \propto m(t)$.

HINT: *what will you change in eqn. 2 or 3?*

Problem: Derive expression for $f_C(t)$ for a reactance tube-based FM modulator. **Solution:** DIY

III. 'Indirect Method' via Narrowband ∠-modulators: Any modulator for conventional AM generation can be easily modified to generate a narrowband ∠-modulated signal.

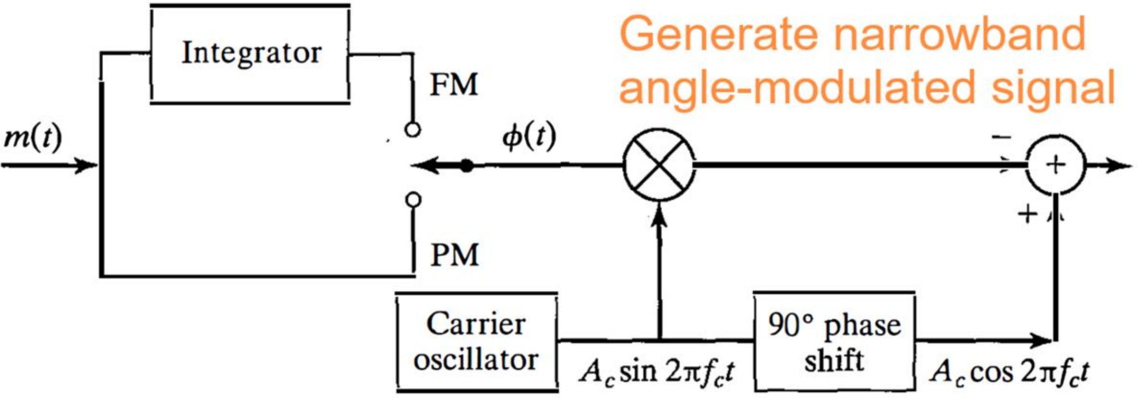
Recall: ∠-modulated signal is $u(t) = A_c \cos[2\pi f_c t + \varphi(t)] \approx A_c \cos(2\pi f_c t) - A_c\varphi(t)\sin(2\pi f_c t)$

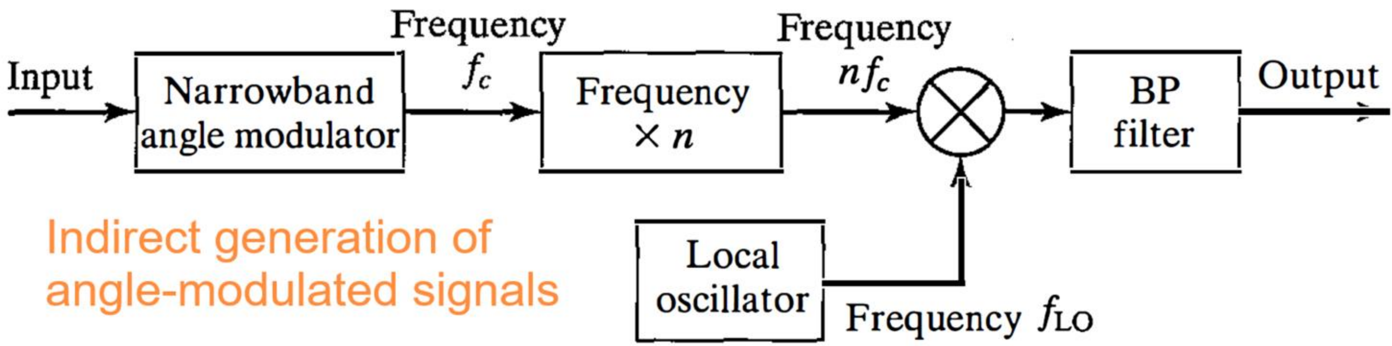
Angle Modulator Implementation [3/4]

III. 'Indirect Method' via Narrowband ∠-modulators:

Step 1: Generate a narrowband ∠-modulated signal as shown in left photo in line with equation on last slide. *Each message source will modulate a different carrier frequency - we do not pursue this idea further here.*

Step 2: Signal is sent to an ∠-multiplier. Thus, $A_c \cos[2\pi f_c t + \varphi(t)] \rightarrow A_c \cos[2\pi n f_c t + n\varphi(t)]$





Step 3: $A_c \cos[2\pi n f_c t + n\varphi(t)]$ is no longer narrowband, i.e., this is $\not\approx$ $A_c \cos(2\pi n f_c t) - A_c \varphi(t) \sin(2\pi n f_c t)$. Thus, we have a wideband signal. However, nf_c may not be the desired central frequency. Thus, the wideband signal and, a local oscillator signal is passed through a mixer (*an electronic device that produces a signal that is sum or difference of the input signals*) to generate the desired shift. Thus, $A_c \cos[2\pi(nf_c - f_{LO})t + n\varphi(t)]$.

Angle Modulator Implementation [4/4]

III. 'Indirect Method' via Narrowband ∠-modulators:

Step 5: This is passed through a bandpass filter to remove undesired frequencies.

By choosing n and f_{LO}, we can generate any modulation index at the desired carrier frequency for a wideband ∠-modulated signal.

I. *Similarities between FM & PM*
II. *PM*
III. *Narrowband Angle Modulation*
IV. *Spectral Characteristics of Angle Modulated Signals*
V. *Angle Modulation by an Arbitrary Message Signal*
VI. *Angle Modulator Implementation*
VII. Angle Demodulator Implementation

Angle Modulator Implementation [1/6]

We will discuss this should time permit.