Amrita Vishwa Vidyapeetham
Amrita School of Engineering, Coimbatore
B.Tech Missed Mid-term – October 10, 2024
5th Semester
Cybersecurity
20CYS301 Digital Communications

Duration: Two hours | Maximum: 50 Marks

Course Outcomes (COs):

CO	Course Outcomes
CO01	Understand the fundamental principles of digital modulation and demodulation methods
CO02	Identify and list various issues present in the design of a communication system
CO03	Apply the time domain and frequency domain concepts of signals in data communication
CO04	Design suitable error detection and error correction algorithms to achieve error free data communication
CO05	
CO06	

NOTE to exam invigilator

- Please do NOT permit anyone to leave the desk until the last 30 minute of the exam.

NOTE to students

- Answer ALL questions
- When answering non-problem/proof-based questions, write in bullet points.
- Rough work should be struck out to ensure an evaluator is able to distinguish rough work from solutions.
- Use any colored pen or pencil except red

Formulae

1. Message bandwidth is 3,000 bytes per second. It is used to AM-modulate a carrier at 1 MHz frequency.

What is the min distance between carrier frequencies in case of DSB SC AM (express in KHz)? Justify.

Draw the diagram for DSB SC AM.

What is the min distance between carrier frequencies in case of DSB Conventional AM (express in KHz)? Justify.

Draw the diagram for DSB Conventional AM.

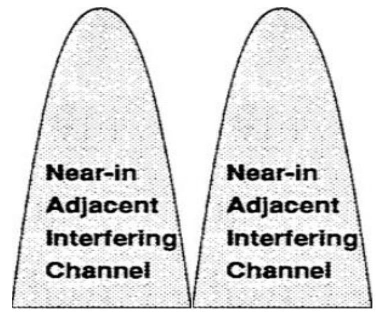
The filter used to select between adjacent channels is not ideal. It needs 10% more bandwidth than the signal bandwidth on the lower side band and 12% more bandwidth than signal bandwidth on the upper side band. What is the filter bandwidth?

[5.0] [C01] [BTL2]

3,000 bytes/sec is 3,000*8 = 24,000 bits/sec. This is 24 KHz.

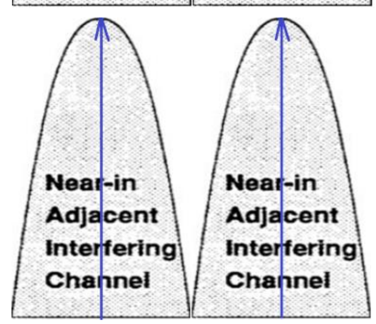
Assuming ideal filters, the distance between adjacent carrier bands must be at least as much as that required to just separate the USB of the lower channel from the LSB of the upper channel.

This is illustrated in the figure on the right for DSC SC AM.



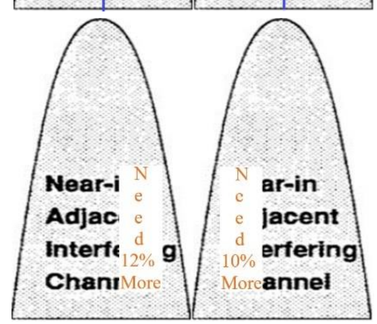
USB size = LSB size = 24 KHz $\therefore$ min distance between adjacent carrier frequencies is 2*24 = **48 KHz.**

The diagram to the right shows DSC Conventional AM. The lines with blue arrows show the carrier.



The minimum distance between adjacent channels does not change. It is **48 KHz.** This is because DSC Conventional AM differs from DSB SC AM only in that that the latter has 2 extra delta functions. Delta functions consume an infinitesimally narrow bandwidth.

On the lower side, we need 10% more. Thus, we need, 110%*(24K) = 26.4 KHz.
On the upper side, we need 12% more. Thus, we need, 120%*(24K) = 26.88 KHz.
$\therefore$ minimum required bandwidth of the non-ideal filter is (26.4+28.8) KHz = **53.28 KHz**



2. State the reason(s) why antenna size for mobile phones has reduced over the past 2 decades.

[5.0] [C02] [BTL3]

Two reasons:

1. Over time, improvements in manufacturing processes and antenna design have led to better coupling of the generated electrical energy and the radiated electromagnetic energy.
2. The operating frequency of mobile phones has increased from c900 MHz to c3.6 GHz. As, for efficient coupling, antenna length must be $> \frac{1}{10}\lambda$ (inversely proportional to frequency) antenna length of mobile phones has decreased.

3. An antenna is mounted at a height of 140 meters. It transmits a signal 100 GHz at 20 dB. Signals at 100 GHz travel line-of-sight (LOS). Find the max distance over which signal can be received by a receptor in a building at a height of 100 meters above the ground.

when there is: a) *No rain* b) *Light rain* c) *Heavy rain*

Minimum signal strength needed at receiver is $12\ dB$

You will find these useful:

line-of-sight radius is $d = \sqrt{15h}\ km$ where h is the height at which the antenna is placed above the local ground

Note that attenuation in air: $\frac{0.02dB}{km}$ attenuation in light rain: $\frac{0.1dB}{km}$ attenuation in heavy rain: $\frac{6dB}{km}$

[5.0] [C02] [BTL3]

The antenna height is 140m above the ground.

However, the receptor itself is 100m above the ground. This has the same effect as having the antenna at 140+100 = 240m above the ground with the receptor at the ground level instead of at 100m above the ground.

With this argument, the rest of the computations are as below:

LOS distance is $d = \sqrt{15h} = \sqrt{15 * 240} = 60km$

Without attention directed towards LOS limitation, signal transmission distance is $\frac{\left(Strength_{Tx} - Min_{strength_{Rx}}\right)}{Attenuation}$ which

in case a) is $\frac{(20-12)}{0.02} = 400km$

in case b) is $\frac{(20-12)}{0.1} = 80km$

in case c) is $\frac{(20-12)}{6} = 1.33km$

As max distance is $MIN(LOS, distance\ over\ which\ signal\ is\ strong\ enough\ to\ be\ received)$,

in case a) it is $MIN(60, 400) = 60km$

in case b) it is $MIN(60, 80) = 60km$

in case c) it is $MIN(60, 1.33) = 1.33km$

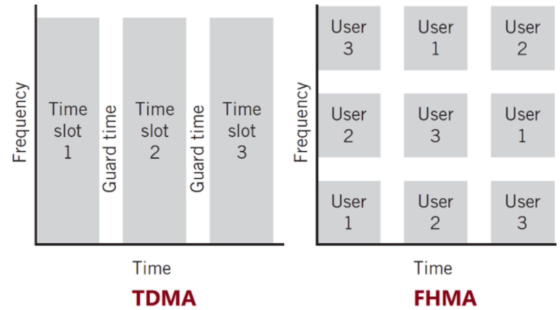
4. Time slots assigned to a user are in multiples of 30 millisecond (ms) width.

Frequency slots are in multiples of 0.9 MHz width.

Guard bands along time axis are 20 ms wide.

Guard bands along frequency axis are 100 KHz wide.

Find the max number of users that can be supported under FHMA during 1 second if the channel bandwidth is 500 MHz.



[5.0] [C02] [BTL2]

The max number of users will correspond to the representation in the figure above on the right.

Along the time axis, one can support N time slots where $N * (30 * 10^{-3}) + (N - 1) * 20 * 10^{-3} = 1$, i.e., $N = \frac{1+20*10^{-3}}{50*10^{-3}} = 1.002 * \frac{10^3}{50} = 20.04$ which must be rounded down to the nearest integer, i.e., 20.

Along the frequency axis, one can support n simultaneous channels where $n * 0.9 + (n - 1) * 0.1 = 500$, i.e., $n = \frac{500.1}{1}$ which must be rounded down to the nearest integer, i.e., 500.

Thus, the max number of users = $N * n$ which is $20 * 500 = \mathbf{1,000}$.

5. Message signal is causal and given by $m(t) = ae^{f_m t}$.

Assume it is periodic over a period $T = 6$ seconds.

The message signal a) frequency modulates and b) phase modulates a carrier $c(t) = A_c cos(2\pi f_c t)$.

What is the modulated signal?

You may find it useful that frequency modulated signal is **FM:** $\varphi(t) = T \cdot k_f \int_{-\infty}^{t} m(\tau)d\tau$

[5] [C03] [BTL2]

For PM: $\varphi(t) = k_p m(t) = k_p ae^{f_m t}$.

Thus, $u_{PM}(t) = A_c cos[2\pi f_c t + k_p ae^{f_m t}]$

For FM: $\varphi(t) = T \cdot k_f \int_{-\infty}^{t} m(\tau)d\tau = T \cdot k_f \int_{-\infty}^{t} ae^{f_m \tau}d\tau = \frac{a \cdot 6.k_f}{f_m} \cdot e^{f_m \tau}|_{-\infty}^{t}$

Thus, $\varphi(t) = \frac{a \cdot 6.k_f}{f_m} \cdot e^{f_m t}$

Thus, $u_{FM}(t) = A_c cos\left[2\pi f_c t + \frac{6a.k_f}{f_m} \cdot e^{f_m t}\right]$

6. Derive the expression for power of DSB-SC AM message signal $A_c \cdot m(t) \cdot cos(2 \cdot \pi \cdot f_c \cdot t)$. Signal power is $P[u(t)] = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u^2(t)dt$

[7.5] [C01] [BTL3]

$$P[u(t)] = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} u^2(t)dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} {A_c}^2 m^2(t) C^2(2\pi f_c t)dt$$

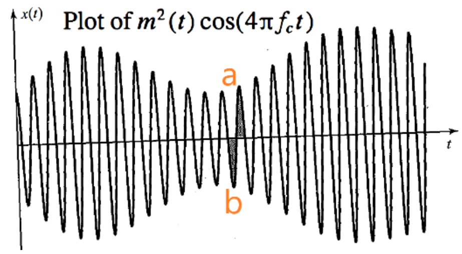
$$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{{A_c}^2}{2} m^2(t)[1 + cos(4\pi f_c t)]dt \ldots \because C^2(t) = \frac{[1+C(2T)]}{2}$$

$$= \frac{{A_c}^2}{2}\left[\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t)dt\right] + \frac{{A_c}^2}{2}\left[\cancel{\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t)cos(4\pi f_c t)dt}\right] \approx \frac{{A_c}^2}{2} P_m$$

$\frac{{A_c}^2}{2}\left[\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} m^2(t)cos(4\pi f_c t)dt\right] \to 0$ because:

a. *The two areas 'a' and 'b' are nearly equal as because $f_m \ll f_c$, the two nearly cancel each other over the period and,*

b. *The term is divided by $T \to \infty$.*

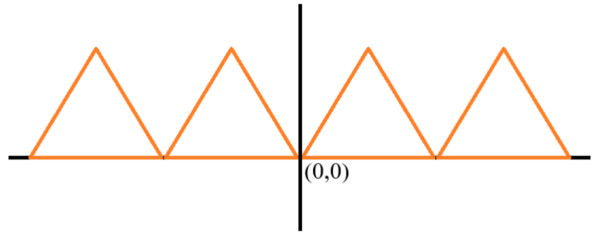


7. Derive the expression for fourier transform for the waveform shown.

Waveform period is 2 sec.

Waveform amplitude is 2 volt.

Plug in the values of n=0 and n=6 into the final equation and compute.

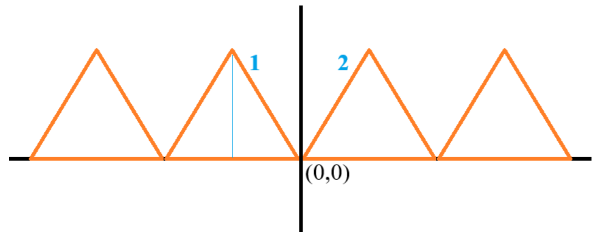


$\sin x = x - x^3/3! + x^5/5! \pm O(x^7)$ and $\cos x = 1 - x^2/2! + x^4/4! \pm O(x^6)$

FT for continuous periodic waveform is $X(n) = \frac{1}{T}\int_T x(t)e^{-jn\pi t}d\tau$

You may find it useful to know that $\int te^{a}dt = \frac{t\cdot e^{at}}{a} + \int e^{at}dt$

[7.5] [C03] [BTL4]



Consider the function over the time axis over one period, $T = \{-1, +1\}$. Over the negative half of this interval, equation of the line is $y - y_1 = m \cdot (x - x_1)$. $m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(2-0)}{(-1-0)} = -2$

Thus, for $t = \{-1,0\}$, $y - y_1 = m \cdot (x - x_1)$, i.e.,

$y - 0 = -2 \cdot (x - 0)$, i.e., for $t = \{-1,0\}$, $y = -2x$

Similarly, the equation of line 2, i.e., for $t = \{0, +1\}$, $y = 2x$

Thus, $X(n) = \frac{1}{T}\int_{-1}^{+1} x(t)e^{-jn\pi t}dt = \frac{1}{2}\int_{-1}^{0} -2te^{-jn\pi t}dt + \frac{1}{2}\int_{0}^{1} 2te^{-jn\pi t}dt$

$= -\int_{-1}^{0} te^{-jn\pi t}dt + \int_{0}^{1} te^{-jn\pi t}dt$

Given that $\int te^{a}dt = \frac{t\cdot e^{at}}{a} + \int e^{at}dt$, $\int te^{-jn\pi t}dt = \frac{-t\cdot e^{-jn\pi t}}{jn\pi} + \frac{e^{-jn\pi t}}{-jn\pi} = -\frac{e^{-jn\pi t}}{jn\pi}(1+t)$

Thus, $-\int_{-1}^{0} te^{-jn\pi t}dt + \int_{0}^{1} te^{-jn\pi t}dt$ is $-\left[-\frac{e^{-jn\pi t}}{jn\pi}(1+t)\right]_{-1}^{0} + \left[-\frac{e^{-jn\pi t}}{jn\pi}(1+t)\right]_{0}^{1}$

which is $\left[\frac{e^{-jn\pi t}}{jn\pi}(1+t)\right]_{-1}^{0} - \left[\frac{e^{-jn\pi t}}{jn\pi}(1+t)\right]_{0}^{1}$

which is $\left[\frac{e^{-jn\pi\cdot 0}}{jn\pi}(1+0) - \cancel{\frac{e^{-jn\pi\cdot -1}}{jn\pi}(1-1)}\right] - \left[\frac{e^{-jn\pi\cdot 1}}{jn\pi}(1+1) - \frac{e^{-jn\pi\cdot 0}}{jn\pi}(1-0)\right]$

which is $\left[\frac{1}{jn\pi}\right] - \left[\frac{2\cdot e^{-jn\pi}}{jn\pi} - \frac{1}{jn\pi}\right] = \frac{2}{jn\pi}\cdot\left[1 - e^{-jn\pi}\right]$

This is $\frac{2e^{-\frac{jn\pi}{2}}}{jn\pi}\cdot\left[e^{\frac{jn\pi}{2}} - e^{-\frac{jn\pi}{2}}\right] = \frac{2e^{-\frac{jn\pi}{2}}}{jn\pi}\cdot\left[\cancel{C(\theta)} + jS(\theta) - \cancel{C(-\theta)} - jS(-\theta)\right]$

Which is $\frac{2e^{-\frac{jn\pi}{2}}}{jn\pi}\cdot 2jS\left(\frac{n\pi}{2}\right)$ which is $\frac{4e^{-\frac{jn\pi}{2}}\cdot S\left(\frac{n\pi}{2}\right)}{n\pi}$ which is $2e^{-\frac{jn\pi}{2}}\cdot\frac{S\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2}}$ which is $2e^{-\frac{jn\pi}{2}}\cdot sinc\left(\frac{n\pi}{2}\right)$

As $\sin x = x - x^3/3! + x^5/5! \pm O(x^7)$, $\lim_{n\to 0} sinc\left(\frac{n\pi}{2}\right) = \lim_{n\to 0}\left[\frac{\frac{n\pi}{2} - \frac{\left(\frac{n\pi}{2}\right)^3}{3!} + \frac{\left(\frac{n\pi}{2}\right)^5}{5!} \pm \cdots}{\frac{n\pi}{2}}\right] = \lim_{n\to 0}\left[1 - \frac{\left(\frac{n\pi}{2}\right)^2}{3!} + \frac{\left(\frac{n\pi}{2}\right)^4}{5!} \pm \cdots\right]$

Which is 1.

Thus, when n is 0, the FT is $2e^{-\frac{j\cdot 0.\pi}{2}} \cdot 1 = 2[C(0) + jS(0)] = 2[1] = \mathbf{2}$ and

when n is 6, the FT is $2e^{-\frac{j\cdot 6.\pi}{2}} \cdot \frac{S\left(\frac{6\pi}{2}\right)}{\frac{6\pi}{2}} = 2[C(-3\pi) + jC(-3\pi)] \cdot \frac{S(3\pi)}{3\pi} = 2[-1 + j \cdot 0] \cdot \frac{0}{3\pi}$

$= -2 \cdot 0 = \mathbf{0}$

8. Derive the mathematical expressions associated with demodulation of a Quadrature Carrier Multiplexed (QCM) signal. $u(t) = A \cdot m_1(t) \cdot cos(2 \cdot \pi \cdot f_c \cdot t) + A \cdot m_2(t) \cdot sin(2 \cdot \pi \cdot f_c \cdot t)$

What is the phase angle between the two message signals?

What is the optimum phase angle between the signals if we want to extend this modulation approach to 3 messages, m_1, m_2 & m_3 instead of just m_1 & m_2? Why?

[10] [C01] [BTL4]

Multiply $u(t) = A_c m_1(t)cos(2\pi f_c t) + A_c m_2(t)sin(2\pi f_c t)$ by $cos(2\pi f_c t)$.

$\therefore u(t)cos(2\pi f_c t) = A_c m_1(t)cos^2(2\pi f_c t) + A_c m_2(t)sin(2\pi f_c t)cos(2\pi f_c t)$

$= \frac{A_c}{2} m_1(t) \cdot [1 + cos(4\pi f_c t)] + \frac{A_c m_2(t)sin(4\pi f_c t)}{2}$

$= \frac{A_c}{2} m_1(t) + \frac{A_c}{2} m_1(t)cos(4\pi f_c t) + \frac{A_c}{2} m_2(t)\,sin(4\pi f_c t)$

Low pass filter will remove the highlighted term, leaving only the envelope of $m_1(t)$.

To recover $m_2(t)$, multiply $u(t) = A_c m_1(t)cos(2\pi f_c t) + A_c m_2(t)sin(2\pi f_c t)$, with $sin(2\pi f_c t)$.

Thus, $u(t)cos(2\pi f_c t) = A_c m_1(t)cos(2\pi f_c t)sin(2\pi f_c t) + A_c m_2(t)sin^2(2\pi f_c t)$

$= \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + A_c m_2(t)sin^2(2\pi f_c t)$

$= \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + A_c m_2(t)[1 - cos^2(2\pi f_c t)]$

$= \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + A_c m_2(t)\left[1 - \frac{1+cos(4\pi f_c t)}{2}\right] = \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + A_c m_2(t)\,\frac{1-cos(4\pi f_c t)}{2}$

$= \frac{A_c}{2} m_1(t)sin(4\pi f_c t) + \frac{A_c}{2} m_2(t) - \frac{A_c}{2} m_2(t)\,cos(4\pi f_c t)$

A low pass filter eliminates the highlighted terms, leaving just the envelop of m_2.

90° $\because$ one message has been put on a cos function and another on a synced sine function and that the phase angle between these two functions is 90°.

The optimum phase angle in case one wants to extend QCM to 3 messages is **120°.** The reason for this is that this angle will achieve maximal separation between the three messages which helps reduce transmission errors.

****

Course Outcome /Bloom's Taxonomy Level (BTL) Mark Distribution Table

CO	Marks	BTL	Marks
CO01	**22.5**	BTL 1	**0.0**
CO02	**15.0**	BTL 2	**15.0**
CO03	**12.5**	BTL 3	**17.5**
CO04	**0.0**	BTL 4	**17.5**
CO05		BTL 5	
CO06		BTL 6	

Bloom's Taxonomy Levels (attached for reference)

Level 1 – Remember | Level 2 – Understand | Level 3 – Apply | Level 4 – Analyze |

Level 5 – Evaluate | Level 6- Create