Sorting Algorithms
using
Divide and Conquer Technique

1. Merge Sort
2. Quick Sort

Sorting

- Insertion sort
  - Design approach: incremental
  - Sorts in place: Yes
  - Best case: $\Theta(n)$
  - Worst case: $\Theta(n^2)$

- Bubble Sort
  - Design approach: incremental
  - Sorts in place: Yes
  - Running time: $\Theta(n^2)$

Sorting

- Selection sort
  - Design approach: incremental
  - Sorts in place: Yes
  - Running time: $\Theta(n^2)$

- Merge Sort
  - Design approach: divide and conquer
  - Sorts in place: No
  - Running time: Let's see!!

Divide-and-Conquer

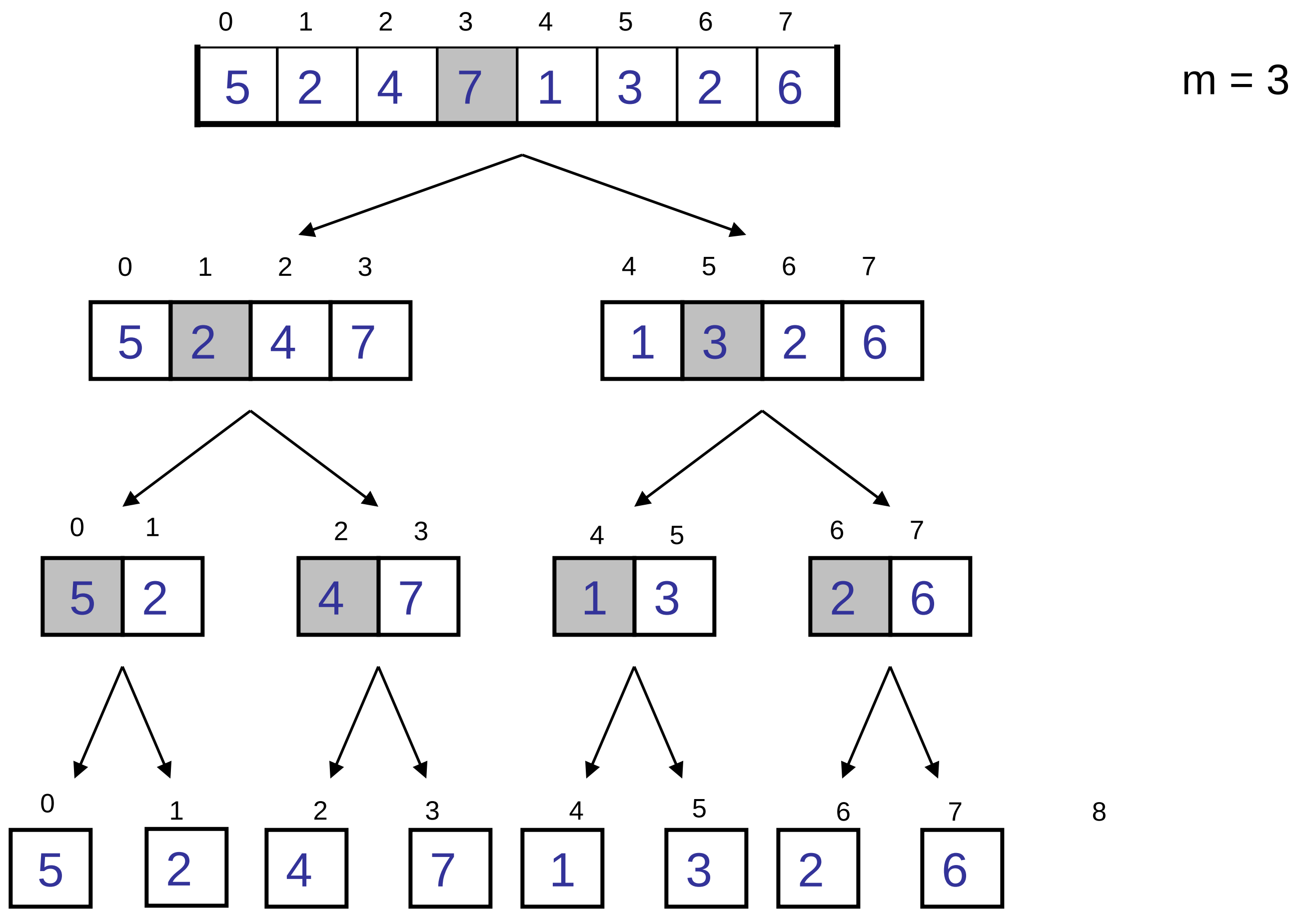
- **Divide** the problem into a number of sub-problems
  - Similar sub-problems of smaller size
- **Conquer** the sub-problems
  - Solve the sub-problems <u>recursively</u>
  - Sub-problem size small enough $\Rightarrow$ solve the problems in straightforward manner
- **Combine** the solutions of the sub-problems
  - Obtain the solution for the original problem

Merge Sort Approach

- To sort an array $A[l \,.\,.\, r]$:
- **Divide**
  – Divide the n-element sequence to be sorted into two subsequences of $n/2$ elements each
- **Conquer**
  – Sort the subsequences recursively using merge sort
  – When the size of the sequences is 1 there is nothing more to do
- **Combine**
  – Merge the two sorted subsequences

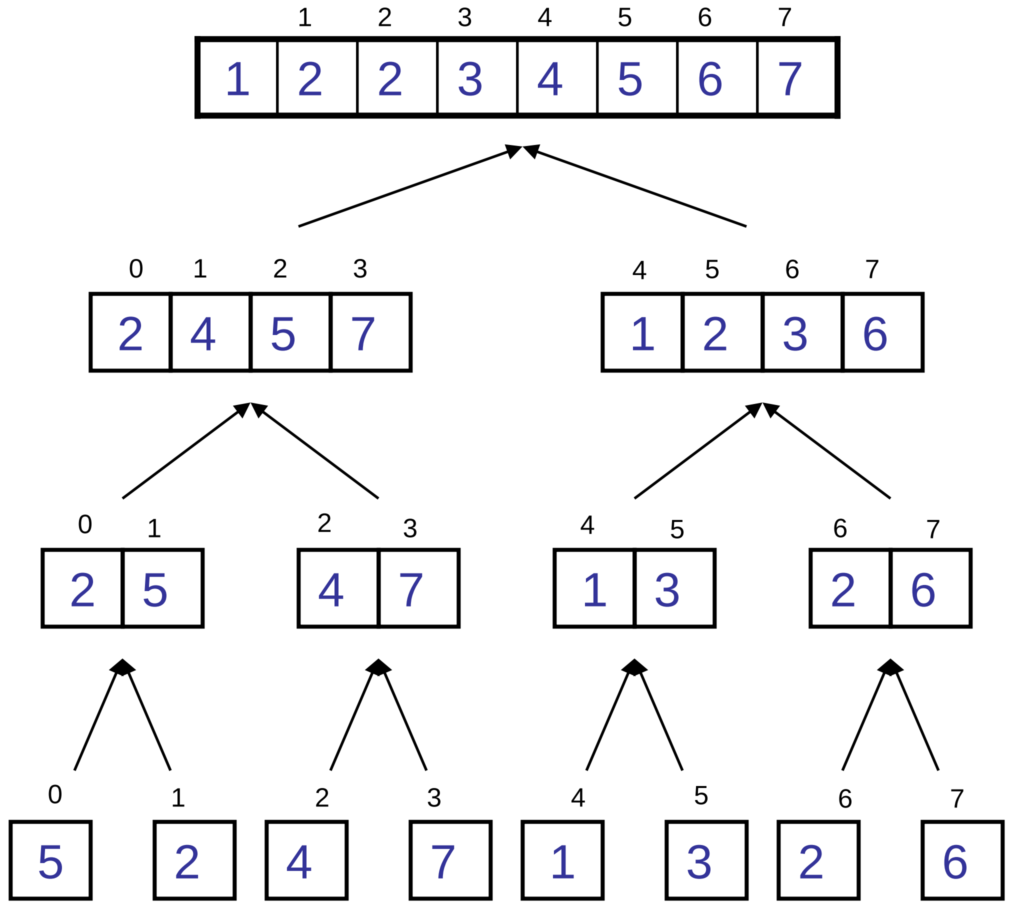
Example – n Power of 2

Divide

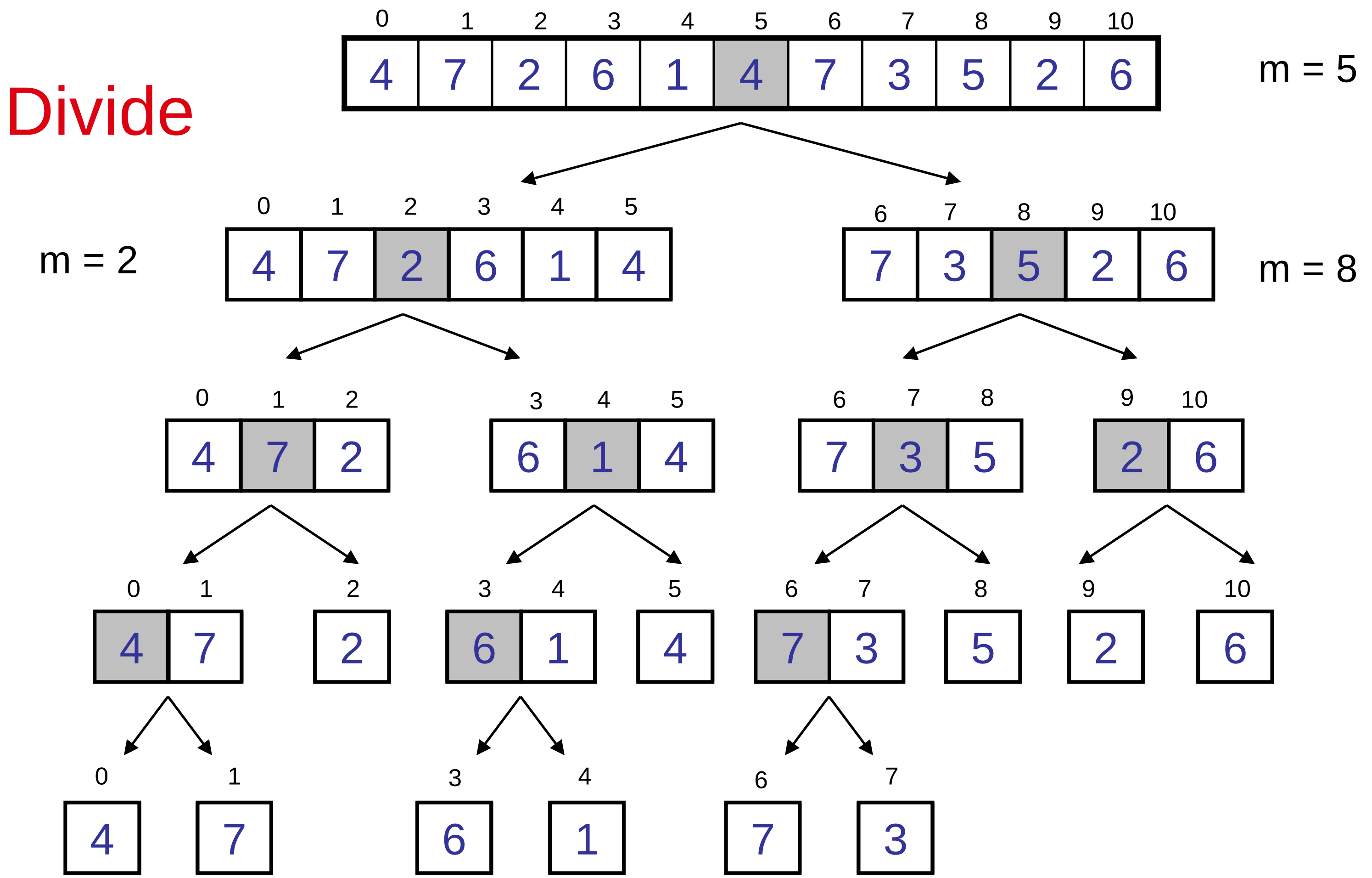


Example – n Power of 2

Conquer and Merge

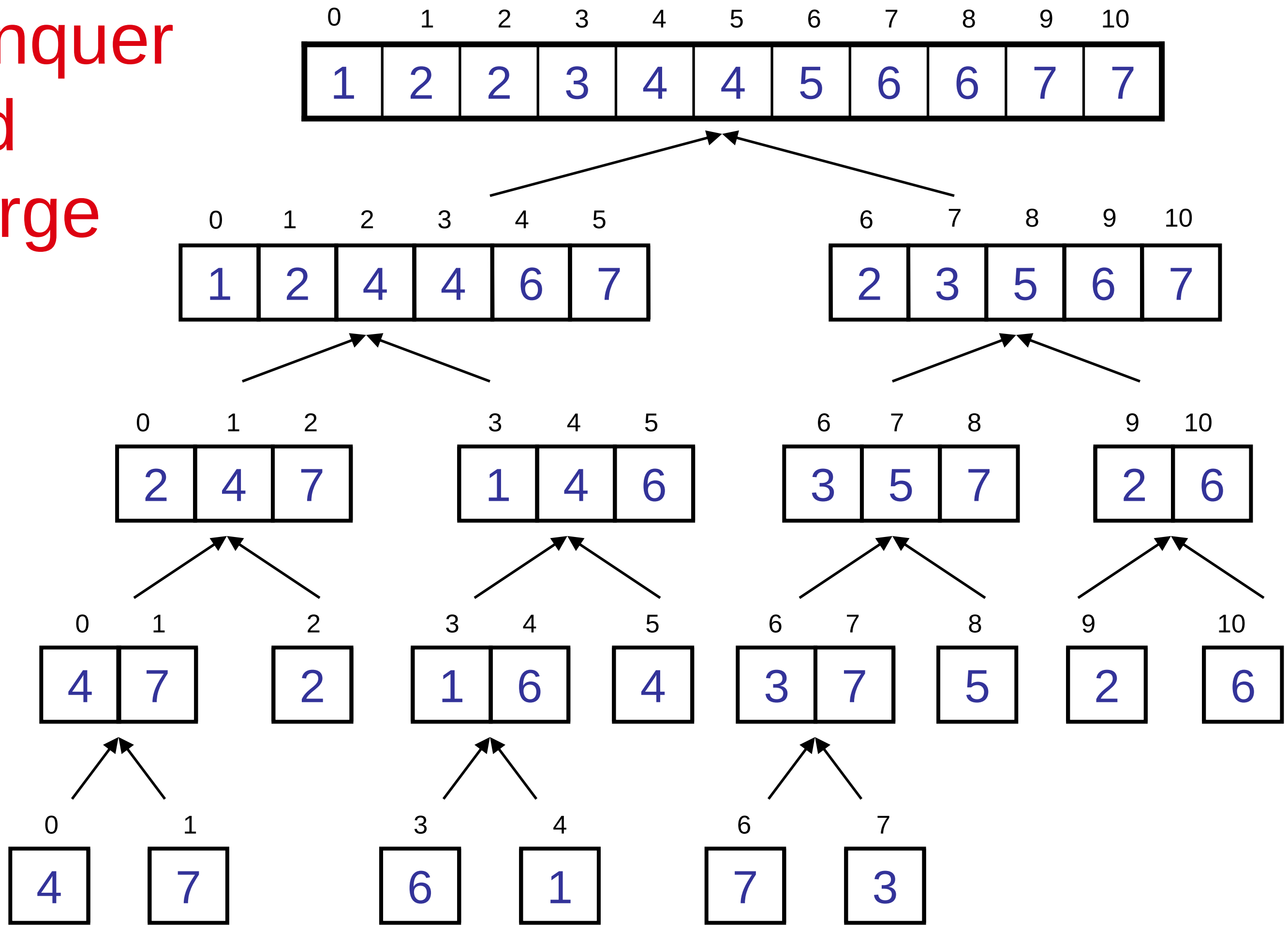


Example – n Not a Power of 2



Example – n Not a Power of 2

Conquer and Merge



Merge Sort

l			m				r
0	1	2	3	4	5	6	7
5	2	4	7	1	3	2	6

Alg.: MERGE-SORT( )

- Initial call:

Merge Sort

l			m				r
0	1	2	3	4	5	6	7
5	2	4	7	1	3	2	6

Alg.: MERGE-SORT(A, l, r)

if $l < r$

Check for base case

$m \leftarrow \lfloor (l + r)/2 \rfloor$ #

Divide

MERGE-SORT(A, l, m) #

Conquer

MERGE-SORT(A, $m + 1$, r) # Conquer

MERGE(A, l, m, r)

Combine

Merging

l			m				r
0	1	2	3	4	5	6	7
5	2	4	7	1	3	2	6

- **Input:** Array A and indices l, m, r such that $l \le m < r$
  - Subarrays $A[l \,..\, m]$ and $A[m + 1 \,..\, r]$ are sorted
- **Output:** One single sorted subarray $A[l \,..\, r]$

Merging

l			m				r
0	1	2	3	4	5	6	7
5	2	4	7	1	3	2	6

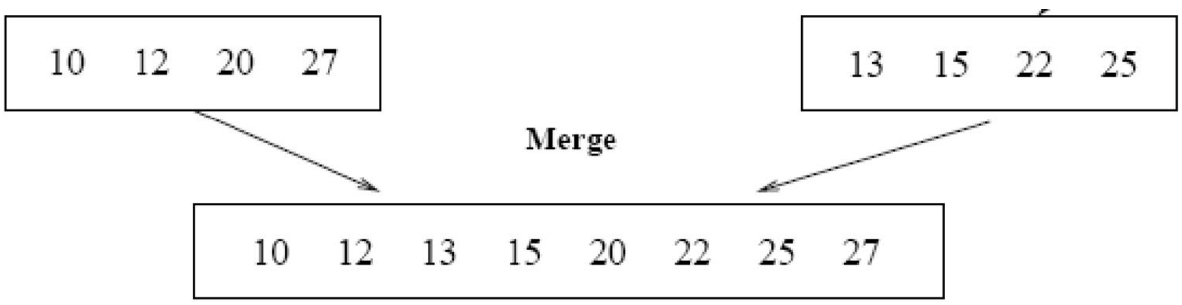
Merge - Pseudocode

```
merge(a,l,m,r):
   i=l,    j=m+1,    k=0
   while i<=m and j<=r:
      if a[i]<=a[j]:
         b[k] = a[i]
               k += 1
         i += 1
      else:
         b[k] = a[j]
               k += 1
         j += 1
```

```
   while i<=m:
         b[k] = a[i]
               k += 1
         i += 1
   while j<=r:
         b[k] = a[j]
               k += 1
         j += 1
   for (i=0 to k):
         a[l+i]=b[i]
```

Running Time of Merge (assume last **for** loop)

- Merging into temporary array:
  - $\Theta(n)$
- Copying the elements from temporary to the final array:
  - n iterations, $\Rightarrow \Theta(n)$
- Total time for Merge:
  - $\Theta(n)$



MERGE-SORT Running Time

- **Divide:**
  - compute m as the average of l and r: $D(n) = \Theta(1)$
- **Conquer:**
  - recursively solve 2 subproblems, each of size $n/2$
    $\Rightarrow 2T(n/2)$
- **Combine:**
  - MERGE on an n-element subarray takes $\Theta(n)$ time
    $\Rightarrow C(n) = \Theta(n)$

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ 2T(n/2) + \Theta(n) & \text{if } n > 1 \end{cases}$$

Solve the Recurrence

$$T(n) = \begin{cases} c & \text{if } n = 1 \\ 2T(n/2) + cn & \text{if } n > 1 \end{cases}$$

Use Master's Theorem:

Compare n with $f(n) = cn$

Case 2: $T(n) = \Theta(n\lg n)$

Merge Sort - Discussion

- Advantages:
  - Guaranteed to run in $\Theta(n \log n)$

- Disadvantage
  - Requires extra space $\approx N$