AMRITA VISHWA VIDYAPEETHAM
श्रद्धावान् लभते ज्ञानम्
DEEMED TO BE UNIVERSITY

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Digital Communications
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Digital Modulation Techniques in Presence of AWGN: ASK, PSK, FSK & QAM

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I. Geometric Representation of Signal Waveforms

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   b. *Binary Orthogonal Signaling*

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IV. *Amplitude Shift Keying* (ASK)

V. *Phase Shift Keying* (PSK)

VI. *Frequency Shift Keying* (FSK)

VII. *Quadrature Amplitude Modulation* (QAM)

1. Geometric Representation of Signal Waveforms [1/5]

In digital communications the message is a sequence of binary information bits.

The modulator receives the message bits and maps them to distinct waveforms $s_1(t)$ & $s_2(t)$.

In **binary modulation,** the modulator output is only (a sequence) of two distinct waveforms.

M-*ary* modulation: modulator transmits 3 bits (k) at a time using 8 waveforms (M); $M = 2^k$.

Geometric representation aka vector representation is a compact representation for digital signal transmission and it simplifies the analysis of the digital communication system.

Grahm-Schmidt Orthogonalization Procedure: Assume that the waveforms $s_1(t)$, $s_2(t)$, …, $s_k(t)$ have energies $\mathcal{E}_1, \mathcal{E}_2, \dots, \mathcal{E}_k$.

Normalized energy of the 1st waveform is $\Psi_1(t) = \frac{s_1(t)}{\sqrt{\mathcal{E}_1}}$

The 2nd waveform is obtained as sum of its projection onto the 1st waveform and residue. Thus: $d_2(t) = s_2(t) - c_{21}(t)\Psi_1(t)$ where $c_{21}(t) = \int_{-\infty}^{\infty} s_2(t)\Psi_1(t)dt$

$\mathcal{E}_2$ is the energy of the residual of the 2nd waveform, not of the 2nd waveform. Thus:

1. Geometric Representation of Signal Waveforms [2/5]

Thus: $\Psi_2(t) = \frac{d_2(t)}{\sqrt{\mathcal{E}_2}}$

$\mathcal{E}_2 = \int_{-\infty}^{\infty} d_2^2(t)dt$

Similarly, to compute $\mathcal{E}_k$, we first remove all projections of $s_k(t)$ onto $\Psi_1(t)$, $\Psi_2(t)$, … $\Psi_{k-1}(t)$.

Thus: $d_k(t) = s_k(t) - \sum_{i=1}^{k-1} c_{ki}(t)\Psi_i(t)$

$\Psi_k(t) = \frac{d_k(t)}{\sqrt{\mathcal{E}_k}}$

$c_{ki}(t) = \int_{-\infty}^{\infty} s_k(t)\Psi_i(t)dt$

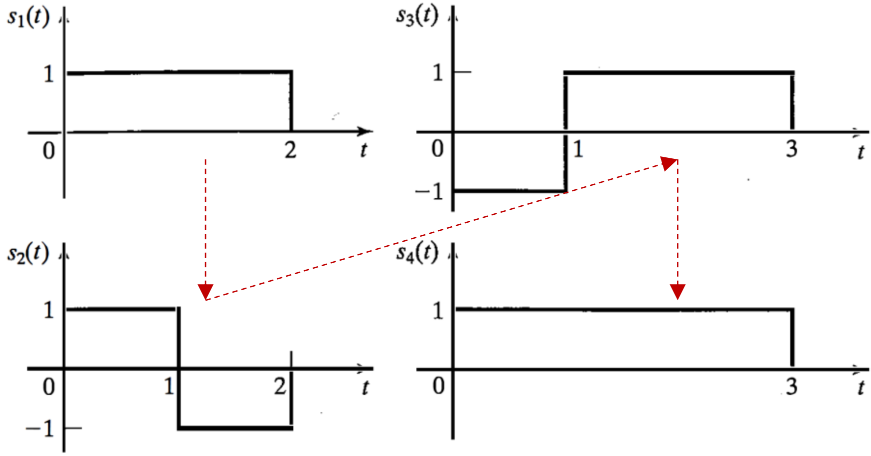
$\mathcal{E}_k = \int_{-\infty}^{\infty} d_k^2(t)dt$

The N orthogonal waveforms $\{\Psi_N(t)\}$ form an orthonormal basis in N-*dimensional* signal space.

$N = K$ if none of the signal waveforms is a linear combination of the other signal waveforms.

1. Geometric Representation of Signal Waveforms [3/5]

Problem: The 4 top figures are signal waveforms. Compute Grahm-Schmidt normalization.



Solution: As $s_1(t)$ has energy $\mathcal{E}_1$, $\Psi_1(t) = \frac{s_1(t)}{\sqrt{2}}$

$$c_{21}(t) = \int_{-\infty}^{\infty} s_2(t)\Psi_1(t)dt$$

$$= \int_0^1 1 \cdot 1dt + \int_1^2 -1 \cdot 1dt = (-1-0) + (2-1) = 0,$$

i.e., $s_2(t)$ & $\Psi_1(t)$ are already orthonormal. Thus: $d_2(t) = s_2(t) - \sum_{i=1}^{2-1} c_{21}(t)\Psi_1(t) = s_2(t) - \sum_{i=1}^{1} 0 \cdot \Psi_1(t) = s_2(t)$

$$\mathcal{E}_2 = \int_{-\infty}^{\infty} d_2^2(t)dt = \int_0^1 1^2 dt + \int_{-1}^{2} (-1)^2 dt = 2$$

Thus, $\Psi_2(t) = \frac{d_2(t)}{\sqrt{\mathcal{E}_2}} = \frac{s_2(t) - c_{21}(t)\Psi_1(t)}{\sqrt{2}} = \frac{s_2(t) - 0 \cdot \Psi_1(t)}{\sqrt{2}} = \frac{s_2(t)}{\sqrt{2}}$

X 1. Geometric Representation of Signal Waveforms [4/5]

The equation $d_3(t) = s_3(t) - \sum_{i=1}^{3-1} c_{3i}(t)\Psi_i(t)$ requires estimation of $c_{31}(t)$, & $c_{32}(t)$

$$c_{31}(t) = \int_{-\infty}^{\infty} s_3(t)\Psi_1(t)dt = \int_0^1 -1\cdot 1dt + \int_1^2 1\cdot(-1)dt + \int_2^3 0\cdot 1dt = 0$$

$c_{32}(t) = \int_{-\infty}^{\infty} s_3(t)\Psi_2(t)dt = \int_0^1 -1\cdot 1dt + \int_1^2 1\cdot(-1)dt = +\int_2^3 1\cdot 0dt = -2.$ As $d_k(t) = s_k(t) - \sum_{i=1}^{k-1} c_{ki}(t)\Psi_i(t)$, $d_3(t) = s_3(t) - c_{31}(t)\Psi_1(t) - c_{32}(t)\Psi_2(t) = s_3(t) - 0 + 2\cdot\Psi_2(t)$

Thus, $d_3(t) = s_3(t) + 2\cdot\frac{s_2(t)}{\sqrt{2}} = s_3(t) + \sqrt{2}\cdot s_2(t)$

$$\mathcal{E}_3 = \int_{-\infty}^{\infty} d_3^2(t)dt = \int_{-\infty}^{\infty}\left[s_3^2(t) + 2s_2^2(t) + 2\sqrt{2}\cdot s_2(t)\cdot s_3(t)\right]dt = 3 + 4 + 0 = 7$$

Thus, $\Psi_3(t) = \frac{d_3(t)}{\sqrt{\mathcal{E}_3}} = \frac{s_3(t) - c_{21}(t)\Psi_1(t) - c_{32}(t)\Psi_2(t)}{\sqrt{7}} = \frac{s_3(t) - 0\cdot\Psi_1(t) + 2\cdot\Psi_2(t)}{\sqrt{7}} = \frac{s_3(t) + 2\cdot\frac{s_2(t)}{\sqrt{2}}}{\sqrt{7}} = \frac{s_3(t) + \sqrt{2}s_2(t)}{\sqrt{7}}$

1. Geometric Representation of Signal Waveforms [5/5]

The equation $d_3(t) = s_3(t) - \sum_{i=1}^{3-1} c_{3i}(t)\Psi_i(t)$ requires estimation of $c_{31}(t)$, & $c_{32}(t)$

$$c_{41}(t) = \int_{-\infty}^{\infty} s_4(t)\Psi_1(t)dt = \int_0^1 1\cdot 1dt + \int_1^2 1\cdot 1dt + \int_2^3 1\cdot 0dt = 2$$

$$c_{42}(t) = \int_{-\infty}^{\infty} s_4(t)\Psi_2(t)dt = \int_0^1 1\cdot 1dt + \int_1^2 1\cdot(-1)dt = +\int_2^3 1\cdot 0dt = 0$$

$$c_{43}(t) = \int_{-\infty}^{\infty} s_4(t)\Psi_3(t)dt = \int_0^1 1\cdot(-1)dt + \int_1^2 1\cdot 1dt = +\int_2^3 1\cdot 1dt = 1$$

As $d_k(t) = s_k(t) - \sum_{i=1}^{k-1} c_{ki}(t)\Psi_i(t)$, $d_4(t) = s_4(t) - c_{41}(t)\Psi_1(t) - c_{32}(t)\Psi_2(t) - c_{43}(t)\Psi_3(t) = s_4(t) - 2\cdot\Psi_1(t) - 0\cdot\Psi_2(t) - 1\cdot\Psi_3(t)$. Thus, $d_3(t) = s_4(t) - 2\cdot\frac{s_1(t)}{\sqrt{2}} - \frac{s_3(t)+\sqrt{2}s_2(t)}{\sqrt{7}}$

$$\mathcal{E}_4 = \int_{-\infty}^{\infty} d_4^2(t)dt = k$$

Thus, $\Psi_4(t) = \frac{d_4(t)}{\sqrt{\mathcal{E}_4}} = \frac{s_4(t)-2\cdot\frac{s_1(t)}{\sqrt{2}}-\frac{s_3(t)+\sqrt{2}s_2(t)}{\sqrt{7}}}{k}$

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2. Binary Modulation Schemes [1/4]

We consider 2 classes of binary modulation schemes:

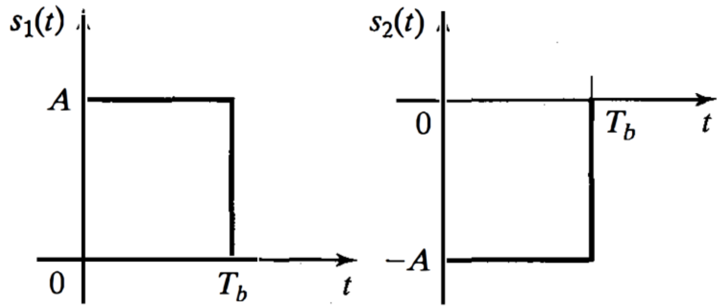
1) Binary antipodal signaling and

2) Binary orthogonal signaling

Binary pulse amplitude modulation (PAM), Binary amplitude shift keying (ASK), Binary pulse position modulation (PPM) and Binary frequency shift keying (FSK) are special cases of these two classes of binary modulations.

In the analysis we assume binary data is to be transmitted @ R_b bps.

Binary Antipodal Signaling Logical 1 is $p(t)$ & logical 0 is $-p(t)$, each with pulse duration $T_b = \frac{1}{R_b}$.



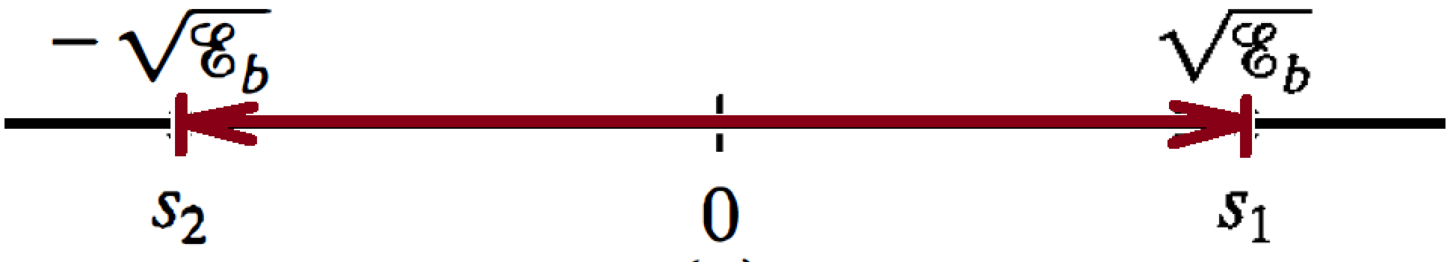
1.1. Binary PAM signal waveforms are expressed as

$s_m(t) = A_m g_T(t)$, $0 \leq t \leq T_b$, $m = 1,2$ & $A_1 = A$, $A_2 = -A$

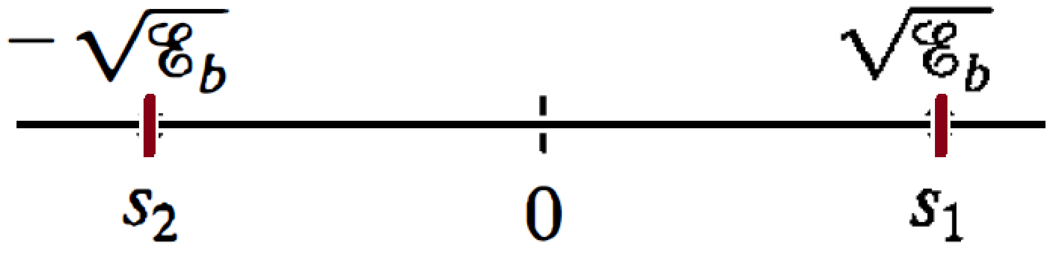
Signal energy of each of the 2 waveforms: $\mathcal{E}_m = \int_0^{T_b} s_m^2(t)dt = A^2 T_b$. This is the energy per bit.

2. Binary Modulation Schemes [2/4]

Thus, the Binary PAM signal can be uniquely represented geometrically in one dimension (on the real line) as two vectors, and each has the amplitude $\sqrt{\mathcal{E}_b}$.



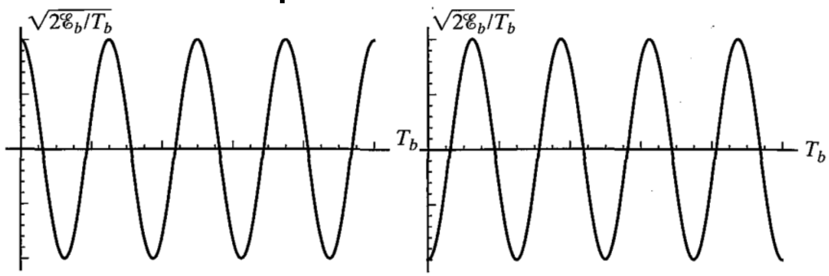
(a)



(b)

1.2. Binary ASK signal The two baseband signals, $\pm p(t)$, is used to amplitude modulate a sinusoidal carrier, $A_c \cos(2\pi f_c t)$. Thus,

$$s_1(t) = p(t)\, A_c \cos(2\pi f_c t) = A_c p(t) \cos(2\pi f_c t) = A_c \sqrt{\frac{2\mathcal{E}_b}{T_b}} \cos(2\pi f_c t)$$



$$s_2(t) = -p(t)\, A_c \cos(2\pi f_c t) = -A_c p(t) \cos(2\pi f_c t) = -A_c \sqrt{\frac{2\mathcal{E}_b}{T_b}} \cos(2\pi f_c t)$$

Binary ASK waveforms

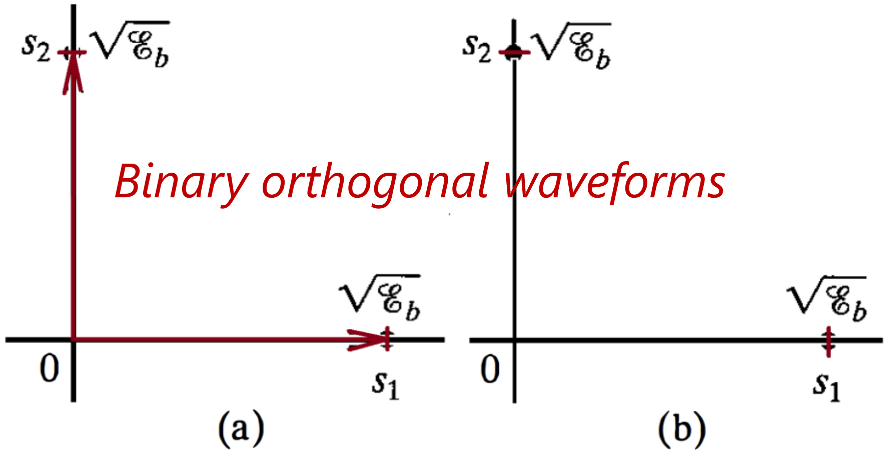
The unit energy signal is $\Psi(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$

Main difference between PAM & ASK is that PAM is baseband whereas, ASK is centered at $\pm f_c$.

2. Binary Modulation Schemes [3/4]

Binary Orthogonal Signaling In binary orthogonal signaling, the two signals are orthogonal, not antipodal, i.e., $\mathcal{E}_b = \int_0^{T_b} s_1^2(t)\,dt = \int_0^{T_b} s_2^2(t)\,dt$ & $\int_0^{T_b} s_1(t)s_2(t)\,dt = 0$

As the signals are already orthogonal, the task of finding an orthonormal basis is simply, the task of normalizing them: $\Psi_1(t) = \frac{s_1(t)}{\sqrt{\mathcal{E}_b}}$ & $\Psi_2(t) = \frac{s_2(t)}{\sqrt{\mathcal{E}_b}}$



Binary orthogonal waveforms

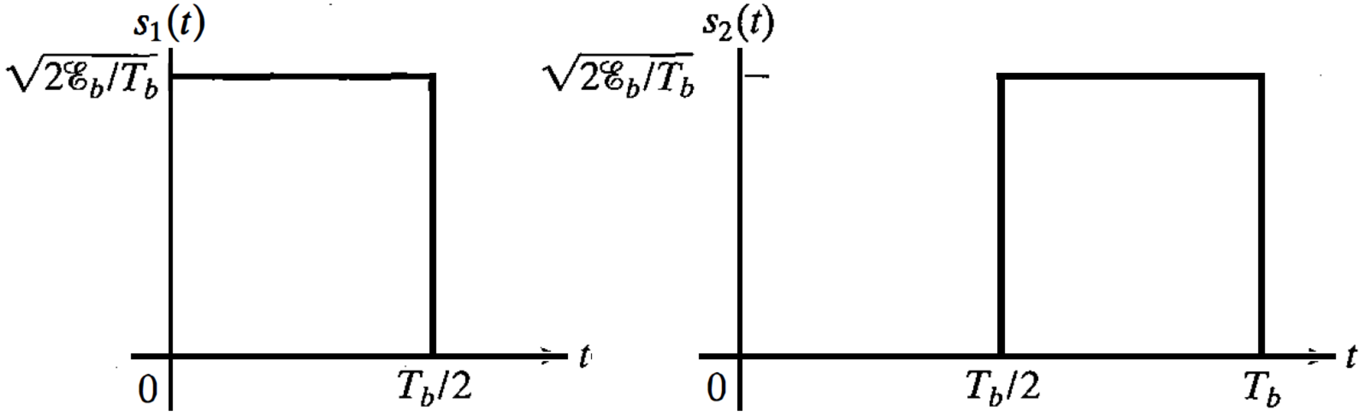
$s_1(t) = \sqrt{\mathcal{E}_b}\Psi_1(t) + 0\Psi_2(t)$ &

$s_2(t) = 0\Psi_1(t) + \sqrt{\mathcal{E}_b}\Psi_2(t)$ i.e.,

$s_1(t) = \left(\sqrt{\mathcal{E}_b}, 0\right)$ & $s_2(t) = \left(0, \sqrt{\mathcal{E}_b}\right)$

only images

1.1. Binary Pulse Position Modulation (PPM) signal Binary PPM is an example of binary orthogonal signaling. In binary PPM, we employ two pulses that are different only in their location. These pulses are shown on the *right*.

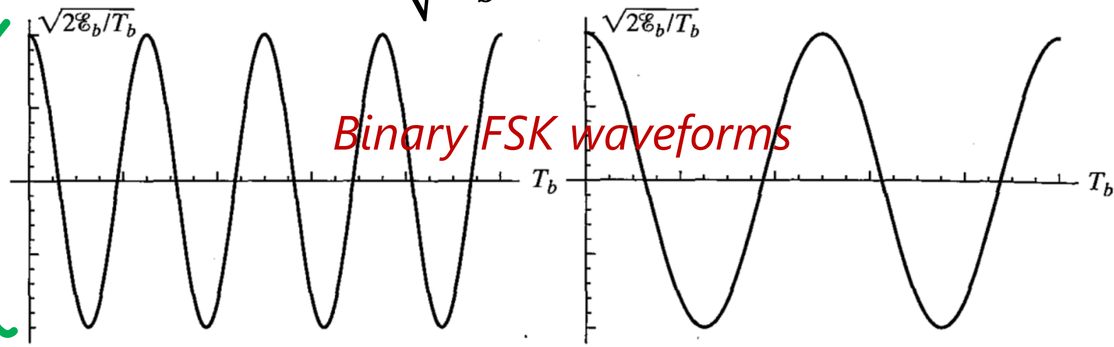


2. Binary Modulation Schemes [4/4]

1.2. Binary Frequency Shift Keying (FSK) signal Another example of orthogonal signal is BFSK. The two signal waveforms are, for $0 \le t \le T_b$ & $f_1 \ne f_2$, $s_1(t) = \sqrt{\frac{2\mathcal{E}_b}{T_b}}\cos(2\pi f_1 t)$ &

$s_2(t) = \sqrt{\frac{2\mathcal{E}_b}{T_b}}\cos(2\pi f_2 t)$

$\Psi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_1 t)$ & $\Psi_2(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_2 t)$



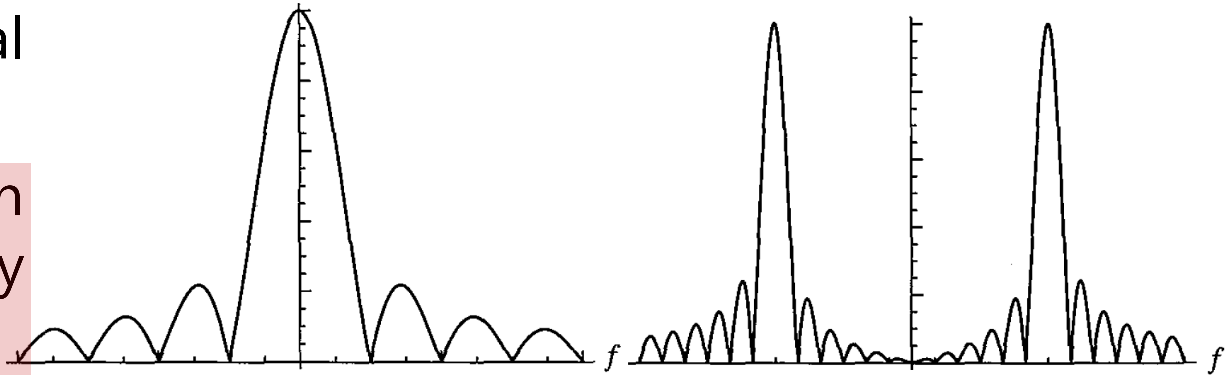
Binary FSK waveforms

Thus, $s_1(t) = \sqrt{\mathcal{E}_b}\Psi_1(t)$ & $s_2(t) = \sqrt{\mathcal{E}_b}\Psi_2(t)$. Thus, binary FSK waveforms have the same geometric representation as on previous slide.

BFSK vs BPPM: The basic difference between BFSK & BPPM signals is their spectral characteristics as shown in the figure.

Despite these differences, their performance in channel corrupted by AWGN is identical ∵ they have identical geometric representations.

Spectral characteristics of binary PPM & FSK signals



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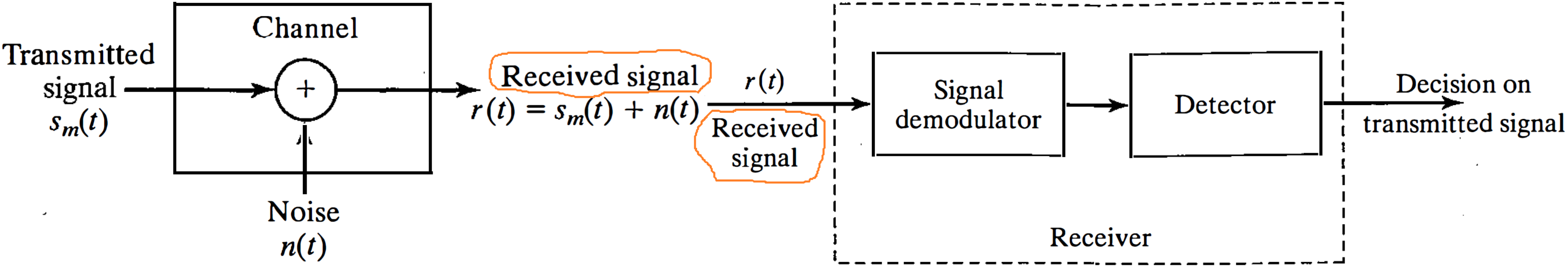
1. Optimum Receiver for Binary Modulated Signals in AWGN [1/5]

The received signal captured in a duration T_b is shown in the figure. Note: because it is a binary signal, m is either 1 or 2. AWGN psd is $S_n(f) = N_o/2\ Watt/Hz$.



By optimum receiver we mean a receiver that minimizes the probability of making an error.

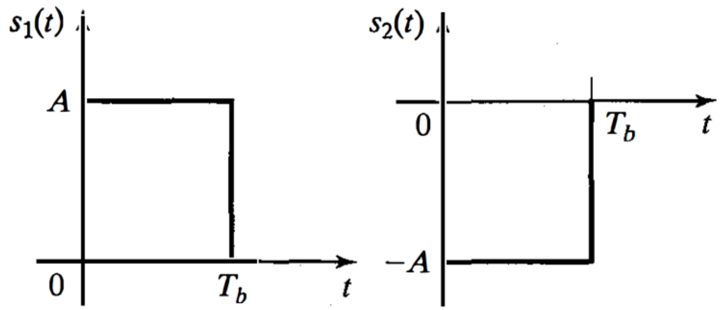
It is convenient to subdivide the receiver into two parts, the signal demodulator and the detector (*right half of the figure*).

The demodulator strips the signal off the carrier wave. The detector decides which of the two waveform was transmitted.

The demodulator may be based on a correlator or, a matched filter.

1. Optimum Receiver for Binary Modulated Signals in AWGN [2/5]

1.1. Correlation-Type Demodulator for Binary Antipodal Signals: The antipodal signals are $s_m(t) = s_m\Psi(t)$, $m = either\ 1\ or\ 2.$



$\Psi(t) = \frac{p(t)}{\sqrt{\mathcal{E}_p}}$ is the unit energy rectangular pulse.

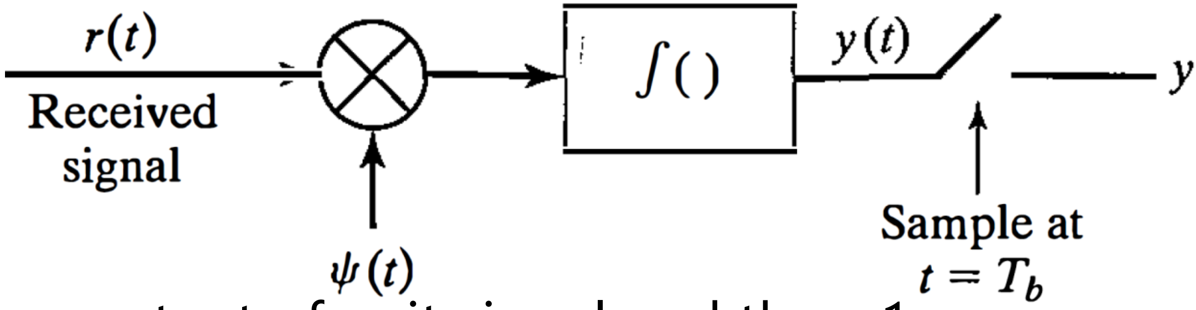
For a unit energy pulse, $s_1 = \sqrt{\mathcal{E}_b}$ & $s_2 = -\sqrt{\mathcal{E}_b}$. $\therefore$ received signal is:

$r(t) = s_m\Psi(t) + \eta(t)$, $0 \le t \le T_b$, $m = 1, 2$

In a correlation-type demodulator, the received signal $r(t)$ is multiplied by the signal waveform $\Psi(t)$ and the product is integrated over the interval $0 \le t \le T_b$.

$y(t) = \int_0^t r(\tau)\Psi(\tau)\,d\tau = \int_0^t [s_m\Psi(t) + \eta(t)]\Psi(\tau)\,d\tau$

$= s_m \int_0^t \Psi^2(t)\,d\tau + \underbrace{\int_0^t \eta(t)\,\Psi(\tau)d\tau}$



The 1st integral term over the period is simply energy content of unit signal and thus, 1.

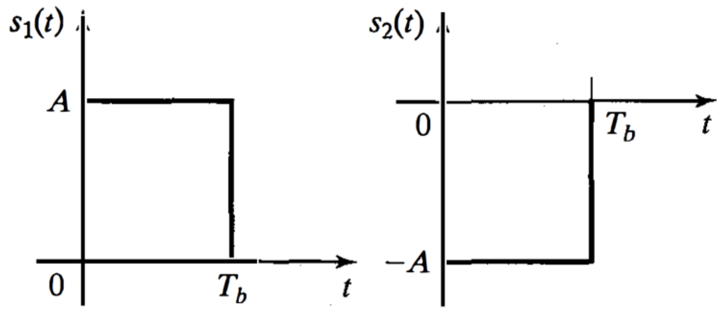
Thus, $y(T_b) = s_m + n$

1. Optimum Receiver for Binary Modulated Signals in AWGN [3/5]

$$\sigma_n^2 = E(n^2) = \int_0^{T_b}\int_0^{T_b} E[n(t)n(\tau)\Psi(t)\Psi(\tau)]\,dtd\tau$$

$$= \int_0^{T_b}\int_0^{T_b} \frac{N_0}{2}\delta(t-\tau)\Psi(t)\Psi(\tau)\,dtd\tau = \frac{N_0}{2}\int_0^{T_b}\Psi^2(t)\,dt = \frac{N_0}{2}\cdot 1 = \frac{N_0}{2}$$

The output of the correlator is a Gaussian random variable with mean s_m & variance $\frac{N_0}{2}$, i.e. $f(y|s_m) = \frac{1}{\pi N_0}e^{-(y-s_m)^2/N_0}$, m is either 1 or 2



$$\sigma_n^2 = E(n^2) = \int_0^{T_b}\int_0^{T_b} E[n(t)n(\tau)\Psi(t)\Psi(\tau)]\,dtd\tau$$

$$= \int_0^{T_b}\int_0^{T_b} \frac{N_0}{2}\delta(t-\tau)\Psi(t)\Psi(\tau)\,dtd\tau = \frac{N_0}{2}\int_0^{T_b}\Psi^2(t)\,dt$$
$$= \frac{N_0}{2}\cdot 1 = \frac{N_0}{2}$$

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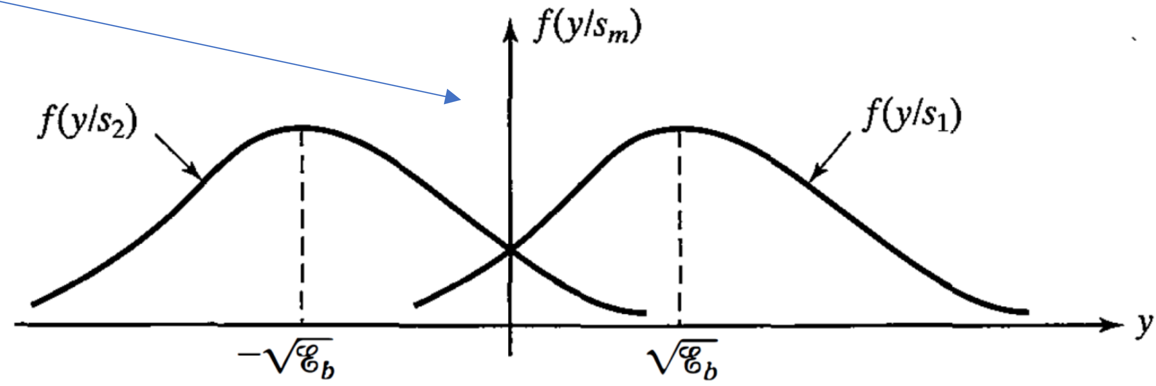


The probability density function of the correlator output for binary antipodal signaling

only graphs

1. Optimum Receiver for Binary Modulated Signals in AWGN [4/5]

1.2. Correlation-Type Demodulator for Binary Orthogonal Signals: TBD

Phase Shift Keying: is a digital modulation process which conveys data by changing (modulating) the phase of a constant frequency carrier wave.

PSK uses a finite number of phases, each assigned a unique pattern of binary digits. Usually, each phase encodes an equal number of bits.

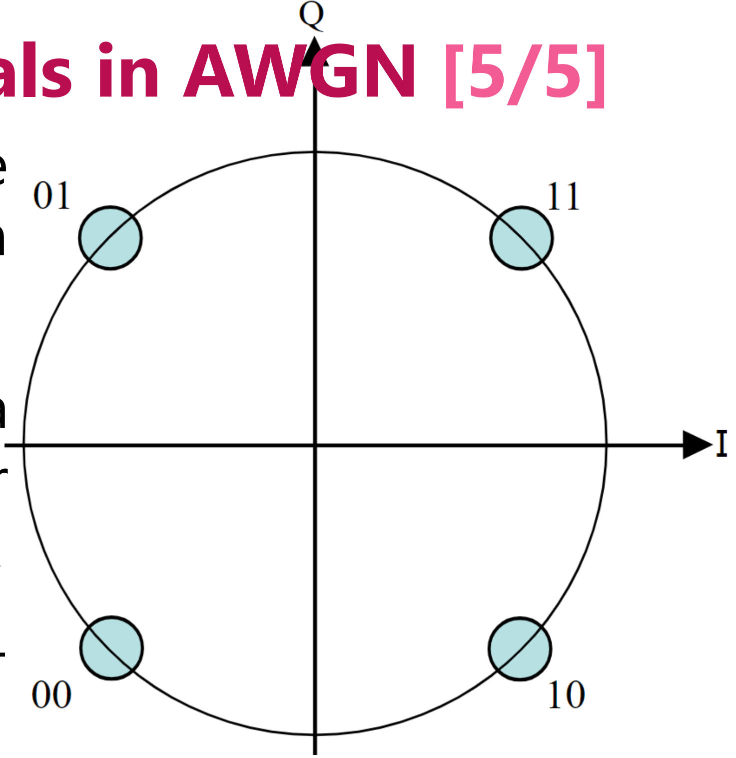
The demodulator, which is designed specifically for the symbol-set used by the modulator, determines the phase of the received signal and maps it back to the symbol it represents, thus recovering the original data.

Precise synchronization is required between the clocks at the transmitter and receiver. It may use a reference signal and thus, BPSK is known as *coherent*-BPSK (CBPSK).

Alternatively, the phase shift of each symbol sent can be measured with respect to the phase of the previous symbol sent. $\because$ the symbols are encoded in the difference in phase between successive samples, this is called differential phase-shift keying (DPSK). DPSK is significantly simpler to implement than ordinary PSK.

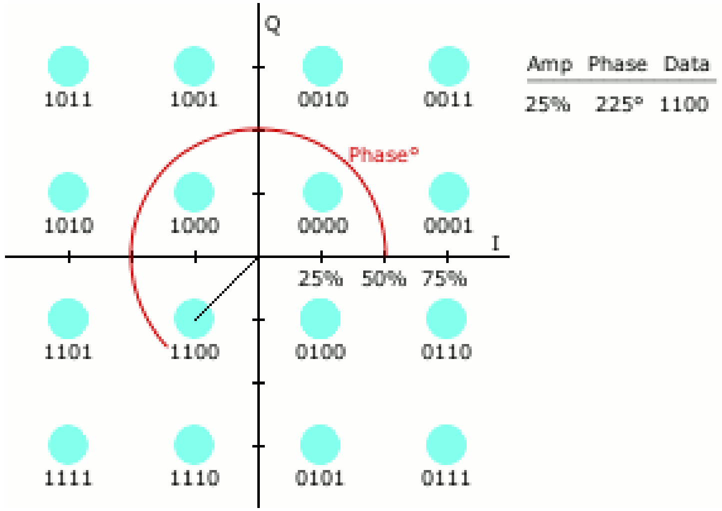
1. Optimum Receiver for Binary Modulated Signals in AWGN [5/5]

Quadrature Phase Shift Keying: QPSK uses four points on the diagram, equispaced around a circle. With four phases, QPSK can encode two bits per symbol, shown with Gray coding.



QPSK can be used either to double the data rate compared with a BPSK system while maintaining the same bandwidth of the signal, or to maintain the data-rate of BPSK but halving the bandwidth needed.

Just as we have differentially-encoded BPSK, we have differentially-encoded QPSK.



Quadrature Amplitude Modulation (QAM): in QPSK all symbols lie on the unit circle and in QAM the symbols lie on straight line that form a square. Here you see 16-QAM.