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Impact of Noise on Analog Communications

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I. Effect of Noise on Amplitude Modulation Systems

II. Effect of Noise on ∠-Modulation Systems

III. Comparison of Analog-Modulation Systems

IV. Transmission Loss & Noise Impact in Analog Communication Systems

Effect of Noise on AM Systems [1/9]

We analyze as follows:

Step 1: Estimate signal-to-noise ratio (SNR) for a baseband signal, i.e., a non-modulated signal.

Step 2: Estimate SNR for a receiver that implements a demodulator for an AM signal where the amplitude is modulated using that baseband signal as the message signal.

SNR for Baseband Signal: There is no carrier. Thus, receiver requires only an ILPF (ideal lowpass filter) with bandwidth W.

AWGN (Additive White Gaussian Noise) is:

why you're doing and whatcha doing is useful

- *additive* $\because$ *the noise is added to the system output which is assumed noiseless*
- *white* $\because$ *it has a uniform power spectral density across all frequency bands*
- *Gaussian* $\because$ *at any time, the magnitude of noise is randomly drawn from a time-invariant Gaussian distribution*

Power of AWGN noise at receiver: $P_{n_o} = \int_{-\infty}^{+\infty} \frac{N_0}{2} df$. As the receiver comprises an ILPF of bandwidth W, $P_{n_o} = \int_{-W}^{+W} \frac{N_0}{2} df = N_0 W$. P_R = signal power at the receiver $\therefore$ SNR is $\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W}$.

Effect of Noise on AM Systems [2/9]

Problem: Transmitted power is 1 KW. Channel attenuation is 10^{-12}. Bandwidth is 5 kHz. Mean noise is $N_0 = 10^{-14} Watt/Hz$. Find $\left(\frac{S}{N}\right)_b$ in dB.

Solution: $\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = \frac{10^3 \times 10^{-12}}{10^{-14} \times 5000} = 20$. In dB this is $10 log_{10}(20) \approx 13dB$.

SNR for DSB-SC AM: Transmitted signal is $u(t) = A_c m(t) \cos(2\pi f_c t)$ … (1)

As the noise is additive, $r(t) = u(t) + \eta(t)$ … (2)

Note that

- *the noise is filtered; bandlimited by the same bandpass filter that selects the modulated signal*
- *filtered noise, like any signal, can be expressed in terms of in-phase & quadrature components*

Thus, $r(t) = A_c m(t) \cos(2\pi f_c t) + [n_c(t) \cos(2\pi f_c t) - n_s(t) \sin(2\pi f_c t)]$ … (3)

We demodulate $r(t)$ at the receiver by first multiplying by a locally-generated sinusoid $\cos(2\pi f_c t + \varphi)$ … (4) Thus, from (2) & (4):

$r(t) \cos(2\pi f_c t + \varphi) = A_c m(t) \cos(2\pi f_c t) \cos(2\pi f_c t + \varphi) + n(t) \cos(2\pi f_c t + \varphi)$ … (5)

Effect of Noise on AM Systems [3/9]

$r(t)\,cos(2\pi f_c t+\varphi) = A_c m(t)\,cos(2\pi f_c t)\,cos(2\pi f_c t+\varphi) + n(t)cos(2\pi f_c t+\varphi)$ ... (5)

As $cosAcosB = \frac{1}{2}[cos(A+B)+cos(A-B)]$ and, from (2) & (3), LHS $= \frac{1}{2}A_c m(t)\,cos(4\pi f_c t+\varphi)+\frac{1}{2}A_c m(t)\,cos(\varphi) + \underbrace{[n_c(t)\,cos(2\pi f_c t) - n_s(t)\,sin(2\pi f_c t)]\cdot cos(2\pi f_c t+\varphi)}$

$$n_c\,cos^2(2\pi f_c t)\,cos(\varphi) - n_c(t)\,cos(2\pi f_c t)\,sin(2\pi f_c t)\,sin(\varphi)$$
$$-n_s(t)\,sin(2\pi f_c t)\,cos(2\pi f_c t)\,cos(\varphi) + n_s(t)\,sin^2(2\pi f_c t)\,sin(\varphi)$$

$$= \frac{1}{2}n_c[1+cos(4\pi f_c t)]\,cos(\varphi) - \frac{1}{2}n_c(t)\,sin(4\pi f_c t)\,sin(\varphi) - \frac{1}{2}\,n_s(t)\,sin(4\pi f_c t)\,cos(\varphi) + \frac{1}{2}n_s(t)\,[1-cos(4\pi f_c t)]\,sin(\varphi)$$

$$= \frac{1}{2}[n_c\,cos(\varphi)+n_s(t)sin(\varphi)] + \frac{1}{2}n_c[cos(4\pi f_c t)\,cos(\varphi) - sin(4\pi f_c t)\,sin(\varphi)] - \frac{1}{2}n_s[sin(4\pi f_c t)\,cos(\varphi)+cos(4\pi f_c t)\,sin(\varphi)]$$

$$= \frac{1}{2}[n_c\,cos(\varphi)+n_s(t)sin(\varphi)] + \frac{1}{2}n_c\,cos(4\pi f_c t+\varphi) - \frac{1}{2}n_s\,sin(4\pi f_c t+\varphi)$$

Effect of Noise on AM Systems [4/9]

Thus, $r(t)\,cos(2\pi f_c t + \varphi) = \frac{1}{2} A_c m(t)\, cos(4\pi f_c t + \varphi) + \frac{1}{2} A_c m(t)\, cos(\varphi)$
$+ \frac{1}{2}[n_c\, cos(\varphi) + n_s(t) sin(\varphi)] + \frac{1}{2} n_c\, cos(4\pi f_c t + \varphi) - \frac{1}{2} n_s\, sin(4\pi f_c t + \varphi)$ ... (6)

Equation (6) is the output of the 1st step of the demodulation process in the presence of AWGN.

The 2nd step is to pass the sinusoid-multiplied AWGN-corrupted received signal through an ILPF. The resulting signal is: $\frac{1}{2} A_c m(t)\, \cancel{cos(4\pi f_c t + \varphi)} + \frac{1}{2} A_c m(t)\, cos(\varphi) + \frac{1}{2}[n_c\, cos(\varphi) + n_s(t) sin(\varphi)] + \frac{1}{2} n_c\, \cancel{cos(4\pi f_c t + \varphi)} - \frac{1}{2} n_s\, \cancel{sin(4\pi f_c t + \varphi)}$, i.e.,

$\frac{1}{2} A_c m(t)\, cos(\varphi) + \frac{1}{2}[n_c\, cos(\varphi) + n_s(t) sin(\varphi)]$ ... (7)

The signal component of the message is just the first term $\frac{1}{2} A_c m(t)\, cos(\varphi)$ ... (8)

$\therefore$ SNR is $\frac{\frac{1}{4} {A_c}^2 P_M\, cos^2(\varphi)}{\frac{1}{4} P_n}$ Thus, $\left(\frac{S}{N}\right)_b = \frac{{A_c}^2 P_M\, cos^2(\varphi)}{P_n}$. Earlier we noted $P_n = N_0 W$

Thus, SNR is $\left(\frac{S}{N}\right)_b = \frac{{A_c}^2 P_M\, cos^2(\varphi)}{N_0 W} = \frac{P_R}{N_0 W}$ ... (9)

from slide 1 to here

Effect of Noise on AM Systems [5/9]

From slides 3 & 6, it is clear that moving from baseband to DSB SC AM does not improve SNR.

Problem: Would SNR over baseband improve if you used SSB SC AM? Justify. shld be able to answer this

Solution: DYI. *For a mathematical proof, look at* Ch06, pp. 258-259, John G. Proakis & Masoud Salehi, "*Fundamentals of Communications Systems*", 2nd Edition, June 2013. argue w logic - no eqns

SNR for Conventional AM: Transmitted signal is $u(t) = A_c[1 + am(t)]\, cos(2\pi f_c t)$ … (1)

As the noise is additive, $r(t) = u(t) + \eta(t)$ … (2)

be aware of what's gng on and why

Thus, $r(t) = A_c[1 + am(t)]\, cos(2\pi f_c t) + [n_c(t)\, cos(2\pi f_c t) - n_s(t)\, sin(2\pi f_c t)]$ … (3)

When you go through the entire process that you adopted for DSC SC AM you will find that SNR is $\left(\frac{S}{N}\right)_{c-AM} = \frac{a^2 P_M}{(1+a^2 P_M)}\left(\frac{P_R}{N_0 W}\right)$ which is $= \frac{a^2 P_M}{(1+a^2 P_M)}\left(\frac{S}{N}\right)_b$. *See* pp. 259-260 *in the book above*. Thus, SNR for Conventional AM is ≪ that for the baseband. This can be analytically arrived at with the reasoning that a substantial power of conventional AM is not in the message signal but is concentrated in the carrier. If $P_M = 0.1$, such as for speech communications, a is typically 0.8-0.9. $a = 0.8$ causes a 11.1 dB loss in SNR over baseband. **[Problem:** show this!]

Effect of Noise on AM Systems [6/9]

We studied the impact of AWGN on the received signal at a synchronous demodulator for DSB SC AM, SSB SC AM & conventional AM . Now we repeat it for an envelope detector. We work with approximations as envelope detector is non-linear.

Impact of AWGN on Envelope Detector: The envelope of signal received at the receptor is $r(t) = A_c[1 + am(t)]\, cos(2\pi f_c t) + [n_c(t)\, cos(2\pi f_c t) - n_s(t)\, sin(2\pi f_c t)]$ … (1)

$= [A_c[1 + am(t)] + n_c(t)]\, cos(2\pi f_c t) - n_s(t)\, sin(2\pi f_c t)$

$|r(t)| = V_r(t) = \sqrt{[A_c[1 + am(t)] + n_c(t)]^2 + n_s(t)^2}$ … (2)

ignore

If noise is much stronger than the message signal … (3) then

$\sqrt{A_c{}^2\left(1 + am(t)\right)^2 + 2n_c(t)A_c\left(1 + am(t)\right) + n_c(t)^2 + n_s(t)^2} \approx$

$\sqrt{2n_c(t)A_c[1 + am(t)] + n_c(t)^2 + n_s(t)^2} = \sqrt{[n_c(t)^2 + n_s(t)^2]} \cdot \sqrt{1 + \frac{2n_c(t)A_c[1+am(t)]}{n_c(t)^2+n_s(t)^2}}$ As for small ϵ, $\sqrt{1+\epsilon} = 1 + \frac{\epsilon}{2}$, this is $n(t) \cdot \left[1 + \frac{n_c(t)}{n(t)^2} A_c[1 + am(t)]\right] = 1 + \frac{n_c(t)}{n(t)} A_c[1 + am(t)]$… (4)

Effect of Noise on AM Systems [7/9]

(4) shows that for envelope detectors in the presence of strong noise, the AWGN is no longer additive but multiplicative. This makes it nearly impossible to define any meaningful SNR.

If the AWGN is weak then, from (2), $V_r(t) = \sqrt{[A_c[1 + am(t)] + n_c(t)]^2 + n_s(t)^2} \approx$

$$\sqrt{[A_c{}^2[1 + am(t)]^2 + 2A_c[1 + am(t)]n_c(t)]} = A_c[1 + am(t)]\sqrt{1 + \frac{2A_c[1+am(t)]n_c(t)}{A_c{}^2[1+am(t)]^2}}$$

$= A_c[1 + am(t)]\sqrt{1 + \frac{2n_c(t)}{A_c[1+am(t)]}}$ As $n_c(t) \ll A_c[1 + am(t)]$, using the approximation $\sqrt{1+\epsilon} = 1 + \frac{\epsilon}{2}$, $V_r(t) \approx A_c[1 + am(t)] \cdot \left[1 + \frac{n_c(t)}{A_c[1+am(t)]}\right]$

ignore

$= A_c[1 + am(t)] + n_c(t)$. The non-signal component, A_c, is irrelevant.

$\therefore V_r(t) = A_c am(t) + n_c(t)$ ... (5)

(5) shows that under small amounts of AWGN, the noise for envelope detector is additive, just as in the case of synchronous demodulators.

Effect of Noise on AM Systems [8/9]

imp question

Problem: Assume the message is $R_M(t) = 16sinc^2(10^4 t)$. $\max\{|m(t)|\} = 6$. The message is transmitted over a channel that imposed a 50 dB attenuation. The AWGN is $2 \times 10^{-12}\ Watt/Hz$. Required SNR at the demodulator is $\geq 50\ dB$. What is the transmission power required for *a*) DSB SC AM, *b*) SSB SC AM & *c*) Conventional AM with modulation index of 0.8?

Solution: First determine the bandwidth of the message using Fourier representation. $\Im[R_M(t)] = \frac{16}{10^4} \wedge \left(\frac{f}{10^4}\right)$. This is a band-limited signal with $-10^4 \leq f \leq 10^4$.

Thus, signal bandwidth is $10^4 Hz$ … (1)

Baseband SNR is $\left(\frac{S}{N}\right)_b = \frac{P_R}{N_0 W} = \frac{P_R}{2\times 10^{-12}\times 10^4} = 5 \times 10^7 P_R$ … (2)

As power is attenuated by $\geq 50\ dB$ between Tx & Rx, $10log_{10}\left(\frac{P_T}{P_R}\right) = 50$ i.e., $P_R = 10^{-5} P_T$ … (3)

From (2) & (3), $\left(\frac{S}{N}\right)_b = 5 \times 10^7 \times 10^{-5} P_T = 500 P_T$ … (4)

As, for **DSB SC AM,** SNR is the same as that for the baseband, SNR is $500 P_T$.

Effect of Noise on AM Systems [9/9]

As this must be $\geq 50\,dB$, $10log_{10}(500P_T) \geq 50$ or, $500P_T \geq 100000$ or, the minimum transmitted power, P_T must be $\geq 200\,Watt$ … (5)

DSB SC AM bandwidth is $2 \times W = 2 \times 10{,}000 = 20\,kHz$.

For **SSB SC AM,** P_T and the required bandwidth are half as that for DSB SC AM … (6)

For Conventional AM: SNR is $\frac{a^2 P_M}{(1+a^2 P_M)} \cdot \left(\frac{S}{N}\right)_b = \frac{0.8^2 P_M}{(1+0.8^2 P_M)} \cdot 500P_T$

Normalized signal is $P_{M_n} = \frac{P_M}{[\max\{|m(t)|\}]^2} = \frac{P_M}{6^2} = \frac{P_M}{36}$

no derviation, formulae will be available

Maximum value of the signal occurs at $t = 0$, which is 16.

Thus, $P_{M_n} = \frac{16}{36}$. Thus, SNR for Conventional AM is: $\frac{0.8^2 \cdot \frac{16}{36}}{\left(1+0.8^2 \cdot \frac{16}{36}\right)} \cdot 500P_T$

As SNR must be $\geq 50\,dB$, $10log_{10}(0.22 \cdot 500P_T) \geq 50$, i.e.,$P_T \geq 909\,Watt$

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Effect of Noise on ∠-Modulation Systems [1/8]

be aware

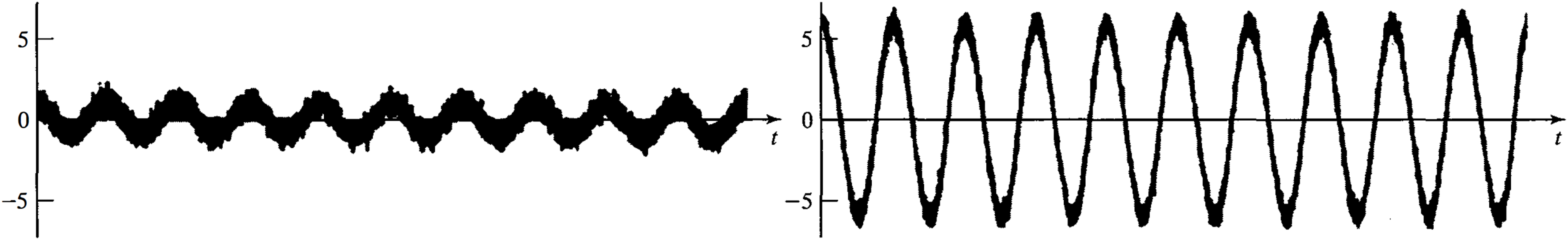
Just as we analyzed the impact of AWGN on AM, we will now analyze its impact on ∠-modulated systems.

In AM, the carrier amplitude is modulated. The noise is simply added to this amplitude. However, for ∠-modulated systems, the message information is in the signal angle instead. Thus, noise is relevant to the extent that it changes the frequency of the message signal.

The frequency is estimated via changes in number of zero crossings.

how noise is impacting sine wave

- additive



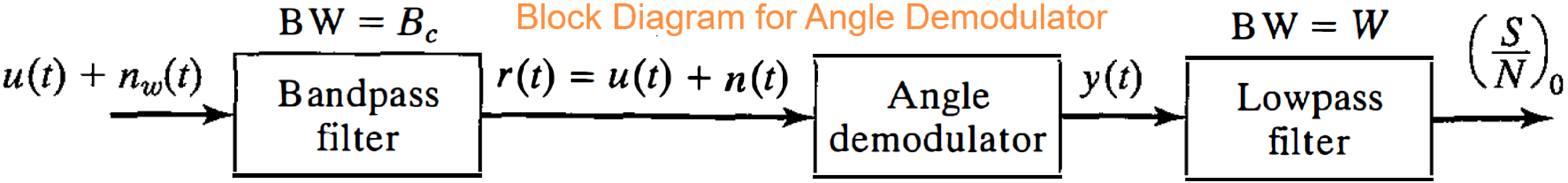
Effect of noise on a low-power vs a high-power FM signal

Problem: ~~How will you determine the number of zero crossings?~~ ~~**Solution:** DYI. Hint: *apply your knowledge of image processing or DSP here. There are many answers*~~ *…*

Effect of Noise on ∠-Modulation Systems [2/8]

For an ∠-modulated signal: $u(t) = A_c m(t)\, cos(2\pi f_c t + \varphi)$ … (1) φ *is different for* FM *and* PM

$$u(t) = \begin{cases} A_c m(t) cos\left(2\pi f_c t + k_p m(t)\right), & PM \\ A_c m(t) cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau) d\tau\right), & FM \end{cases} \quad \ldots (2)$$



Block Diagram for Angle Demodulator

In the diagram, $n_w(t)$ is AWGN. $n(t)$ is band-limited noise.

Received signal in presence of AWGN and, upon bandpass filtering, is $r(t) = u(t) + \eta(t)$ … (3)

$$r(t) = u(t) + \eta_c(t)\, cos(2\pi f_c t) - \eta_s(t) sin(2\pi f_c t) \ \ldots (4)$$

Effect of Noise on ∠-Modulation Systems [3/8]

We will analyze the case for **signal power ≫ noise, i.e., $A_c{}^2 m(t) \gg \eta_c{}^2(t) + \eta_s{}^2(t)$:**

Note $\eta_c(t)\,cos(2\pi f_c t) - \eta_s(t) sin(2\pi f_c t) = \sqrt{\eta_c{}^2(t) + \eta_s{}^2(t)}\,\cos\left(2\pi f_c t + tan^{-1}\left(\frac{\eta_s(t)}{\eta_c(t)}\right)\right)$ … (4)

(4) follows from the facts that the:

- *1st term is the magnitude of noise*
- *1st part of the ∠ is represents central frequency as noise is bandlimited by the ILPF*
- *2nd part of the ∠ is instantaneous term*

Thus, $\eta(t) = V_n(t)\cos\left(2\pi f_c t + \Phi_n(t)\right)$ … (5) *where $V_n(t)$ & $\Phi_n(t)$ are the envelope and phase of the bandpass noise process.*

$r(t) \approx \left[A_c + V_n(t)\,cos\left(\Phi_n(t) - \varphi(t)\right)\right] \times \cos\left[2\pi f_c t + \varphi(t) + tan^{-1}\left(\frac{V_n(t)\,sin\left(\Phi_n(t) - \varphi(t)\right)}{A_c + V_n(t) cos\left(\Phi_n(t) - \varphi(t)\right)}\right)\right]$ … (6)

(6) follows from the phasor diagram of the ∠-modulated signal … *see next slide.*

Effect of Noise on ∠-Modulation Systems [4/8]

Note $\eta_c(t)\,cos(2\pi f_c t) - \eta_s(t) sin(2\pi f_c t) = \sqrt{{\eta_c}^2(t) + {\eta_s}^2(t)}\cos\left(2\pi f_c t + tan^{-1}\left(\frac{\eta_s(t)}{\eta_c(t)}\right)\right)$ … (4)