20CYS301

Digital Communications

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Amplitude Modulation [AM]

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I. DSB-SC AM

II. DSB-SC AM Demodulation

III. Conventional AM

IV. Conventional AM Demodulation

V. Quadrature Carrier Multiplexing

DSB-SC AM [1/7]

Amplitude Modulation is modulation of an analog carrier using analog data. Examples of such data include speech, music, images or videos.

We discuss:

1. how amplitude modulation can be constructed mathematically (*the analog message signals change the amplitude of an analog carrier*)
2. methods for demodulation of the carrier-modulated signal to recover the analog information

Treatment of system performance under various noise conditions & power efficiency will be considered after we discuss angle modulation.

$m(t)$ is the analog signal to be transmitted. It is assumed to be a lowpass signal with bandwidth W. Thus, if $|f| > W$, $M(f) = 0$.

Power of the signal is $P_m = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |m(t)|^2 dt$

Carrier is $c(t) = A_c cos(2\pi f_c t + \varphi_c)$

Modulation changes $m(t)$ from lowpass to bandpass, in the neighborhood of carrier frequency.

DSB-SC AM [2/7]

Types of AMs:

1. Double-sideband-suppressed-carrier DSB-SC AM
2. Conventional double-sideband AM
3. Single-sideband SSB AM
4. Vestigial-sideband VSB AM

DSB-SC AM: message signal is $m(t)$. $c(t) = m(t)A_c cos(2\pi f_c t + \varphi_c)$ is carrier signal. Ignore Tx-Rx synchronization issues. Thus, φ_c can be set to 0. Thus, Tx signal is: $u(t) = A_c m(t) cos(2\pi f_c t)$.

Note: multiplication in time-domain is convolution in frequency domain.

Frequency Spectrum of DSB-SC AM Waveform:

Problem: Prove that $\Im[u(t)] = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$

Solution: $\Im[u(t)] = \Im[A_c m(t) cos(2\pi f_c t)] = \frac{A_c}{2}\Im\left[m(t)\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)\right]$

$= \frac{A_c}{2}\Im\left[m(t)e^{j2\pi f_c t}\right] + \frac{A_c}{2}\Im\left[m(t)e^{-j2\pi f_c t}\right]$

DSB-SC AM [3/7]

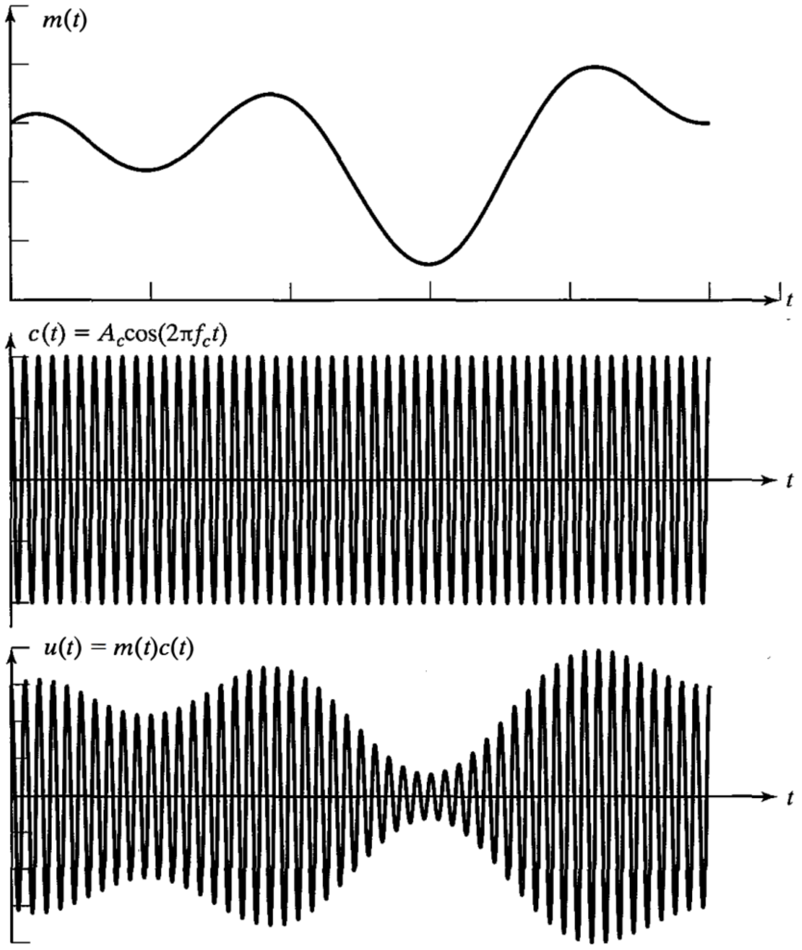
$$= \frac{A_c}{2}\Im\left[m(t)e^{j2\pi f_c t}\right] + \frac{A_c}{2}\Im\left[m(t)e^{-j2\pi f_c t}\right]$$

$$= \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{j2\pi f_c t}e^{-j2\pi f t}dt + \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{-j2\pi f_c t}e^{-j2\pi f t}dt$$

$$= \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{j2\pi f_c t}e^{-j2\pi f t}dt + \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{-j2\pi f_c t}e^{-j2\pi f t}dt$$

$$= \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{-j2\pi(f-f_c)t}dt + \frac{A_c}{2}\frac{1}{T}\int_0^T m(t)e^{-j2\pi(f+f_c)t}dt$$

$$= \frac{A_c}{2}M(f-f_c) + \frac{A_c}{2}M(f+f_c) = \frac{A_c}{2}[M(f-f_c) + M(f+f_c)]$$

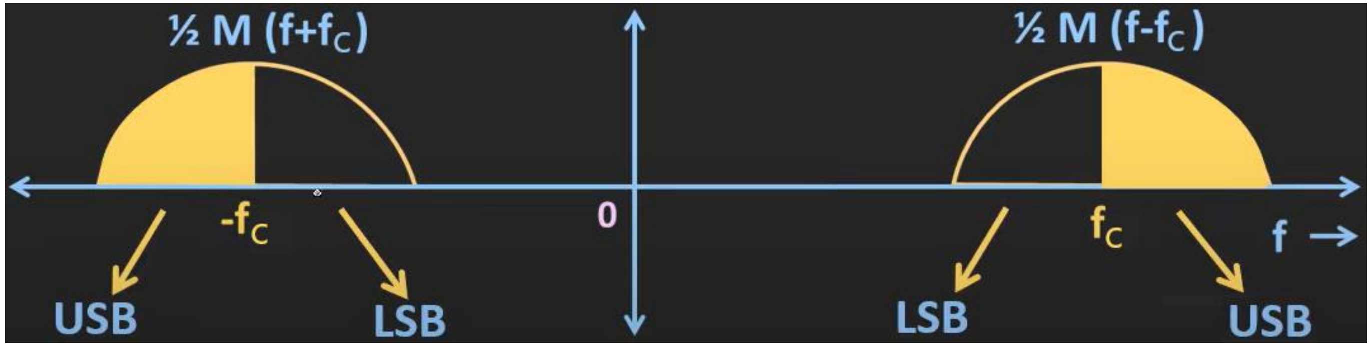


The spectrum of the message signal is shifted by f_c. If the message signal width was W then it occupies $2W$ after amplitude modulation (AM). Thus, channel bandwidth needed is $B_c = 2W$.

The frequency content of the modulated signal $u(t)$ in the frequency band $|f| > f_c$ is upper sideband of $U(f)$ and that in $|f| < f_c$ is lower sideband of $U(f)$. As $U(f)$ comprises both USB and LSB, it is called a double-sideband (DSB) AM signal.

DSB-SC AM [4/7]

$U_f = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$ upper side band (USB) & lower side band (LSB) of U_f.



Either of the two bands have all frequencies in $M(f)$. Message is shifted by f_c and channel bandwidth is $B_c = 2W$.

Note, the carrier is not transmitted. Thus, that bit of power is conserved.

Problem: How do we know from U_f or the chart that the carrier was not transmitted?

Solution: Had the carrier been transmitted, you will see $\delta(f \pm f_c)$ terms also in U_f.

Thus, the transmission is double side band (DSB) suppressed carrier (SC) amplitude modulated (AM) signal or, DSB SC AM signal, or, simply, DSBSC signal.

Problem: Express the product of 2 signals $u_t = Acos(2\pi f_m t)cos(2\pi f_c t)$ as sum of two signals.

Solution: As $A \cdot C[2\pi(f_m + f_c)t] = A \cdot C(2\pi f_m t)C(2\pi f_c t) - A \cdot S(2\pi f_m t)S(2\pi f_c t)$& $A \cdot C[2\pi(f_m - f_c)t] = A \cdot C(2\pi f_m t)C(2\pi f_c t) + A \cdot S(2\pi f_m t)S(2\pi f_c t)$,

$\frac{1}{2}A \cdot C[2\pi(f_m + f_c)t] + \frac{1}{2}A \cdot C[2\pi(f_m - f_c)t] = A \cdot cos(2\pi f_m t)cos(2\pi f_c t)$

DSB-SC AM [5/7]

Problem: In the previous problem, what if the signal is $A \cdot cos(2\pi f_m t + \theta) cos(2\pi f_c t + \varphi)$?

Solution: DIY!

Problem: Find fourier transform of $u_t = A \cdot cos(2\pi f_c t)$.

Solution: $\mathfrak{J}[u_t] = \mathfrak{J}[A \cdot C(2\pi f_c t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} A \cdot C(2\pi f_c t) e^{-j2\pi f_m t} dt$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A \cdot \frac{C(2\pi f_c t) + jS(2\pi f_c t) + C(2\pi f_c t) - jS(2\pi f_c t)}{2} e^{-j2\pi f_m t} dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A \cdot \left[\frac{C(2\pi f_c t) + jS(2\pi f_c t)}{2} + \frac{C(2\pi f_c t) - jS(2\pi f_c t)}{2}\right] e^{-j2\pi f_m t} dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} A \cdot \left[\frac{e^{j(2\pi f_c)t} + e^{-j(2\pi f_c)t}}{2}\right] e^{-j2\pi f_m t} dt = \frac{A}{4\pi}\int_{-\infty}^{\infty} e^{j2\pi(f_c - f_m)t} + e^{-j2\pi(f_c + f_m)t} dt$$

$$= \frac{A}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j2\pi(f_m - f_c)t} dt\right] + \frac{A}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-j2\pi(f_c + f_m)t} dt\right]$$

$$= \frac{A}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} 1 \cdot e^{-j2\pi(f_m - f_c)t} dt\right] + \frac{A}{2}\left[\frac{1}{2\pi}\int_{-\infty}^{\infty} 1 \cdot e^{-j2\pi(f_c + f_m)t} dt\right]$$

$$= \frac{A}{2}\delta(f - (f_m - f_c)) + \frac{A}{2}\delta(f - (f_m + f_c))$$

Thus, the Fourier Transformation of $A \cdot cos(2\pi f_c t)$ are two delta functions of half the amplitude.

DSB-SC AM [6/7]

Problem: Message signal is $m(t) = sinc(10^4 t)$. Find DSB-SC-AM signal and its bandwidth if the carrier is a sinusoid of frequency 1 MHz, i.e., $c(t) = cos(2\pi 10^6 t)$.

Solution: $u(t) = m(t)c(t)$ If we can get the Fourier transform of $m(t)$ then, as we know the signal is DSB-SC-AM signal, bandwidth of the signal will be 2 times the bandwidth of $m(t)$.

$\therefore \mathfrak{F}[m(t)] = \mathfrak{F}[sinc(10^4 t)] = \frac{1}{|10^4|}\sqcap\left[\frac{f}{10^4}\right] \because \mathfrak{F}[at] = \frac{1}{|a|}F\left[\frac{f}{a}\right]$ and $F[f]$ is rectangle function $\sqcap$.

The rectangle function has a bandwidth of 10^4 Hz, centered at 0. Thus, DSB-SC-AM signal bandwidth is $10KHz$.

What about "Find DSB-SC-AM signal"?

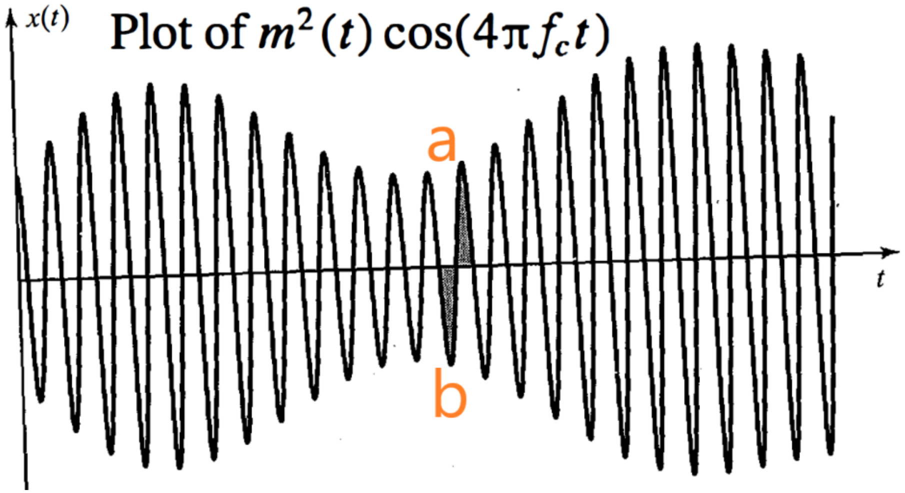
Problem: What is the power of DSB-SC-AM signal?

Solution: $\mathrm{P}[u(t)] = \lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} u^2(t)dt = \lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} {A_c}^2 m^2(t) C^2(2\pi f_c t)dt$

$= \lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{{A_c}^2}{2} m^2(t)[1 + cos(4\pi f_c t)]dt \ldots \because C^2(t) = \frac{[1+C(2T)]}{2}$

$= \frac{{A_c}^2}{2}\left[\lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} m^2(t)dt\right] + \frac{{A_c}^2}{2}\left[\lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} m^2(t)cos(4\pi f_c t)dt\right] \approx \frac{{A_c}^2}{2}P_m$

DSB-SC AM [7/7]



$\frac{A_c{}^2}{2}\left[\lim_{T \to \infty} \frac{1}{T}\int_{-T/2}^{T/2} m^2(t)cos(4\pi f_c t)dt\right] \to 0$ because:

a. *The two areas 'a' and 'b' are nearly equal as because $f_m \ll f_c$, the two nearly cancel each other over the period and,*

b. *The term is divided by $T \to \infty$.*

Problem: What is the power of USB-SC-AM and LSB-SC-AM signal?

Solution: Do it yourself!

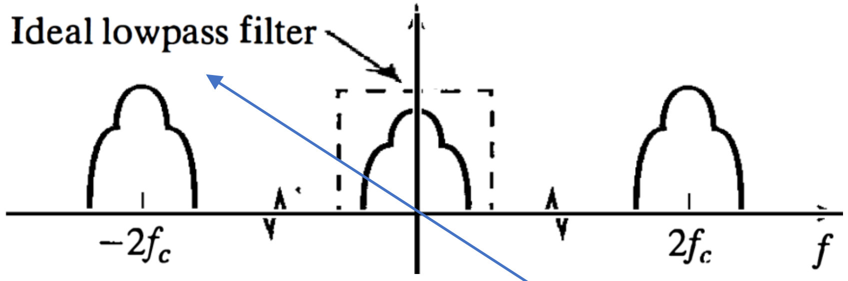
DSB-SC AM Demodulation [1/2]

If a DSB-SC AM signal $u(t)$ is transmitted through an ideal channel then the received signal, $r(t) = u(t) = A_c m(t)\, cos(2\pi f_c t)$.

Ideal channel: *it may be defined as an impulse function with a sampling frequency at least twice of the maximum frequency in* $m(t)$.

To demodulate, we multiply $r(t)$ with a locally generated sinusoid.

Assume the receiver crystal operates at the same frequency as that of the carrier, f_c, but is out of sync in phase with the transmitter by φ.



Thus, $r(t)\, cos(2\pi f_c t + \varphi) = [A_c m(t) cos(2\pi f_c t)] \cdot cos(2\pi f_c t + \varphi)$.

As $cos\, A\, cos\, B = \frac{1}{2}[cos(A + B) + cos(A - B)]$, we get:

$\frac{1}{2} A_c m(t)\, cos(\cancel{4\pi f_c t + \varphi}) + \frac{1}{2} A_c m(t)\, cos(\varphi)$ … the 1st term can be filtered out by a low pass filter.

This is possible in a cost-effective manner only because signal bandwidth $W \ll f_c$.

The output of an ideal DSB-SC AM lowpass filter is $\frac{1}{2} A_c m(t) \cos(\varphi)$.

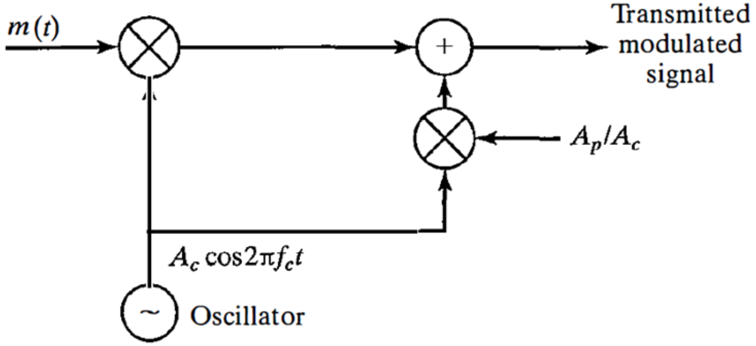
DSB-SC AM Demodulation [2/2]

Note that the demodulated signal is weaker. Its power decreases by a factor of $cos^2(\varphi)$.

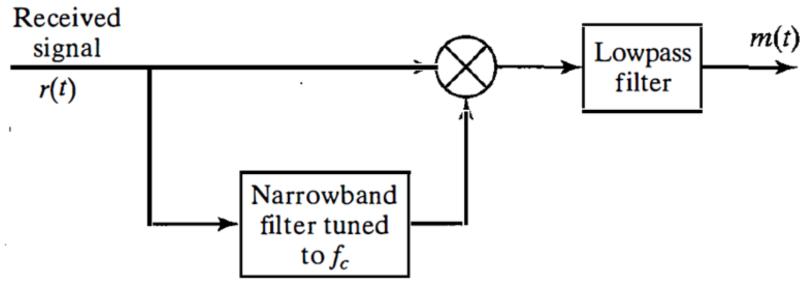
If Rx is $45°$ out of phase with the Tx then, the power of demodulated signal is $cos^2(45°) = (1/\sqrt{2})^2 = 50\%$ of the received signal.

This can be addressed in two ways:

1. *Adding a carrier component, called a pilot tone, to the carrier component whose magnitude, A_p (power is ${A_p}^2/2$), is much smaller than that of the modulated signal. This introduces a DC component and the signal is no longer a DSB-SC AM signal.*
2. *Generating a phase-locked sinusoidal carrier from the received signal $r(t)$ without the need of a pilot signal by using a phase-locked loop* (TBD *later*).



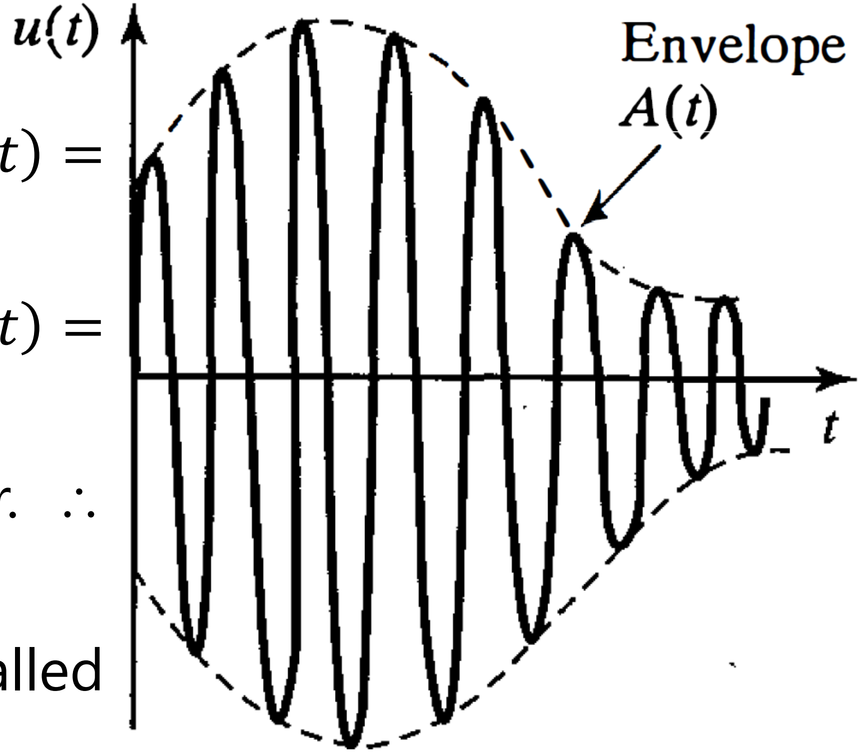
Addition of pilot tone to a DSB-AM signal



Use of a pilot tone to demodulate a DSB-AM signal

I. *DSB-SC AM*
II. *DSB-SC AM Demodulation*
III. Conventional AM
IV. *Conventional AM Demodulation*
V. *Quadrature Carrier Multiplexing*

Conventional AM [1/4]



Recall that the received signal for DSB-SC AM was $r(t) = u(t) = A_c m(t)\, cos(2\pi f_c t)$.

In case of conventional AM, i.e., just AM, it is $r(t) = u(t) = A_c[1 + m(t)]\, cos(2\pi f_c t)$ where $|m(t)| \le 1$.

If $|m(t)| \le 1$, then $A_c[1 + m(t)]$ is +ve & demodulation is easier. $\therefore$ commercial broadcasters use this kind of modulation.

If $|m(t)| > 1$, then $A_c[1 + m(t)]$ is –ve. This signal is called overmodulated & its demodulation is complex.

$m(t) = a m_n(t)$; $-1 \le m_n(t) \le 1$ and $0 \le a \le 1$. Thus, $r(t) = u(t) = A_c[1 + a m_n(t)]\, cos(2\pi f_c t)$.

Spectrum of AM Signal: $\Im[u(t)] = U(f) = \Im[A_c[1 + a m_n(t)]\, cos(2\pi f_c t)]$

$$= \Im[A_c a m_n(t)\, cos(2\pi f_c t)] + \Im[A_c\, cos(2\pi f_c t)]$$

this was computed before

$$= \frac{A_c a}{2}[M_n(f - f_c) + M_n(f + f_c)] + \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

Note: Conventional AM is same as DSB-SC where $m(t)$ is replaced by $[1 + a m_n(t)]$

Conventional AM [2/4]

Like in the case of DSB-SC AM, conventional AM will occupy twice the bandwidth of the message signal $m(t)$.

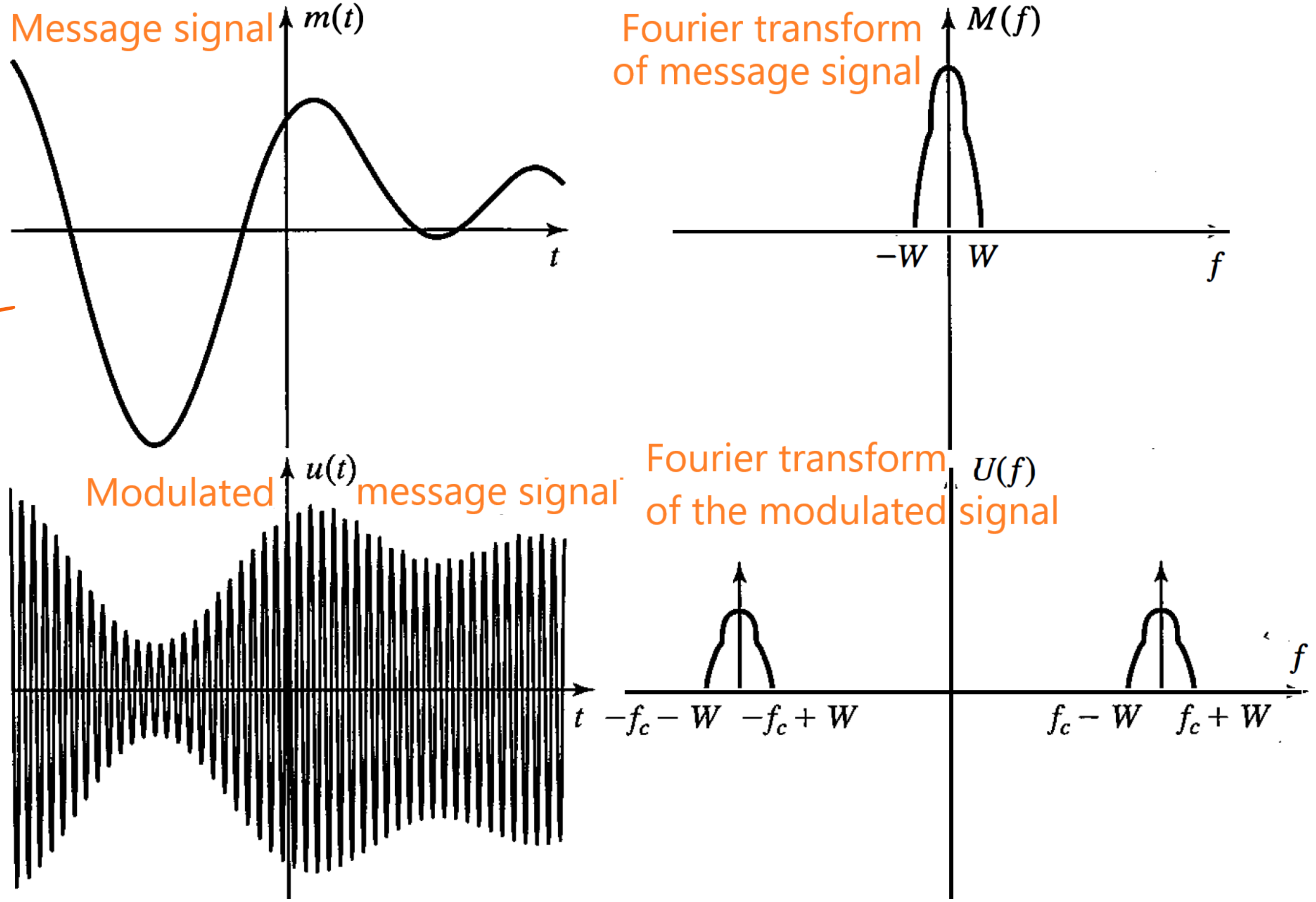
If message signal $m(t) = cos(2\pi f_m t)$; $\cancel{f_m \ll f_c}$
then, $u(t) = A_c[1 + a\, cos(2\pi f_m t)]\, cos(2\pi f_c t)$

$$= A_c cos(2\pi f_c t) + A_c a\, cos(2\pi f_m t)\, cos(2\pi f_c t)$$

$$u(t) = A_c cos(2\pi f_c t) + \frac{A_c a}{2} cos[2\pi(f_c - f_m)t] + \frac{A_c a}{2} cos[2\pi(f_c + f_m)t]$$

LSB component: $u_l(t) = \frac{A_c a}{2} cos[2\pi(f_c - f_m)t]$

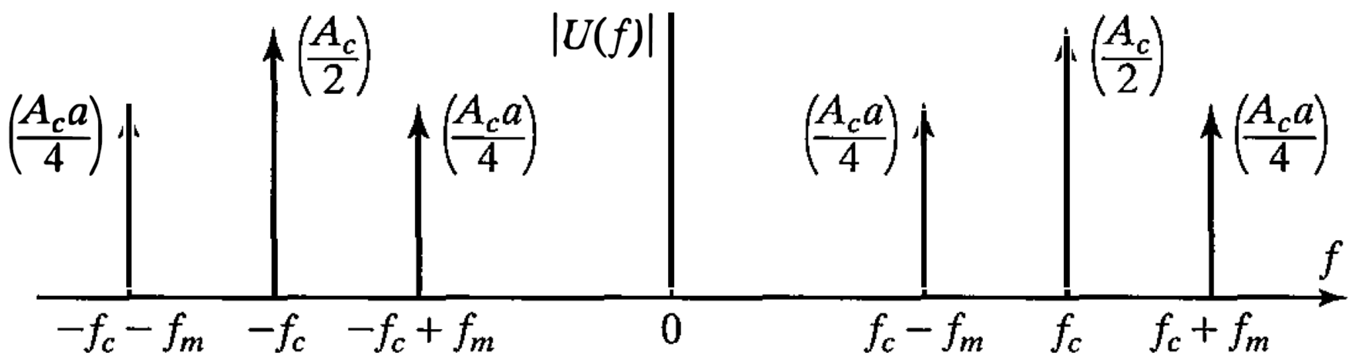
USB component: $u_l(t) = \frac{A_c a}{2} cos[2\pi(f_c + f_m)t]$



Conventional AM [3/4]

Problem: Compute power of conventional AM signal using $u(t)$ derived on last slide. Is it the same if $U(f)$ is used? **Solution:** DIY.

The answer is ${A_c}^2/2 + a^2{A_c}^2/4$. Note that carrier power $\geq 2x$ of message signal.



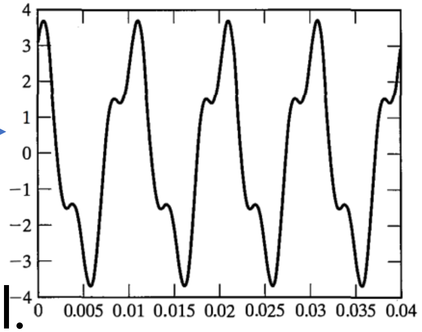
Depicted in the figure is the spectrum of conventional AM signal, i.e., DSB AM signal.

Recall that received power of DSB-SC AM is $\frac{{A_c}^2}{2}P_m$. Compare this with received power of DSB AM of $\frac{{A_c}^2}{2} + \frac{a^2{A_c}^2}{4}$ if the message signal is $cos(2\pi f_m t)$ or, $\frac{{A_c}^2}{2} + \frac{a^2{A_c}^2}{2}P_m$ for a general case.

$$\frac{Efficiency_{DSB\ AM}}{Efficiency_{DSB-SC\ AM}} = \frac{\cancel{\frac{{A_c}^2}{2}} + \frac{a^2{A_c}^2}{2}P_m}{\frac{{A_c}^2}{2}P_m} = \cancel{\frac{1}{P_m}} \cancel{+} a^2 \ldots \ldots$$ why did we cancel out the terms?

As a is typically quite small, power efficiency of conventional demodulators is much smaller than that of DSB-SC AM transmissions. Still, the former is preferred due to ease in demodulation, which in this case, makes receiver demodulators materially cheaper.

Conventional AM [4/4]



Problem: Message signal $m(t) = 3\,cos(200\pi t) + sin(600\pi t)$.
Carrier signal $c(t) = cos(2 \times 10^5 t)$. Modulation index $a = 0.85$.
Determine the power of sidebands & carrier components of the modulated signal.

Solution: Step 1: find extremum of the signal. To do this, take derivative and set it to 0.

Thus, $\frac{d(m(t))}{dt} = 3 \cdot 200\pi\,[-sin(200\pi t)] + 600\pi \cdot cos(600\pi t) = 0.$ $\therefore\ cos(600\pi t) = sin(200\pi t)$.

As $sin(\theta) = cos\left(\frac{\pi}{2} - \theta\right)$, $cos(600\pi t) = cos\left(\frac{\pi}{2} - 200\pi t\right)$ $\therefore\ cos^{-1}[cos(600\pi t)] = cos^{-1}\left[cos\left(\frac{\pi}{2} - 200\pi t\right)\right]$, i.e., $600\pi t = \frac{\pi}{2} - 200\pi t$, i.e., $t = \frac{1}{1600}$.

$$m(t)_{|t=\frac{1}{1600}} = 3\,cos\left(200\pi \cdot \frac{1}{1600}\right) + sin\left(600\pi \cdot \frac{1}{1600}\right) = 3.696.$$

Thus, maximum value of normalized message signal is $m_n(t) = \frac{3\,cos(200\pi t) + sin(600\pi t)}{3.696} = 0.82\,cos(200\pi t) + 0.27\,sin(600\pi t)$.

Power in sum of 2 sinusoids is the sum of powers of each sinusoid, i.e., $P_{m_n} = \frac{1}{2}[0.82^2 + 0.27^2] = 0.37$. Carrier power: $\frac{{A_c}^2}{2} = 0.5$ & sideband power: $\frac{{A_c}^2 \cdot a^2 \cdot P_{m_n}}{2} = \frac{1}{2} \times 0.85^2 \times 0.37 = 0.133$.

Conventional AM [explanatory note for last slide]

In these slides on slide "Conventional AM [4/4]", the numerical is indeed correct.

The integration of the cross term, 0.44 cos(200πt)*sin(600πt) indeed goes to zero when you integrate it.

∴ the statement "Power in sum of 2 sinusoids is the sum of powers of each sinusoid" is correct.

One can convince oneself of this using 3 methods:

- do it via actual integration or
- graphically assess it as on slide "DSB-SC AM [7/7]" or,
- by looking at the figure on slide "Conventional AM [3/4]“

In the exam, you do not need to show how the integration of the cross terms go to zero.

I. DSB-SC AM

II. DSB-SC AM Demodulation

III. Conventional AM

IV. Conventional AM Demodulation

V. Quadrature Carrier Multiplexing

Conventional AM Demodulation [1/1]

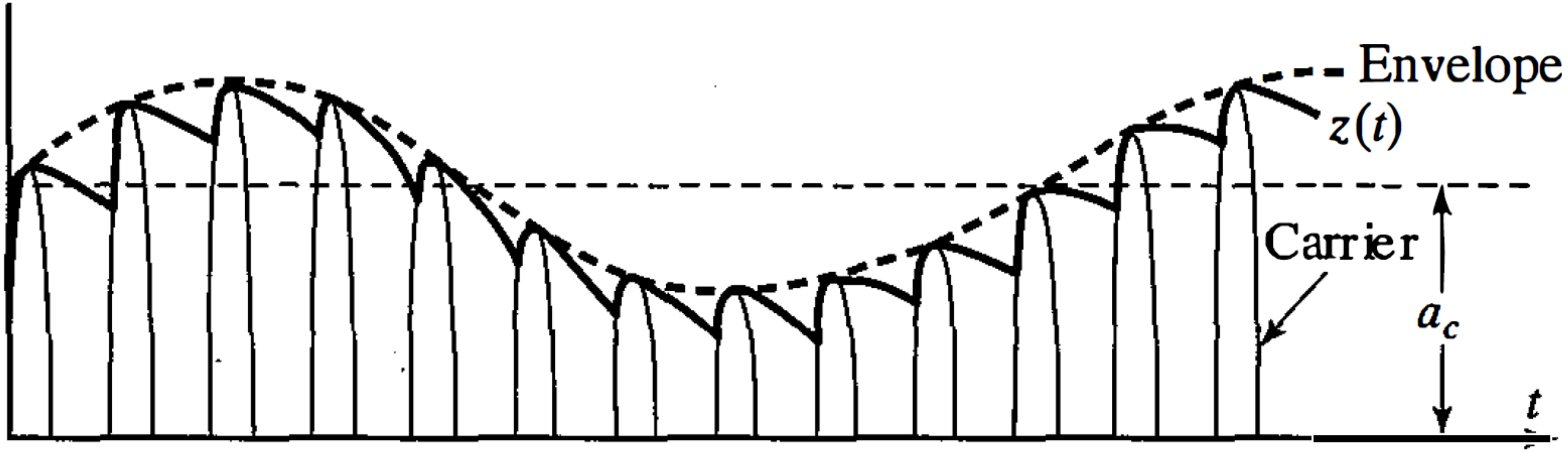
Demodulation of Conventional DSB-AM Signal: The demodulation is easier than that of DSB-SC AM as there is no need to synchronize demodulator.

As long as $|m(t)| < 1$, the envelope $1 + m(t) > 0$.

If we rectify the signal then the negative values are eliminated.

The rectified signal is then passed through a low-pass filter whose bandwidth matches that of the message signal.

Rectifier followed by lowpass filter is called *envelop detector.* The envelope then must be passed through a DC blocker to get the message signal.



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V. Quadrature Carrier Multiplexing

Quadrature Carrier Multiplexing [1/3]

Signal Multiplexing: If we wish to send ≥ 2 messages simultaneously then you could modulate each with a different carrier frequency.

Problem: Is this an example of time division or frequency division multiplexing? Justify.

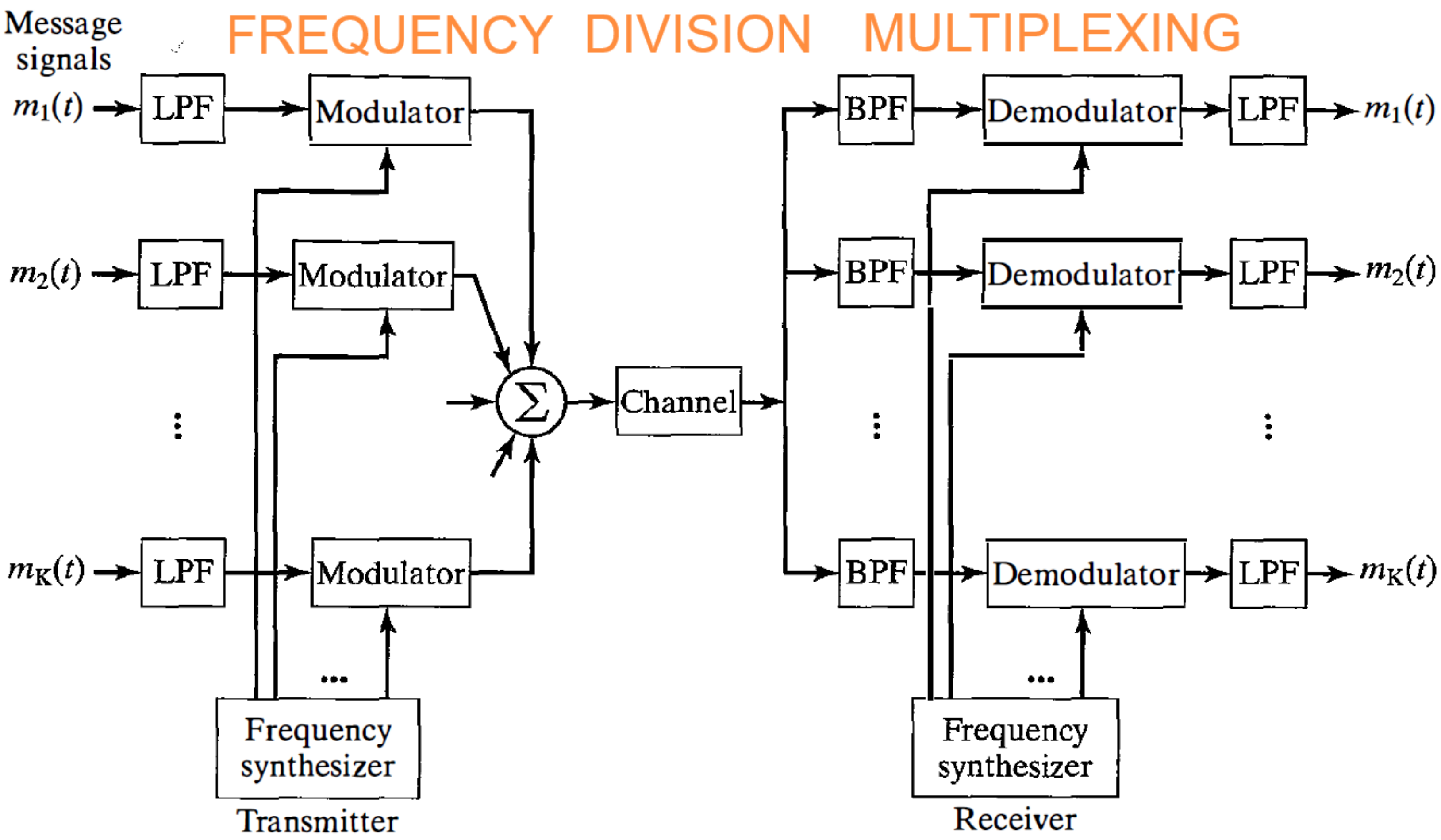
Solution: DIY.

Problem: What is the min distance between carrier frequencies for SSB? for DSB? Justify.

Solution: DIY.

Time division multiplexing is typically not used for transmitting analog information.

FREQUENCY DIVISION MULTIPLEXING



Quadrature Carrier Multiplexing [2/3]

Quadrature Carrier Multiplexing (QCM): We can instead, send the data on two carriers: $A_c cos(2\pi f_c t)$ and $A_c sin(2\pi f_c t)$. Thus, $u(t) = A_c m_1(t) cos(2\pi f_c t) + A_c m_2(t) sin(2\pi f_c t)$.

Problem: Bandwidth efficiency of QCM is higher or lower or the same as that of SSB?

Solution: DIY

A Balanced Modulator is a (de)modulator that implements DSB SC modulation.

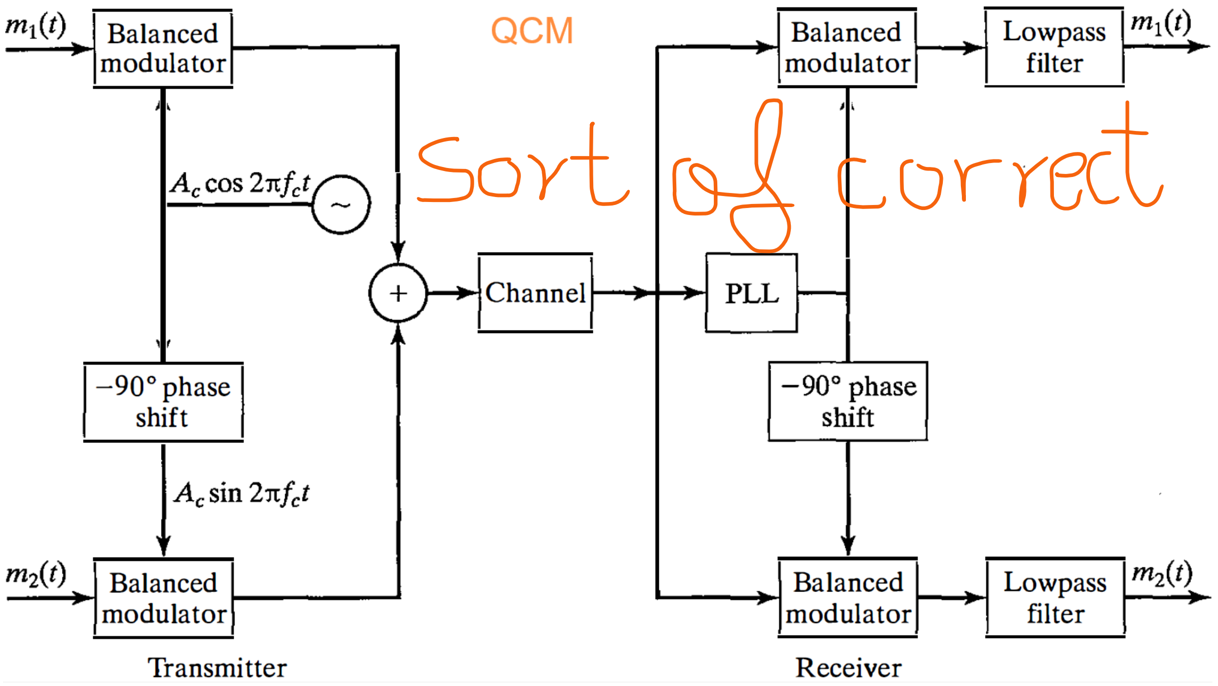
PLL is phase-locked loop.

Demodulation: multiply by $cos(2\pi f_c t)$.

$\therefore\ u(t)cos(2\pi f_c t) = A_c m_1(t) cos^2(2\pi f_c t) + A_c m_2(t) sin(2\pi f_c t) cos(2\pi f_c t)$

$$= \frac{A_c}{2} m_1(t) \cdot [1 + cos(4\pi f_c t)] + \frac{A_c m_2(t) sin(4\pi f_c t)}{2}$$

$$= \frac{A_c}{2} m_1(t) + \frac{A_c}{2} m_1(t) cos(4\pi f_c t) + \frac{A_c}{2} m_2(t)\, sin(4\pi f_c t)$$



AMRITA VISHWA VIDYAPEETHAM

Quadrature Carrier Multiplexing [3/3]

Low pass filter will remove the highlighted term, leaving only the envelope of $m_1(t)$.

Problem: How do we recover $m_2(t)$?

Solution: Multiply $u(t) = A_c m_1(t) cos(2\pi f_c t) + A_c m_2(t) sin(2\pi f_c t)$ with $sin(2\pi f_c t)$.

Thus, $u(t) cos(2\pi f_c t) = A_c m_1(t) cos(2\pi f_c t) sin(2\pi f_c t) + A_c m_2(t) sin^2(2\pi f_c t)$

$$= \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + A_c m_2(t) sin^2(2\pi f_c t)$$

$$= \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + A_c m_2(t)[1 - cos^2(2\pi f_c t)]$$

$$= \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + A_c m_2(t) \left[1 - \frac{1+cos(4\pi f_c t)}{2}\right] = \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + A_c m_2(t) \frac{1-cos(4\pi f_c t)}{2}$$

$$= \frac{A_c}{2} m_1(t) sin(4\pi f_c t) + \frac{A_c}{2} m_2(t) - \frac{A_c}{2} m_2(t)\ cos(4\pi f_c t)$$

A low pass filter eliminates the highlighted terms, leaving just the envelop of m_2.