20CYS301

Digital Communications

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A2D Conversion

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I. Sampling of Signals and Signal Reconstruction from Samples

II. Scalar Quantization

III. Vector Quantization

IV. Pulse Code Modulation (PCM)

V. Uniform PCM

VI. Nonuniform PCM

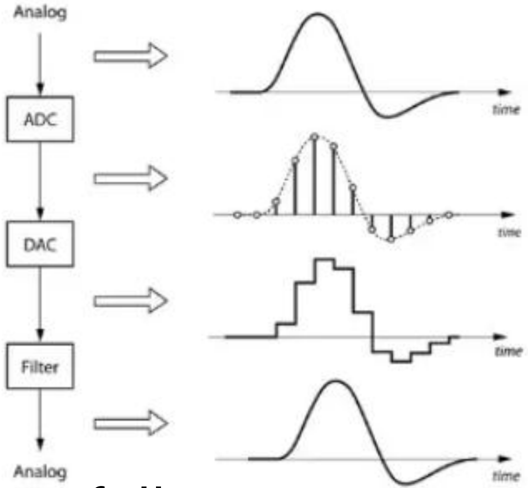
VII. Differential PCM

Sampling of Signals and Signal Reconstruction from Samples [1/6]

A2D Conversion Steps:

quantisation
how to sampling
sample
discrete

blink - delta function - world



1. Sample the signal (*continuous time to discrete time*)
2. Quantize the signal (*infinitely quantized to discretely quantized*)
3. Encode (*convert the sampled signal into logic levels*)

Is AM a linear modulation method? Justify. **Solution:** DIY i can ask

Sampling Theorem: For a band-limited signal with maximum frequency f, to fully reconstruct the signal from its digitized version, it is sufficient to sample it at twice the maximum frequency.

Nyquist Sampling Rate (NSR) is the corresponding sampling rate. In practice, sampling is done at a higher rate than Nyquist Sampling Rate. **Problem:** Why?

Problem: A bandlimited signal has a bandwidth of 3,000 Hz. It has a guard band of 1,000 Hz. What is the Nyquist Sampling Rate?

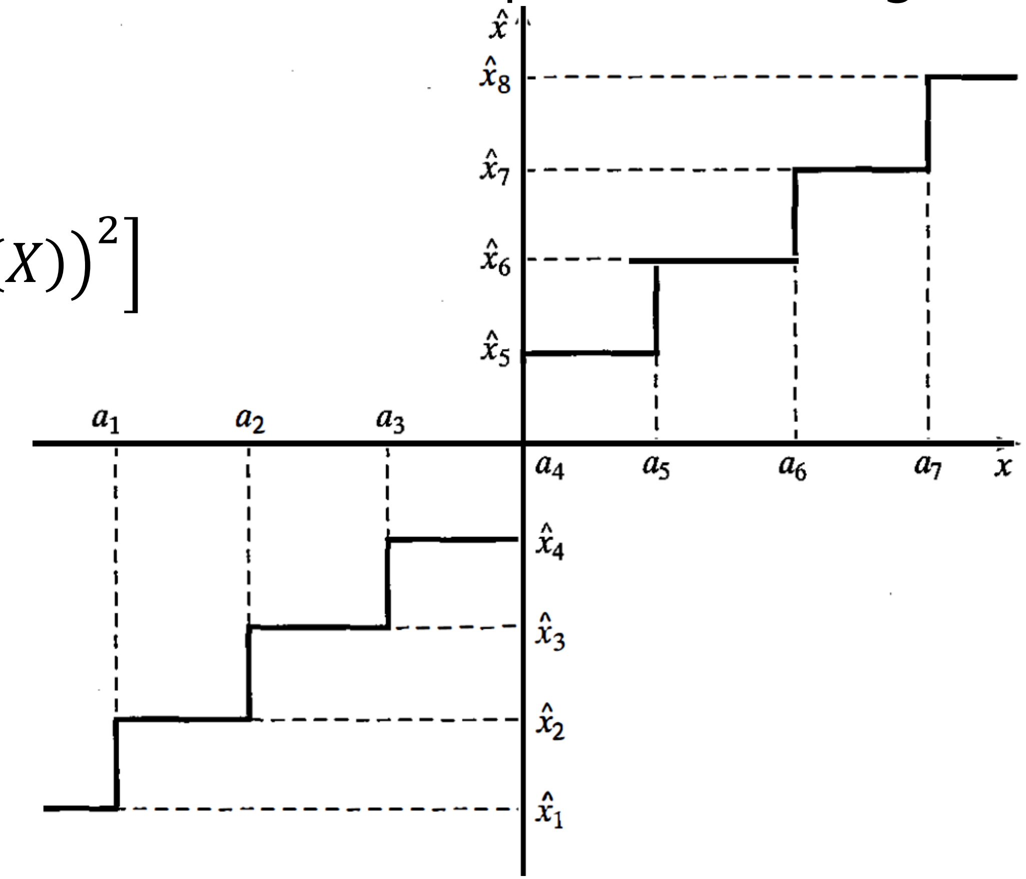
Solution: The Nyquist Sampling Rate is $f = 2W_S + W_G = 2 \cdot 3000 + 1000 = 7KHz.$

Problem: Does the NSR depend on whether the signal is SSB, DBS, FM or PM?

Solution: No.

Sampling of Signals and Signal Reconstruction from Samples [2/6]

Quantization: As transmission of real numbers requires infinite bits, we quantize the signal upon discretization.



1.1. Scalar quantization: uniform; needs $log_2(N)$ bits

Mean squared error (MSE): $D = E\left[d\left(X, \hat{X}\right)\right] = E\left[\left(X - Q(X)\right)^2\right]$

$$= \frac{\left(x_1 - Q(x_1)\right)^2 + \left(x_2 - Q(x_2)\right)^2 + \cdots + \left(x_N - Q(x_N)\right)^2}{N}$$

If quantization errors are uncorrelated with values of the signal being quantized, time or the discretization buckets then one may assume that the random variable, $\tilde{x} = x - \tilde{x}$, is *independently and identically distributed with a mean* μ *and standard deviation* σ.

Alternatively, $\tilde{x}$ is drawn from a Gaussian distribution.

Sampling of Signals and Signal Reconstruction from Samples [3/6]

Problem: Assume that $X(t)$ is a stationary Gaussian source with $\mu = 0$ and a power spectral density (psd) $S_x(f) = \begin{cases} 2 \; if \; |f| < 100 \; Hz \\ 0 \qquad otherwise \end{cases}$. Source is sampled at Nyquist rate.

1. *What is the sampling rate?* 2. *What is the variance?* 3. *What is the bit rate?*

An 8-level quantizer is used. Ref figure on the last slide:

$\{a_1, a_2, a_3, a_4, a_5, a_6, a_7\} = \{-60, \; -40, \; -20, 0, 20, 40, 60\}$

$\{\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4, \hat{x}_5, \hat{x}_6, \hat{x}_7, \hat{x}_8\} = \{-70, \; -50, \; -30, -10, 10, 30, 50, 70\}$

4. *What is distortion gain in dB if quantization results in distortion power declining to 33.38 dB?*

Solution: The PSD function is 0 for $|f| > 100 \, Hz$. This means, the max frequency in the message is $100 \, Hz$. As source is sampled at Nyquist rate, the sampling rate is $2 * 100 = 200 \, Hz$.

Variance is $E\left[\left(X - Q(X)\right)^2\right]$. However, as mean is zero, it is is $E[(X)^2]$. But PSD is 2. $\therefore$ $\int_{-\infty}^{\infty} 2 \, df = \int_{-100}^{100} 2 \, df = 2[100 - (-100)] = 400 = E[(X)^2] = \sigma^2$=Variance.

As sampling rate is 200 Hz and each sample is represented by 3 bits, $log_2(\#of \; quantization \; levels) = log_2(8) = 3$, Bit rate is $3 * 200 = 600 \, Hz$

Sampling of Signals and Signal Reconstruction from Samples [4/6]

Distortion gain is $10log_{10}\left(\frac{400}{33.38}\right) = 10log_{10}(11.98) = 10.78\ dB$

Signal-to-noise quantization ratio (SQNR): another measure of distortion. In the last slide, we defined mean squared error as a measure of distortion. This was $E\left[\left(X - Q(X)\right)^2\right]$.

SQNR is $\frac{E[(X)^2]}{E\left[\left(X-Q(X)\right)^2\right]}$ which is simply $\frac{signal\ power}{quantization\ noise\ power} = \frac{\lim_{T\to\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}} E\left[\left(X(t)-Q\left(X(t)\right)\right)^2\right]dt}{\lim_{T\to\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}} E\left[\left(X(t)\right)^2\right]dt}$

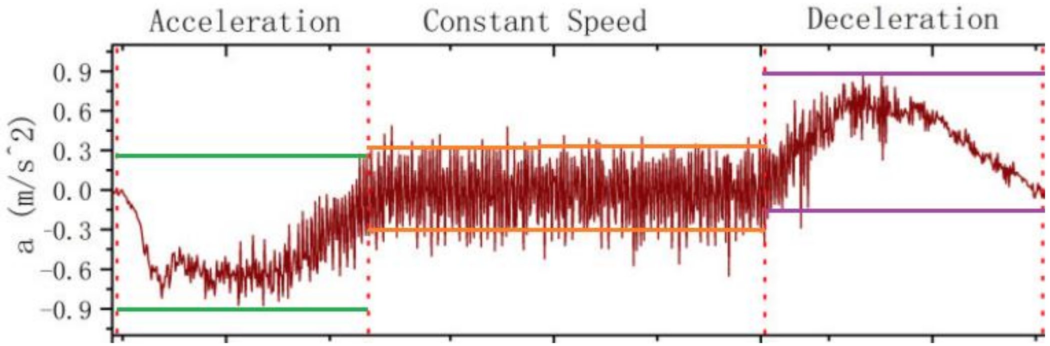
Problem: Is a **non-uniform quantizer** always at least as good as a uniform quantizer? Justify.

Solution: If we relax the condition that quantization regions be of equal length then we are minimizing distortions with one less constraint. $\therefore$ the resulting quantization will be better than a uniform quantizer for the same number of levels.

See graph on next page.

Sampling of Signals and Signal Reconstruction from Samples [5/6]

The fig. shows vibrations of a rope-connected lift platform. The colored horizontal lines in each phase capture the 99% of the range of vibrations.



As #(*quantization levels*) is fixed (say, 16), a large chunk of them (say, 14), must fall within the colored lines and not span -1.0 to 1.0 at equal intervals along y-axis.

Vector Quantizer: optimal quantization across multiple samples.

It divides a large set of points (vectors) into groups having approximately the same number of points closest to them.

Each group is represented by its centroid point, as in k-means and some other clustering algorithms.

In simpler terms, vector quantization chooses a set of points to represent a larger set of points.