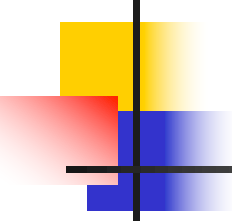


ADIP: Mid-Semester -2018 : Solutions



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Q.1

- Consider the following homography.

$$H = \begin{bmatrix} 3 & 4 & -6 \\ 0 & 2 & -8 \\ 0 & 0 & 1 \end{bmatrix}$$

- Answer the following questions w.r.t. H .



Q. 1(a) Show that parallel line remains parallel.

$$H^{-T} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & \frac{1}{2} & 0 \\ \frac{10}{3} & 4 & 1 \end{bmatrix}$$

$$H^{-T} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- \rightarrow Line at infinity in the transformed space is the vanishing line containing all points of intersections of parallel lines of the original space. Hence the property.
- *You may also prove it by showing transformation of intersection of any arbitrary two parallel lines.*



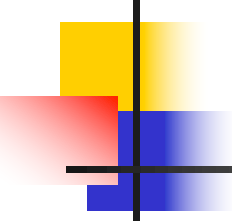
Q.1(b) Given a circle of radius 5 with center at $(-3,2)$ in \mathbb{R}^2 , find the transformed conic in \mathbb{P}^2 .

- Equation of a circle: $(x+3)^2+(y-2)^2=5^2$
- In the projective form, the conic C :

$$C = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 3 & -2 & -12 \end{bmatrix}$$

Now, the transformed conic

$$\begin{aligned} C' &= H^{-T} C H^{-1} \\ &= \frac{1}{36} \begin{bmatrix} 4 & -8 & -4 \\ -8 & 25 & 44 \\ -4 & 44 & -752 \end{bmatrix} \end{aligned}$$



Q1(c): Define the Conic dual at infinity (C_α) identifying the set of straight lines in this conic. Compute the transformation under H .

- $C_\alpha = IJ^T + JI^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- where $I = \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix}$ and $J = \begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix}$ and lines passing through these points (including line at infinity) lie on the dual conic.

- Transformed dual conic at infinity:

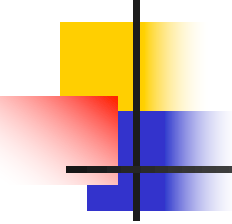
- $C'_\alpha = HC_\alpha H^T = \begin{bmatrix} 25 & 8 & 0 \\ 8 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$



Q.1 (d): Compute a nontrivial fixed point of H .

- Compute x such that $Hx = \lambda x$, where λ is an Eigen value. There are 3 Eigen values. Compute corresponding Eigen vector for any one of them. That is a fixed point.

- $\rightarrow x = \begin{bmatrix} -13 \\ 8 \\ 1 \end{bmatrix}$



Q. 1(e) Compute the transformed point of intersection of the straight lines given by the equations $3x+4y+2=0$, and $4x+3y+5=0$ in \mathbb{R}^2 .

- A point of intersection p is computed as follows:
 - $p = l_1 \times l_2$, where $l_1 = [3 \ 4 \ 2]^T$ and $l_2 = [4 \ 3 \ 5]^T$.
 - $p = [14 \ -7 \ -7]^T \rightarrow [-2 \ 1]^T$ in \mathbb{R}^2 .
 - Transformed point:
 $p' = Hp = [-8 \ -6 \ 1]^T \rightarrow [-8 \ -6]^T$ in \mathbb{R}^2 .



Q. 2

- Consider the following projection matrix P of an optical camera based imaging system.

$$\begin{bmatrix} -6 & 1 & 1 & -15 \\ 2 & -7 & 3 & 25 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- Answer the following w.r.t. P .



Q. 2(a) Camera center in \mathbb{R}^3 (world coordinates).

- $P = [M \mid p_4]$

- $M^{-1} = -\frac{1}{40} \begin{bmatrix} 7 & 1 & -10 \\ 2 & 6 & -20 \\ 0 & 0 & -40 \end{bmatrix}$

- Camera Center at $\tilde{C} = -M^{-1}p_4 = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$



Q. 2(b): The equation of the image plane in \mathbb{R}^3 .

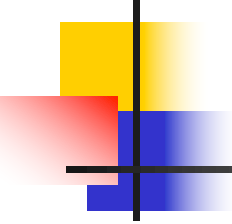
- Principal plane: Normal from first three elements of the third row: $[0 \ 0 \ 1]^T$, which is parallel to Z -axis, i.e. parallel to XY plane.
- As z -coordinate of camera center is 0, the principal plane containing the center is the XY -plane.
- Image plane is at a distance of focal length f from XY plane, i.e. $Z=f$ (for the canonical imaging system, $Z=1$).



Q. 2(c): The vanishing point in the image coordinates (R^2) of the line in the world coordinate system (R^3) with direction ratio 1:1:1.

- Vanishing point: Image of a point lying at infinity along that direction, e.g. a point, $[1 \ 1 \ 1 \ 0]^T$.

- Vanishing point $= M \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} -4 \\ -2 \end{bmatrix}$

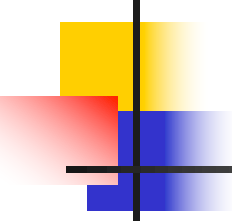


Q. 2(d): Describe an algorithm for finding the focal length of the camera from P given the pixel resolutions (dots per cm) in horizontal and vertical directions.

- As $M=KR$, where K is the camera calibration matrix and R is the rotational matrix, **perform RQ-decomposition** of M , where R is the upper triangular matrix (corresponding to K) and Q is the orthogonal matrix (corresponding to R).
- The diagonal elements of K , $K(1,1)$ and $K(2,2)$ provide the product of resolution and focal length (f)
→

$$f = (K(1,1)/m_x + K(2,2)/m_y)/2$$

- Where m_x and m_y are the resolutions in dots per cm along horizontal and vertical directions.
- *You may estimate f from any one of these elements.*



Q. 2(e): Given an image point (5,-8) of a 3D point p, which lies in a plane parallel to the image plane at a distance of 35 unit, compute the 3-D world coordinate.

- $X(\mu) = \mu \begin{bmatrix} M^{-1}x \\ 0 \end{bmatrix} + \begin{bmatrix} \tilde{C} \\ 1 \end{bmatrix},$

- where $x = \begin{bmatrix} 5 \\ -8 \\ 1 \end{bmatrix}$ and $\tilde{C} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

- $\Rightarrow X(\mu) = \begin{bmatrix} -\frac{17}{40}\mu - 2 \\ \frac{58}{40}\mu + 3 \\ \mu \\ 1 \end{bmatrix}$



Q. 2(e) (Contd.)

- Distance: $\widehat{mr}_3.(\tilde{X} - \check{C})$

$$= [0 \quad 0 \quad 1] \begin{bmatrix} -\frac{17}{40}\mu - 2 \\ \frac{58}{40}\mu + 3 \\ \mu \end{bmatrix}$$
$$= \mu$$

$$\text{Now, } \mu=35, \Rightarrow X(35) = \begin{bmatrix} -\frac{17}{40} \times 35 - 2 \\ \frac{58}{40} \times 35 + 3 \\ 35 \end{bmatrix} = \begin{bmatrix} -16.875 \\ 53.75 \\ 35 \end{bmatrix}$$



Q. 3

- Consider a pair of images of a plane $ax+by+cz+d=0$ in \mathbb{R}^3 captured by two cameras given by their projection matrices P_1 and P_2 , respectively. Prove that there exists a homography H from image of P_1 to that of P_2 , such that $H=(P_2T)(P_1T)^{-1}$, where T is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{a}{c} & -\frac{b}{c} & -\frac{d}{c} \\ 0 & 0 & 1 \end{bmatrix}$$



Q. 3 (ans.)

- $\mathbf{x} = P_1 \mathbf{X}$, and $\mathbf{x}' = P_2 \mathbf{X}$.

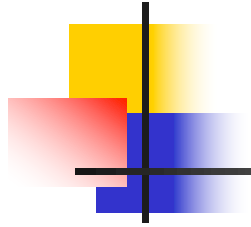
- When \mathbf{X} is in the plane,

$$z = -(a/c)x - (b/c)y - (d/c)$$

$$\rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{a}{c} & -\frac{b}{c} & -\frac{d}{c} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = T \mathbf{x}_p$$

Now, $\mathbf{x} = P_1 \mathbf{X} = P_1 T \mathbf{x}_p$, and $\mathbf{x}' = P_2 \mathbf{X} = P_2 T \mathbf{x}_p$

Hence, $\mathbf{x}' = (P_2 T)(P_1 T)^{-1} \mathbf{x}$.



Thank you!