# ADIP: Mid-Semester -2018 : Solutions

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# Q.1

- Consider the following homography.

$$H = \begin{bmatrix} 3 & 4 & -6 \\ 0 & 2 & -8 \\ 0 & 0 & 1 \end{bmatrix}$$

- Answer the following questions w.r.t. H.

# Q. 1(a) Show that parallel line remains parallel.

$$H^{-T} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & \frac{1}{2} & 0 \\ \frac{10}{3} & 4 & 1 \end{bmatrix}$$

$$H^{-T} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

- → Line at infinity in the transformed space is the vanishing line containing all points of intersections of parallel lines of the original space. Hence the property.
- *You may also prove it by showing transformation of intersection of any arbitrary two parallel lines.*

## Q.1(b) Given a circle of radius 5 with center at (-3,2) in $R^2$, find the transformed conic in $P^2$.

- Equation of a circle: $(x+3)^2+(y-2)^2=5^2$
- In the projective form, the conic $C$ :

$$C = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 3 & -2 & -12 \end{bmatrix}$$

Now, the transformed conic

$$C' = H^{-T} C H^{-1}$$

$$= \frac{1}{36} \begin{bmatrix} 4 & -8 & -4 \\ -8 & 25 & 44 \\ -4 & 44 & -752 \end{bmatrix}$$

## Q1(c): Define the Conic dual at infinity ($C_\alpha$) identifying the set of straight lines in this conic. Compute the transformation under H.

- $C_\propto = IJ^T + JI^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- where $I = \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix}$ and $J = \begin{bmatrix} 1 \\ -i \\ 0 \end{bmatrix}$ and lines passing through these points (including line at infinity) lie on the dual conic.
- Transformed dual conic at infinity:
  - $C'_\propto = HC_\propto H^T = \begin{bmatrix} 25 & 8 & 0 \\ 8 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

## Q.1 (d): Compute a nontrivial fixed point of H.

- Compute $x$ such that $H\boldsymbol{x} = \lambda\boldsymbol{x}$, where $\lambda$ is an Eigen value. There are 3 Eigen values. Compute corresponding Eigen vector for any one of them. That is a fixed point.
- → $\boldsymbol{x} = \begin{bmatrix} -13 \\ 8 \\ 1 \end{bmatrix}$

## Q. 1(e) Compute the transformed point of intersection of the straight lines given by the equations $3x+4y+2=0$, and $4x+3y+5=0$ in $R^2$.

- A point of intersection p is computed as follows:
  - $p=l_1 \times l_2$, where $l_1=[3\ \ 4\ \ 2]^T$ and $l_2=[4\ \ 3\ \ 5]^T$.
  - $p=[14\ \ -7\ \ -7]^T \Rightarrow [-2\ \ 1]^T$ in $R^2$.
  - Transformed point:

    $p'=Hp=[-8\ \ -6\ \ 1]^T \rightarrow [-8\ \ -6]^T$ in $R^2$.

# Q. 2

- Consider the following projection matrix P of an optical camera based imaging system.

$$\begin{bmatrix} -6 & 1 & 1 & -15 \\ 2 & -7 & 3 & 25 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

- Answer the following w.r.t. P.

## Q. 2(a) Camera center in $R^3$ (world coordinates).

- $P = [M \mid p_4]$
- $M^{-1} = -\frac{1}{40}\begin{bmatrix} 7 & 1 & -10 \\ 2 & 6 & -20 \\ 0 & 0 & -40 \end{bmatrix}$
- Camera Center at $\tilde{C} = -M^{-1}p_4 = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

## Q. 2(b): The equation of the image plane in $R^3$.

- Principal plane: Normal from first three elements of the third row: $[0\ 0\ 1]^T$, which is parallel to $Z$-axis, i.e. parallel to $XY$ plane.
- As $z$-coordinate of camera center is 0, the principal plane containing the center is the $XY$-plane.
- Image plane is at a distance of focal length $f$ from $XY$ plane, i.e. $Z=f$ (for the canonical imaging system, $Z=1$).

## Q. 2(c): The vanishing point in the image coordinates ($R^2$) of the line in the world coordinate system ($R^3$) with direction ratio 1:1:1.

- Vanishing point: Image of a point lying at infinity along that direction, e.g. a point, $[\,1\ \ 1\ \ 1\ 0]^T$.
- Vanishing point=$M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}-4\\-2\\1\end{bmatrix}\equiv\begin{bmatrix}-4\\-2\end{bmatrix}$

## Q. 2(d): Describe an algorithm for finding the focal length of the camera from P given the pixel resolutions (dots per cm) in horizontal and vertical directions.

- As M=KR, where K is the camera calibration matrix and R is the rotational matrix, **perform RQ-decomposition** of M, where R is the upper triangular matrix (corresponding to K) and Q is the orthogonal matrix (corresponding to R).
- The diagonal elements of K , K(1,1) and K(2,2) provide the product of resolution and focal length (f) →

$$f = (K(1,1)/m_x + K(2,2)/m_y)/2$$

- Where $m_x$ and $m_y$ are the resolutions in dots per cm along horizontal and vertical directions.
- *You may estimate* f *from any one of these elements.*

## Q. 2(e): Given an image point (5,-8) of a 3D point p, which lies in a plane parallel to the image plane at a distance of 35 unit, compute the 3-D world coordinate.

- $X(\mu) = \mu \begin{bmatrix} M^{-1}x \\ 0 \end{bmatrix} + \begin{bmatrix} \tilde{C} \\ 1 \end{bmatrix}$,
- where $x = \begin{bmatrix} 5 \\ -8 \\ 1 \end{bmatrix}$ and $\tilde{C} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$
- ➔$X(\mu) = \begin{bmatrix} -\frac{17}{40}\mu - 2 \\ \frac{58}{40}\mu + 3 \\ \mu \\ 1 \end{bmatrix}$

## Q. 2(e) (Contd.)

- Distance: $\widehat{mr}_3 . \left(\tilde{X} - \check{C}\right)$

$$= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{17}{40}\mu - 2 \\ \frac{58}{40}\mu + 3 \\ \mu \end{bmatrix}$$

$$=\mu$$

Now, $\mu$=35, ➔ $X(35) = \begin{bmatrix} -\frac{17}{40}\times 35 - 2 \\ \frac{58}{40}\times 35 + 3 \\ 35 \end{bmatrix} = \begin{bmatrix} -16.875 \\ 53.75 \\ 35 \end{bmatrix}$

# Q. 3

- Consider a pair of images of a plane $ax+by+cz+d=0$ in R$^3$ captured by two cameras given by their projection matrices $P_1$ and $P_2$, respectively. Prove that there exists a homography $H$ from image of $P_1$ to that of $P_2$, such that $H=(P_2T)(P_1T)^{-1}$, where T is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{a}{c} & -\frac{b}{c} & -\frac{d}{c} \\ 0 & 0 & 1 \end{bmatrix}$$

# Q. 3 (ans.)

- $\mathbf{x}=P_1\mathbf{X}$, and $\mathbf{x}'=P_2\mathbf{X}$ .
- When $\mathbf{X}$ is in the plane,

$$z=-(a/c)x-(b/c)\,y-(d/c)$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{a}{c} & -\frac{b}{c} & -\frac{d}{c} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = T\boldsymbol{x_p}$$

Now, $\mathbf{x}=P_1\mathbf{X}=P_1T\boldsymbol{x_p}$ , and $\mathbf{x}'=P_2\mathbf{X}=P_2T\boldsymbol{x_p}$

Hence, $\mathbf{x}'=(P_2T)(P_1T)^{-1}\,\mathbf{x}$ .

Thank you!