Week3Prob

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[1]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

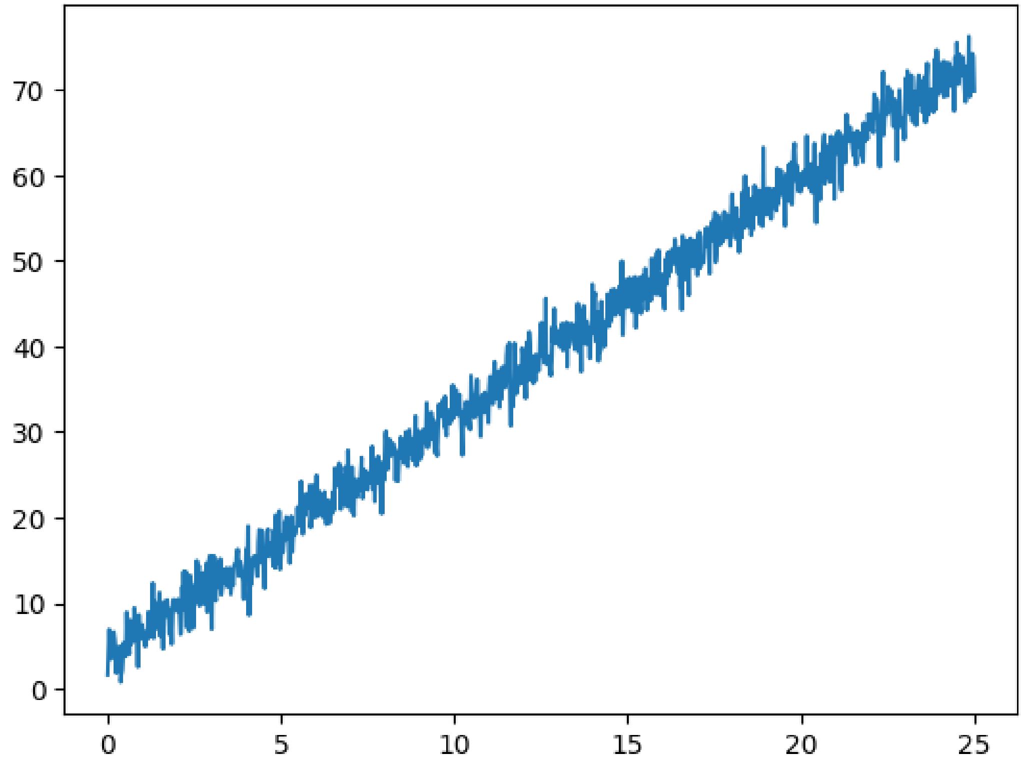
Imported all the useful libraries

1 Dataset-1

[2]:
```
x1=[]
y1=[]
filename = 'dataset1.txt'
with open(filename, "r") as file:
    data=file.readlines()
    for l in data:
        l=l.split()
        x1.append(float(l[0]))
        y1.append(float(l[1]))

x1=np.asarray(x1)
y1=np.asarray(y1)
plt.plot(x1, y1)
```

[2]:
```
[<matplotlib.lines.Line2D at 0x1b78318ce50>]
```



- This cell reads the data from the file dataset.txt and stores the values of x and y in the lists x1 and y1. Then converts them to numpy arrays and plots the graph
- This graph looks similar to straight line with noise.

[3]:
```
def stline(x, m, c):
    return m * x + c
```

Here a function stline is defined which takes x, m = the slope of the straight line and c = intercept as arguments and returns the y value.

[4]:
```
M = np.column_stack([x1, np.ones(len(x1))])
(m1, c1), _, _, _ = np.linalg.lstsq(M, y1, rcond=None)
print(f"The slope and intercept of the straight line respectively ar m1 = {m1}␣
↪and c1 = {c1}\nThe estimated equation is y = {m1} x + {c1}")
%timeit np.linalg.lstsq(M, y1, rcond=None)
```

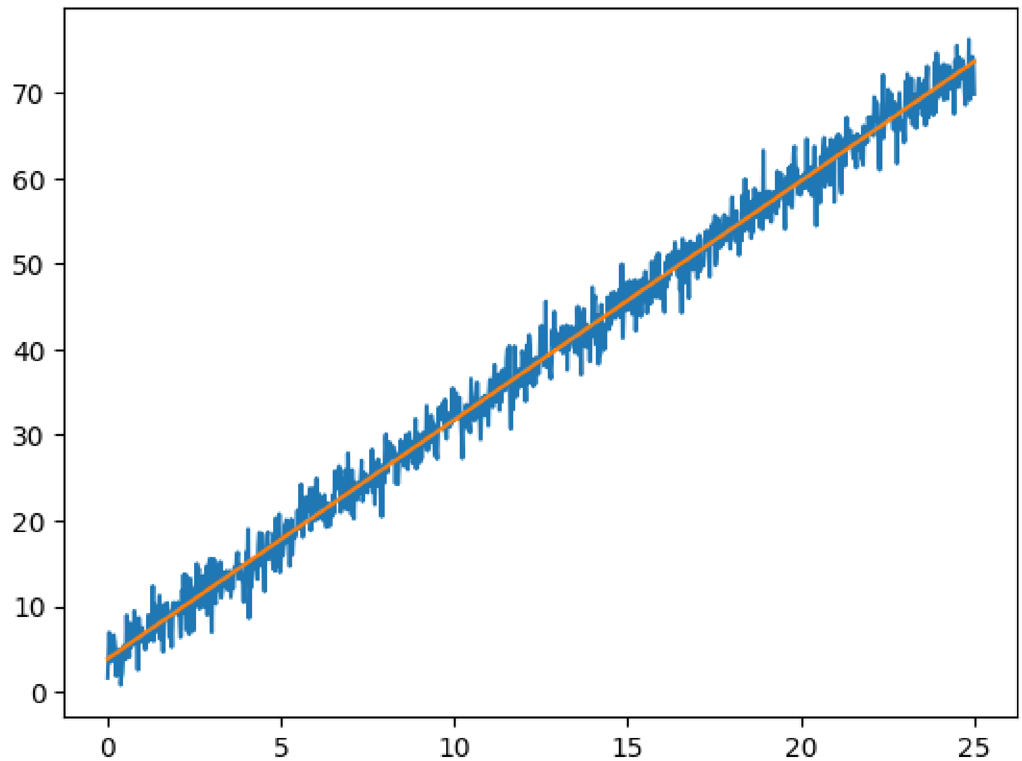
```
The slope and intercept of the straight line respectively ar m1 =
2.791124245414918 and c1 = 3.8488001014307436
The estimated equation is y = 2.791124245414918 x + 3.8488001014307436
19.6 µs ± 1.19 µs per loop (mean ± std. dev. of 7 runs, 100,000 loops each)
```

- As the plot of the given dataset looks like a straight line, here the least square function is used to estimate the values of the slope and intercept and the estimated values of slope and intercept are m1 = 2.791124245414918 and c1 = 3.8488001014307436.

- The time taken by leastsquare function to estimate this values is 19.6 μ s

[5]:
```
y1_l=stline(x1,m1,c1)
plt.plot(x1, y1, x1, y1_l)
```

[5]:
```
[<matplotlib.lines.Line2D at 0x1b7853c6110>,
 <matplotlib.lines.Line2D at 0x1b7853c6170>]
```



Here we are plotting both the given dataset(blue) and estimated fit(orange) got using lstsq

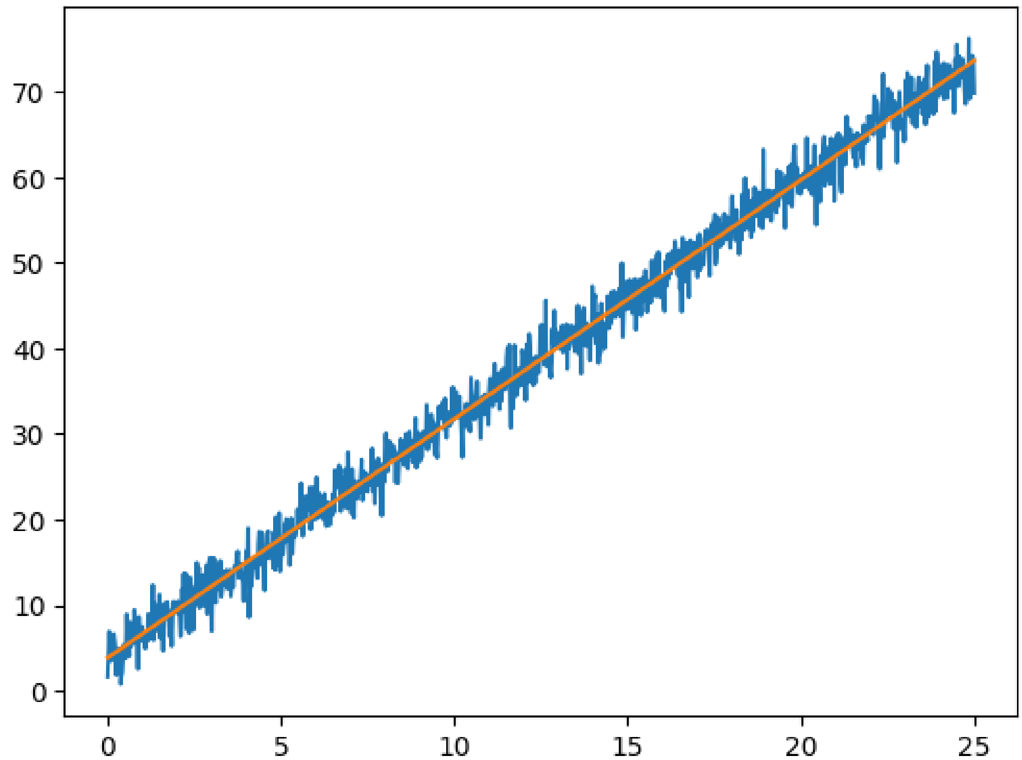
[6]:
```
from scipy.optimize import curve_fit
(m2, c2), pcov = curve_fit(stline, x1, y1)
print(f"The slope and intercept of the straight line respectively ar m2 = {m2}␣
↪and c2 = {c2}\nThe estimated equation is y = {m2} x + {c2}")
%timeit curve_fit(stline, x1, y1)
```

```
The slope and intercept of the straight line respectively ar m2 =
2.7911242448201588 and c2 = 3.848800111263445
The estimated equation is y = 2.7911242448201588 x + 3.848800111263445
247 µs ± 40.3 µs per loop (mean ± std. dev. of 7 runs, 1,000 loops each)
```

- The curve_fit function from scipy is used to get the estimates for slope and intercept and they are m2 = 2.7911242448201588 and c2 = 3.848800111263445
- The time taken to estimate this values is 247 μ s

[7]:
```
y1_c=stline(x1,m2,c2)
plt.plot(x1, y1, x1, y1_c)
```

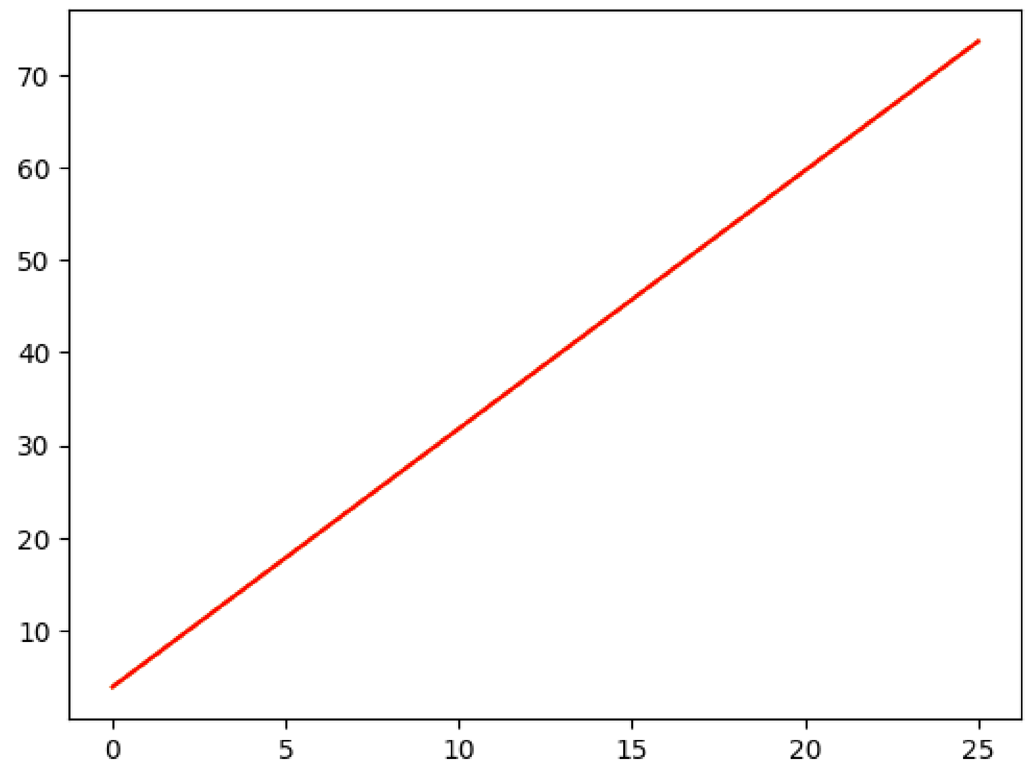
[7]:
```
[<matplotlib.lines.Line2D at 0x1b7938a7e50>,
 <matplotlib.lines.Line2D at 0x1b7938a7eb0>]
```



Here we are plotting both the given dataset(blue) and estimated fit(orange) got using curve_fit

[8]:
```
plt.plot(x1, y1_c, 'y', x1, y1_l, 'r')
# plt.plot(x1, y1_l, 'r', x1, y1_c, 'y')
```

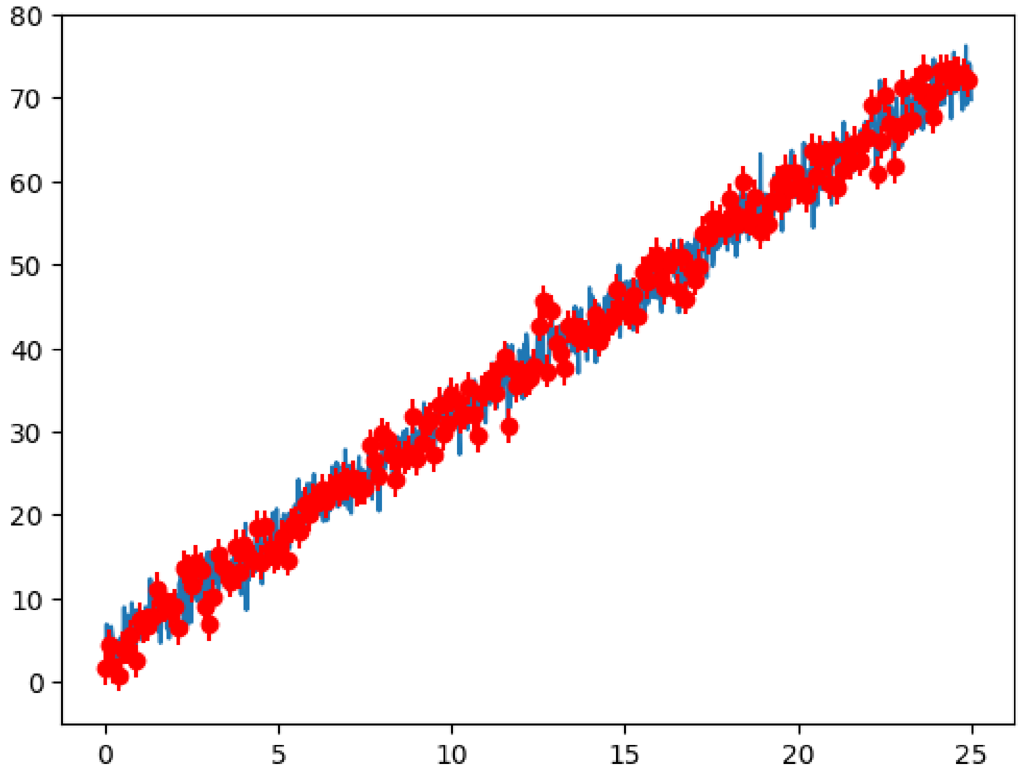
[8]:
```
[<matplotlib.lines.Line2D at 0x1b796a1a890>,
 <matplotlib.lines.Line2D at 0x1b796a1a8c0>]
```



Here i am plotting both the estimates i got using lstsq and curve_fit.

[9]:
```
plt.plot(x1,y1)
plt.errorbar(x1[::5], y1[::5], np.std(y1-y1_l), fmt='ro')
```

[9]:
```
<ErrorbarContainer object of 3 artists>
```



Here we are plotting the error bars with yerr as standard deviation of the noise in the given data set.

[10]:
```
size_of_errorbars=1.96*np.std(y1-y1_l)
print(f'size of errorbars: {1.96*np.std(y1-y1_l)}')
```

```
size of errorbars: 3.9118636225219166
```

The size of error bar will be 1.96 times the error used to plot the error bars

[11]:
```
diff=np.square(np.subtract(y1_l, y1_c)).mean()
print(diff)
```

```
2.4212178149379354e-17
```

Here i am taking the difference between each value of y we got using lstsq and curve_fit and calculating the mean of it.

1.0.1 Observations

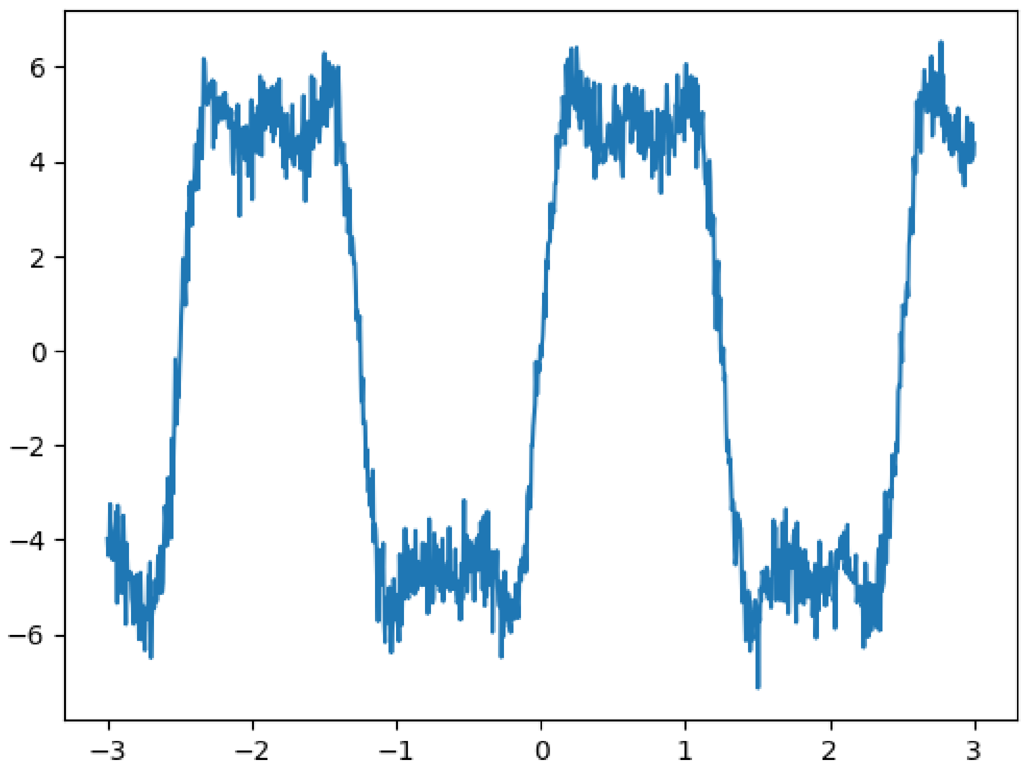
- From the plot of the given data set we can see that this is a straight line
- so i have used lstsq to get the estimate but then i have also checked with curve_fit and calculated the difference as shown in the above cell
- we can see that there is a very small difference between them, so i think using anyone of them will be fine but according to time taken curve_fit takes more time.

2 Dataset-2

[12]:
```python
x2=[]
y2=[]
filename = 'dataset2.txt'
with open(filename, "r") as file:
    data=file.readlines()
    for l in data:
        l=l.split()
        x2.append(float(l[0]))
        y2.append(float(l[1]))

x2=np.asarray(x2)
y2=np.asarray(y2)
plt.plot(x2, y2)
```

[12]:
```
[<matplotlib.lines.Line2D at 0x1b78311fdf0>]
```



This is a sum of 3 sine waves 1st, 3rd and 5th harmonics i.e. the frequencies of the sin waves will be w, 3w and 5w and the amplitudes will be a1>a2>a3 and w will be equal to the frequency of the given function. So we will use curve_fit to get the estimate

[13]:
```
def func(x,w,a1,a2,a3):
    return a1*np.sin(w*x)+a2*np.sin(3*w*x)+a3*np.sin(5*w*x)
```

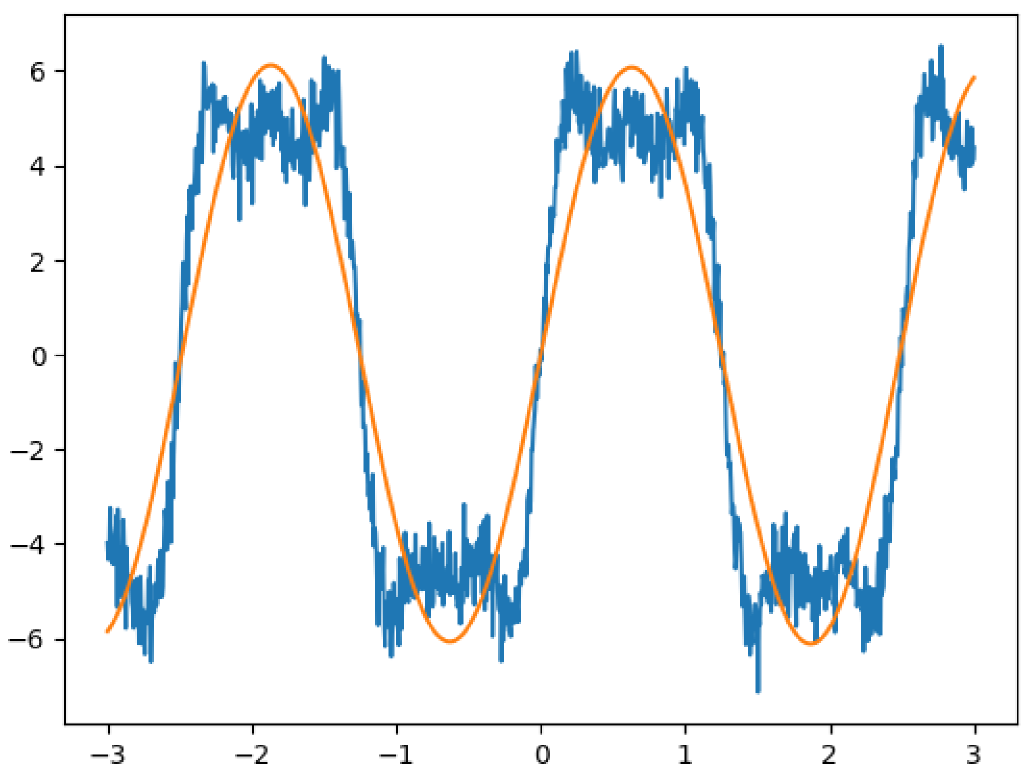
Here we are defining that function i have mentioned before

[14]:
```
from scipy.optimize import curve_fit
(w, a1,a2,a3), pcov = curve_fit(func, x2, y2)
print(f"The wavelenths of 1st, 2nd, and 3rd harmonics are w = {w}, 3w = {3*w},
 ↪5w = {5*w} and the amplitudes of 1st, 2nd, 3rd harmonics are a1 = {a1}, a2 =
 ↪{a2}, a3 = {a3}\nThe estimated equation is y = {a1} sin({w} x) + {a2}
 ↪sin({3*w} x)+ {a3} sin({5*w} x)")
```

```
The wavelenths of 1st, 2nd, and 3rd harmonics are w = 0.840458258714396, 3w =
2.5213747761431877, 5w = 4.20229129357198 and the amplitudes of 1st, 2nd, 3rd
harmonics are a1 = 0.036390745839994514, a2 = 6.082552583736292, a3 =
-0.06688134106152409
The estimated equation is y = 0.036390745839994514 sin(0.840458258714396 x) +
6.082552583736292 sin(2.5213747761431877 x)+ -0.06688134106152409
sin(4.20229129357198 x)
```

[16]:
```
y2_c = func(x2,w,a1,a2,a3)
plt.plot(x2,y2,x2,y2_c)
```

[16]:
```
[<matplotlib.lines.Line2D at 0x1b796b8fd60>,
 <matplotlib.lines.Line2D at 0x1b796b8fdc0>]
```



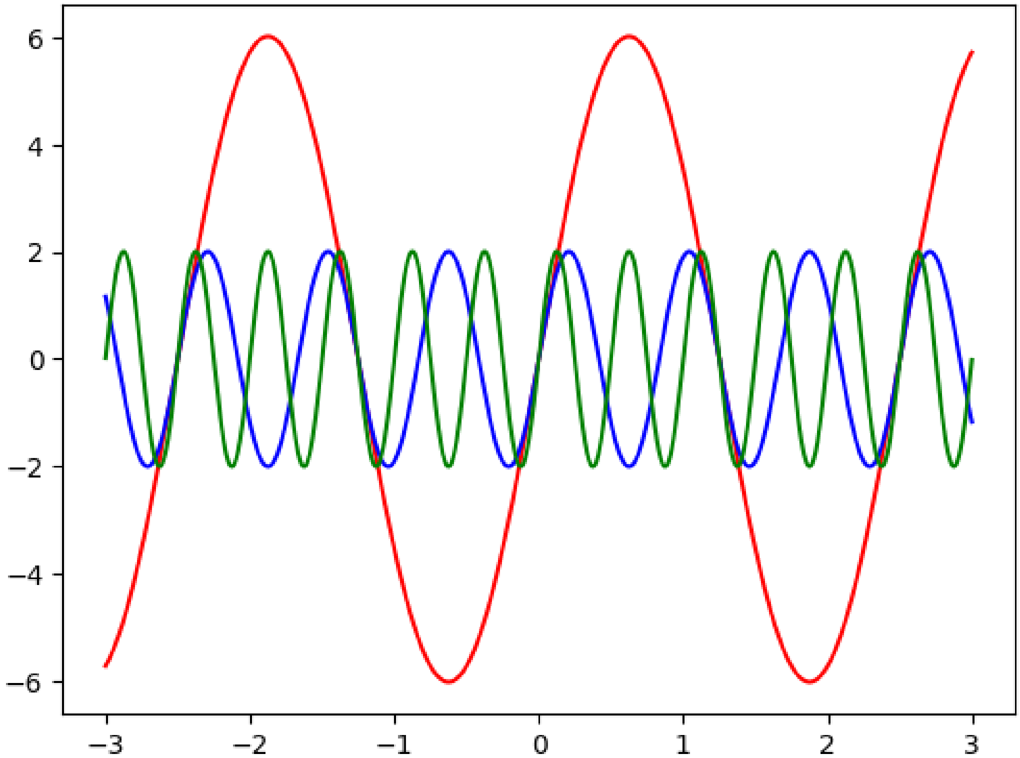
[18]:
```
from scipy.optimize import curve_fit
(w, a1,a2,a3), pcov = curve_fit(func, x2, y2, p0=(2.5,1,1,1))
print(f"The wavelenths of 1st, 2nd, and 3rd harmonics are w = {w}, 3w = {3*w},␣
↪5w = {5*w} and the amplitudes of 1st, 2nd, 3rd harmonics are a1 = {a1}, a2 =␣
↪{a2}, a3 = {a3}\nThe estimated equation is y = {a1} sin({w} x) + {a2}␣
↪sin({3*w} x)+ {a3} sin({5*w} x)")
```

```
The wavelenths of 1st, 2nd, and 3rd harmonics are w = 2.512734463620896, 3w =
7.538203390862687, 5w = 12.56367231810448 and the amplitudes of 1st, 2nd, 3rd
harmonics are a1 = 6.011119946742541, a2 = 2.001459363576296, a3 =
0.9809052713126482
The estimated equation is y = 6.011119946742541 sin(2.512734463620896 x) +
2.001459363576296 sin(7.538203390862687 x)+ 0.9809052713126482
sin(12.56367231810448 x)
```

Here in the curve_fit we are using p0 which takes the initial guesses for the estimates and from the original dataset graph we can see that w is approximately 2.5. If i do not use the p0 curve_fit is estimating the curve to a normal sine wave kind of graph.

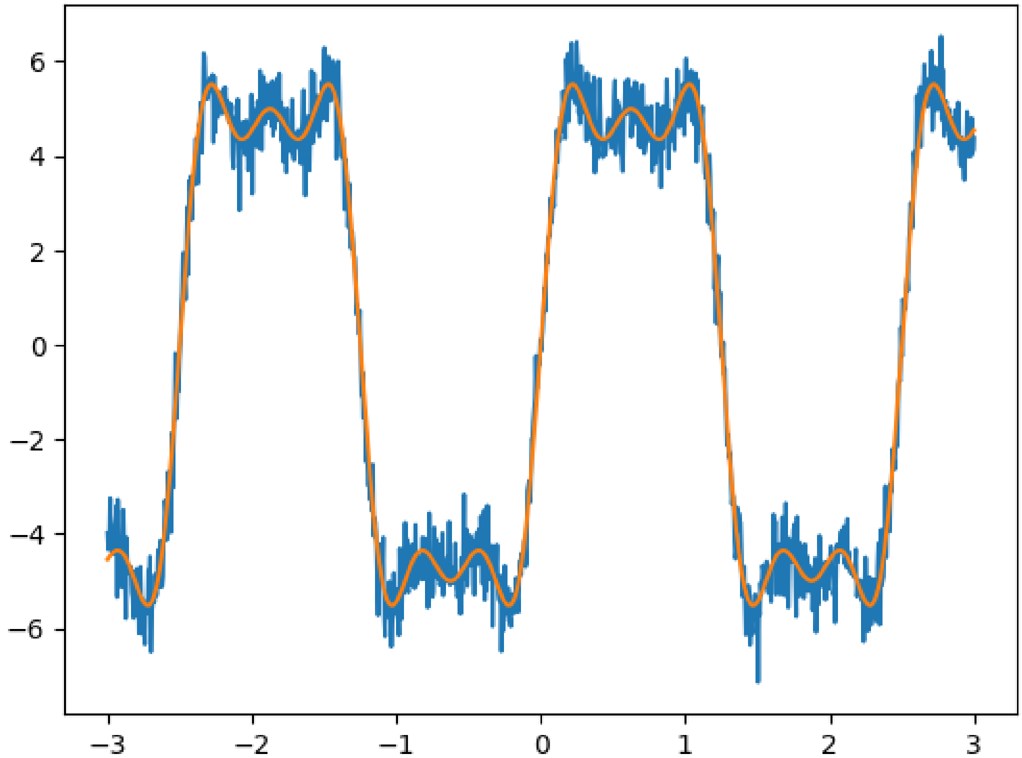
[19]:
```
sin_1=a1*np.sin(w*x2)
sin_2=a2*np.sin(3*w*x2)
sin_3=a2*np.sin(5*w*x2)
plt.plot(x2,sin_1,'r', x2,sin_2,'b', x2,sin_3, 'g')
```

[19]:
```
[<matplotlib.lines.Line2D at 0x1b796de93c0>,
 <matplotlib.lines.Line2D at 0x1b796de9270>,
 <matplotlib.lines.Line2D at 0x1b796de92a0>]
```



[20]:
```
y2_c = func(x2,w,a1,a2,a3)
plt.plot(x2,y2,x2,y2_c)
```

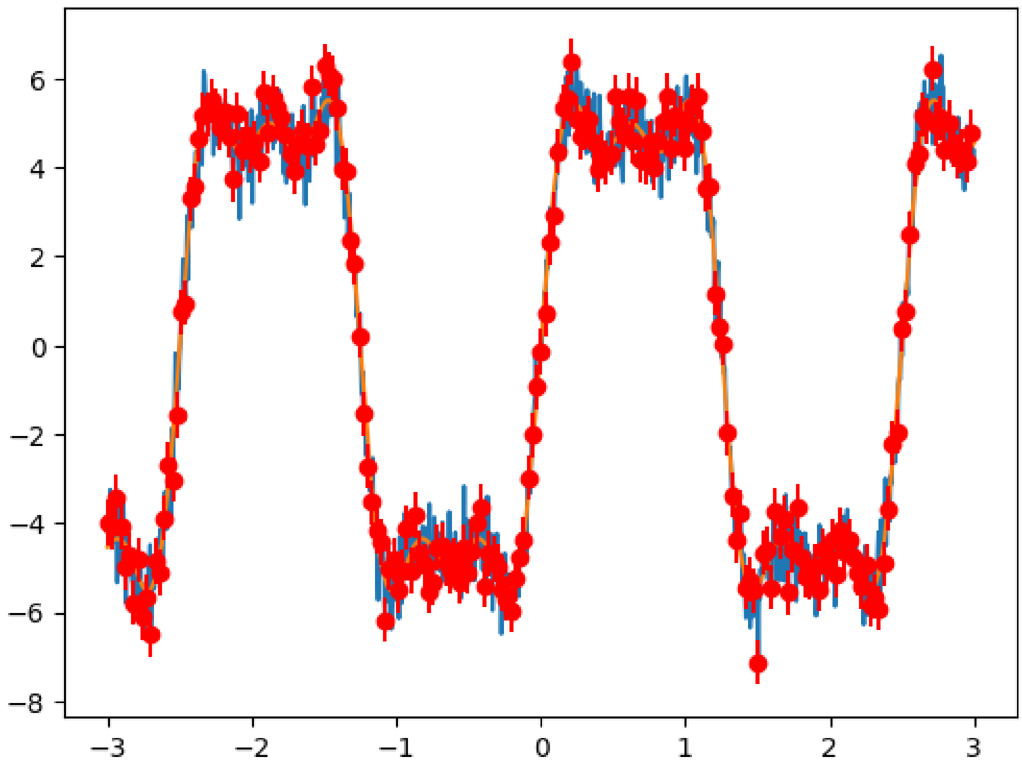
[20]:
```
[<matplotlib.lines.Line2D at 0x1b796c235e0>,
 <matplotlib.lines.Line2D at 0x1b796c23640>]
```



Here we are plotting the estimate

[21]:
```
plt.plot(x2,y2,x2,y2_c)
plt.errorbar(x2[::5], y2[::5], np.std(y2-y2_c), fmt='ro')
```

[21]: <ErrorbarContainer object of 3 artists>



Here we are plotting the error bars with yerr as standard deviation of the noise in the given data set.

[22]:
```
size_of_errorbars=1.96*np.std(y2-y2_c)
print(f'size of errorbars: {1.96*np.std(y2-y2_c)}')
```

```
size of errorbars: 0.9855517243513262
```

Here we are plotting the error bars with yerr as standard deviation of the noise in the given data set.

2.0.1 Observations

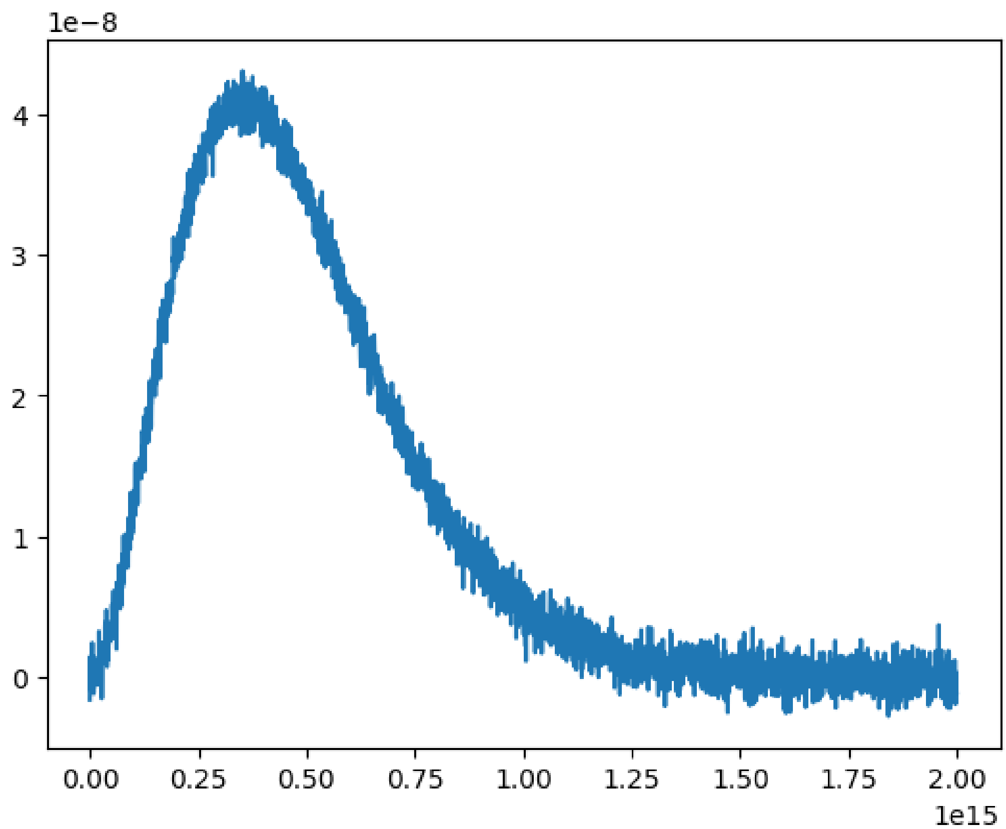
- From the plot of the given dataset we can observe that it is a summation of 3 sine waves
- And they are the 1st 3rd and 5th harmonics of the fundamental frequency w=2.512734463620896 and the amplitudes will be a1=6.011119946742541, a2=2.001459363576296, a3=0.9809052713126482

3 Dataset-3

[23]:
```python
x3=[]
y3=[]
filename = 'dataset3.txt'
with open(filename, "r") as file:
    data=file.readlines()
    for l in data:
        l=l.split()
        x3.append(float(l[0]))
        y3.append(float(l[1]))

x3=np.asarray(x3)
y3=np.asarray(y3)
plt.plot(x3, y3)
```

[23]:
```
[<matplotlib.lines.Line2D at 0x1b795997e20>]
```



[24]:
```python
k=1.38e-23
c=3.0e8
```

```python
def planck(v, T, h):
    return 2*h*((v)**3)/((c**2)*(np.exp((h*(v))/(k*T))-1))
```

Here we are defining the planck's law which takes frequency, temperature and planck's constant as arguments and returns the radition.

[25]:
```python
from scipy.optimize import curve_fit
(T, h), pcov = curve_fit(planck, x3, y3)
print(f"The estimated values of Temperature and Planck's constant are T = {T}␣
 ↪and h = {h}")
```

```
The estimated values of Temperature and Planck's constant are T = 1.0 and h =
1.0
```

```
C:\Users\gvvpr\AppData\Local\Temp\ipykernel_984\4284930493.py:4: RuntimeWarning:
overflow encountered in exp
  return 2*h*((v)**3)/((c**2)*(np.exp((h*(v))/(k*T))-1))
c:\Python310\lib\site-packages\scipy\optimize\_minpack_py.py:906:
OptimizeWarning: Covariance of the parameters could not be estimated
  warnings.warn('Covariance of the parameters could not be estimated',
```

Here i am using the curve_fit to get the estimates of T and h but here we are get a runtime warning that overflow is encountered and also the values of T and h we got are improper.

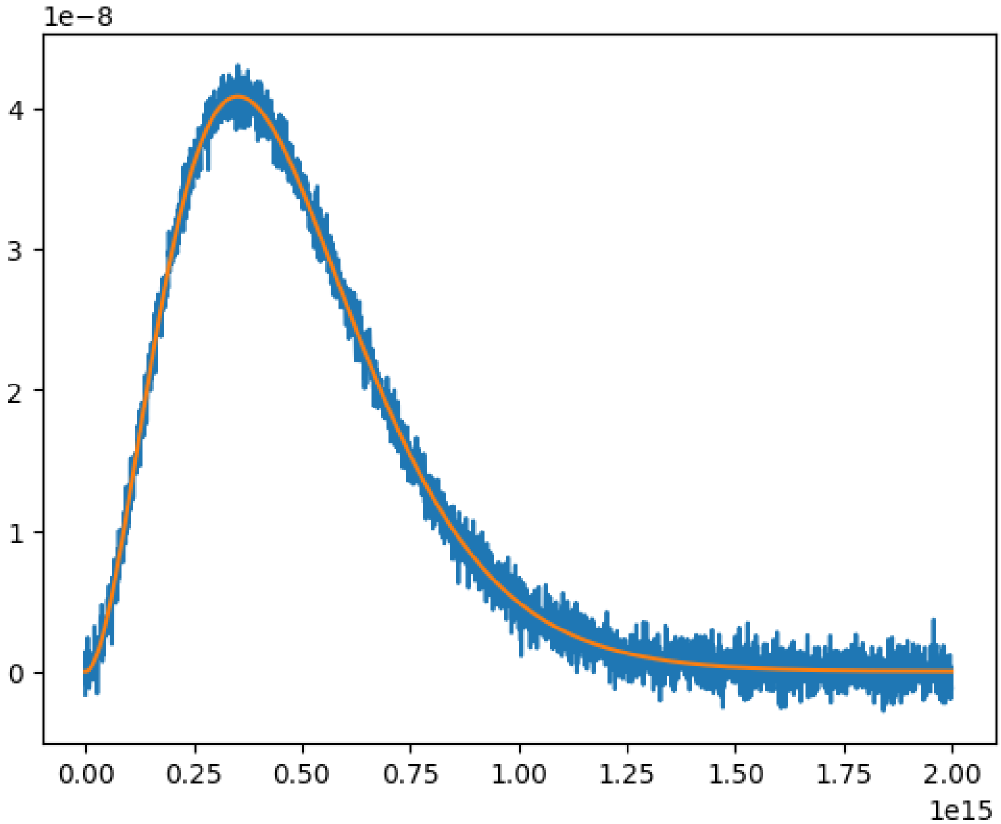
[26]:
```python
from scipy.optimize import curve_fit
(T, h), pcov = curve_fit(planck, x3, y3,p0=(100,1e-34))
print(f"The estimated values of Temperature and Planck's constant are T = {T}␣
 ↪and h = {h}")
```

```
The estimated values of Temperature and Planck's constant are T =
6011.361511513504 and h = 6.643229743118619e-34
```

So i have used the p0 and guessed that T will be in the order of 100 and h will be in the order of e-34 and we got the proper estimates.

[27]:
```python
y3_c=planck(x3,T,h)
plt.plot(x3,y3,x3,y3_c)
```

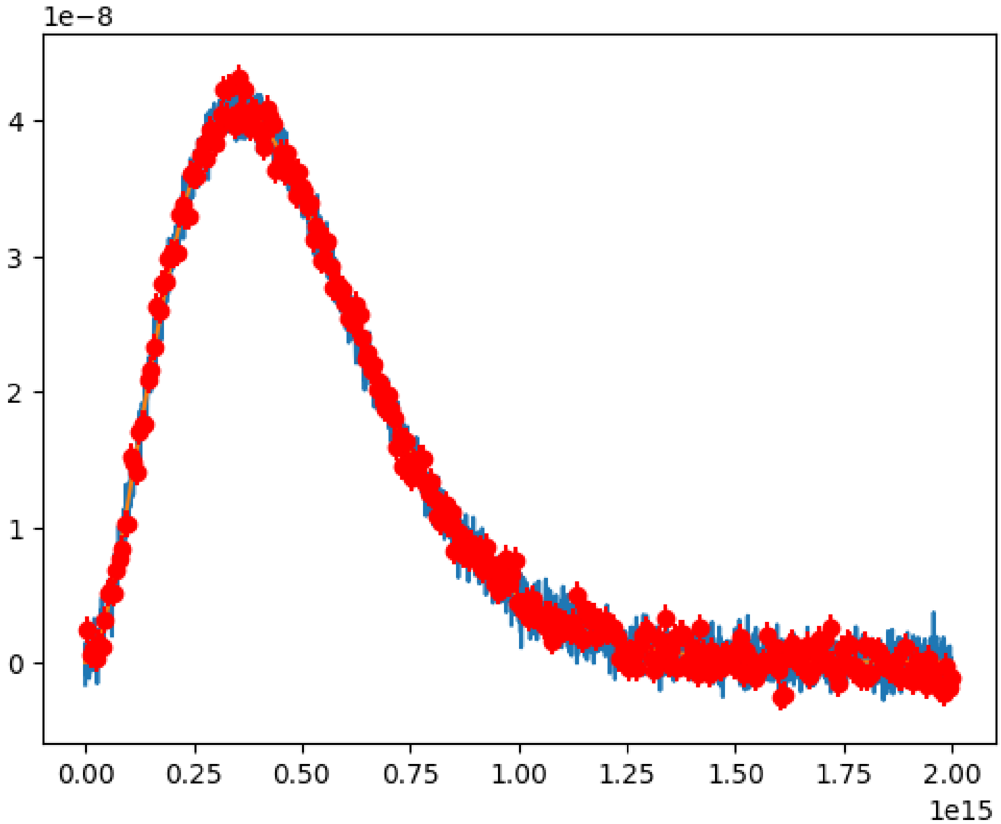
[27]:
```
[<matplotlib.lines.Line2D at 0x1b7980d0ca0>,
 <matplotlib.lines.Line2D at 0x1b7980d0d00>]
```



Here we are plotting the estimate

[28]:
```
plt.plot(x3,y3,x3,y3_c)
plt.errorbar(x3[::10], y3[::10], np.std(y3-y3_c), fmt='ro')
```

[28]: `<ErrorbarContainer object of 3 artists>`



[29]:
```
size_of_errorbars=1.96*np.std(y3-y3_c)
print(f'size of errorbars: {1.96*np.std(y3-y3_c)}')
```

```
size of errorbars: 1.9442476116397922e-09
```

Here we are plotting the error bars with yerr as standard deviation of the noise in the given data set.

3.0.1 Observations

- So this a plot of radiation vs frequency of planck's law and we are estimating the temperature and planck's constant from this
- The stimated values of T and h we got are 6011.361511513504 and 6.643229743118619e-34 respectively.

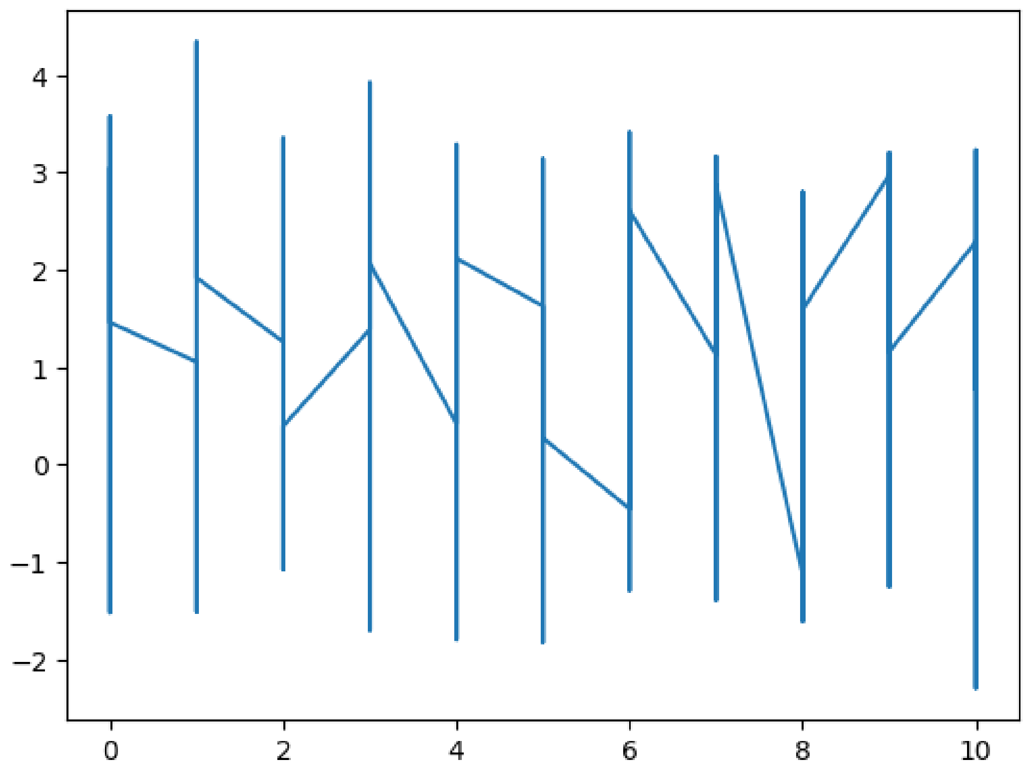
4 Dataset-4

[30]:
```
x4=[]
y4=[]
filename = 'dataset4.txt'
with open(filename, "r") as file:
```

```
    data=file.readlines()
    for l in data:
        l=l.split()
        x4.append(float(l[0]))
        y4.append(float(l[1]))

x4=np.asarray(x4)
y4=np.asarray(y4)
plt.plot(x4, y4)
```

[30]: [<matplotlib.lines.Line2D at 0x1b79803d7b0>]

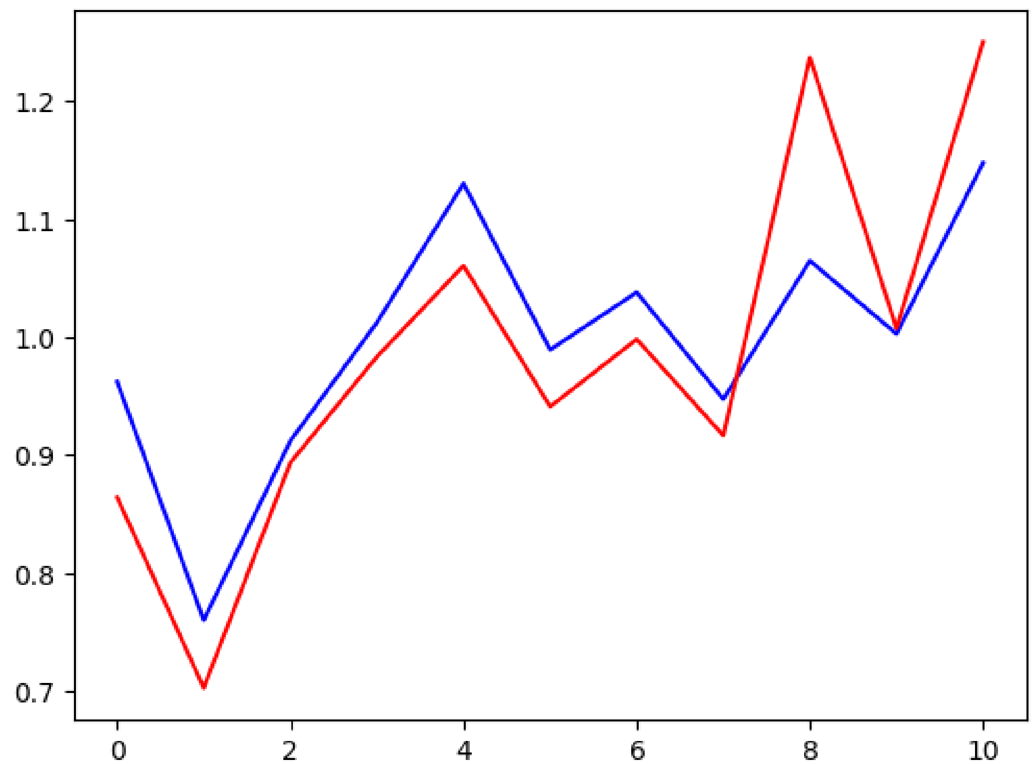


[31]:
```
a=[]
for i in range(11):
    a.append([y4[b] for b in range(len(y4)) if x4[b]==float(i)])
    # print(a[i])
y4_a_1=[np.mean(a[i]) for i in range(11)]
y4_a_2=[np.median(a[i]) for i in range(11)]
x4_a=np.unique(x4)
print(y4_a_1)
print(y4_a_2)
plt.plot(x4_a,y4_a_1,'b',x4_a,y4_a_2,'r')
```

```
[0.962239959304362, 0.7599583941221197, 0.9120059461643524, 1.0124390255027198,
1.1304311636644533, 0.9893082607856816, 1.0380387234220483, 0.9473934169581565,
1.0647663276347632, 1.002810241625165, 1.1478677609323475]
[0.8641702634914421, 0.7025556501631175, 0.8934988183766779, 0.9834280927364801,
1.0606541798130986, 0.9410311446901526, 0.9982410356751026, 0.916530291901234,
1.2371438082159272, 1.0074353586680458, 1.2504184009167236]
```

[31]:

```
[<matplotlib.lines.Line2D at 0x1b7982e9090>,
 <matplotlib.lines.Line2D at 0x1b7982e90c0>]
```



As the dataset has different y values for the same x values i have took the mean and median of y values for each x value and plotted them.

[32]:

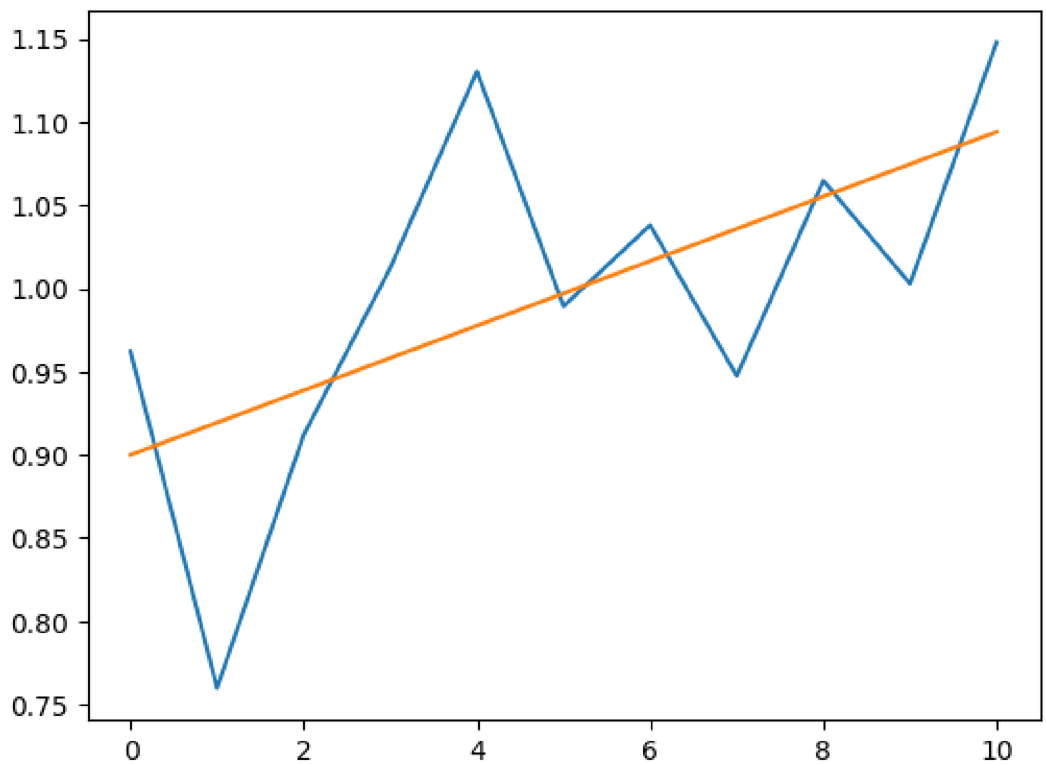
```
M_1 = np.column_stack([x4_a, np.ones(len(x4_a))])
(m_a1, c_a1), _, _, _ = np.linalg.lstsq(M_1, y4_a_1, rcond=None)
print(f"The estimated equation is {m_a1} t + {c_a1}")
```

```
The estimated equation is 0.019412217138470963 t + 0.8999624797727518
```

[33]:

```
y_mean=stline(x4_a,m_a1,c_a1)
plt.plot(x4_a,y4_a_1,x4_a,y_mean)
```

[33]: [<matplotlib.lines.Line2D at 0x1b79834b4c0>,
 <matplotlib.lines.Line2D at 0x1b79834b520>]



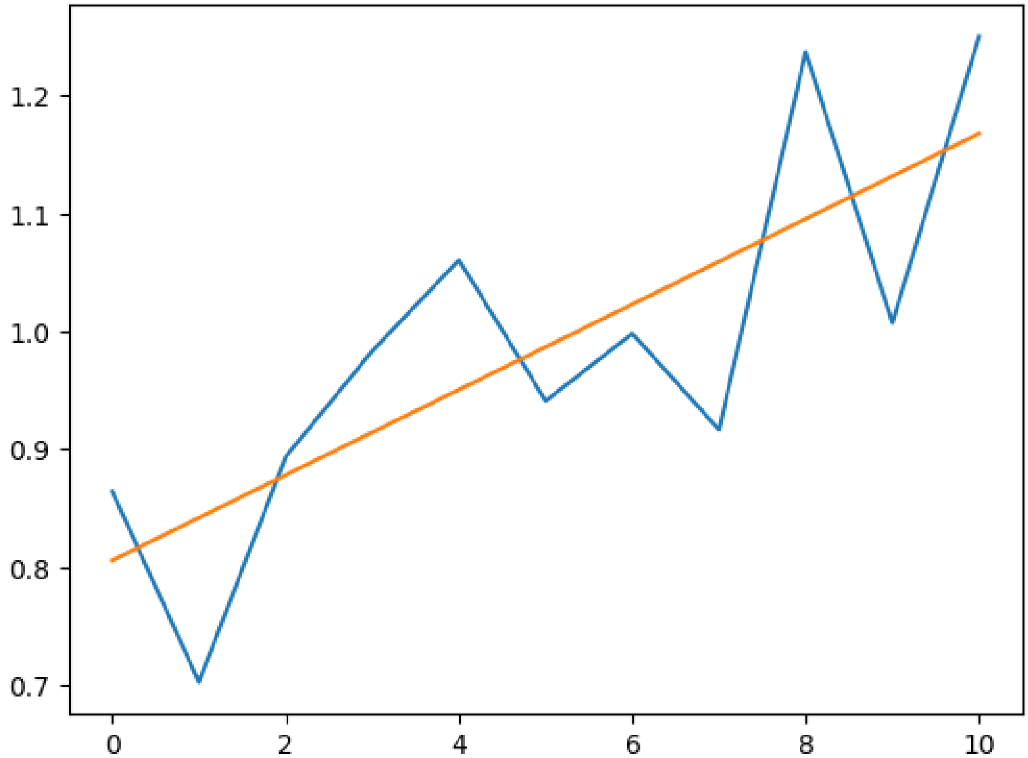
I couldn't estimate what function is it so i have used the lstsq to get a linear fit and this graph is the linear fit for the mean values.

[34]:
```
M_2 = np.column_stack([x4_a, np.ones(len(x4_a))])
(m_a2, c_a2), _, _, _ = np.linalg.lstsq(M_2, y4_a_2, rcond=None)
print(f"The estimated equation is {m_a2} t + {c_a2}")
```

```
The estimated equation is 0.036231688589594344 t + 0.8056694702018469
```

[35]:
```
y_median=stline(x4_a,m_a2,c_a2)
plt.plot(x4_a,y4_a_2,x4_a,y_median)
```

[35]: [<matplotlib.lines.Line2D at 0x1b7993c4190>,
 <matplotlib.lines.Line2D at 0x1b7993c41f0>]

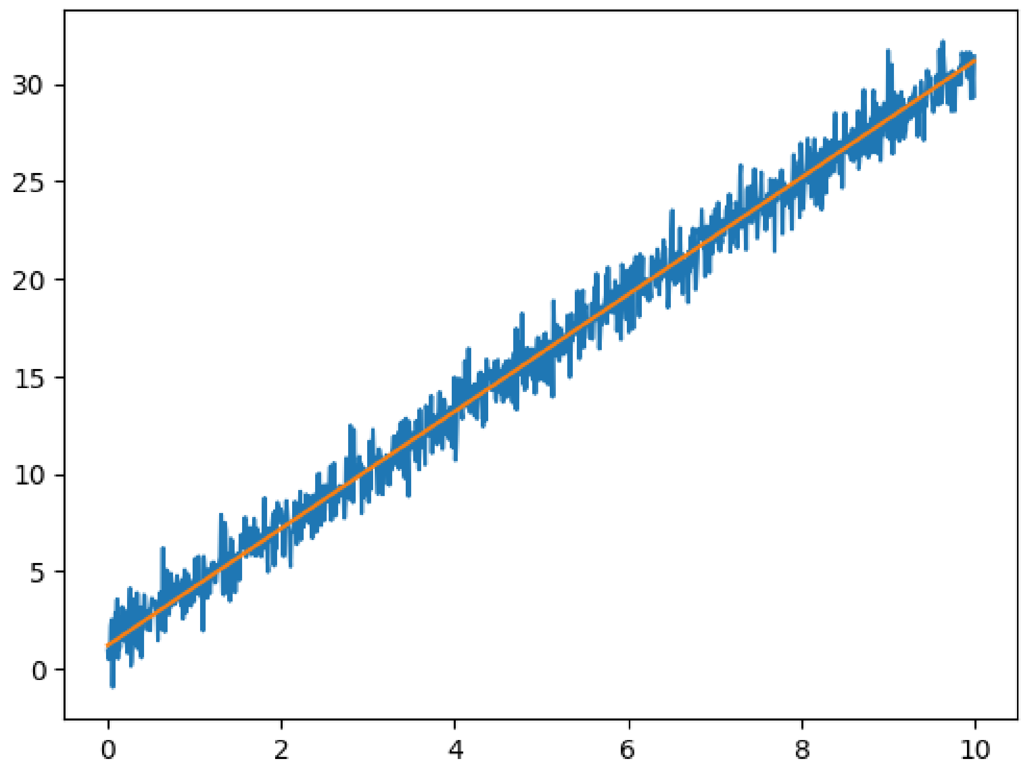


This the linear fit for the median values

5 Example

[43]:
```
t = np.arange(0, 10, 0.01)
y = stline(t, 3, 1.2)
n = 1 * np.random.randn(len(t))
yn = y + n
plt.plot(t, yn, t, y)
```

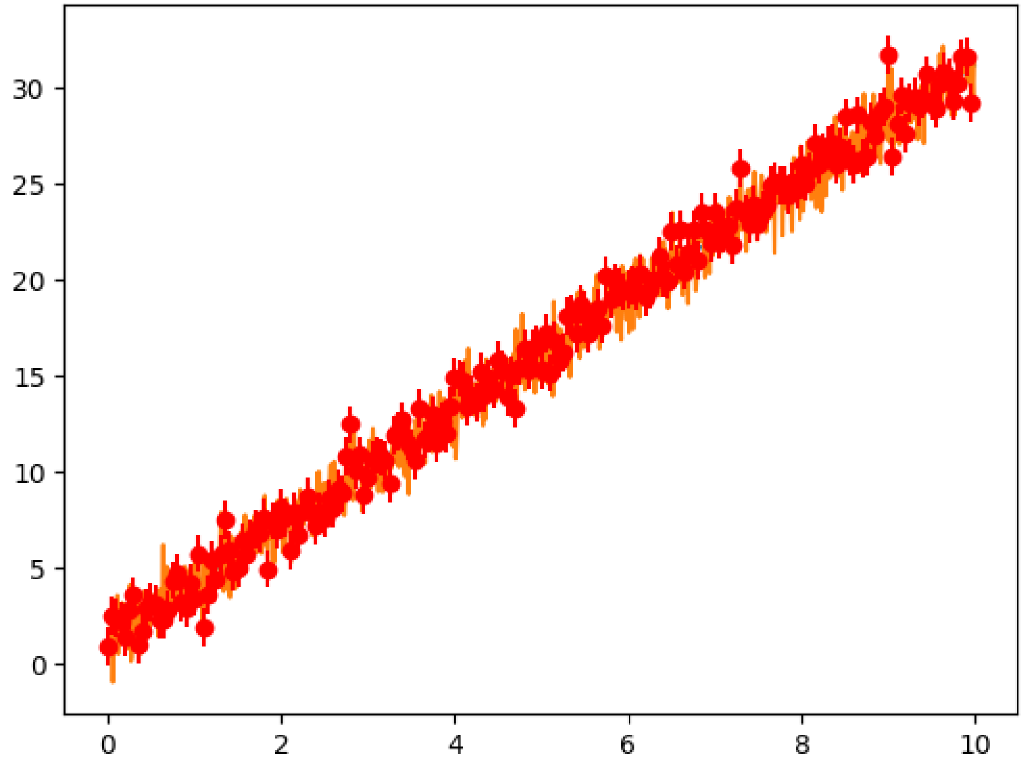
[43]:
```
[<matplotlib.lines.Line2D at 0x1b79967b0d0>,
 <matplotlib.lines.Line2D at 0x1b79967b130>]
```



This is the example dataset of straight line given in the presentation.

[44]:
```
plt.plot(t, y, t, yn)
plt.errorbar(t[::5], yn[::5], np.std(n), fmt='ro')
```

[44]: <ErrorbarContainer object of 3 artists>

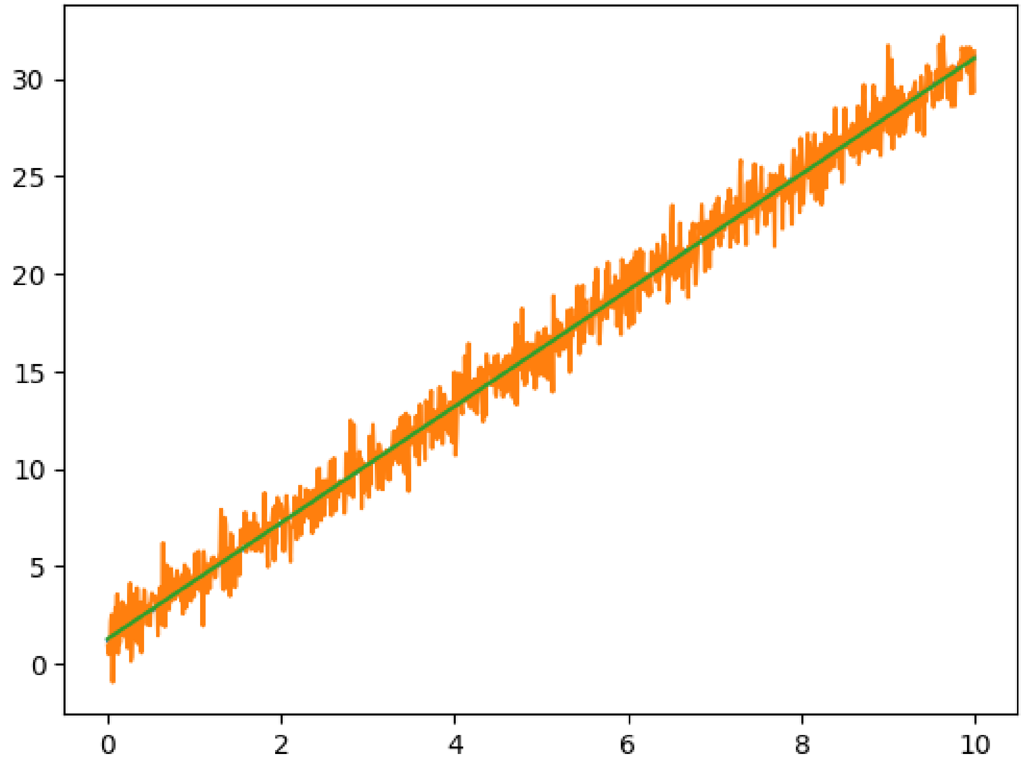


[45]:
```
M = np.column_stack([t, np.ones(len(t))])
(p1, p2), _, _, _ = np.linalg.lstsq(M, yn, rcond=None)
print(f"The estimated equation is {p1} t + {p2}")
%timeit np.linalg.lstsq(M, yn, rcond=None)
```

```
The estimated equation is 2.9799177887425503 t + 1.282468245341636
21.8 µs ± 1.86 µs per loop (mean ± std. dev. of 7 runs, 10,000 loops each)
```

[46]:
```
y_l = stline(t, p1, p2)
plt.plot(t, y, t, yn, t, y_l)
```

[46]:
```
[<matplotlib.lines.Line2D at 0x1b79949d0c0>,
 <matplotlib.lines.Line2D at 0x1b79949d120>,
 <matplotlib.lines.Line2D at 0x1b79949d150>]
```

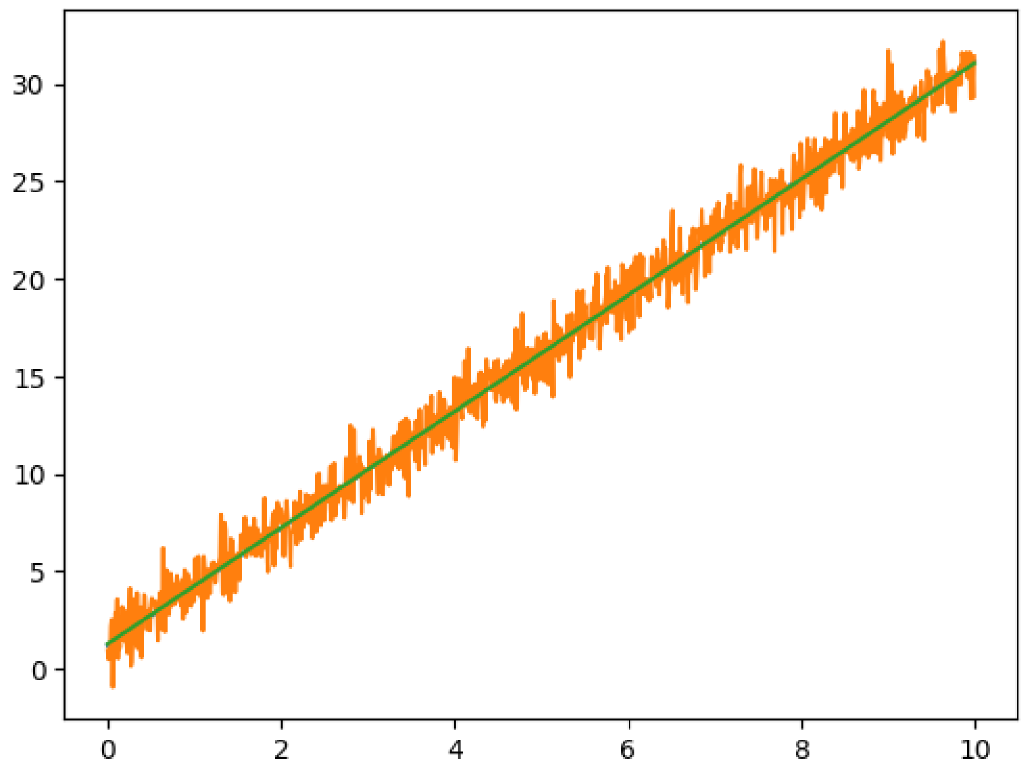


[47]:
```
from scipy.optimize import curve_fit
(q1, q2), pcov = curve_fit(stline, t, yn)
print(f"The estimated equation is {q1} t + {q2}")
%timeit curve_fit(stline, t, yn)
```

```
The estimated equation is 2.9799177896569358 t + 1.282468248201741
244 µs ± 24.7 µs per loop (mean ± std. dev. of 7 runs, 1,000 loops each)
```

[48]:
```
y_c = stline(t, q1, q2)
plt.plot(t, y, t, yn, t, y_c)
```

[48]:
```
[<matplotlib.lines.Line2D at 0x1b79950ae00>,
 <matplotlib.lines.Line2D at 0x1b79950ae60>,
 <matplotlib.lines.Line2D at 0x1b79950ae90>]
```



[49]:

```
error_l=np.square(np.subtract(y_l, y)).mean()
print(error_l)
error_c=np.square(np.subtract(y_c, y)).mean()
print(error_c)
```

```
0.0036791412812574296
0.0036791407101623326
```

time c>l accuracy c>l

5.0.1 Observations

- Here we have found the estimates using both lstsq and curve_fit and we can see that curve_fit takes 10 times more time compared to lstsq.
- I have calculated the meansquare error of y values which i got using lstsq with the original values without noise and named it as error_l and similarly error_c for curve_fit.
- So we can see here that error_c is less than error_l so we can say that accuracy of curve_fit is slightly higher than lstsq.