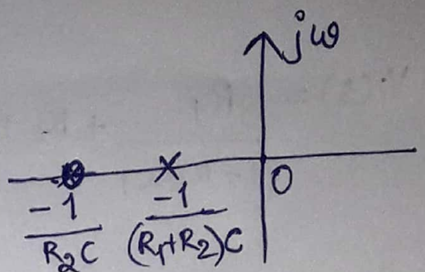
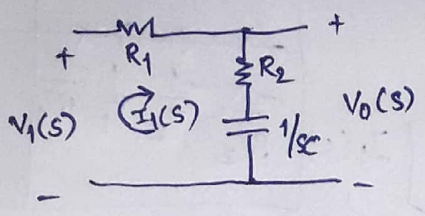
Control system Lab



1.
a) **Lag compensator**

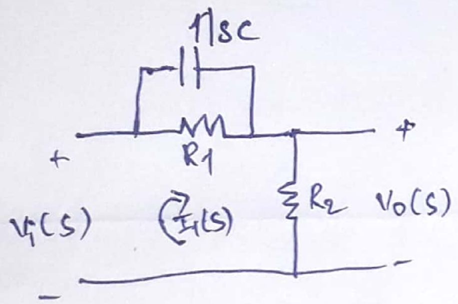


$$T.F = \frac{V_0(s)}{V_1(s)} = \frac{\left(R_2+\frac{1}{sc}\right)I_1(s)}{R_1 \cdot I_1(s) + \left(R_2+\frac{1}{sc}\right)I_1(s)}$$

$$T.F = \frac{1+R_2SC}{R_1SC+R_2SC+1}$$

$$T.F = \frac{1+R_2SC}{1+(R_1+R_2)SC}$$

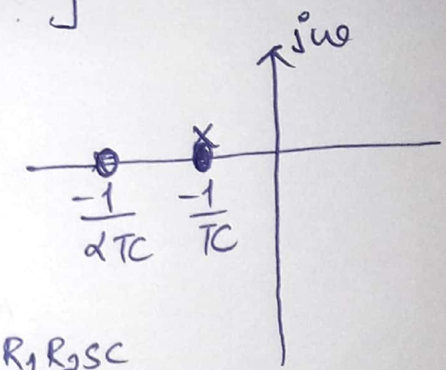
Lead compensator



$$V_i(s) = \left[\frac{\frac{R_1}{SC}}{R_1+\frac{1}{SC}} + R_2\right] I_1(s)$$

$$= \left[\frac{R_1}{1+R_1SC} + R_2\right] I_1(s)$$

$$V_0(s) = R_2 \cdot I_1(s)$$



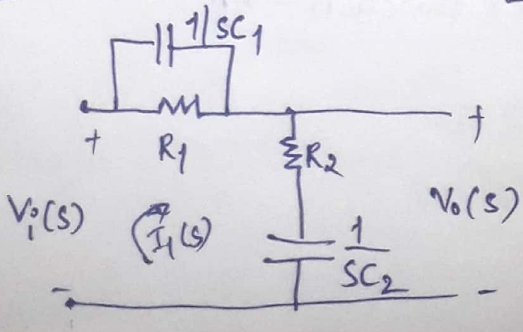
$$T.F = \frac{V_0(s)}{V_i(s)} = \frac{R_2 \cdot I_1(s)}{\left[R_2 + \frac{R_1}{1+R_1SC}\right] \cdot I_1(s)}$$

Let
$\boxed{\alpha = \frac{R_1+R_2}{R_2}}$ $\boxed{\alpha T = R_1}$
$\boxed{T = \frac{R_1R_2}{R_1+R_2}}$

$$= \frac{R_2(1+R_1SC)}{R_2(1+R_1SC)+R_1} = \frac{R_2+R_1R_2SC}{R_1+R_2+R_1R_2SC}$$

$$= \frac{R_2(1+R_1SC)}{R_2\left[\left(\frac{R_1+R_2}{R_2}\right)+R_1SC\right]} = \frac{1+R_1SC}{\frac{R_1+R_2}{R_2}\left[1+\frac{SCR_1R_2}{R_1+R_2}\right]}$$

Lag-Lead Compensator



$$V_0(s) = \left[R_2 + \frac{1}{SC_2}\right] I_1(s)$$

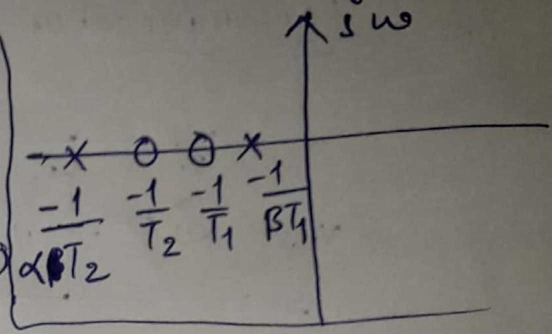
Scanned with CamScanner

$$\alpha = \frac{R_1+R_2}{R_2} = \frac{1}{\beta}$$

$$T_1 = R_1C_1$$

$$T_2 = R_2C_2$$

$$V_i(s) = \left[\frac{R_1}{1+R_1SC_1} + R_2 + \frac{1}{SC_2}\right] I_1(s)$$

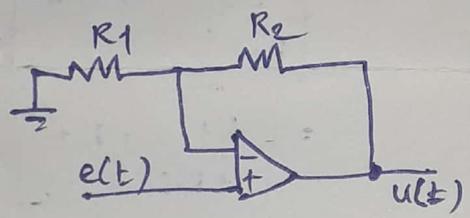


$$T.F = \frac{V_0(s)}{V_i(s)} = \frac{\left[R_2 + \frac{1}{SC_2}\right] I_1(s)}{\left[\frac{R_1}{1+R_1SC_1} + R_2 + \frac{1}{SC_2}\right] I_1(s)}$$

$$T.F = \frac{(1+R_1SC_1)(1+R_2SC_2)}{R_1SC_2 + R_2(1+R_1SC_1)\cdot SC_2 + (1+R_1SC_1)}$$

$$= \frac{(1+sT_1)(1+sT_2)}{(1+\beta T_1 s)(1+\alpha T_2 s)}$$

P– controller



e(t) → error signal

$$u(t) \propto e(t)$$

$$u(t) = K_p \cdot e(t)$$

Applying laplace transform

$$U(s) = K_p E(s)$$

$$\frac{U(s)}{E(s)} = K_p$$

Transfer function – K_p

PI - controller

$$u(t) = e(t) \cdot K_p + K_I \int e(t)\,dt$$

Applying laplace transform

$$U(s) = \left(K_p + \frac{K_I}{s}\right) E(s)$$

$$T.F = \frac{U(s)}{E(s)} = K_p + \frac{K_I}{s}$$

PD - controller

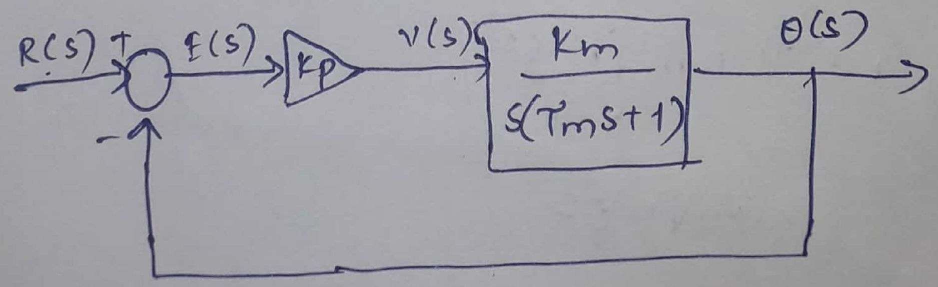
$$u(t) = e(t) \cdot K_p + K_D \cdot \frac{de(t)}{dt}$$

Applying laplace transform

$$U(s) = \left[K_p + K_D \cdot s\right] \cdot E(s)$$

$$T.F = \frac{U(s)}{E(s)} = \underline{K_p + K_D \cdot s}$$

Servo motor



$$T.F = \frac{\theta(s)}{R(s)} = \frac{\dfrac{K_p \cdot K_m}{s(T_m s+1)}}{1 + \dfrac{K_p \cdot K_m}{s(T_m s+1)}}$$

$$T.F = \frac{K_p \cdot K_m}{s(\tau_m s + 1) + K_p \cdot K_m}$$

Time domain specifications of above model

$$T.F = \frac{K_p \cdot K_m}{s^2 \tau_m + s + K_p \cdot K_m}$$

$$= \frac{\frac{K_p \cdot K_m}{\tau_m}}{s^2 + \frac{1}{\tau_m} \cdot s + \frac{K_p \cdot K_m}{\tau_m}}$$

comparing to $\frac{\omega_n^2}{s^2 + 2\xi\omega_n + \omega_n^2}$

$$\omega_n^2 = \frac{K_p \cdot K_m}{\tau_m}$$

$$\boxed{\omega_n = \sqrt{\frac{K_p \cdot K_m}{\tau_m}}}$$

$$2\xi\omega_n = \frac{1}{\tau_m}$$

$$\xi = \frac{1}{2\tau_m} \sqrt{\frac{\tau_m}{K_p \cdot K_m}}$$

$$\boxed{\xi = \frac{1}{2\sqrt{\tau_m \cdot K_p K_m}}}$$

Rise time $t_r = \dfrac{\pi - \tan^{-1}\left[\frac{\sqrt{1-\xi^2}}{\xi}\right]}{\omega_n \sqrt{1-\xi^2}}$

$$= \frac{\pi - \tan^{-1}\left[\sqrt{4 K_m K_p \tau_m - 1}\right]}{2 K_p \cdot K_m}$$

delay time $t_d = \dfrac{1+0.7\xi}{\omega_n}$

$$\dfrac{1+\dfrac{0.35}{\sqrt{T_m \cdot K_p \cdot K_m}}}{\sqrt{\dfrac{K_p K_m}{T_m}}} = \dfrac{\sqrt{T_m K_p K_m}+0.35}{K_p \cdot K_m}$$

Peak time $t_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}}$

$$= \dfrac{\pi}{2K_p \cdot K_m}$$

Peak overshoot $M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$

$$= e^{\frac{-\pi}{\sqrt{4T_m K_p K_m - 1}}}$$

settling time

$t_s = \dfrac{4}{\xi\omega_n}$ (for 2% error)

$= \dfrac{4}{(1/2T_m)} = 8T_m$

$t_s = \dfrac{3}{\xi\omega_n}$ (for 5% error)

$t_s = 6T_m$

b) In a transfer function the system order is the degree of the polynomial

First order system

when input changes, output also changes but not immediately. The system takes some delay but without oscillation.

Second order system

when input changes, output changes with some delay and with oscillation.