

Introduction to Quantum Information and Communication

Theory Assignment-1

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Question 1

To Prove: Any $n + 1$ vectors belonging to an n dimensional vector space must be linearly dependent

Proof:

Let V be an n dimensional vector space

Assume $A = \{v_1, v_2, v_3, \dots, v_{n+1}\}$ is a set of linearly independent vectors where $v_i \in V$

Let $B = A \setminus \{v_{n+1}\} = \{v_1, v_2, v_3, \dots, v_n\}$. Since $B \subset A$, B is also a set of linearly independent vectors.

Now, since V is n dimensional and $|B| = n$, $\text{span}(B) = V$ by the definition of n dimensional vector space.

Therefore, every vector $v \in V$ can be expressed as a linear combination of vectors in B

$\therefore v_{n+1} = a_1 v_1 + a_2 v_2 + a_3 v_3 + \dots + a_n v_n$, where $a_i \in \mathbb{F}$ (field over which V is defined)

$\therefore V$ is not linearly independent. This is a contradiction

Any set A of $n + 1$ vectors belonging to an n dimensional vector space must be linearly dependent.