

Introduction to Quantum Information and Communication

Theory Assignment-2

Moida Praneeth Jain, 2022101093

Exercise 4.1.3

Given:

- A is a square operator acting on Hilbert space \mathcal{H}_S
- I_R is the identity operator acting on a Hilbert space \mathcal{H}_R isomorphic to \mathcal{H}_S
- $|\Gamma\rangle_{RS}$ is the unnormalized maximally entangled vector.

To Prove:

$$\text{Tr}\{A\} = \langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS}$$

Proof:

In the computational basis

$$|\Gamma\rangle_{RS} = \sum_{i=0}^{d-1} |i\rangle_R |i\rangle_S$$

$$\langle \Gamma |_{RS} = \sum_{i=0}^{d-1} \langle i |_R \langle i |_S$$

$$\langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS} = \left(\sum_{i=0}^{d-1} \langle i |_R \langle i |_S \right) (I_R \otimes A_S) \left(\sum_{j=0}^{d-1} |j\rangle_R |j\rangle_S \right)$$

$$\langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS} = \left(\sum_{i=0}^{d-1} \langle i |_R \langle i |_S \right) \left(\sum_{j=0}^{d-1} (I_R \otimes A_S) (|j\rangle_R \otimes |j\rangle_S) \right)$$

$$\langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS} = \left(\sum_{i=0}^{d-1} \langle i |_R \langle i |_S \right) \left(\sum_{j=0}^{d-1} (I_R |j\rangle_R) \otimes (A_S |j\rangle_S) \right)$$

$$\langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS} = \left(\sum_{i=0}^{d-1} \langle i |_R \langle i |_S \right) \left(\sum_{j=0}^{d-1} |j\rangle_R \otimes (A_S |j\rangle_S) \right)$$

$$\langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS} = \sum_{i,j=0}^{d-1} (\langle i |_R \otimes \langle i |_S) (|j\rangle_R \otimes (A_S |j\rangle_S))$$

$$\langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS} = \sum_{i,j=0}^{d-1} (\langle i | j \rangle_R \otimes \langle i |_S A_S | j \rangle_S)$$

$$\langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS} = \sum_{i,j=0}^{d-1} (\delta_{i,j} \otimes \langle i |_S A_S | j \rangle_S)$$

$$\langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS} = \sum_{i=0}^{d-1} \langle i |_S A_S | i \rangle_S$$

$$\langle \Gamma |_{RS} I_R \otimes A_S | \Gamma \rangle_{RS} = \text{Tr}\{A\}$$

Hence, proven.