Science-1

Assignment-1

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Question 1

(a)

$n \approx 4$

(b)

Average pace of a human is $v = 1.4\ \frac{m}{s}$

Assuming the distance to travel is $d = 4 * 100 = 400m$

$v = \frac{d}{t}$

$1.42 = \frac{400}{t}$

$t = \frac{400}{1.42}$

$t \approx 282s$

The time required is approximately 4 minutes and 42 seconds

(c)

Assuming the drift velocity of an electron on average is 0.000023 $\frac{m}{s}$

Assuming the distance to travel is $d = 4 * 100 = 400m$

$v = \frac{d}{t}$

$0.000023 = \frac{400}{t}$

$t \approx 17391304s$

The time required is approximately 201 days and 6 hours

(d)

Speed of electron in ground state of hydrogen atom = $2180000\frac{m}{s}$

$v = \frac{d}{t}$

$2180000 = \frac{d}{17391304}$

$d = 37913042720000m$

Circumference of first bohr orbit in hydrogen = 0.0000000003322

Number of rounds = $\frac{d}{\text{circumference}}$ = $\frac{37913042720000}{0.0000000003322}$ = 114127160505719450000000 rounds

(e)

Assuming standard copper wires with $\rho = 0.00000001724$ ohm m and area $A = 0.000015\ m^2$

$R = \rho\frac{l}{A}$ = 0.46 Ω

Assuming a current of 5 A

Power $P = I^2R$ = 11.5 W

Question 2

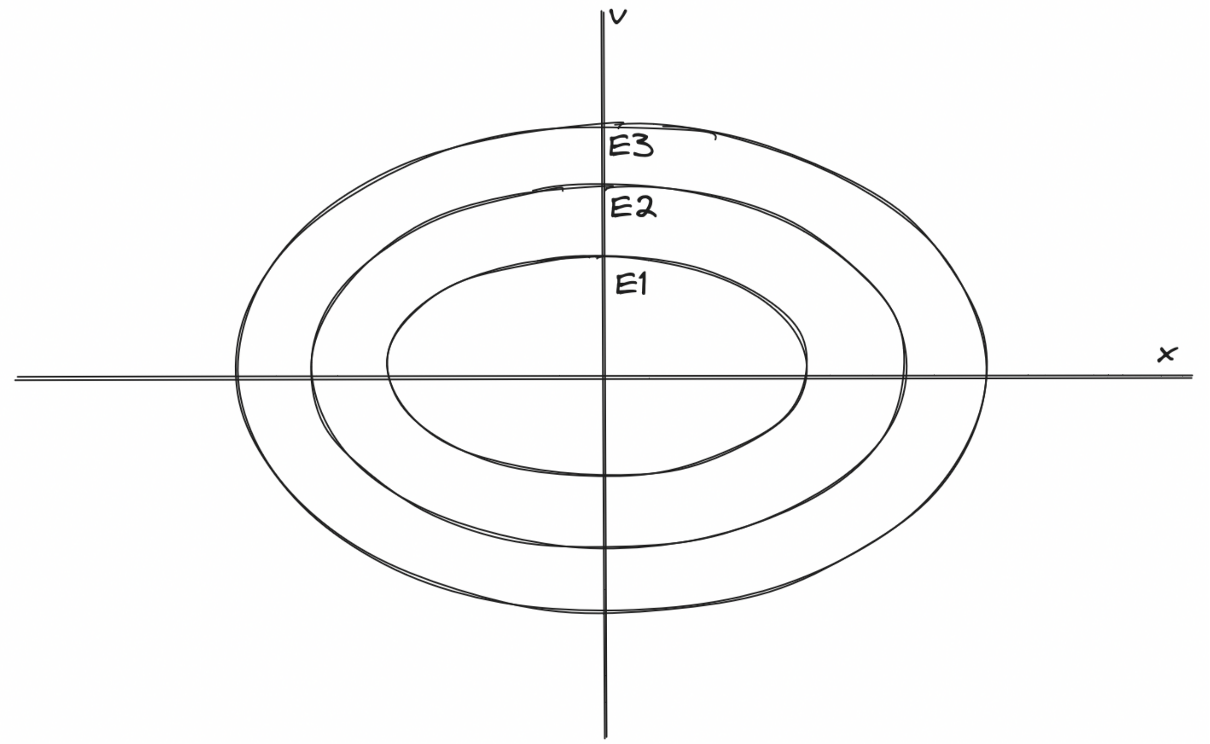
(i)

Consider a spring mass system performing SHM with total energy E

$\mathrm{PE} + \mathrm{KE} = E$

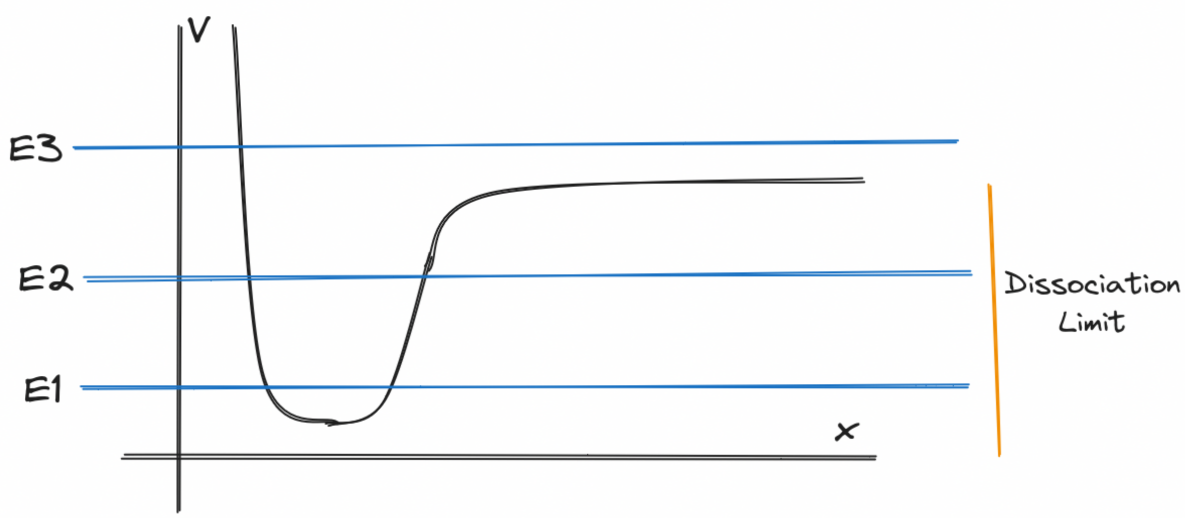
$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = E$

This is an equation of ellipse. Varying total energy will result in concentric ellipses

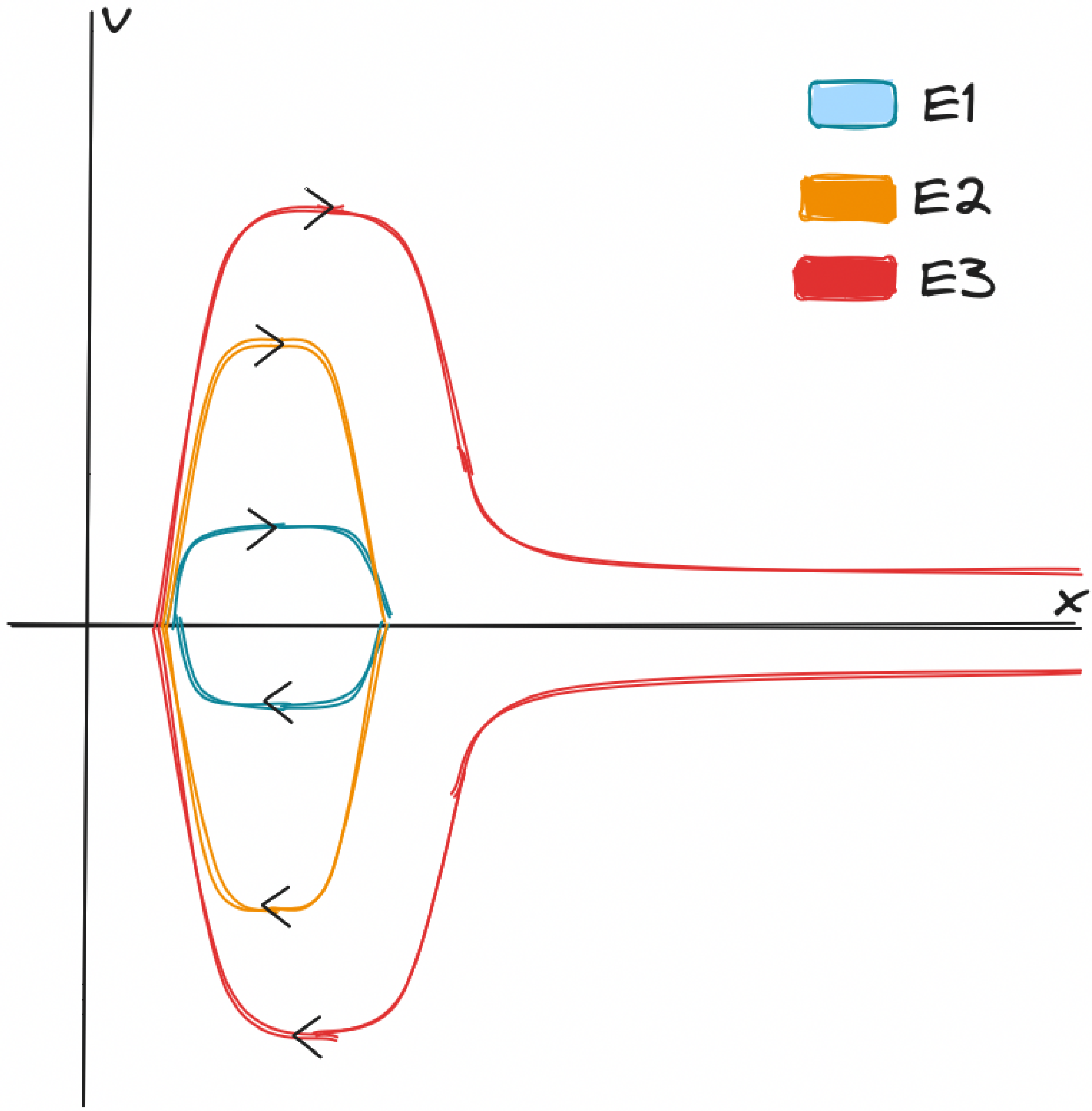


(ii)

Consider a system with anharmonic oscillations with the following potential graph



The phase space trajectories for each of the total energies would be

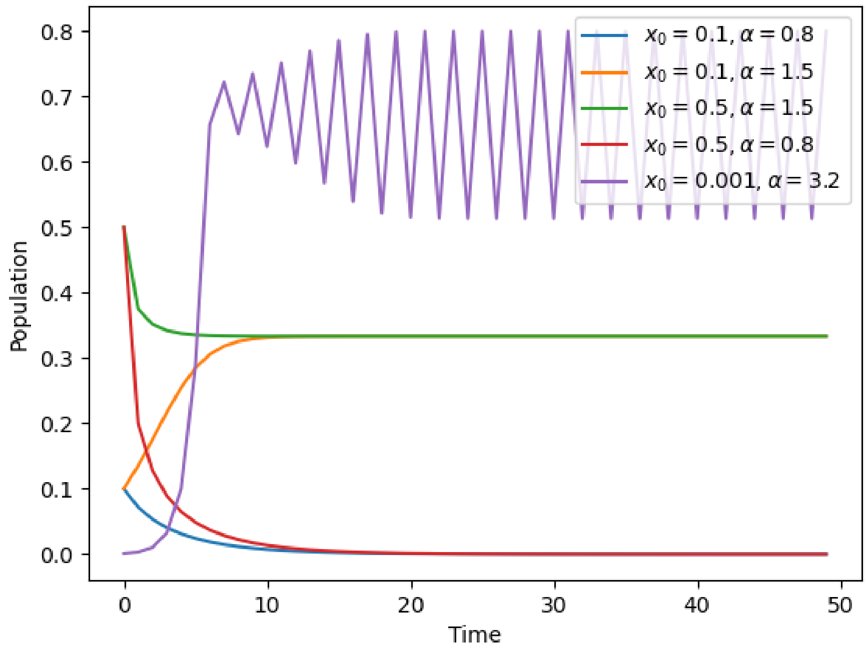

v
x
E1
E2
E3

Question 3

The logistic map $x_{n+1} = \alpha x_n(1 - x_n)$ is usually studied for the growth parameter, α, in the range $0 \leq \alpha \leq 4$. This is because $\forall \alpha > 4$, the values taken by x diverge. For α lying within the range, the logistic map yields very interesting results like converging to one value, or even converging to two values (oscillating between the two), and some phenomenon like period doubling can be observed. Due to such properties, it is usually studied within this range.

Python code to plot logistic map

```python
import matplotlib.pyplot as plt


def step(x: float, alpha: float) -> float:
    return alpha * x * (1 - x)


def generate(x0: float, alpha: float, generations: int = 50) -> list[float]:
    result = [x0] * generations
    for i in range(1, generations):
        result[i] = step(result[i - 1], alpha)
    return result


if __name__ == "__main__":
    plt.plot(generate(x0=0.1, alpha=0.8), label=r"$x_0 = 0.1, \alpha=0.8$")
    plt.plot(generate(x0=0.1, alpha=1.5), label=r"$x_0 = 0.1, \alpha=1.5$")
    plt.plot(generate(x0=0.5, alpha=1.5), label=r"$x_0 = 0.5, \alpha=1.5$")
    plt.plot(generate(x0=0.5, alpha=0.8), label=r"$x_0 = 0.5, \alpha=0.8$")
    plt.plot(generate(x0=0.001, alpha=3.2), label=r"$x_0 = 0.001, \alpha=3.2$")

    plt.xlabel("Time")
    plt.ylabel("Population")
    plt.legend()

    plt.savefig("plot.png")
```



$x_0 = 0.1, \alpha = 0.8$
$x_0 = 0.1, \alpha = 1.5$
$x_0 = 0.5, \alpha = 1.5$
$x_0 = 0.5, \alpha = 0.8$
$x_0 = 0.001, \alpha = 3.2$
Population
Time
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
10
20
30
40
50

Question 4

$$P(N|n) = \binom{N}{n} p^n q^{N-n} \text{ where } p+q=1, p \geq 0, q \geq 0$$

$$P(N|n) = \frac{N!}{n!(N-n)!} p^n q^{N-n}$$

$$\ln(P(N|n)) = \ln(N!) - \ln(n!) - \ln((N-n)!) + n\ln(p) + (N-n)\ln(q)$$

Using Stringling approximation ($\ln(N!) = N\ln(N) - N$)

$$\ln(P(N|n)) \approx N\ln(N) - N - (n\ln(n) - n) - ((N-n)\ln(N-n) - (N-n)) + n\ln(p) + (N-n)\ln(q)$$

$$\ln(P(N|n)) = N\ln(N) - N - n\ln(n) + n - (N-n)\ln(N-n) + (N-n) + n\ln(p) + (N-n)\ln(q)$$

$$\ln(P(N|n)) = N\ln(N) - n\ln(n) - (N-n)\ln(N-n) + n\ln(p) + (N-n)\ln(q)$$

Let $\ln(P(N|x)) = f(x)$ $f(x) = N\ln(N) - x\ln(x) - (N-x)\ln(N-x) + x\ln(p) + (N-x)\ln(q)$

$$f^1(x) = -1 - \ln(x) + \frac{N}{N-x} - \frac{x}{N-x} + \ln(N-x) + \ln(p) - \ln(q)$$

$$f^1(x) = \ln(N-x) - \ln(x) + \ln\left(\frac{p}{q}\right)$$

Consider maxima at $x = x_0$

$$f^1(x_0) = \ln(N-x_0) - \ln(x_0) + \ln\left(\frac{p}{q}\right)$$

$$0 = \ln\left(\frac{N}{x_0} - 1\right) + \ln\left(\frac{p}{q}\right)$$

$$\ln\left(\frac{q}{p}\right) = \ln\left(\frac{N}{x_0} - 1\right)$$

$$x_0 = N\frac{p}{p+q}$$

$$x_0 = Np$$

$$f(x_0) = N\ln(N) - Np\ln(Np) - Nq\ln(Nq) + Np\ln(p) + Nq\ln(q)$$

$$f(x_0) = N\ln\left(\frac{Np^pq^q}{N^pp^pN^qq^q}\right)$$

$$f(x_0) = N\ln\left(\frac{N}{N^{p+q}}\right)$$

$$f(x_0) = 0$$

$$f^2(x) = -\frac{1}{N-x} - \frac{1}{x}$$

$$f^2(x) = -\frac{N}{N-x}$$

$$f^2(x_0) = -\frac{N}{N - Np}$$

$$f^2(x_0) = -\frac{1}{q}$$

Expanding $f(x)$ around x_0 using taylor expansion

$$f(x) \approx f(x_0) + (x - x_0) f^1(x)|_{x=x_0} + \frac{1}{2}(x - x_0)^2 f^2(x)|_{x=x_0}$$

$$f(x) = 0 + (x - Np)(0) + \frac{1}{2}(x - Np)^2 \left(-\frac{1}{q}\right)$$

$$\ln(P(N|n)) = -\frac{1}{2q}(n - Np)^2$$

$$P(N|n) = e^{-\frac{1}{2q}(n-Np)^2}$$

$$P(N|n) = c_1 e^{-c_2 (n-Np)^2}$$

Therefore, as N grows large, the binomial distribution tends to the Gaussian distribution. QED