A quick review of Transition Systems

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What is a transition system?

A labelled transition system (LTS) is a three tuple

$$S = (X, U, \rightarrow)$$

- X is a set of *states* of S.
- U is a set of labels, also called the *actions* of S.
- $\rightarrow$ is a partial function from states and actions to states called the *dynamics* of S.
- $x \xrightarrow[S]{u} x'$ means x moves to x' on the action u in the system S.

Why transition systems?

1. **Formality**: Notional machines best formalised as labelled transition systems.

1. **Generality and wide applicability**: LTS's are extremely general in the applicability. They can be used to describe phenomena in a wide variety of domains (Computer science, hardware, engineering systems, etc.)

Runs of an LTS

- **Finite Run**:

$$x_0 \xrightarrow[S]{u_0} x_1 \xrightarrow[S]{u_1} x_2 \ldots x_{n-1} \xrightarrow[S]{u_{n-1}} x_n$$

- **Infinite Run**:

$$x_0 \xrightarrow[S]{u_0} x_1 \xrightarrow[S]{u_1} x_2 \ldots x_{i-1} \xrightarrow[S]{u_{i-1}} x_i \ldots$$

Terminal state

1. A state is **terminal** if no transition is possible from that state. $x \not\rightarrow$.

A run is **terminating** if it reaches a terminal state.

$$2 \xrightarrow[C]{\text{dn}} 1 \xrightarrow[C]{\text{dn}} 0 \underset{C}{\not\rightarrow}$$

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An up-down counter

$$C = (X, U, \rightarrow)$$

where

- $X = \mathbb{N}$
- $U = \{\text{up}, \text{dn}\}$
- Dynamics:

$$x \xrightarrow[C]{\text{up}} x + 1 \qquad \text{if } x \neq 0$$
$$x \xrightarrow[C]{\text{dn}} x - 1 \qquad \text{if } x \neq 0$$

Runs of the counter machine

A factorial system

$$F = (X, U, \rightarrow)$$

where

- $X = \mathbb{N} \times \mathbb{N}$
- $U = \text{next}$
- $\underset{F}{\rightarrow}$:

$$(i, a) \xrightarrow[F]{\text{next}} (i-1, a*i) \qquad \text{if } i > 0$$

- To compute $n!$, consider the finite run starting from $(n, 1)$ at terminating at $(0, n!)$.

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The notional machine as an LTS

1. In the rest of the workshop, we will describe a family of notional machines as LTS's.
2. The program to be executed is a parameter to the notional machine (the machine refers to the program but does not change it.)
3. The run of the notional machine corresponds to the execution of the program.

