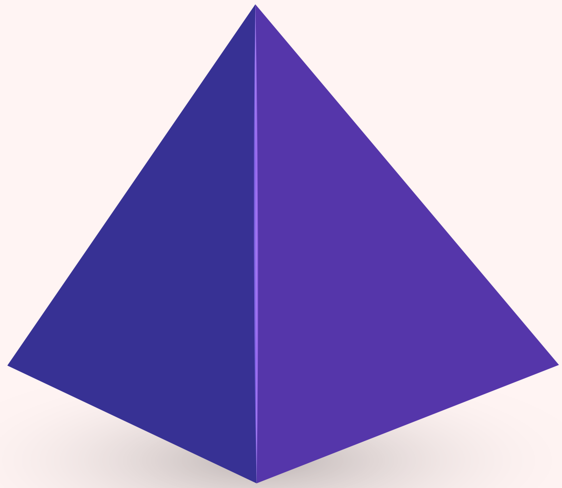
COMPLETE CONCEPTS OF GEOMETRY TRIANGLES

FROM BASICS TO ADVANCED LEVEL



FOR DIFFERENT SSC EXAMS

Complete Geometry Notes On Triangles For SSC Exams

Importance of Geometry in SSC exams

Geometry for any SSC exam is a very important topic. Let us tell you how vital is geometry in SSC exams by taking one of the SSC exams 'SSC CGL'. This exam consist two rounds of objective type exam Tier-I and Tier-II. In Tier-I around 3-6 questions of geometry was asked out of 25 questions whereas in Tier-II around 10-15 geometry questions were asked out of 100 question.

Importance of triangle in Geometry for SSC exams

In SSC exams there are basically three topics in Geometry – Triangles, Circles and Polygon. If we take SSC CGL Tier-I around 1-3 questions are from triangle and in Tier-II around 4-6 questions are from triangle. Triangle is one of the simplest topics in the preparation of any SSC exam. Let's jump to some tricks that will be useful for the preparation.

Condition for the formation of a triangle:

There are only two conditions for the formation of a triangle
- The sum of any two sides of a triangle must be greater than the third side of the triangle.
- The difference of any two sides of a triangle must be smaller than the third side of the triangle.

Types of triangle

On the basis of side

1. Scalene triangle- A triangle in which all three sides are unequal is Scalene triangle.

2. Isosceles triangle- A triangle with two equal sides and one unequal side is an Isosceles triangle.

3. Equilateral triangle- A triangle with all three sides equal is an Equilateral triangle.

On the basis of Angle

1. Acute angle triangle- A triangle with all three angles less than 90° is an acute angle triangle. In this triangle the sum of square of any two sides is more than the square of the third side.

2. Right angle triangle- A triangle where one of the angles is 90° is called right angle triangle. Pythagoras theorem is applicable in this triangle, where sum of square of the two sides is equal to the square of the third side which is opposite to 90°.

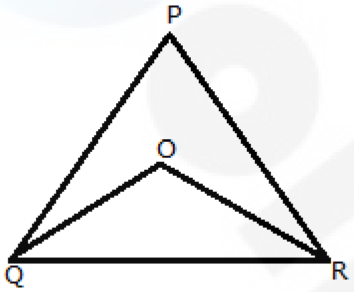
3. Obtuse angle triangle- A triangle with one angle more than 90° is an obtuse angle triangle. In this triangle the sum of the square of two smaller sides is less than the square of the largest side.

Properties of a triangle

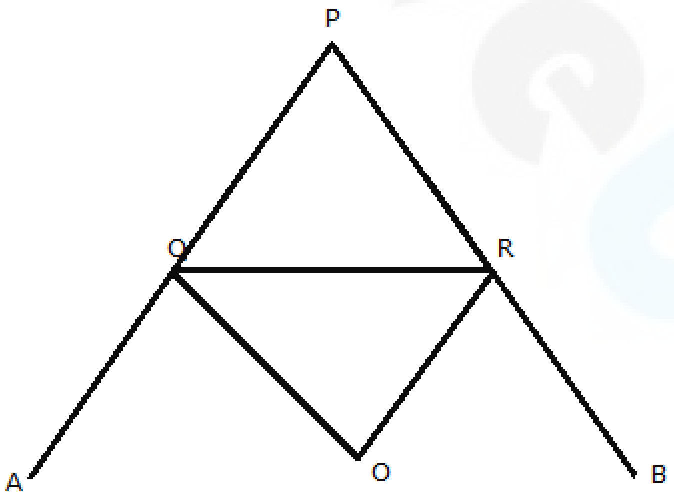
1. Angle sum property: The sum of all three internal angles of a triangle will always be 180°

2. Exterior angle property: An exterior angle of a triangle will always be equal to the sum of opposite interior angles.

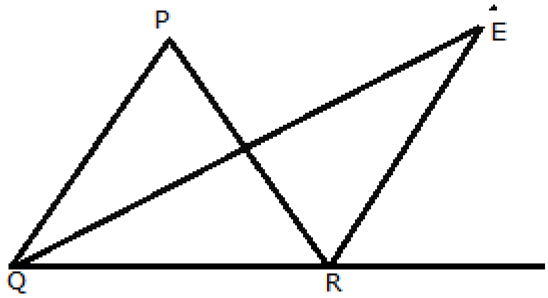
3. If the bisector of ∠PQR and ∠PRQ of triangle PQR meets at O then ∠QOR= (90° + (∠P/ 2))



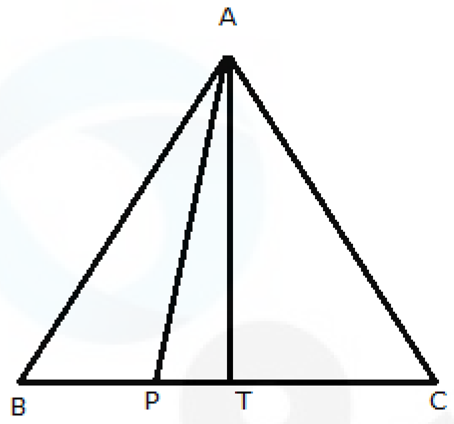
4. If the line PQ and PR are produced to point A and B respectively and if the bisector of ∠AQR and ∠BRQ Meets at point O then ∠QOR = (90° - ((∠P / 2))



5. The angle between the interior angle bisector of base angle and the exterior angle bisector of another base angle of the triangle is equal to the half of the vertical angle. ∠QER = (∠QPR/2)



6. In the given figure AP is the angle bisector of ∠BAC and AT is perpendicular to BC then ∠PAT = (|∠B - ∠C|)/2



Congruent triangles

Two Triangles can be congruent only when one triangle superimpose the other triangle or we can say when both the triangle cover each other completely they are congruent. One triangle is the mirror image of the other triangle.
If Δ ABC and Δ ZYX are congruent then
AB = ZY, BC= YX, CA= ZX and ∠A = ∠Z, ∠B = ∠Y, and ∠C = ∠X.

- The point to remember here is the order if, AB = ZY, BC= YX, CA= ZX, then we write Δ ABC And Δ ZYX are congruent But if AB = YL, BC= LT, CA= YT then we will write Δ ABC And Δ YLT are congruent.
- If all three sides of a triangle are equal to the corresponding sides of the other triangle then we can say the triangles are congruent but if all three angles are equal to the corresponding angles we cannot say the triangles are congruent.

Conditions for congruence of the triangles.

- **SAS (Side – Angle – Side):** Two triangles are congruent when two sides and the included angle are equal to the corresponding sides and the included angle of the other triangle. The thing to remember here is the included angle.

- **ASA (Angle – Side – Angle):** Two triangles are congruent when two angles and the included side are equal to the corresponding angle and the included side of the other triangle.

- **AAS (Angle – Angle – Side):** Two triangles are congruent if the two angles and the non included side of one triangle is equal to the corresponding two angles and the non included side of the other triangle.

- **SSS (Side – Side- Side):** Two triangles are congruent **i**f all three sides of a triangle are equal to the corresponding sides of the other triangle.

- **RHS (Right angle – Hypotenuse – side):** As the name suggest two right angle triangles are congruent if hypotenuse and one other side are equal to the corresponding Hypotenuse and the other side of the other triangle.

Similarity of Triangles

Two triangles are similar either when Corresponding angles have same measure or when the corresponding sides are in proportion.

Condition for similarity of triangle

(i) AAA or AA (ii) SAS (iii) SSS

When two triangles Δ ABC and Δ XYZ are similar then simply every corresponding ratio is equal

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX} = \frac{Perimeter\ of\ ABC}{Perimeter\ of\ XYZ} = \frac{H1}{H2} = \frac{M1}{M2} = \frac{R1}{R2} = \frac{r1}{r2} = \frac{\sqrt{Area\ of\ ABC}}{\sqrt{Area\ of\ XYZ}}$$

Here H1, M1, R1, r1 are the height, median, Circumradius and in-radius respectively of triangle ABC and H2, M2, R2, r2 are the height, median, Circumradius and in-radius respectively of triangle XYZ.

- Every congruent triangle is a similar triangle but the vice versa is not true

Some important theorems

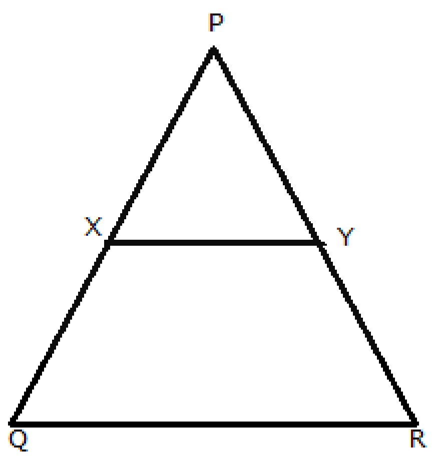
1. Mid-point theorem: The line joining any two mid points of the side of the triangle is always parallel to the third side and also half of the third side and the converse of it is also true.

2. Basic proportionality theorem: If XY is parallel to QR we can derive the following conclusions

$$\frac{PX}{PQ} = \frac{PY}{PR} = \frac{XY}{QR}$$

And

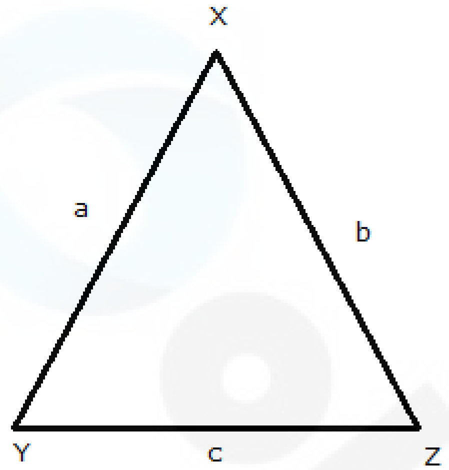
$$\frac{PX}{XQ} = \frac{PY}{YR}$$



Both Midpoint theorem and Basic proportionality theorem can be derived from similarity.

3. Inequality relation in a triangle:

If XYZ is a scalene triangle where side XY=a, XZ=b and YZ=c

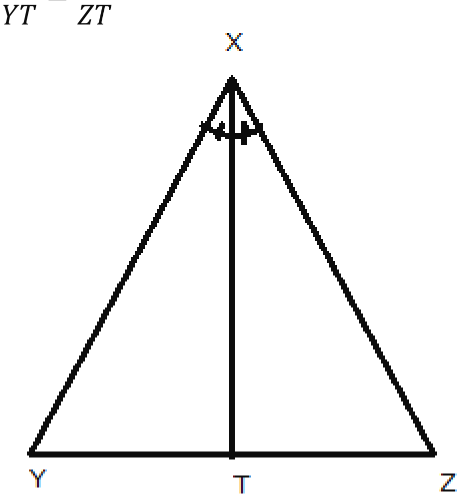


If ∠Z = ∠Y, then a = b
∠Z > ∠Y, then b > a
∠Z < ∠Y, then b < a

4. Interior angle bisector theorem:

In triangle XYZ, XT is the angle bisector of ∠YXZ then

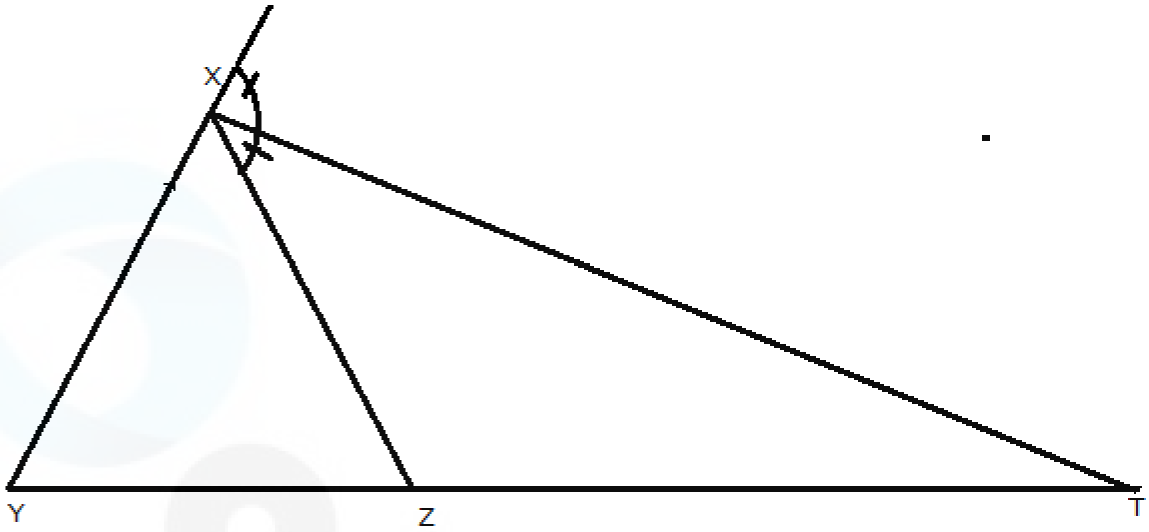
$\frac{XY}{YT} = \frac{XZ}{ZT}$



5. Exterior angle bisector theorem:

In triangle XYZ, XT is the angle bisector of ∠YXZ then

$\frac{XY}{YT} = \frac{XZ}{ZT}$

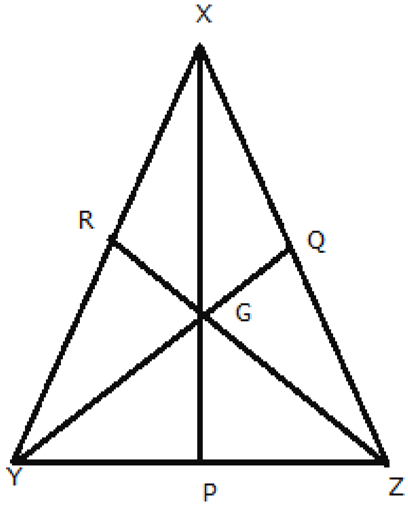


Centers of triangle

1. Centroid: It is the point of intersection of three medians. A line that joins the vertex and the midpoint of the opposite side is called median.

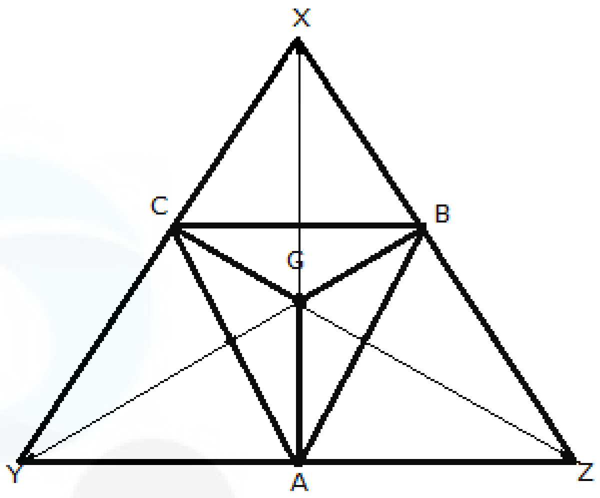
- Centroid of the triangle divides the median in 2:1.
- Median divides the area of triangle in equal parts. If we draw 3 medians in a triangle the area will be divided into 6 equal parts.
- Centroid always lies inside the triangle
- Centroid divides the triangle in three equal parts

Some important key results



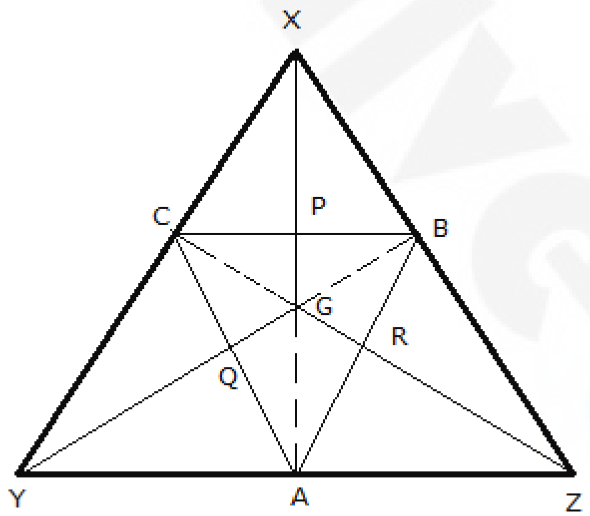
(I) 4(XP+YQ+ZR) › 3(XY+YZ+ZX)
(II) $4(XP^2+YQ^2+ZR^2) = 3(XY^2+YZ^2+ZX^2)$
(III)Area of triangle XYZ = $\frac{4}{3}$ (Area of triangle formed by median YQ, ZR and XP)
(IV) Area of triangle formed by joining the mid points of any two sides to the centroid is $\frac{1}{12th}$ of the area of triangle.

Area of ΔABG = Area of ΔCBG = Area of ΔACG = $\frac{1}{12}$th of Area of ΔXYZ



(V) The line joining the vertex and the centroid is divided by the line joining the mid points of two sides in the ratio 3:1

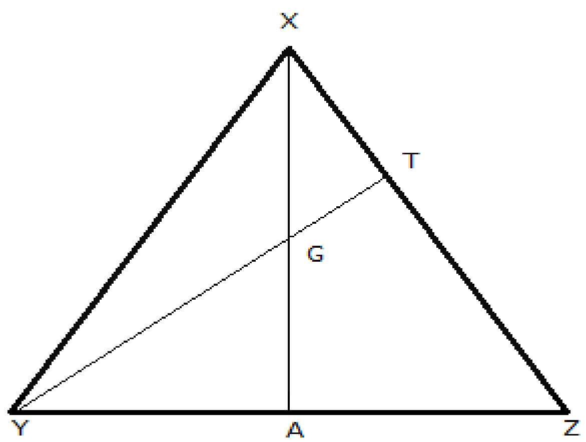
XP: PG = YQ: QG = ZR: RG = 3: 1



(VI) The line joining the mid-point of the median to the vertex divides the opposite side in the ratio of 1:2

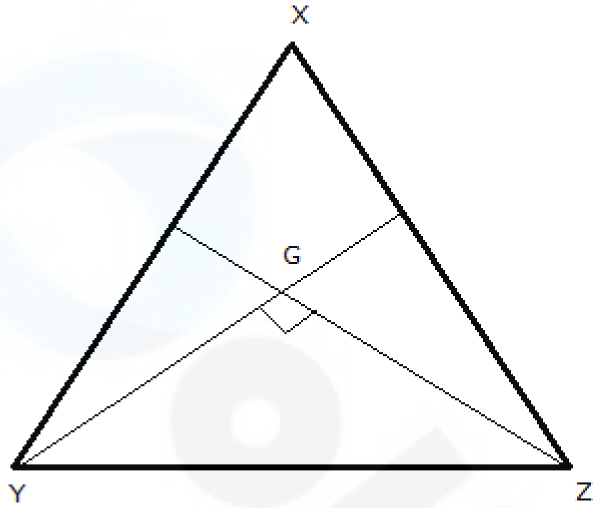
XA is a median and G is the midpoint of the median then

XT : TZ = 1: 2



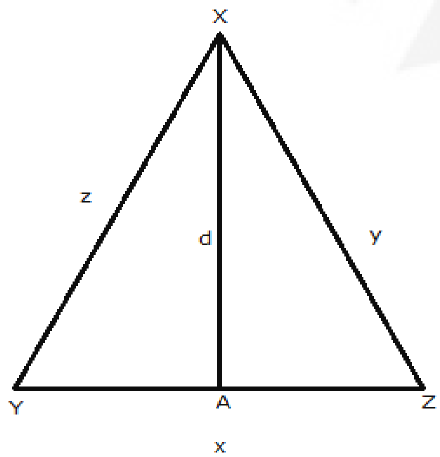
(VII) If the medians intersect at 90° then 5 times the square of the common side is equal to the sum of the squares of the other two sides.

$5(YZ)^2 = XY^2 + XZ^2$



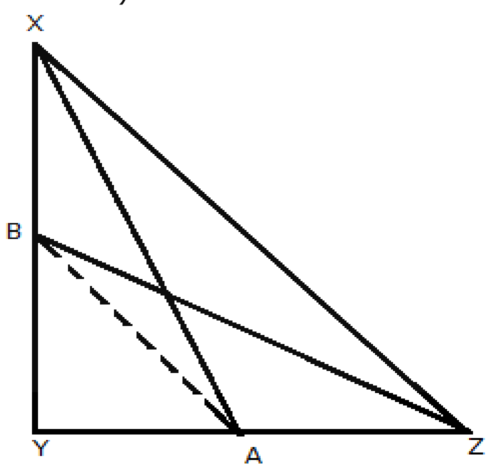
(VIII) **Apollonius theorem:** The length of median can be obtained by using this theorem

$4(d)^2 = 2z^2 + 2y^2 - x^2$



(IX) Relation of medians in Right angle triangle can be determined by using the following key result

$4\ (XA^2 + ZB^2) = 5XZ^2$

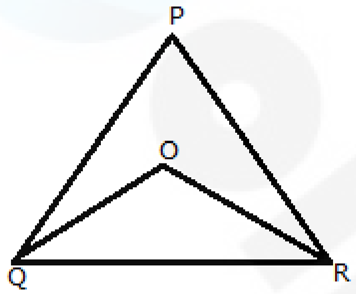


2. In-center: The point of intersection of all the three angle bisectors of a triangle is known as In-center. An angle bisector is a line which divides angle into two equal parts. If we draw a circle inside a triangle the center of the circle is called In-center and the distance between the in-center and any point on that circle is In-radius.

- In-center always lies inside the triangle
- Generally angle bisector doesn't intersect the opposite side perpendicularly.
- The perpendicular distance from the in-center to the side is in-radius.

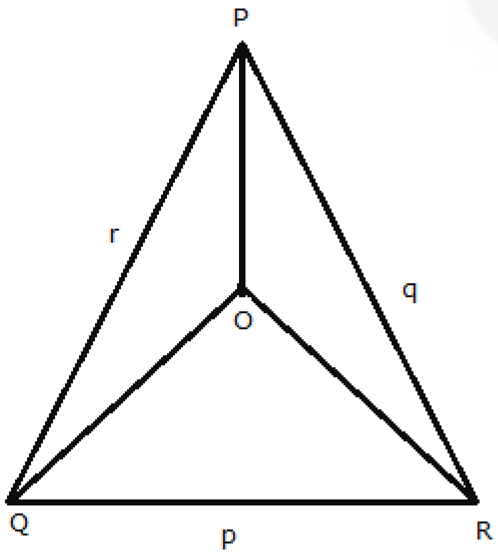
Some Important key results

(I) If the bisector of ∠PQR and ∠PRQ of triangle PQR meets at O then ∠QOR= (90° + (∠P/ 2))



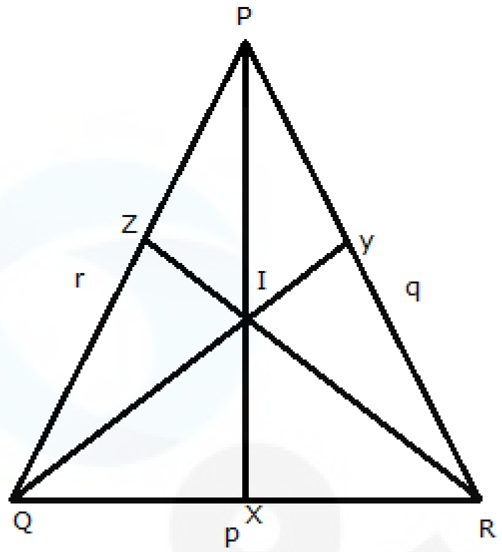
(II)The ratio of the sides of the triangle is equal to the area formed by the corresponding sides and the In-center.

$\frac{p}{Area\ of\ \Delta QOR} = \frac{q}{Area\ of \Delta POR} = \frac{r}{Area\ of\ \Delta POQ} = 1 : 1 : 1$



(III) The length of angle bisector is divided by the in-center in the ratio of length of sum of two adjacent sides and the opposite side.

PI: IX = (q+r) : p, QI: IY = (r+p) : q, RI : IZ= (p+q) : r



(IV) The area of any triangle can be determined by multiplying the in-radius and the semi perimeter of the triangle.
Δ = r*s (r is the in-radius, s is the semi perimeter of triangle and Δ is the area of triangle)

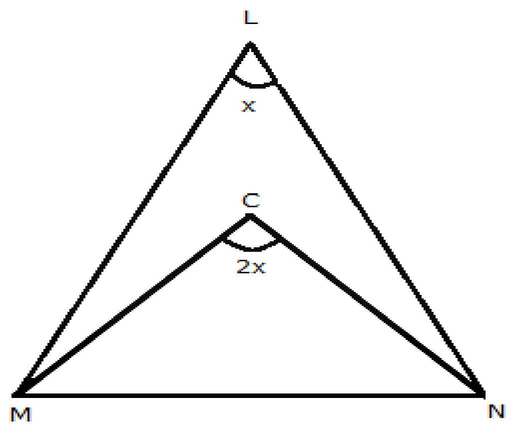
(V) In-radius in any right-angled triangle can be determined by subtracting hypotenuse from the sum of perpendicular and base and then by dividing the result by 2.
In-radius for any right-angled triangle = $\frac{P+B-H}{2}$ (P, B and H are perpendicular, base and hypotenuse respectively)

3. Circumcenter: The intersection point of all three perpendicular bisector of an triangle is known as circumcenter. It is also the center of triangle's circumcircle.

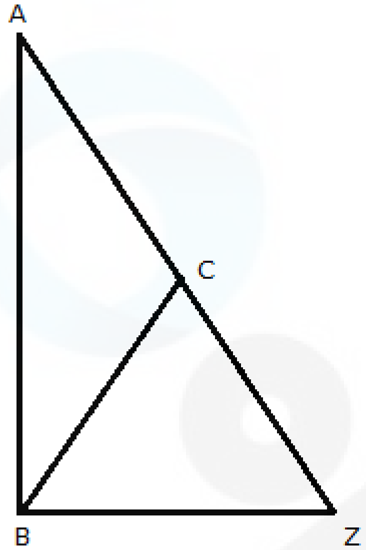
- In an acute angle triangle the circum center lies inside the triangle
- In an obtuse angle triangle the circum center lies outside the triangle
- In an right angled triangle the circumcenter lies on the midpoint of the hypotenuse. So for a right angle triangle circumradius is half of Hypotenuse which is equal to the line joining right angle vertex to the circumcenter.
- The length of all three vertices from the circumcenter is equal.

Some Important key results

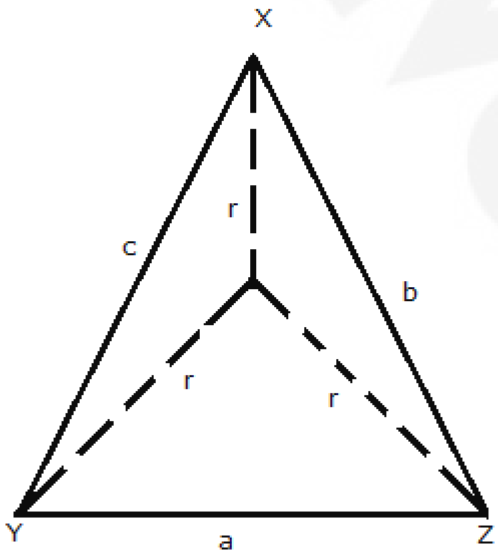
(I) The angle between the line segment joining the circumcenter and any two vertices of the triangle is double of the angle made on the third vertex.



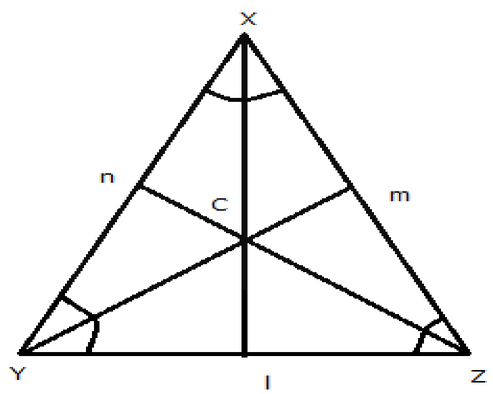
(II) In a right-angle triangle circumradius is half of Hypotenuse which is equal to the line joining right angle vertex to the circumcenter.
BC = CA = CZ = circumradius



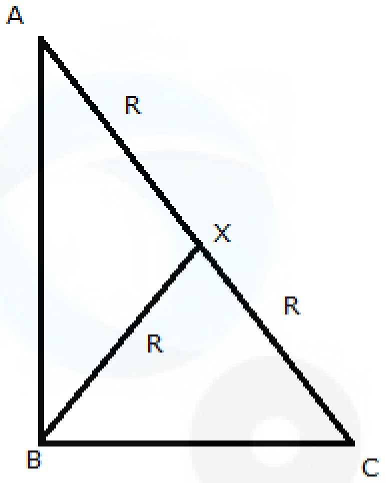
(III) Circumradius of any triangle can be determined as
R = $\frac{a*b*c}{4*area\ of\ \Delta XYZ}$



(IV) Angle side relation in circumradius:
$\frac{YZ}{SinX}$ =$\frac{XZ}{Sin\ Y}$ = $\frac{XY}{Sin\ Z}$ = 2R (R is the circumradius CX, CY and CZ)



(V) Area of any right angle triangle where circumradius is R and semi-perimeter is S = S(S – 2R)



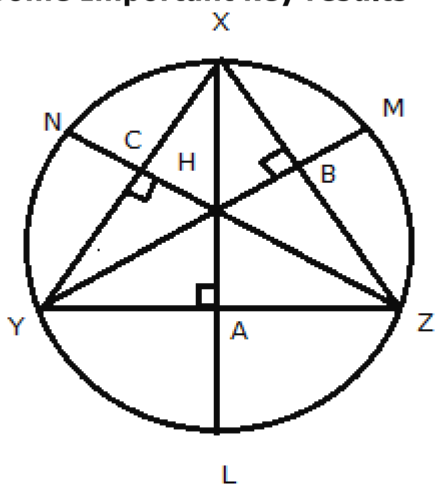
Relation between In-radius and circumradius:

(i) The distance between circumradius(R) and In-radius(r) in any triangle = $\sqrt{((R^2) - 2Rr)}$
(ii) For a right angle triangle the sum of In-radius(r) and circumradius(R) is half of the sum of perpendicular and base
R+r = (P+B)/2 (P and B are perpendicular and base respectively of any right-angled triangle)
(iii) Area of any right-angle triangle where in-radius is r and circumradius is R = $r^2 + 2Rr$

4. Orthocenter: The intersection point of all three altitudes in a triangle is known as Orthocenter.

- Orthocenter lies inside an acute angle triangle
- Orthocenter lies outside an obtuse angle triangle
- Orthocenter lies on the right-angle vertex in a right-angled triangle

Some Important key results



- H is the orthocenter of ΔXYZ
  Z is the orthocenter of ΔXHY
  X is the orthocenter of ΔYHZ
  Y is the orthocenter of ΔXHZ

- $\angle YHZ + \angle X = 180^\circ$
  $\angle XHY + \angle Z = 180^\circ$
  $\angle XHZ + \angle Y = 180^\circ$

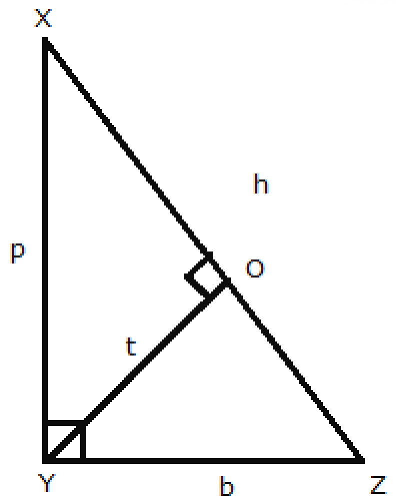
- YH*HB = ZH*HC = XH*HA

- HC = CN
  HB = BM
  HA = AL

- YA*AZ = XA*AH
  ZB*BX = YB*BH
  YC*CX = ZC*CH

- (XY + YZ + ZX) > (XA + YB +ZC)

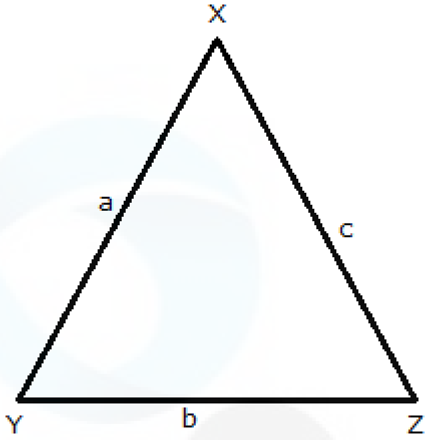
(I) Orthocenter in right angle triangle:



$XO = (p^2 / h)$ $OZ = (b^2 / h)$
$XO : OZ = p^2 : b^2$
$t^2 = ZO * OX$
$t = (p*b)/h$
$1/t^2 = 1/p^2 + 1/b^2$

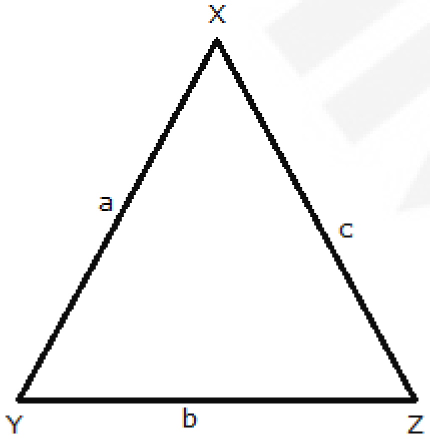
Relation between Centroid, Orthocenter and Circumcenter
In any triangle centroid, orthocenter and circumcenter are always collinear and the ratio between orthocenter to centroid and centroid to Circumcenter is 2: 1

Sine Rule:



$$\frac{YZ}{Sin X} = \frac{XZ}{Sin Y} = \frac{XY}{Sin Z}$$

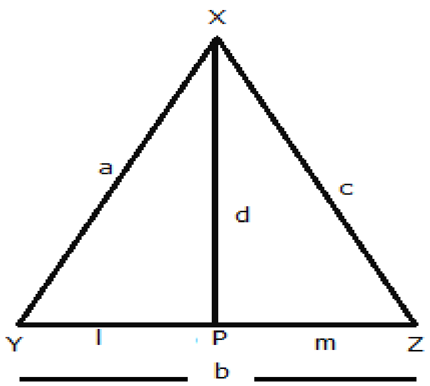
Cosine rule:



Cos X = $(a^2 + c^2 - b^2) / 2ac$
Cos Y = $(a^2 + b^2 - c^2) / 2ab$
Cos Z = $(b^2 + c^2 - a^2) / 2bc$

Stewarts Theorem

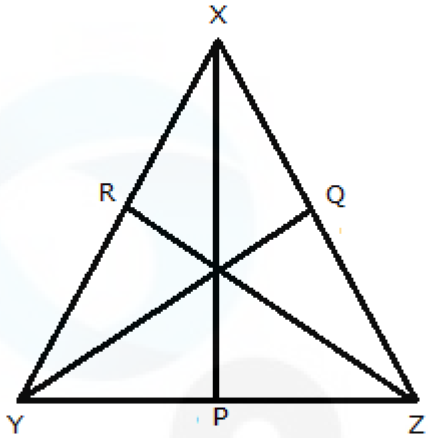
This theorem helps in establishing relationship between the side of a triangle and the cevian of the triangle. Cevian is line joining the vertex to the opposite side.



$a^2m + c^2l = b(d^2 + mn)$

Ceva's Theorem:

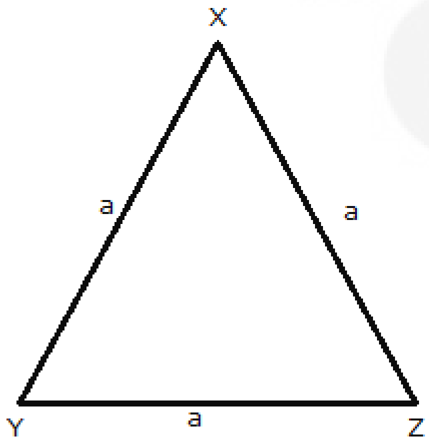
If we join three cevians of the triangle then we can derive the following conclusion



$\frac{YP}{PZ} \times \frac{ZQ}{QX} \times \frac{XR}{RY} = 1$

Equilateral Triangle:

- In this triangle all three sides are equal and all three angles are equal to 60°.
- All the centers (Centroid, in-center, circumcenter and orthocenter) in this triangle lie at a same point.
- The angle bisector, median, perpendicular bisector and altitude are the same line.



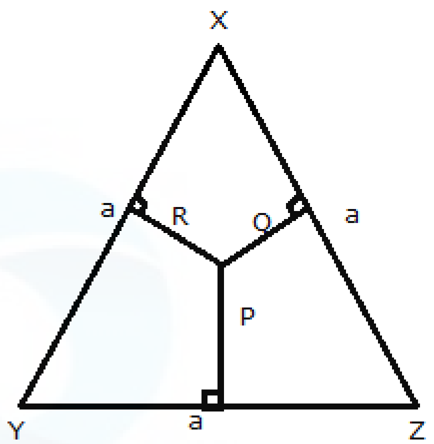
Perimeter = 3a

Area = $\frac{\sqrt{3}}{4} \times a^2$

Height = $\frac{\sqrt{3}}{2} \times a$

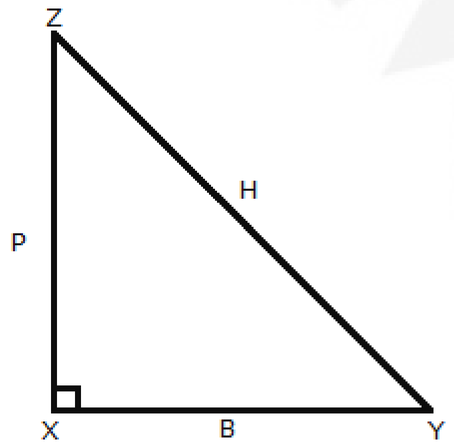
Circum radius = $(\frac{a}{\sqrt{3}})$ In-radius = $(\frac{a}{2\sqrt{3}})$

If from any given point we draw perpendicular on the sides of equilateral triangle then the side of the triangle is



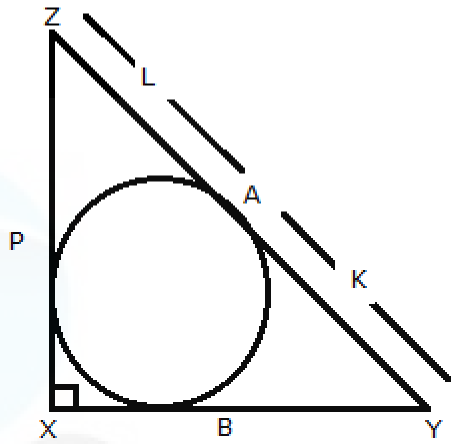
Side = $2/\sqrt{3}$ (P + Q + R)
And P +Q +R = height of the equilateral triangle

Right angle triangle:
In this triangle one angle is 90° and the sum of rest 2 angles is 90°.
Let's take a standard right angle triangle where base XY is 'B', Perpendicular XZ is ' P' and hypotenuse is 'H'. In-radius is 'r', circumradius is 'R', semi-perimeter is 'S' and the shortest median is 'm'.



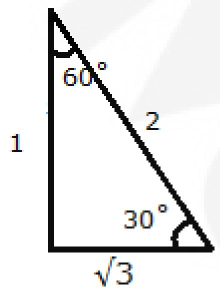
Area of right angle triangle:-

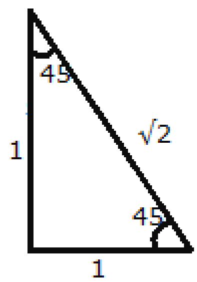
- $\frac{1}{2}$ * P * B
- S(S – H)
- S(S – 2R)
- S(S-2M)
- If in a right angle triangle we draw an in-circle which divides the hypotenuse say in K and L then the area of triangle = K * L

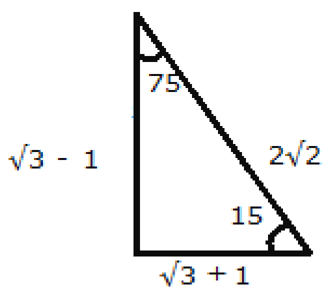


A right-angle triangle always follow Pythagoras theorem, some standard results:
(3, 4, 5) (5, 12, 13) (7, 24, 25) (8, 15, 17) (9, 40, 41) (11, 60, 61) (12, 35, 37) (13, 84, 85) (16, 63, 65) (20, 21, 29)

$((X^2 - Y^2), 2XY, (X^2 + Y^2)), ((M^2 - 1), (2M), (M^2 + 1))$

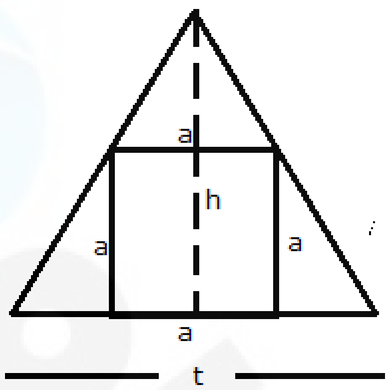




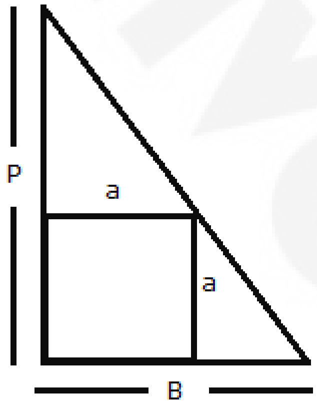


Square in triangle:

If we insert the maximum size square in a triangle whose side is a ,then
The side of the square $= \frac{h*t}{h+t}$



If we insert the maximum size square in a right-angle triangle then there are two possible ways of inserting it
1. Here the side of the square a = (P*B)/(P+B)



2. Here the side of the square a = (H * p)/ (H + p)