Page No.
Date

Pranet Singh kalra
102103747 3CO26

Assignment-6

Q1 $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$x_1, x_2, x_3 --- x_n \Rightarrow$ sample of size n

$L(x_1, x_2 --- x_n) = f(x_1) \cdot f(x_2) --- f(x_n)$

$\Rightarrow \left(\frac{1}{2\pi\sigma^2} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right)\left(\frac{1}{2\pi\sigma^2} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) ----$

taking ln on both sides

$\ln(L) = -\frac{n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{2\sigma^2}$ — (1)

Take partial derivative wrt $\mu$

$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\frac{(2(x_i-\mu)}{2\sigma^2} = 0$

$n\bar{X} - n\mu = 0$

$\bar{X} = \mu$

$Q_1 = \bar{x_1}$ is therefore sample mean

Taking derivative wrt $\sigma^2$ of (1)

$$\frac{\partial \ln(L)}{\partial \sigma^2} = -\frac{n}{2\sigma^2} + \sum_{i=1}^{n} -\frac{(x_i-\mu)^2}{2(\sigma^2)^2} =$$

$$n = \sum_{i=1}^{n} \frac{(x_i-\mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n}\left( \sum_{1}^{n} \frac{(x_i-\mu)^2}{\sigma^2} \right.$$

$$\text{hence } \theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2$$

Page No.
Date

(2) Binomial: ${}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

Take log on both sides

$$\log L = \sum_{i=1}^{n} \left( \log({}^{n}C_{x_i}) + \log(\theta^{x_i}) + \log(1-\theta)^{n-x_i} \right)$$

differentiate wrt $\theta$.

$$\frac{\partial \log(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \Sigma x_i = \frac{1}{1-\theta} \Sigma (n - x_i)$$

$$\frac{1}{\theta} \Sigma x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \Sigma x_i = 0$$

$$\boxed{\theta = \frac{\Sigma x_i}{n^2}}$$