05. Ratio, Proportion, Variation and Alligation

1) The ratio of Tom's and Harry's age is $3:5$ and the sum of their ages is 80 years. What is the ratio of their ages after 10 years?

Answer:
- Let the common multiple be x
- Tom's age $= 3x$; Age after 10 years $3x + 10$
- Harry's age $= 5x$; Age after 10 years $5x + 10$
- $3x + 5x = 80; x = 10$
- Solution $= (3x + 10):(5x + 10) = 40:60 = 2:3$

2) In a mixture of 35 litres, the ratio of milk and water is $4:1$. If 1 litre of water is added to the mixture, what will be the new ratio of milk and water?

Answer:
- $4x + 1x = 35$
- $x = 7$
- Milk $= 4x = 28$
- Water $= 1x = 7$
- New water content $= 7 + 1 = 8$
- New ratio $= 28:8 = 7:2$

3) Two brothers, P and Q have their annual income in the ratio of $8:5$ and their annual expenditure in the ratio of $5:3$. If they save $1200 and $1000 per annum respectively, find their total annual income.

Answer:
- Let x be the common multiple of income and y be the common multiple of expenditure
- P –   Income $= 8x$;   Expenditure $= 5y$;   Saving $= 8x - 5y = 1200$
- Q –   Income $= 5x$;   Expenditure $= 3y$;   Saving $= 5x - 3y = 1000$
- $x = 1400$; $y = 2000$
- Total annual income $= 8x + 5x = 13x = 18,200$

4) The sum of two numbers is 40 and their difference is 4. Find the ratio of two numbers.

Answer:
- $x + y = 40$
- $x - y = 4$
- $x = 22$ and $y = 18$
- $x:y = 11:9$

5) A mixture of milk and honey has a ratio 4:5 in the first vessel and a ratio of 5:1 in the second vessel. In what ratio should the mixture be extracted from each vessel and poured into the third vessel, so that the ratio of milk and honey becomes 5:4 in the third vessel.
Answer:
- Mixture A – $M:H = 4:5 = \frac{400}{9}:\frac{500}{9} = 8:10 \rightarrow (8 + 10 = 18)$
- Mixture B – $M:H = 5:1 = \frac{500}{6}:\frac{100}{6} = 15:3 \rightarrow (15 + 3 = 18)$
- Mixture C – $M:H = 5:4 = \frac{500}{9}:\frac{400}{9} = 10:8 \rightarrow (10 + 8 = 18)$
- Use Alligation
- For Milk - $Mixture\ A : Mixture\ B = \left(\frac{500}{6} - \frac{500}{9}\right):\left(\frac{500}{9} - \frac{400}{9}\right) = 5:2$
- For Milk - $Mixture\ A : Mixture\ B = (15 - 10):(10 - 8) = 5:2$

6) The amounts of time that three workers worked on a special job are in the ratio of $1:2:5$. If they worked a combined total of 104 hours, how many hours did the worker who worked the longest spend on the project?
Answer:
- $1x + 2x + 5x = 104$
- $x = 13$
- $5x = 65$

7) In a certain game, 3 nurbs are equal to 2 zimps and 6 clabs are equal to 1 zimp. 4 clabs are equal to how many nurbs?
Answer:
- $3n = 2z$
- $6c = 1z$
- $3n = 2(6c)$
- $3n = 12c$
- $4c = 1n$

8) In a certain company the ratio of officers to workers is $5:72$. If 8 additional workers were to be hired, the ratio of the officers to workers would be $5:74$. How many officers does the company have?
Answer:
- Let number of officers be O and worker be W
- $\frac{O}{W} = \frac{5}{72}$ ...(1)
- If 8 workers are added the ratio will be $5:74$
- New ratio: $\frac{O}{W+8} = \frac{5}{74}$ ...(2)
- 2 equation 2 variables
- $O = 20$

9) If the ratio of boys and girls in the class is $3: 5$ and the class contain 32 students, how many additional boys would have to enroll to make the ratio of boys to girls $1: 1$?

Answer:

- Let the common multiple be x
- $3x + 5x = 32$
- $x = 4$
- Boys $= 3 \times 4 = 12$
- Girls $= 5 \times 4 = 20$
- Solution: $20 - 12 = 8$

10) Three men A, B and C agreed to share the expenses of a trip in the ratio $2: 3: 4$. A paid \$250 to porters, B paid \$700 for the stuff and C paid \$490 for travelling expenses. How much should A and C together pay to B to settle their agreed share?

Answer:

- Total expense $= 250 + 700 + 490 = 1440$
- Let the common multiple be x
- $2x + 3x + 4x = 1440$
- $x = 160$
- B should have paid $3x = 480$
- Solution: $700 - 480 = 220$

11) A grey hound follows a deer and takes <u>6 leaps for every 7 leaps of the deer</u> but <u>5 leaps of the grey hound are equal to 6 leaps of the deer</u>. Compare the ratio of the hound and the deer?

Answer:

- Speed of grey hound $= 6\frac{GHL}{T}$ {GHL = Grey Hound Leap, T = Time}
- Speed of deer $= 7\frac{DL}{T}$ {DL = Deer Leap, T = Time}
- $5\,GHL = 6\,DL$
- Speed of grey hound $= 6\frac{GHL}{T} = \left(6 \times \frac{6}{5}\right)\frac{DL}{T} = \frac{36}{5}\frac{DL}{T}$
- Ratio of speeds $= \frac{36}{5}: 7 = 36: 35$

12) A man wishes to divide his monthly savings of \$846 among his two sons and a daughter in the ratio $\frac{1}{4}: \frac{1}{5}: \frac{1}{3}$ respectively. How much should he give to his daughter?

Answer:

- Let the common multiple be x
- $\frac{1}{4}x + \frac{1}{5}x + \frac{1}{3}x = 846$
- $x = 1080$
- Daughter $= \frac{1}{3}x = \frac{1080}{3} = 360$

13) Three numbers bear a ratio of 2: 3: 4 to one another. If the sum of the reciprocals of the first and the third number exceeds the reciprocal of the second number by $\frac{5}{12}$, find the second number.

Answer:
- Let the common multiple be x
- So, the numbers are $2x$, $3x$ and $4x$
- $\frac{1}{2x} + \frac{1}{4x} = \frac{1}{3x} + \frac{5}{12}$
- $x = 1$
- Second number $= 3x = 3$

14) $1087 is divided among A, B and C so that if $10, $12 and $15 are diminished form the share of A, B and C respectively, the remainder will be in ratio of 5: 7: 9. What is the share of B?

Answer:
- Let the common multiple be x
- After Deduction   Before Deduction
- $A = 5x$ $A = 5x + 10$
- $B = 7x$ $B = 7x + 12$
- $C = 9x$ $C = 9x + 15$
- $A + B + C = 1087$
- $(5x + 10) + (7x + 12) + (9x + 15) = 1087$
- $x = 50$
- $B = 7x + 12 = 362$

15) What quantity of sugar costing Rs 6. 10 per kg must be mixed with 126 kg of sugar priced at Rs 2. 85 per kg so that 20% may be gained by selling the mixture at Rs 4. 80 per kg?

Answer:
- Let quantity of sugar costing Rs 6. 10 per kg be x
- Total cost price $= (6.10x) + (2.85{\times}126)$
- Total selling price $= 4.80{\times}(126 + x)$
- Percentage profit $= \frac{SP-CP}{CP}{\times}100 = 20$
- Rearranging we get: $1.2{\times}CP = SP$
- $1.2[(6.10x) + (2.85{\times}126)] = 4.80{\times}(126 + x)$
- $x = 69$

16) A mixture of 70 liters of alcohol and water contains 10% of water. How much water must be added to make the water 37% in the resulting mixture?

Answer:
- Let the amount of water required be x
- Original: Water $- \frac{10}{100} \times 70 = 7\ liters$ Alcohol $- \frac{90}{100} \times 70 = 63\ liters$
- New: Water $- (7 + x)\ liters$ Alcohol $- 63\ liters$
- New percentage water $= \frac{7+x}{63+7+x} \times 100 = 37\%$
- $x = 30$

17) The force of attraction between two bodies of mass m_1 and m_2 respectively varies directly as the product of their masses and inversely as the square of the distance between them. If the masses of the bodies and the distance between them are doubled, the force of attraction will become.

Answer:
- $Original\ Force\ \alpha \frac{m_1 m_2}{r^2}$
- Masses and distance both double
- $New\ Force\ \alpha \frac{(2m_1)(2m_2)}{(2r)^2}$
- $New\ Force\ \alpha \frac{4(m_1)(m_2)}{4(r)^2} = \frac{m_1 m_2}{r^2}$
- So, the Force remains **unchanged** {Original Force = New Force}

18) What is the least whole number which when subtracted from the terms which are in the ratio of 6: 7 gives a ratio less than 16: 21?

Answer:
- Let the number subtracted be x
- $\frac{6-x}{7-x} < \frac{16}{21}$
- $x > 2.8$
- $x = 3$

19) A certain test consists of 8 section with 25 questions numbered from 1 to 25, in each section. If a student answered all of the even-numbered questions correctly and $\frac{3}{4}$ of the odd-numbered questions correctly, what was the total number of questions be answered correctly?

Answer:
- Questions numbered from 1 to 25 in each section and total 8 sections
- Even numbered questions in each section 12
- Total even numbered questions $= 12 \times 8 = 96$
- Odd numbered questions in each section 13
- Total odd numbered questions $= 13 \times 8 = 104$
- Correctly answered $= Even + \frac{3}{4} Odd = 96 + \frac{3}{4} \times 104 = 174$

20) Lou and Selma were hired to paint a room for a total of $72. They completed the job with Lou working 3 $hrs.$ and 20 $mins.$ and Selma working 2 $hrs.$ and 40 $mins.$ If they decided to split the $72 in proportion to the amount of time each spent on the job, how much did Lou receive?
Answer:
- Lou worked for 3 hrs 20 $mins = 200$ $mins$
- Selma worked for 2 hrs 40 $mins = 160$ $mins$
- Ratio of their time is $Lou: Selma = 200: 160 = 5: 4$
- $5x + 4x = 72$
- $x = 8$
- $Lou = 5x = 5(8) = 40$

21) Of the 40 schools invited to participate in a research study, $\frac{7}{8}$ agreed to participate. If 60 questionnaires were sent to each of the participating schools and $\frac{4}{5}$ of these questionnaires were completed and returned, what was the total number of questionnaires completed and returned?
Answer:
- Number of schools participating $= \frac{7}{8} \times 40 = 35$
- Total number of questionnaires sent $= 35 \times 60 = 2100$
- Number of questionnaires completed and returned $= 2100 \times \frac{4}{5} = 1680$

22) Which three of the following fractions are equivalent?
v=5/80, w=0.05/0.08, x=0.5/8.0, y=0.05/0.8 and z=0.05/0.008
Answer:
- $v = \frac{5}{80}$
- $w = \frac{0.05}{0.08} = \frac{50}{80}$
- $x = \frac{0.5}{8.0} = \frac{5}{80}$
- $y = \frac{0.05}{0.8} = \frac{5}{80}$
- $z = \frac{0.05}{0.008} = \frac{500}{80}$
- Solution: **v, x and y** are equal

23) Of the people who responded to a market survey, 120 preferred Brand X and the rest preferred Brand Y. If the respondents indicated a preference for Brand X over Brand Y by a ratio 3: 1, how many people responded to the survey?
Answer:
- Let the common multiple be z
- Number of people who preferred Brand X: $3z = 120$
- $z = 40$
- Number of people who preferred Brand Y: $1z = 40$
- Total $= 120 + 40 = 160$

24) The current ratio of men to women on a certain board of trustees is 2 to 5. If 4 men were added to the board, the ratio of men to women would be 2 to 3. How many men are currently on the board?

Answer:
- Let M be number of men and W be number of women
- $\frac{M}{W} = \frac{2}{5}$ $\qquad\qquad \therefore 5M = 2W$
- If 4 men were to be added
- $\frac{(M+4)}{W} = \frac{2}{3}$ $\qquad \therefore 3M + 12 = 2W$
- 2 equation 2 variable
- $M = 6$

25) Tom and Carlos begin to play a series of 4 games with 400 chips each. At the end of each game, there is a loser who must surrender half of his chips to the winner. If Tom wins only the first and third games, how many chips does he have after the pay off at the end of the fourth games?

Answer:

	Tom	Carlos
Initial chips	400	400
After 1st game	600	200
After 2nd game	300	500
After 3rd game	550	250
After 4th game	**275**	525