

02. Algebra

- 1) What is the value of $x^2 + 12x + 36$ when $x = 994$?

Answer:

- $x^2 + 12x + 36 = (x + 6)(x + 6) = (x + 6)^2$
- now substitute $x = 994$
- $(994 + 6)^2 = 1000^2 = \mathbf{1000000}$

- 2) If $x^2 - y^2 = 28$ and $x - y = 8$, what is the average of x and y ?

Answer:

- $x^2 - y^2 = (x + y)(x - y) = 28$
- $(x + y) = \frac{28}{8}$
- $\frac{x+y}{2} = \frac{28}{16} = \mathbf{1.75}$

- 3) Vidya and Vandana solved a quadratic equation. In solving it, Vidya made a mistake in the constant term and got the roots as 6 and 2, while Vandana made a mistake in the co-efficient of x only and obtained the roots as -7 and -1 . The correct roots of the equation are

Answer:

- Let the quadratic equation be $x^2 + bx + c$
- According to Vidya, $(x - 6)(x - 2) = x^2 - 8x + 12$, so $b = -8$
- According to Vandana, $(x + 7)(x + 1) = x^2 + 8x + 7$, so $c = 7$
- So, the equation is
- $x^2 - 8x + 7 = (x - 1)(x - 7)$
- Roots are $\{1, 7\}$

- 4) If $4y - 3x = 5$, what is the smallest integer value of x for which $y > 100$?

Answer:

- $y = \frac{3x+5}{4} \rightarrow y > 100 \rightarrow \frac{3x+5}{4} > 100$
- So, we are looking for the smallest integer value of x which satisfies the above inequality
- $3x + 5 > 400 \rightarrow 3x > 395 \rightarrow x > 131.66$
- $x = \mathbf{132}$

- 5) A jar contains only red, white and blue marbles. The number of red marbles is $\frac{4^{th}}{5}$ the number of white ones, and the number of white ones is $\frac{3^{th}}{4}$ the number of blue ones. If there are 470 marbles in all, how many of them are blue?

Answer:

- $r = \frac{4w}{5}$
- $w = \frac{3b}{4}$
- $r + w + b = 470 \rightarrow \frac{4w}{5} + w + b = 470 \rightarrow \frac{9w}{5} + b = 470 \rightarrow \frac{27b}{20} + b = 470$
- $\frac{47b}{20} = 470$
- $b = \mathbf{200}$

- 6) What is the greater of the two numbers whose product is 900 and whose sum exceeds their difference by 30?

Answer:

- $xy = 900$
- $x + y = x - y + 30 \rightarrow 2y = 30 \rightarrow y = 15 \rightarrow x = \frac{900}{15}$
- $x = \mathbf{60}$

- 7) A shop owner charges \$150 per T.V. and \$90 per refrigerator. Last week she sold 5 more T.V. than refrigerators. If her total sale for these two items were \$2910, what was the number of T.V.s and refrigerators that she sold?

Answer:

- Number of T.V.s sold be x
- Number of refrigerators sold $x - 5$
- $150(x) + 90(x - 5) = 2910$
- $x = 14 \rightarrow x - 5 = 9$
- $Total = \mathbf{23}$

- 8) In a bag containing black and white balls, half the number of white ones equals one-third the number of black balls and twice the whole number of balls, exceeds 3 times the number of black balls by four. How many balls does the bag contain?

Answer:

- $\frac{w}{2} = \frac{b}{3}$
- $2(w + b) - 4 = 3b$
- $w = 8, b = 12$
- $w + b = 8 + 12 = \mathbf{20}$

- 9) Rs. 38 is divided among A , B and C such that B has Rs. 5 more than A and C has Rs. 10 more than B . How much did C get?

Answer:

- $A, B = A + 5, C = B + 10 = A + 15$
- $A + B + C = 38 \rightarrow A + (A + 5) + (A + 15) = 38$
- $A = 6$
- $C = \mathbf{21}$

- 10) Find the number such that if 5, 15 and 35 are added to it, the product of the first and the third result is equal to the square of the second?

Answer:

- Let number be x
- $(x + 5)(x + 35) = (x + 15)^2$
- $x = 5$

- 11) A man is 5 times as old as his son. The sum of the squares of their ages is 2106. Find the age of the man.

Answer:

- $m = 5s$
- $m^2 + s^2 = 2106$
- $\frac{26m^2}{25} = 2106$
- $m = 45$

- 12) The sum of the reciprocals of two positive consecutive numbers is $\frac{15}{56}$. Find the smaller number.

Answer:

- $\frac{1}{a} + \frac{1}{a+1} = \frac{15}{56}$
- $a = 7$

- 13) A fraction becomes $\frac{1}{2}$ when 1 is subtracted from the numerator and 2 is added to its denominator, and it becomes $\frac{1}{3}$ when 7 is subtracted from the numerator and 2 from the denominator. What is the fraction?

Answer:

- Substitute options
- Solution: $\frac{15}{26}$

- 14) In a class when 4 students sit on each bench, 3 benches are left vacant. If 3 students sit on each bench, 3 students are left standing. Find the number of students in the class.

Answer:

- $4(b - 3) = s$
- $3b + 3 = s$
- $s = 48$

- 15) A number consists of two digits whose product is 30. If the numbers are interchanged the new number is more than the original number by 9, the number is

Answer:

- Let the number be $10a + b$

- $a \times b = 30$
- $10b + a = 10a + b + 9$
- 2 equations 2 variables
- $a = 5, b = 6$, so, the number is **56**

16) If the roots of the equation $9x^2 + 3ax + 4 = 0$ are equal, then the value of a can be

Answer:

- $(Ax + B)^2 = A^2x^2 + 2ABx + B^2$
- $A^2 = 9, A = \pm 3$
- $B^2 = 4, B = \pm 2$
- $2AB = \pm 12 = 3a$
- $a = \pm 4$

17) Dick travelled four seventh of a journey by train, $\frac{5^{th}}{6}$ of the balance by bus and he walked the remaining distance of 3 kms. What is the total distance travelled by Dick?

Answer:

- Total Distance be x
- Train = $\frac{4x}{7}$
- Bus = $\frac{5}{6}\left(x - \frac{4x}{7}\right) = \left(\frac{5}{6}\right)\left(\frac{3x}{7}\right) = \frac{5x}{14}$
- Walk = $x - \frac{4x}{7} - \frac{5x}{14} = \frac{x}{14} = 3 \text{ km}$
- $x = (3)(14) = \mathbf{42}$

18) An instructor scored a student's test of 50 questions by subtracting 2 times the number of incorrect answers from the correct answers. If the students answered all of the questions and received a score of 38, how many questions did the student answered correctly?

Answer:

- Let number of correct answers be c
- Wrong answers $50 - c$
- Marks scored: $c - (2)(50 - c) = 38$
- $c = \mathbf{46}$

19) If a total number of identical disks can be arranged in 8 stacks of equal height or in 12 stacks of equal height, the least possible value of x is

Answer:

- We are looking for the least number which is completely divisible by 8 and 12
- $LCM(8, 12) = \mathbf{24}$

20) On a legislative committee, the number of males is 3 fewer than twice the number of females. If a female replaced one male, there would be an equal number of males and females on the committee. How many members are on the committee?

Answer:

- Males: m
- Females: f
- $m + 3 = 2f$
- $m - 1 = f + 1$
- 2 equations 2 variables
- $m + f = 7 + 5 = \mathbf{12}$

21) In a certain brick wall, each row of brick above the bottom contains one less brick than the row just below it. If there are 5 rows in all and a total of 75 bricks in the wall, how many bricks does the bottommost row contain?

Answer:

- Let bottom most row (1st row) contain x bricks
- 2nd row: $x - 1$
- 3rd row: $x - 2$
- 4th row: $x - 3$
- 5th row: $x - 4$
- Total bricks: $x + (x - 1) + (x - 2) + (x - 3) + (x - 4) = 5x - 10 = 75$
- $x = \mathbf{17}$

22) A kennel sold 3 puppies of breed X and 2 puppies of breed Y for a total of \$690. If each breed Y puppy was sold for 20% less than each breed X puppy, how much did each breed X puppy sell for?

Answer:

- $3x + 2y = 690$
- $y = \frac{80}{100}x$
- 2 equations 2 variables
- $x = \mathbf{150}$

23) Rs. 429 is made up of *one* rupee, 50 paisa and 25 paisa coins. The number of these coins is in the proportion of 5: 6: 7. Total number of coins is

Answer:

- Let multiplicative constant be x
- Number of 1-rupee coins: $5x$
- Total amount from 1-rupee coins: $(5x)(1) = 5x$
- Number of 50 paisa coins: $6x$
- Total amount from 50 paisa coins: $(6x)(0.5) = 3x$
- Number of 25 paisa coins: $7x$
- Total amount from 25 paisa coins: $(7x)(0.25) = \frac{7x}{4}$
- Total amount: $5x + 3x + \frac{7x}{4} = 429$
- $x = 44$
- Total number of coins: $5x + 6x + 7x = \mathbf{792}$

- 24) Kim bought a total of \$2.65 worth of postage stamps in four denominations. If she bought an equal number of 5-cent and 25-cent stamps and twice as many 10-cent stamps as 5-cent stamps, what is the least number of 1-cent stamps she could have bought?

Answer:

- Let number of 5 cent stamps be x
- Let number of 1 cent stamps be y
- Number of 25 cent stamps = x
- Number of 10 cent stamps = $2x$
- Total amount from 5 cent stamps: $5x$
- Total amount from 10 cent stamps: $20x$
- Total amount from 25 cent stamps: $25x$
- Total amount from 1 cent stamps: y
- Total amount: $5x + 20x + 25x + y = 265$
- $y = 265 - 50x$
- Since x and y can only take non negative integer values, min value of y is **15** (for $x = 5$)

- 25) A certain truck, travelling at 55 mph gets 4.5 miles per gallon of diesel fuel consumed. Travelling at 60 mph, the truck gets 3.5 miles per gallon. On a 500-mile trip, if the truck used a total of 120 gallons of diesel fuel and travelled part of the trip at 55 miles per hour and the rest at 60 mph, how many miles did the truck travel at 55 mph?

Answer:

- Let truck travel x miles at 55 mph
- Distance traveled at 60 mph: $500 - x$
- Fuel consumed at 55 mph: $\frac{x}{4.5}$
- Similarly, Fuel consumed at 60 mph: $\frac{500-x}{3.5}$
- Net Fuel consumed: $\frac{x}{4.5} + \frac{500-x}{3.5} = 120$
- $x = \mathbf{360}$