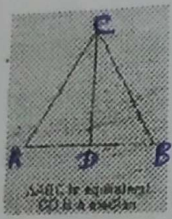
$$\text{length of minor arc } BE = \frac{90}{360} \times 2\pi(3)$$
$$= \frac{1}{4} \times 6\pi$$
$$= \frac{6\pi}{4}$$
$$= \frac{3\pi}{2}$$

EXERCISE

AABC is an equilateral. Δ. CD is the median.

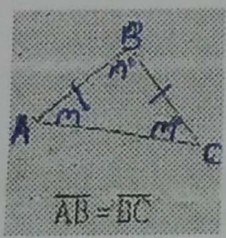


AABC is an equilateral Δ, $CD = \frac{\sqrt{3}}{2} \times AC$

Soln: ∵ CD is the median of an equilateral Δ, $\therefore 3 = \frac{\sqrt{3}}{2} \times AC \therefore AC = \frac{6}{\sqrt{3}} \therefore A(\triangle ABC) = \frac{\sqrt{3}}{4} \times AC^2 = \frac{\sqrt{3}}{4} \times \left(\frac{6}{\sqrt{3}}\right)^2$

1. If CD = 3, then the area of triangle ACB is

a) 6 b) 3 c) 3/2 d) $3\sqrt{3}$ e) $(3\sqrt{3}) / 2$



Soln: $2m + n = 180°$

Possibilities for m and n → $m > n \rightarrow 2(70) + 40 = 180°$

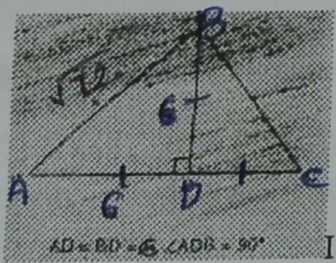
$n > m \rightarrow 2(40) + 100° = 180°$

$m = n \rightarrow 2(60) + 60 = 180$

2. Which of the following CAN be true? SELECT ALL THAT APPLY

i) n>m ii) m>n iii) m=n

$\frac{x + 36}{\sqrt{72}}$



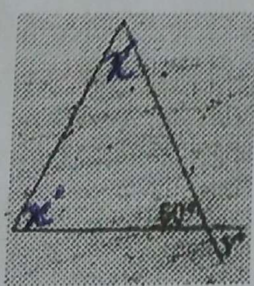
AD=BD=6

3. If triangle DBC is an isosceles right-angle triangle, what is the area of triangle ABC?

Soln ∴ BD = DC = 6 ∴ $A(\triangle ABC) = \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 12 \times 6$

a) 36 b) $30\sqrt{2}$ c) 24 d) 18 e) It can't be determined



Soln: $2x + 60 = 180°$ $\therefore x = 60°$

∴ The given triangle is an equilateral Δ.

∴ Perimeter = 3s = 3(6)

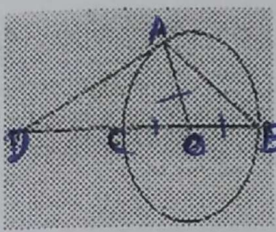
4. If the length of one of the sides of the triangle is 6, what is the perimeter of the given triangle?

a) 36 ~~b) 18~~ c) 12 d) $9\sqrt{3}$ e) It can't be determined

5. Circle O has radius 1 unit. What is the difference between its circumference and area?

a) 2 b) 2π c) π d) $\pi/2$ e) $\pi/4$

Sol^n: $2\pi r - \pi r^2 = 2\pi - \pi$ ($\because r = 1$)

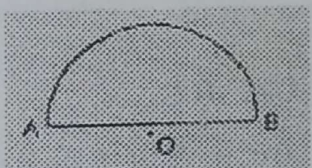


Sol^n: $m\angle AOC = 60°$ $\therefore l(\widehat{AC}) = \frac{60}{360} \times 2 \times \pi \times 3 = \pi$

Perimeter of sector AOC $= l(\widehat{AC}) + AO + OC = \pi + 3 + 3$

6. Measure of angle AOB = 120°. Length of OC = 3 cm. What is the perimeter of sector AOC?

a) $6+2\pi$ ~~b) $6+\pi$~~ c) $3+\pi$ d) $2+\pi$ e) π

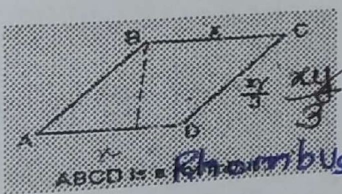


Sol^n: Note: O is NOT the center.

$\therefore$ AB is NOT the diameter $\therefore$ although $l(\widehat{AB})$ is known, $m\angle$ is not known.

7. If the length of arc AB $= 6\pi$, what is the area of the given circle?

a) It can't be determined b) 36π c) 18π d) 9π e) 6π

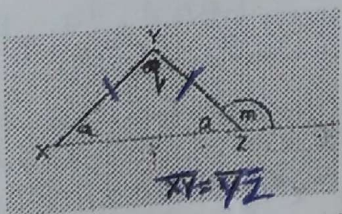


Sol^n: $\because$ ABCD is a rhombus $\therefore BC = DC \Rightarrow x = \frac{xy}{3}$

$\therefore y = 3$ But x can be any value $\therefore$ the relation bet^n x and y can't be known.

8. What is the relation between x and y?

a) $x=y=3$ b) $x<y$ c) $y<x$ d) $x=2y$ ~~e) It can't be determined~~



Sol^n: Let $m\angle YXZ$ and $m\angle YZX$ be $a°$ $\therefore q + 2a = 180°$

$\therefore a = \frac{180-q}{2}$ and $q + a = m$ (Exterior angle theorem)

$\therefore a = m - q$

9. q can be expressed as

a) $2m-180$ b) $m-180$ c) $m+60$ d) $m/2$ e) $m/3$

$\therefore \frac{180-q}{2} = m - q$

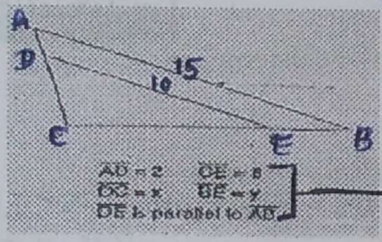
$\therefore 180 - q = 2m - 2q$

$\therefore q = 2m - 180$

Sol^n: $\Delta ACB \sim \Delta DCE$ ($\because \angle C$ is the common angle and $\angle D \cong \angle A$; $\angle E \cong \angle B$ Corresponding angles)

$\therefore \frac{AC}{DC} = \frac{AB}{DE} = \frac{CB}{CE}$ (Corresponding sides)

$\therefore \frac{x+2}{x} = \frac{10}{15} = \frac{8+y}{8}$



$\rightarrow \overline{AD} = 2;\ \overline{CE} = 8;\ \overline{DC} = x;\ \overline{BE} = y \quad \overline{DE} \parallel \overline{AB}$

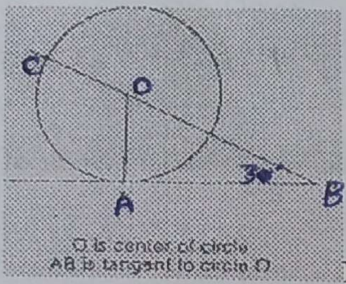
10. What is the value of x and y?

a) 1 b) 2 c) 3 d) 4 e) 5

Sol^n: $\angle OAB = 90°$ $\because$ AB is a tangent to the given circle

$\therefore \Delta OAB$ is a $30°-60°-90°\ \Delta$. $\therefore$ Ratio is: $1:\sqrt{3}:2$ of sides $\rightarrow OA : AB : OB$ $\therefore AB = 5\sqrt{3}$

$\therefore A(\text{sector}AOC) = \frac{120}{360} \times \pi \times 5^2$ and $A(\Delta AOB) = \frac{1}{2} \times 5\sqrt{3} \times 5$



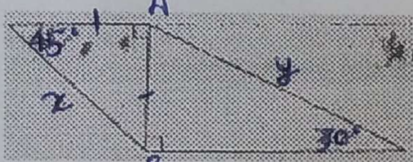
11. If OA = 5, What is the difference between area of the sector AOC and area of triangle AOB?

a) 25 b) 5 c) $(5 - 25\sqrt{3})$ d) $(25\pi)/3 - (25\sqrt{3})/2$ e) None of these

Sol^n: Let the common side be AB

$\therefore AB = \frac{x}{\sqrt{2}}$ ($45°-45°-90°\ \Delta$) and $AB = \frac{y}{2}$ ($30°-60°-90°\ \Delta$)

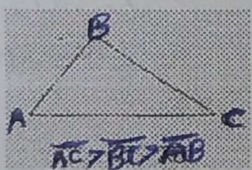
$\therefore AB = \frac{x}{\sqrt{2}} = \frac{y}{2}$ $\therefore \frac{x}{y} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$



12. The ratio between x and y (x:y) is

a) 2:1 b) $1:\sqrt{2}$ c) $1:\sqrt{3}$ d) 1:1 e) It can't be determined

Sol^n: Applying rule no. 4 on pg. 50 i.e. rule of proportion, we get $\angle B > \angle A > \angle C$ $\therefore$ (i) is true and (iv) is out. Next, assuming $\angle C = 60°$ we get $m\angle A + m\angle B = 120°$ $\therefore$ either A & B are 60° or A is more than 60° and B is less than 60° or vice versa. All the 3 cases can't be true since $\angle C = 60°$ $\therefore \angle C$ can't be 60°

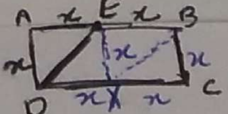


13. Which of the following MUST be true?

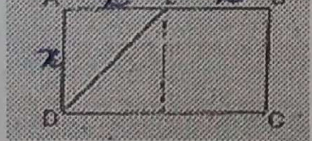
i) C is the smallest angle ii) Measure of angle C < 60°

iii) Measure of angle C = 60° iv) Measure angle A = Measure angle B

Sol^n:



Construct EX and join XB.



14. What is $\frac{\text{Area of DEBC}}{\text{Area of rectangle ABCD}}$?

a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{3}{4}$ d) $\frac{3}{8}$ e) $\frac{2}{7}$

A(DEBC) = Area of 3 congruent Δs.

A(ABCD) = Area of 4 congruent Δs

Note: the triangles $\rightarrow$ AED, DEX, EBX and BXC are congruent since the sides are equal.

$\therefore$ Reqd. ratio $= \frac{3}{4}$

$\boxed{OR}$

$A(DEBC) = \frac{1}{2}(x+2x) \times x$ [Area of trapezoid]

$A(ABCD) = x(2x)$ [Area of rectangle]

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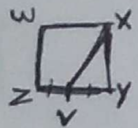
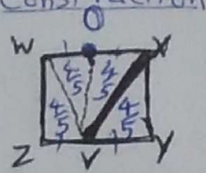
Construction

Solⁿ:  ---->  $A(WXYZ) = 4 \times \frac{4}{5}$

15. In a square WXYZ (not shown), point V is the mid-point of side YZ and the area of triangle XYV is 4/5. What is the area of square WXYZ?

a) 2 b) 8/5 c) 4 ✓d) 16/5 e) 18/5

16. Which of the following could be the lengths of the sides of a triangle? Solⁿ. Refer to rule no. 3 on pg. 50

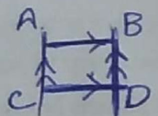
a) (1,2,3) b) (3,6,9) c) (4,8,16) d) (5,10,20) ✓e) (7,8,13)

∴ All the sides satisfy the 3 conditions, only 7, 8, 13 can be a Δ.

$$\begin{cases} 13-8 < 7 < 13+8 \\ 13-7 < 8 < 13+7 \\ 8-7 < 13 < 7+8 \end{cases}$$

17. A, B, C, D and E are all distinct points that lie in the same plane. If seg AB is parallel to seg CD and seg AC is parallel to set BD, which of the following is a set of points all of which could lie on the same line?

a) {A,B,C,E} b) {B,C,D,E} ✓c) {C,D,E} d) {A, C,D} e) {A,B,D}

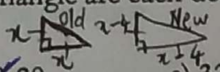
Solⁿ:  Of the given options, only C, D, E can be collinear i.e. lie on the same line.

18. If the length of a rectangle is one-third the perimeter of the rectangle, then the width of the rectangle is what fraction of the perimeter?

Solⁿ: $l = \frac{1}{3}(2l+2b)$ ∴ $3l = 2l + 2b$ ∴ $\underline{l = 2b}$

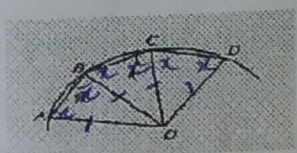
a) 1/3 b) 2/3 ✓c) 1/6 d) 1/12 e) None of these

$$\therefore \frac{b}{2l+2b} = \frac{b}{2(2b+2b)} = \frac{b}{6b} = \frac{1}{6}$$

19. When the base and height of an isosceles right triangle are each decreased by 4, the area decreases by 72. What is the height of the original triangle?  Solⁿ: Let the side of the orig. Δ be x ∴ $A(\text{orig } \Delta) = \frac{1}{2} \times x^2$

a) 4 b) 8 c) 16 ✓d) 20 e) 32

$\therefore A(\text{New } \Delta) = \frac{1}{2}(x-4)^2 = \frac{1}{2}(x^2) - 72$ (According to the given condition).

20.  The given figure represents part of a regular polygon with n sides, inscribed in a circle with center O. In terms of n, what is the measure of ∠OBC?

Solⁿ: $m\angle ABC = \frac{(n-2)180}{n}$ (m of an interior angle of a regular polygon)

a) $\frac{360}{n}$ b) $180 - n$ c) $180n - 360$ d) $180 - \frac{360}{n}$ ✓e) $90 - \frac{180}{n}$

$m\angle ABC = 2m\angle OBC$.

$$\therefore \frac{(n-2)180}{n} = 2m\angle OBC$$

$$\therefore \frac{n-2\,(180)}{2n} = m\angle OBC$$

$$\therefore \frac{180n}{2n} - \frac{360}{2n} = 90 - \frac{180}{n}$$

13. CLOCKS AND CALENDARS AND FUNCTIONS

CLOCKS AND CALENDARS

CLOCK: The face of a clock or a watch is a circle which is divided into 60 minute spaces. The minutes hand passes over 60-minute spaces while the hours hand goes over 5-minute spaces. That is, in 60-minutes, the minutes hand gains 55 minutes on the hour hand.

In every hour:

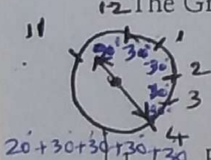
- The minutes hand revolves around the clock and travels the distance equivalent to the circumference of the clock and traces 360 degrees and the hour hand traces 30 degrees and travels the distance equivalent to 1/12th the circumference of the clock every hour.
- The hands coincide once
- They are twice at right angles when the hands are 15 minutes spaces apart.
- They point in the opposite directions once when they are 30 minutes spaces apart. The hands are in the same straight line when they are coincident or opposite to each other.

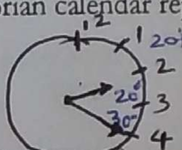
In one minute: The minutes hand makes an angle of 6 degrees and the hour hand makes an angle of ½ degrees.

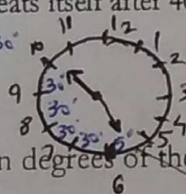
Clock too fast, too slow: If a clock indicates 7:10 when the correct time is 7:00, it is said to be 10 min too fast. If it indicates 6:50, when the correct time is 7:00, it is said to be 10 min slow.

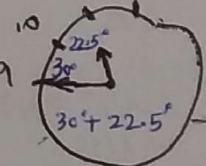
CALENDAR: The following facts should be remembered about a calendar.

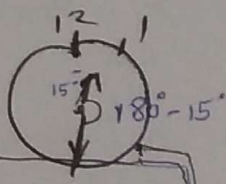
- In an ordinary year, there are 365 days, i.e. 52 weeks + 1 day. Therefore, an ordinary year contains 1 odd day.
- A leap year contains 366 days, i.e. 52 weeks + 2 days. Therefore, a leap year contains two odd days. February has 29 days in a leap year.
- A leap year is divisible by 4. The turn of century is a leap year if it is divisible by 400. For instance, 1900 was not a leap year but 2000 was a leap year.
- The Gregorian calendar repeats itself after 400 years.











EXERCISE

1. Find the measures in degrees of the smaller angle formed by the hour hand and the minute hand of a clock at:

a) 11:20 p.m. → 140°   b) 20 mins past 2 → 50°   c) 10 mins to 6 a.m. → 125°

d) Quarter to 11 → 52.5°   e) 30 mins past mid noon → 165° (180° − 15°)

∴ l(min hand) = 2 l(hour hand) → ∴ l(hour hand) = r   ∴ l(Min hand) = 2r

2. The minute hand is twice as long as the hour hand. What is the ratio of the distance travelled by the minute hand in 3 hours and the distance by the hour hand in 9 hours? Soln: NOTE: in 1 hour → Min hand moves → 2πr; Hour hand moves → $\frac{2\pi r}{12} = \frac{\pi r}{6}$

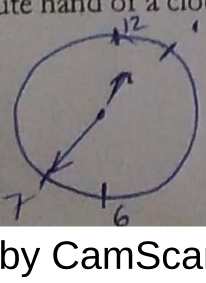
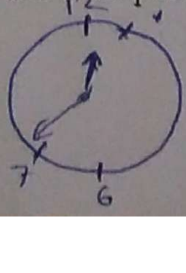
a) 2:1   b) 3:1   c) 8:1   d) 9:1   e) None of these

∴ in 3 hours, min hand moves → $[2\pi(2r)] \times 3 = 12\pi r$ ∴ $12\pi r : \frac{3\pi r}{2}$

∴ in 9 hours, hour hand moves → $\frac{2\pi r}{12} \times 9 = \frac{3\pi r}{2}$

3. What is the difference in the degree measures of the angles formed by the hour hand and the minute hand of a clock at 12:35 and 12:36 p.m.?

Soln:

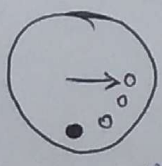



In 1 min → The hour hand moves → 0.5° closer to the min hand and the min hand moves 6° away from hour hand → ∴ Difference → 6° − 0.5°

a) 1 b) 5 ~~c)~~ 5.5 d) 6 e) 30

4. An ultramodern clock in a school has 12 lights in place of numerals from 1 to 12. The clock has no hour hand; instead a flashing light signals the hour. The clock does, however, have a minute hand, which starts in a vertical position pointing up at 00, the start of the hour, and rotates clockwise through 360 degrees in 60 minutes. Here is how the clock shows various times:

Sol^n:



9:30 → 6:15 upon rotation through 90° counter clockwise

At 9:30 in the morning, while the teacher is out of the classroom, some mischievous students rotate the clock through 90 degrees counterclockwise, without touching the hand. In the next instant, before the clock display changes, the teacher reenters the room. She glances at the clock. What time does she see?

~~a)~~ 6:15 b) 3:00 c) 3:45 d) 12:15 e) 12:45

5. How many pairs of days are feasible for the two extra days of the leap year?

a) 2 ~~b)~~ 7 c) 21 d) 42 e) None of these

Sol^n: 365th day and 366th day. 365th day and 366th day.
→ M T
→ T W
→ W Th
→ Th F
F S
→ S SU
→ SU M
} 7 pairs possible for the 365th & 366th day.

FUNCTIONS

A function relates an input to an output. It is like a machine that has an input and an output. And the output is related somehow to the input. Functions have only one output for a given input.

Input ⇒FUNCTION⇒Output For instance: $f(x) = x^2$ We say *"f of x equals x squared"*

What goes **into** the function is put inside parentheses () after the name of the function:
So ***f(x)*** shows us the function is called "***f***", and "***x***" goes **in**
And we usually see what a function does with the input: ***f(x) = x²*** shows us that function "***f***" takes "***x***" and squares it.

Example: with **f(x)** = x²: an input of 4 becomes an output of 16. In fact we can write **f(4) = 16**.

The "x" is Just a Place-Holder! Don't get too concerned about "x", it is just there to show us where the input goes and what happens to it. It could be anything!
So this function: $f(x) = 1 - x + x^2$ is the same function as: $f(q) = 1 - q + q^2$ or $h(A) = 1 - A + A^2$ or $w(\theta) = 1 - \theta + \theta^2$
The variable (x, q, A, etc) is just there so we know where to put the values. Thus, $f(2) = 1 - 2 + 2^2 = 3$

Sometimes There is No Function Name: Sometimes a function has no name, and we see something like: $y = x^2$

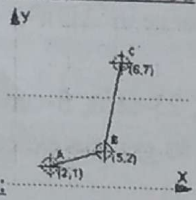
But there is still: an input (x) a relationship (squaring) and an output (y)

A Function is Special: A function has **special rules**:

They are called **ordered** pairs because the input always comes first, and the output second: (input, output).
So it looks like this: (**x, f(x)**). Example: **(4,16)** means that the function takes in "4" and gives out "16".

Set of Ordered Pairs: A function can then be defined as **a set of ordered pairs**: Example: **{(2,4), (3,5), (7,3)}** is a function that says "2 is related to 4", "3 is related to 5" and "7 is related 3".
Also, notice that: the domain is **{2,3,7}** (the input values) and the range is **{4,5,3}** (the output values).

A Benefit of Ordered Pairs:



We can graph them...... because they are also coordinates! So a set of coordinates is also a function (if they follow the rules above, that is)

EXERCISE

→ Sol^n: $g(K) = 2K-3$ and $g(2K-3) = 2(2K-3) - 3$
$\therefore 2K - 3 = 2(2K-3) - 3$
$\therefore K = 3 \quad \therefore g(4K) = g(12) = 2(12) - 3$

1. Let the function g be defined by $g(x) = 2x-3$. If $g(k) = g(2k-3)$, what is the value of $g(4k)$?

a) 3 b) 12 c) 21 ✓ d) It can't be determined e) None of these

2. The table below shows selected values for function *f*. If $f(x) = cr^x$, where c and r are constants. What is $f\left(\frac{3}{2}\right)$?

Sol^n: $f(0) = cr^0 \quad \therefore c = 16$
$f(1) = cr^1 = 16 \times r^1 = 4 \quad \therefore r = \frac{1}{4}$
$\therefore f\left(\frac{3}{2}\right) = 16 \times \left(\frac{1}{4}\right)^{\frac{3}{2}}$
$= 16 \times \frac{1}{8} = 2$

x	*f(x)*
0	16
1	4
2	1
3	¼

a) 1/4 b) 2 ✓ c) 4 d) 16 e) Can't be determined

3. According to the table, for what value of x does $g(f(x)) = -1$?

x	f(x)	g(x)
-1	-2	4
0	0	3
1	2	2
2	4	1
3	6	0
4	8	-1

a) 4 b) 3 c) 2 ✓ d) 0 e) None of these

Sol^n: $g(4) = -1$.
$\therefore f(x) = 4$
$\therefore f(2) = 4$
$\therefore g(f(2)) \quad \therefore x = 2$

4. Let *f(x)* be defined for any positive integer x greater than 2 as he sum of all prime numbers less than *x*. For example $f(4) = 2+3 = 5$ and $f(8) = 7+5+3+2=17$. What is the value of $f(81) - f(78)$?

a) 79 ✓ b) 73 c) 71 d) Can't be determined e) None of these

Sol^n: $f(81) \rightarrow 79 + \cancel{73} + \cancel{71} + \ldots + 2$
$f(78) \rightarrow \cancel{73} + \cancel{71} + \ldots + 2$
$\boxed{79}$

Sol^n: $1-x$ must be greater than 0.

input

5. What is the domain of $f(x) = \frac{x}{\sqrt{1-x}}$?

$\therefore 1-x > 0$ $\therefore x < 1$.

a) $x < 1$ b) $x \le 1$ c) $x > 1$ d) $x \le 0$ e) $x < 0$

14. NUMERIC ENTRY

1. Let A= {all 3-digit positive integers with the digit 1 in the ones place}, and let B= {all 3-digit positive integers with the digit 2 in the tens place}. How many elements are there in A U B?

2. Yan needs \$2.37 in postage to mail a letter. If he has 60-cent, 37-cent, 23-cent, 5-cent and 1-cent stamps, at least 10 of each, what is the smallest number of stamps he can use to get the exact postage he needs?

3. A population of bacteria doubles every 2 hours. What is the percent increase after 4 hours?

4. Six chairs are placed in a row to seat six people. How many different seating arrangements are possible if two of the people insist on sitting next to each other?

5. Let x, y and z be consecutive even integers. If the product of 3 and y is 32 more than the sum of x and z, what is the median of the numbers in set S = $\{x, y, z, 2x, 2y, 2z\}$?

6. If $f(x) = x^2 + 3$ and $g(x) = x-5$, evaluate $f(g(9))$.

7. A line intersects two parallel lines, forming eight angles. If one of the angles has measure a^0, how many of the other seven angles are supplementary to it?

8. A rectangular box with length 22 inches, width 5 inches, and height 5 inches is to be packed with steel balls of radius 2 inches. What is the maximum number of balls that can fit into the box, provided that no balls should protrude from the box?

5

9. Mary has d dollars to spend and goes on a shopping spree. First she spends 2/5 of her money on shoes. Then she spends ¾ of what's left on a few books. Finally she buys a raffle ticket that costs 1/3 of her remaining dollars. What fraction of d is left?

10. Ten pounds of mixed nuts contain 50 percent peanuts. How many pounds of peanuts must be added so that the final mixture has 60 percent peanuts?

11. At John Adams High School, 120 students take programming, and 200 students take statistics. Of these, 50 students take both programming and statistics. An additional 80 students take neither programming nor statistics. If a student at this school is picked at random, what is the probability that he or