01. Basic Arithmetic

1) In how many numbers between 100 & 200 does the number 3 appear?
   Answer:
   - 10 numbers with 3 in units place (i.e. – $103, 113, 123, \ldots 193$)
   - 10 numbers with 3 in tens place (i.e. – $130, 131, 132, \ldots\ 139$)
   - But the number 133 was counted twice.
   - So, $10 + 10 - 1 = \mathbf{19}$

2) The value of $32^{0.16} \times 32^{0.04}$ is
   Answer:
   - $2^5 = 32$
   - $32^{0.16} \times 32^{0.04} = 32^{0.16+0.04} = 32^{0.2} = (2^5)^{0.2} = \mathbf{2}$

3) The square root of 0.9 can be
   Answer:
   - Square root of any positive number less than 1 is greater than that number.
   - Only one option that is greater than 0.9 is $\mathbf{0.94}$

4) The difference between the largest and the smallest of the fractions $\frac{6}{11}, \frac{3}{8}$ and $\frac{11}{16}$ is?
   Answer:
   - $\frac{11}{16}$ is the largest fraction
   - Smallest fraction is $\frac{3}{8}$
   - $\frac{11}{16} - \frac{3}{8} = \mathbf{\frac{5}{16}}$

5) $\sqrt{3} + \sqrt{2} = 3.146$ then $\frac{1}{\sqrt{3}-\sqrt{2}}$ is
   Answer:
   - Rationalize the denominator.
   - $\frac{1}{(\sqrt{3}-\sqrt{2})} = \frac{1}{(\sqrt{3}-\sqrt{2})} \times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} = \frac{(\sqrt{3}+\sqrt{2})}{((\sqrt{3})^2-(\sqrt{2})^2)} = \frac{(\sqrt{3}+\sqrt{2})}{(3-2)} = \frac{(\sqrt{3}+\sqrt{2})}{1} = \mathbf{3.146}$

6) The greatest number that will divide 728 and 901 leaving remainders of 8 and 5 is
   Answer:
   - Effectively we want a number that will divide $(728 - 8)$ and $(901 - 5)$.
   - So GCD$(728 - 8, 901 - 5)$ = $\mathbf{16}$

7) $10^9 - 10$ is not divisible by
Answer:
- $10^2 - 10 = 90$
- $10^3 - 10 = 990$ (divisible by 9, 10 and 11)
- $10^4 - 10 = 9990$
- $10^5 - 10 = 99990$ (divisible by 9, 10 and 11)
- So the expression is divisible by 9, 10 and 11 for all odd powers
- Last 2 numbers of $10^9 - 10 = 90$. And 90 is not divisible by 4, so $10^9 - 10$ is also not divisible by $\mathbf{4}$

8) If seven numbers, each divisible by 4 are added, the sum will be divisible by
Answer:
- Let the numbers be: $4a_1$ $4a_2$ $4a_3$ $4a_4$ $4a_5$ $4a_6$ $4a_7$ (where a_n is any integer)
- Sum: $4a_1 + 4a_2 + 4a_3 + 4a_4 + 4a_5 + 4a_6 + 4a_7$
- Sum: $4 (a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7)$
- The only thing that can be said for certain is the sum is divisible by **4**.

9) How many numbers between 100 and 300 are divisible by both 7 and 11?
Answer:
- Which numbers are divisible by 7 and 11? –> all multiples of their LCM and no other.
- $LCM(7,11) = 7 \times 11 = 77$
- So, we are looking for multiples of 77 between 100 and 300 –> 154 & 231 –> **2** numbers.

10) Three bells begin tolling at the same time and continue to do so at intervals 21, 28 and 30 seconds respectively. After how many seconds will the bells toll together for the first time after they begin tolling?
Answer:
- Bell 1: Rings at 21 seconds interval – Rings at t_1 seconds – $t_1 = 21n_1$ (where n_1 is whole number)
- Bell 2: Rings at 28 seconds interval – Rings at t_2 seconds – $t_2 = 28n_2$ (where n_2 is whole number)
- Bell 3: Rings at 30 seconds interval – Rings at t_3 seconds – $t_3 = 30n_3$ (where n_3 is whole number)
- So, we are looking for $t_1 = t_2 = t_3$, that is $21n_1 = 28n_2 = 30n_3$ (for whole number values of n_1, n_2 and n_3)
- $21n_1 = 28n_2 = 30n_3$ this condition will first occur at the beginning (for n_1, n_2 and n_3 all = 0)
- But we are looking for the next occurrence, which will happen at LCM (21, 28, 30)
- LCM (21, 28, 30) = **420**

11) If x is greater than 0 but less than 1, which of the following is the largest?
Answer:
- The condition must hold for all values of x between 0 and 1.
- So, we can just substitute values and check.
- If for some value two options are giving same value (which is largest), just try some other value of x the resolve the discrepancy. (NOTE: this issue will not occur in this question)
- Solution: $\boldsymbol{\frac{1}{x^2}}$

12) If $x = -1$, which of the following is the largest?
Answer:
- Substitute value
- Solution: $\boldsymbol{x^2}$

13) If n is an integer, which of the following must be even?
Answer:
- Substitute value
- Solution: $\mathbf{2}\boldsymbol{n} + \mathbf{2}$

14) If m, n, o, p and q are integers then $m(n + o)(p - q)$ must be even, when which of the following is even?
Answer:
- Either m OR $(n + o)$ OR $(p - q)$ is even.
- Options only contain $\boldsymbol{m}$.

15) If 9^{32} is divided by 5, then the remainder would be
Answer:
- 9^1 = 9
- 9^2 = 81
- 9^3 = 729
- For all even powers' units place is 1.
- Last digit of 9^{32} is 1. So, remainder would be **1**.

16) The sum of two numbers is 27 and the difference of their squares is 243. What is the difference between the numbers?
Answer:
- 2 equations, 2 variables.
- $x + y = 27$
- $x^2 - y^2 = 243$ –> $(x + y)(x - y) = 243$ –> $x - y = \mathbf{9}$

17) Find the least number which must be added to 15463 so that the resulting number is exactly divisible by 107.
Answer:
- I will use % symbol for modulo operation –> x % y gives the remainder when x is divided by y
- 15463 % 107 is 55
- 55 + x = 107
- x = **52**

18) A number when divided by 221 leaves a remainder 64. What is the remainder when the same number is divided by 13?
Answer:
- N % 221 = 64
- So, we can write N as $221n + 64$
- $(221n + 64)\ \%\ 13\ = ??$
- Because 221n is completely divisible by 13, our equation reduces to $(64)\ \%\ 13\ = ??$
- $64\ \%\ 13\ =\ \mathbf{12}$

19) Find the greatest number which will divide 38, 45 and 52 and leave remainders 2, 3 and 4 respectively.
Answer:
- Same as Question 6
- Solution: **6**

20) If sum of two positive numbers is 24 and the difference of their squares is 48, what is the product of the two integers?
Answer:
- Same as Question 16
- Solution: x=11, y=13, x*y = **143**

21) There are a few cards in the collection. If they are counted out 3 at a time, there are 2 cards left out, but if they are counted out 4 at a time, there is 1 card left out. How many cards are there in the collection?
Answer:
- Let the number be N, so it can be represented as
- N = $3n_1$ + 2 -> n_1 = (N-2)/3 {where n_1 is integer}
- N = $4n_2$ + 1 -> n_2 = (N-1)/4 {where n_2 is integer}
- $n_1 - n_2$ = (N-2)/3 - (N-1)/4 = (N–5)/12
- Since n_1 and n_2 are both integers $n_1 - n_2$ is also an integer, let it be x
- x = (N–5)/12 -> N = (12x+5)
- Check which option is of the form 12x+5, where x is an integer
- Solution: for $x = 8$ -> $12(8) + 5 = \mathbf{101}$
- Alternatively, plug-in all options and check which satisfies both conditions.

22) Mary Lou has 969 dimes in a piggy bank. What is the least number of dimes that she can remove from the bank so that she could divide the remaining dimes equally among 7 people?
Answer:
- $969 \ \% \ 7 = \mathbf{3}$ {read as 969 'modulo' 7} {969 % 7 give the remainder when 969 is divided by 7}

23) For how many of the integers from 10 to 99 is at least one of the two digits 4?
Answer:
- Similar to Question 1
- Solution: $9 + 10 - 1 = \mathbf{18}$

24) The sum of the three consecutive odd integers, x, y and z, in ascending order, is 39. What is the sum of the three consecutive odd integers that immediately follow z?
Answer:
- General form of odd integers $2n + 1$ (where n is any integer)
- So, let $x = 2n + 1$
- $y = 2n + 3, z = 2n + 5$
- $x + y + z = 39$ –> $(2n + 1) + (2n + 3) + (2n + 5) = 39$
- $n = 5, x = 11, y = 13, z = 15$
- $17 + 19 + 21 = \mathbf{57}$

25) There are 125 chips on a table. Of as many of the chips as possible are to be arranged into an equal number of 3-chips and 4-chips stacks and the remaining chips are to be removed, how many of the chips are to be removed?
Answer:
- Let number of stacks be x chips
- Total chips used $= 3x + 4x = 7x$
- $125 \ \% \ 7 = \mathbf{6}$