



Vidyavardhini's College of Engineering and Technology

Department of Artificial Intelligence & Data Science

AY: 2025-26

Class:	TE	Semester:	V
Course Code:	CSC 504	Course Name:	Data warehouse & mining

Name of Student:	Pranita kumbhar
Roll No. :	70
Assignment No.:	04
Title of Assignment:	Clustering
Date of Submission:	
Date of Correction:	

Evaluation

Performance Indicator	Max. Marks	Marks Obtained
Completeness	5	5
Demonstrated Knowledge	3	3
Legibility	2	2
Total	10	10

Performance Indicator	Exceed Expectations (EE)	Meet Expectations (ME)	Below Expectations (BE)
Completeness	5	3-4	1-2
Demonstrated Knowledge Legibility	3	2	1
Legibility	2	1	0

Checked by

Name of Faculty : Ms. Neha Raut

Signature :

Date :

Q.1] Using the following training dataset create classifications Model using Decision tree.

Day	Outlook	Temp	Humidity	Wind	Play tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No



i) Attribute : Outlook.

Values (Outlook) = Sunny, Overcast, Rain.

$$S = [9+, 5-], \text{Entropy}(S) = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

$$S_{\text{sunny}} \leftarrow [2+, 3-], \text{Entropy}(S_{\text{sunny}}) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.971$$

$$S_{\text{overcast}} \leftarrow [4+, 0-], \text{Entropy}(S_{\text{overcast}}) = -\frac{4}{4} \log_2 \frac{4}{4} - \frac{0}{4} \log_2 \frac{0}{4} = 0$$

$$S_{\text{rain}} \leftarrow [3+, 2-], \text{Entropy}(S_{\text{rain}}) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.971$$

$$Gain(S, Outlook) = Entropy(S) - \sum_{v \in \{Sunny, Overcast, Rain\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Outlook) = Entropy(S) - \frac{5}{14} Entropy(S_{Sunny}) - \frac{4}{14} Entropy$$

$$(S_{Overcast}) - \frac{5}{14} Entropy(S_{Rain})$$

$$= 0.94 - \frac{5}{14} 0.971 - \frac{4}{14} 0 - \frac{5}{14} 0.971 = 0.2464.$$

2] Attribute : Temp

Values (Temp) = Hot, Mild, Cool.

$$S = [9+, 5-], Entropy(S) = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

$$S_{Hot} \leftarrow [2+, 2-], Entropy(S_{Hot}) = -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} = 1.0.$$

$$S_{Mild} \leftarrow [4+, 2-], Entropy(S_{Mild}) = -\frac{4}{6} \log_2 \frac{4}{6} - \frac{2}{6} \log_2 \frac{2}{6} = 0.9183.$$

$$S_{Cool} \leftarrow [3+, 1-], Entropy(S_{Cool}) = -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} = 0.8113$$

$$Gain(S, Temp) = Entropy(S) - \frac{4}{14} Entropy(S_{Hot}) - \frac{6}{14} Entropy(S_{Mild})$$

$$Entropy(S_{Mild}) - \frac{4}{14} Entropy(S_{Cool})$$

$$= 0.94 - \frac{4}{14} 1.0 - \frac{6}{14} 0.9183 - \frac{4}{14} 0.8113 = 0.0289$$

3] Attribute : Humidity

Values (Humidity) = High, Normal

$$S = [9+, 5-], Entropy(S) = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

$$S_{high} \leftarrow [3+, 4-], \text{Entropy}(S_{high}) = -\frac{3}{7} \log_2 \frac{3}{7} - \frac{4}{7} \log_2 \frac{4}{7} = 0.9852$$

$$S_{normal} \leftarrow [6+, 1-], \text{Entropy}(S_{normal}) = -\frac{6}{7} \log_2 \frac{6}{7} - \frac{1}{7} \log_2 \frac{1}{7} = 0.5916$$

$$\text{Gain}(S, \text{Humidity}) = \text{Entropy}(S) - \frac{7}{14} \text{Entropy}(S_{high}) - \frac{7}{14}$$

$$\text{Entropy}(S_{normal})$$

$$= 0.94 - \frac{7}{14} 0.9852 - \frac{7}{14} 0.5916 = 0.1516$$

4] Attribute : Wind

Value (Wind) = Strong, Weak.

$$S = [9+, 5-], \text{Entropy}(S) = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

$$S_{strong} \leftarrow [3+, 3-], \text{Entropy} = 1.0$$

$$S_{weak} \leftarrow [6+, 2-], \text{Entropy}(S_{weak}) = -\frac{6}{8} \log_2 \frac{6}{8} - \frac{2}{8} \log_2 \frac{2}{8} = 0.8113$$

$$\text{Gain}(S, \text{Wind}) = \text{Entropy}(S) - \frac{6}{14} \text{Entropy}(S_{strong}) - \frac{8}{14}$$

$$\text{Entropy}(S_{weak})$$

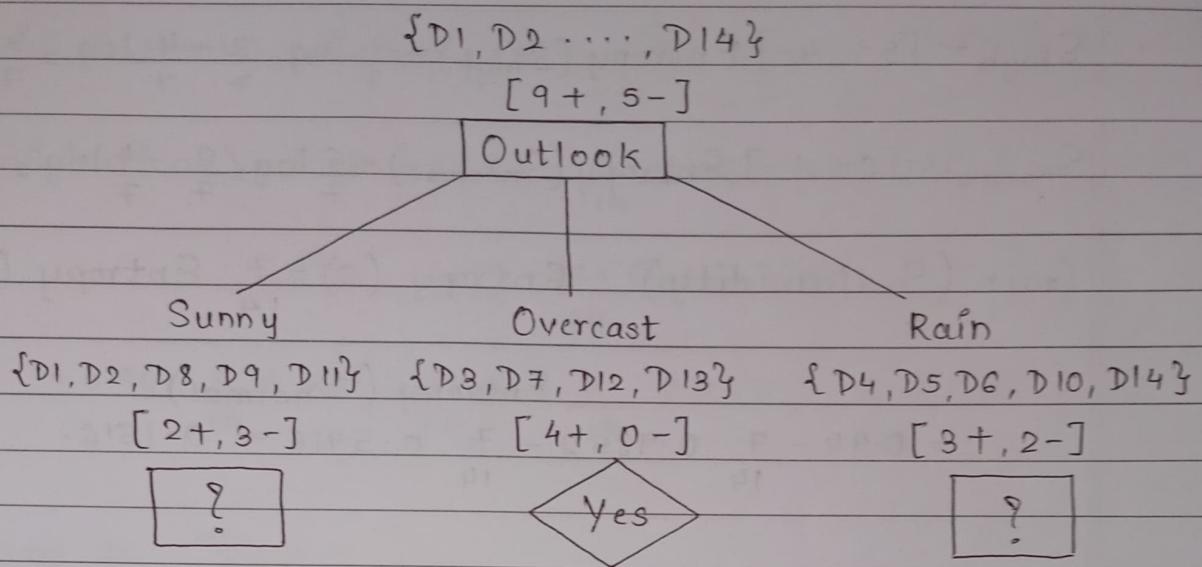
$$= 0.94 - \frac{6}{14} 1.0 - \frac{8}{14} 0.8113 = 0.0478$$

$$\text{Gain}(S, \text{Outlook}) = 0.2464$$

$$\text{Gain}(S, \text{Temp}) = 0.0289$$

$$\text{Gain}(S, \text{Humidity}) = 0.1516$$

$$\text{Gain}(S, \text{Wind}) = 0.0478$$



Day	Temp	Humidity	Wind	Play tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

1) Attribute : Temp

Values (temp) = Hot, Mild, Cool.

$$S_{\text{sunny}} = [2+, 3-], \text{Entropy}(S_{\text{sunny}}) = \frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

$$S_{\text{hot}} \leftarrow [0+, 2-], \text{Entropy}(S_{\text{hot}}) = 0.0$$

$$S_{\text{mild}} \leftarrow [1+, 1-], \text{Entropy}(S_{\text{mild}}) = 1.0$$

$$S_{\text{cool}} \leftarrow [1+, 0-], \text{Entropy}(S_{\text{cool}}) = 0.0$$

$$\text{Gain}(S_{\text{sunny}}, \text{Temp}) = \text{Entropy}(S) - \frac{2}{5} \text{Entropy}(S_{\text{hot}}) - \frac{2}{5}$$

$$\text{Entropy}(S_{\text{mild}}) = \frac{1}{5} \text{Entropy}(S_{\text{cool}})$$

$$= 0.97 - \frac{2}{5} 0.0 - \frac{2}{5} 1 - \frac{1}{5} 0.0 = 0.570$$

2] Attribute : Humidity

Values (Humidity) = High, Normal

$$S_{\text{sunny}} = [2+, 3-], \text{Entropy}(s) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

$$S_{\text{high}} \leftarrow [0+, 3-], \text{Entropy}(S_{\text{high}}) = 0.0$$

$$S_{\text{normal}} \leftarrow [2+, 0-], \text{Entropy}(S_{\text{normal}}) = 0.0$$

$$\text{Gain}(S_{\text{sunny}}, \text{Humidity}) = \text{Entropy}(s) - \frac{3}{5} \text{Entropy}(S_{\text{high}})$$

$$- \frac{2}{5} \text{Entropy}(S_{\text{normal}})$$

$$= 0.97 - \frac{3}{5} 0.0 - \frac{2}{5} 0.0 = 0.97$$

3] Attribute : Wind

Values (Wind) = Strong, Weak

$$S_{\text{sunny}} = [2+, 3-], \text{Entropy}(s) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

$$S_{\text{strong}} \leftarrow [1+, 1-], \text{Entropy}(S_{\text{strong}}) = 1.0$$

$$S_{\text{weak}} \leftarrow [1+, 2-], \text{Entropy}(S_{\text{weak}}) = -1 \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = 0.9188$$

$$\text{Gain}(S_{\text{sunny}}, \text{Wind}) = \text{Entropy}(s) - \frac{2}{5} \text{Entropy}(S_{\text{strong}}) - \frac{3}{5}$$

$$\text{Entropy}(S_{\text{weak}})$$

$$= 0.97 - \frac{2}{5} 1.0 - \frac{3}{5} 0.918 = 0.0192$$

$$\text{Gain}(S_{\text{sunny}}, \text{Temp}) = 0.570$$

$$\text{Gain}(S_{\text{sunny}}, \text{Humidity}) = 0.97$$

$$\text{Gain}(S_{\text{sunny}}, \text{Wind}) = 0.0192$$

Day	Temp	Humidity	Wind	Play tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
D10	Mild	Normal	Weak	Yes
D14	Mild	High	Strong	No

1] Attribute : Temp

Values (temp) = Hot, Mild, Cool.

$$S_{\text{rain}} = [3+, 2-], \text{Entropy}(S_{\text{sunny}}) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.97$$

$$S_{\text{hot}} \leftarrow [0+, 0-], \text{Entropy}(S_{\text{hot}}) = 0.0$$

$$S_{\text{mild}} \leftarrow [2+, 1-], \text{Entropy}(S_{\text{mild}}) = -\frac{2}{3} \log_2 \frac{2}{3} - \frac{1}{3} \log_2 \frac{1}{3} = 0.9183$$

$$S_{\text{cool}} \leftarrow [1+, 1-], \text{Entropy}(S_{\text{cool}}) = 1.0$$

$$\text{Gain}(S_{\text{rain}}, \text{Temp}) = \text{Entropy}(S) - \frac{0}{5} \text{Entropy}(S_{\text{hot}}) - \frac{3}{5}$$

$$\text{Entropy}(S_{\text{mild}}) - \frac{2}{5} \text{Entropy}(S_{\text{cool}})$$

$$= 0.97 - \frac{0}{5} 0.0 - \frac{3}{5} 0.918 - \frac{2}{5} 1.0 = 0.0192$$

2] Attribute : Wind

Values (wind) = Strong, Weak.

$$S_{\text{rain}} = [3+, 2-], \text{Entropy}(S_{\text{sunny}}) = 0.97$$

$S_{\text{strong}} \leftarrow [0+, 2-]$, Entropy (S_{strong}) = 0.0

$S_{\text{weak}} \leftarrow [3+, 0-]$, Entropy (S_{weak}) = 0.0.

Gain ($S_{\text{rain}}, \text{Wind}$) = Entropy (S) - $\frac{2}{5}$ Entropy ($S_{\text{strong}})$ - $\frac{3}{5}$

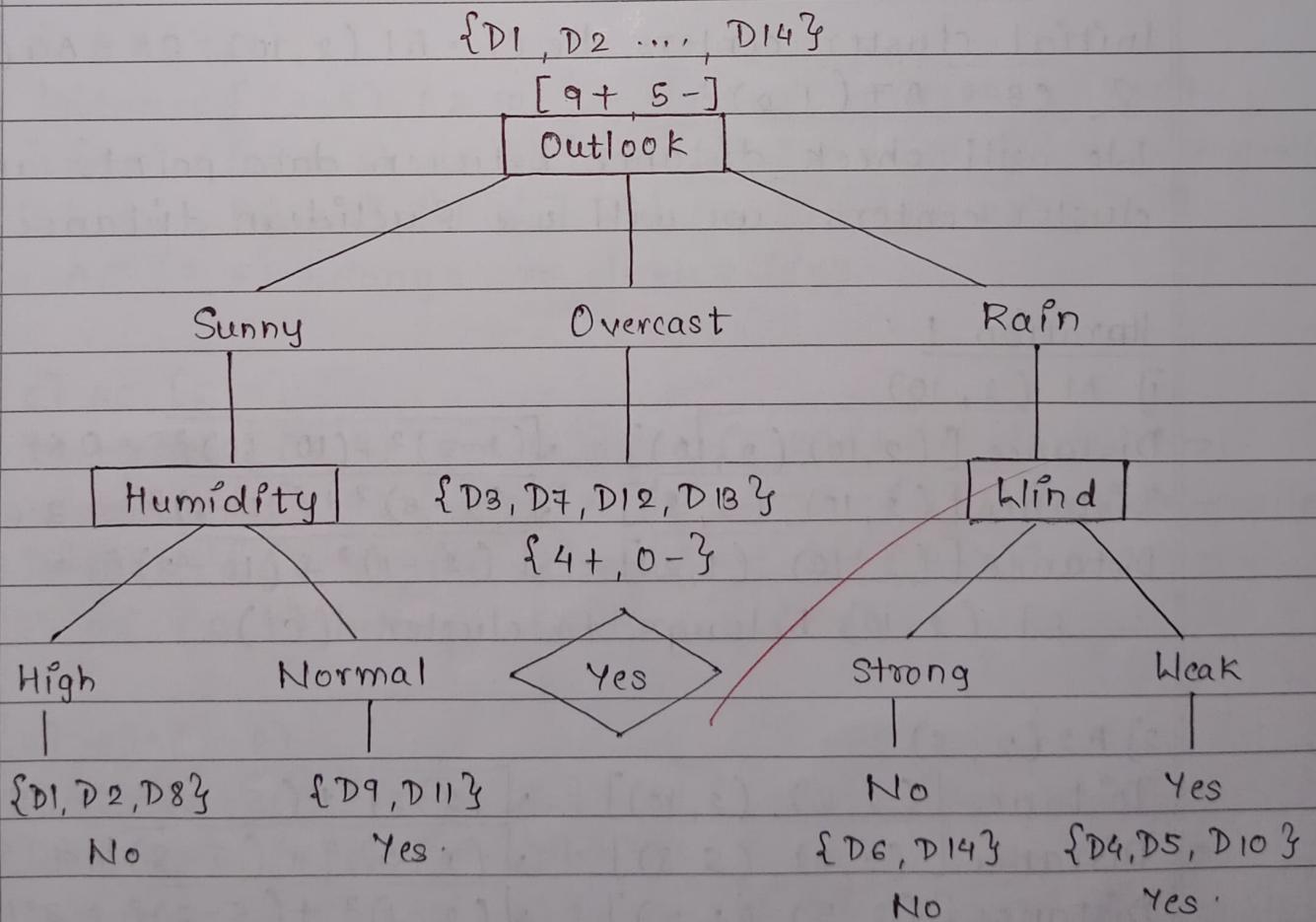
Entropy (S_{weak})

$$= 0.97 - \frac{2}{5} 0.0 - \frac{3}{5} 0.0 = 0.97.$$

Gain ($S_{\text{rain}}, \text{Temp}$) = 0.0192

Gain ($S_{\text{rain}}, \text{Humidity}$) = 0.0192

Gain ($S_{\text{rain}}, \text{Wind}$) = 0.97.



Q. 2] Suppose that the data mining task is to cluster the following eight points (with $(x:y)$ representing location) into three clusters: A1(2, 10), A2(2, 5), A3(8, 4), A4(5, 8), A5(7, 5), A6(6, 4), A7(1, 2), A8(4, 9). The distance function is Euclidean distance. Suppose initially we assign A1, A4 & A7 as the center of each cluster, respectively. Use the K-means algorithm to show only.

- The three cluster centers after the first round of execution and
- The final three clusters.



No. of cluster $k = 3$.

Initial cluster centers be $C_1 = A_1(2, 10)$, $C_2 = A_4(5, 8)$, & $C_3 = A_7(1, 2)$.

We will check distance between data points and all cluster centers we will use Euclidean distance formula.

Iteration 1 :

1] A1(2, 10) :

- Distance $[(2, 10), (2, 10)] = \sqrt{(2-2)^2 + (10-10)^2} = 0 \leftarrow$ smaller
 - Distance $[(2, 10), (5, 8)] = \sqrt{(2-5)^2 + (10-8)^2} = 3.6$.
 - Distance $[(2, 10), (1, 2)] = \sqrt{(2-1)^2 + (10-2)^2} = 8.06$.
- $\therefore A_1(2, 10)$ belongs to cluster (C_1).

2] A2(2, 5) :

- Distance $[(2, 5), (2, 10)] = \sqrt{(2-2)^2 + (5-10)^2} = 5$
 - Distance $[(2, 5), (5, 8)] = \sqrt{(2-5)^2 + (5-8)^2} = 4.2$
 - Distance $[(2, 5), (1, 2)] = \sqrt{(2-1)^2 + (5-2)^2} = 3.16 \leftarrow$ smaller
- $\therefore A_2(2, 5)$ belongs to cluster (C_3).

3] A3 (8, 4)

- Distance $[(8, 4), (2, 10)] = \sqrt{(8-2)^2 + (4-10)^2} = 8.5$
 - Distance $[(8, 4), (5, 8)] = \sqrt{(8-5)^2 + (4-8)^2} = 5 < \text{smaller}$
 - Distance $[(8, 4), (1, 2)] = \sqrt{(8-1)^2 + (4-2)^2} = 7.28$
- $\therefore A3 (8, 4)$ belongs to cluster (C2)

4] A4 (5, 8)

- Distance $[(5, 8), (2, 10)] = \sqrt{(5-2)^2 + (8-10)^2} = 3.6$
 - Distance $[(5, 8), (5, 8)] = \sqrt{(5-5)^2 + (8-8)^2} = 0 < \text{smaller}$
 - Distance $[(5, 8), (1, 2)] = \sqrt{(5-1)^2 + (8-2)^2} = 7.21$
- $\therefore A4 (5, 8)$ belongs to cluster (C2)

5] A5 (7, 5)

- Distance $[(7, 5), (2, 10)] = \sqrt{(7-2)^2 + (5-10)^2} = 7.67$
 - Distance $[(7, 5), (5, 8)] = \sqrt{(7-5)^2 + (5-8)^2} = 3.61 < \text{smaller}$
 - Distance $[(7, 5), (1, 2)] = \sqrt{(7-1)^2 + (5-2)^2} = 6.71$
- $\therefore A5 (7, 5)$ belongs to cluster (C2)

6] A6 (6, 4)

- Distance $[(6, 4), (2, 10)] = \sqrt{(6-2)^2 + (4-10)^2} = 7.21$
 - Distance $[(6, 4), (5, 8)] = \sqrt{(6-5)^2 + (4-8)^2} = 4.12 < \text{smaller}$
 - Distance $[(6, 4), (1, 2)] = \sqrt{(6-1)^2 + (4-2)^2} = 5.38$
- $\therefore A6 (6, 4)$ belongs to cluster (C2)

7] A7 (1, 2)

- Distance $[(1, 2), (2, 10)] = \sqrt{(1-2)^2 + (2-10)^2} = 8.06$
 - Distance $[(1, 2), (5, 8)] = \sqrt{(1-5)^2 + (2-8)^2} = 7.21$
 - Distance $[(1, 2), (1, 2)] = \sqrt{(1-1)^2 + (2-2)^2} = 0 < \text{smaller}$
- $\therefore A7 (1, 2)$ belongs to cluster (C3)

8] A8 (4, 9)

- Distance $[(4, 9), (2, 10)] = \sqrt{(4-2)^2 + (9-10)^2} = 8.24$
 - Distance $[(4, 9), (5, 8)] = \sqrt{(4-5)^2 + (9-8)^2} = 6.08 < \text{smaller}$
 - Distance $[(4, 9), (1, 2)] = \sqrt{(4-1)^2 + (9-2)^2} = 7.62$
- $\therefore A8 (4, 9)$ belongs to cluster (C2).

9] After Iteration 1:

$$\therefore \text{Cluster } C1 = [A1, (2, 10)]$$

$$\therefore \text{Cluster } C2 = [A3 (8, 4), A4 (5, 8), A5 (7, 5), A6 (6, 4), A8 (4, 9)]$$

$$\therefore \text{Cluster } C3 = [A2 (2, 5), A7 (1, 2)]$$

Iteration 2:

$$\text{New cluster } C1 = [2, 10]$$

$$\text{New cluster } C2 = \left[\left(\frac{8+5+7+6+4}{5} \right), \left(\frac{4+8+5+4+9}{5} \right) \right] = (6, 6)$$

$$\text{New cluster } C3 = \left[\left(\frac{2+1}{2} \right), \left(\frac{5+2}{2} \right) \right] = (1.5, 3.5)$$

1] A1 (2, 10)

- Distance $[(2, 10), (2, 10)] = \sqrt{(2-2)^2 + (10-10)^2} = 0 < \text{smaller}$
 - Distance $[(2, 10), (5, 8)] = \sqrt{(2-5)^2 + (10-8)^2} = 3.6$
 - Distance $[(2, 10), (1, 2)] = \sqrt{(2-1)^2 + (10-2)^2} = 8.06$
- $\therefore A1 (2, 10)$ belongs to cluster (C1).

2] A2 (2, 5)

- Distance $[(2, 5), (2, 10)] = \sqrt{(2-2)^2 + (5-10)^2} = 5$
 - Distance $[(2, 5), (6, 4)] = \sqrt{(2-6)^2 + (5-4)^2} = 4.12$
 - Distance $[(2, 5), (1.5, 3.5)] = \sqrt{(2-1.5)^2 + (5-3.5)^2} = 1.58 < \text{smaller}$
- $\therefore A2 (2, 5)$ belongs to cluster (C3)

3] A3 (8, 4)

- distance $[(8, 4), (2, 10)] = \sqrt{(8-2)^2 + (4-10)^2} = 8.49$

- distance $[(8, 4), (6, 6)] = \sqrt{(8-6)^2 + (4-6)^2} = 2.83 \leftarrow \text{smaller}$

- distance $[(8, 4), (1.5, 3.5)] = \sqrt{(8-1.5)^2 + (4-3.5)^2} = 6.52$

$\therefore A3 (8, 4)$ belongs to cluster (C2).

4] A4 (5, 8)

- Distance $[(5, 8), (2, 10)] = \sqrt{(5-2)^2 + (8-10)^2} = 3.61$

- Distance $[(5, 8), (6, 6)] = \sqrt{(5-6)^2 + (8-6)^2} = 2.24 \leftarrow \text{smaller}$

- Distance $[(5, 8), (1.5, 3.5)] = \sqrt{(5-1.5)^2 + (8-3.5)^2} = 5.70$

$\therefore A4 (5, 8)$ belongs to cluster (C2).

5] A5 (7, 5)

- Distance $[(7, 5), (2, 10)] = \sqrt{(7-2)^2 + (5-10)^2} = 7.07$

- Distance $[(7, 5), (6, 6)] = \sqrt{(7-6)^2 + (5-6)^2} = 1.41 \leftarrow \text{smaller}$

- Distance $[(7, 5), (1.5, 3.5)] = \sqrt{(7-1.5)^2 + (5-3.5)^2} = 5.70$

$\therefore A5 (7, 5)$ belongs to cluster (C2).

6] A6 (6, 4)

- Distance $[(6, 4), (2, 10)] = \sqrt{(6-2)^2 + (4-10)^2} = 7.21$

- Distance $[(6, 4), (6, 6)] = \sqrt{(6-6)^2 + (4-6)^2} = 2 \leftarrow \text{smaller}$

- Distance $[(6, 4), (1.5, 3.5)] = \sqrt{(6-1.5)^2 + (4-3.5)^2} = 4.53$

$\therefore A6 (6, 4)$ belongs to cluster (C2).

7] A7 (1, 2)

- Distance $[(1, 2), (2, 10)] = \sqrt{(1-2)^2 + (2-10)^2} = 8.06$

- Distance $[(1, 2), (6, 6)] = \sqrt{(1-6)^2 + (2-6)^2} = 6.40$

- Distance $[(1, 2), (1.5, 3.5)] = \sqrt{(1-1.5)^2 + (2-3.5)^2} = 1.58 \leftarrow \text{smaller}$

$\therefore A7 (1, 2)$ belongs to cluster (C3).

8] A8 (4,9)

- Distance $[(4,9), (2,10)] = \sqrt{(4-2)^2 + (9-10)^2} = 2.24 \leftarrow$ smaller
 - Distance $[(4,9), (6,6)] = \sqrt{(4-6)^2 + (9-6)^2} = 3.61$
 - Distance $[(4,9), (1.5, 3.5)] = \sqrt{(4-1.5)^2 + (9-3.5)^2} = 6.04$
- $\therefore A8 (4,9)$ belongs to cluster (C₁)

* • After Iteration 2 :

$$\text{cluster C1} = [A1(2,10), A8(4,9)]$$

$$\text{cluster C2} = [A3(8,4), A4(5,8), A5(7,5), A6(6,4)]$$

$$\text{cluster C3} = [A7(1,2), A2(2,5)]$$

* Iteration 3 :

$$- C1 = \left[\left(\frac{2+4}{2} \right), \left(\frac{10+9}{2} \right) \right] = (3, 9.5)$$

$$- C2 = \left[\left(\frac{8+5+7+6}{4} \right), \left(\frac{4+8+5+4}{4} \right) \right] = (6.5, 5.25)$$

$$- C3 = \left[\left(\frac{1+2}{2} \right), \left(\frac{2+5}{2} \right) \right] = (1.5, 3.5)$$

1] A1 (2,10)

- Distance $[(2,10), (3,9.5)] = \sqrt{(2-3)^2 + (10-9.5)^2} = 1.1180 \leftarrow$ smaller
 - Distance $[(2,10), (6.5, 5.25)] = \sqrt{(2-6.5)^2 + (10-5.25)^2} = 6.5431$
 - Distance $[(2,10), (1.5, 3.5)] = \sqrt{(2-1.5)^2 + (10-3.5)^2} = 6.5192$
- $\therefore A1 (2,10)$ belongs to cluster (C₁).

2] A2 (2,5)

- Distance $[(2,5), (3,9.5)] = \sqrt{(2-3)^2 + (5-9.5)^2} = 4.6097$
 - Distance $[(2,5), (6.5, 5.25)] = \sqrt{(2-6.5)^2 + (5-5.25)^2} = 4.5069$
 - Distance $[(2,5), (1.5, 3.5)] = \sqrt{(2-1.5)^2 + (5-3.5)^2} = 1.5811 \leftarrow$ smaller
- $\therefore A2 (2,5)$ belongs to cluster (C₃).

3] A3 (8, 4)

- Distance $[(8, 4), (3, 9.5)] = \sqrt{(8-3)^2 + (4-9.5)^2} = 7.4330$
 - Distance $[(8, 4), (6.5, 5.25)] = \sqrt{(8-6.5)^2 + (4-5.25)^2} = 1.952 \leftarrow \text{smaller}$
 - Distance $[(8, 4), (1.5, 3.5)] = \sqrt{(8-1.5)^2 + (4-3.5)^2} = 6.5192$
- $\therefore A3 (8, 4)$ belongs to cluster (C2)

4] A4 (5, 8)

- Distance $[(5, 8), (3, 9.5)] = \sqrt{(5-3)^2 + (8-9.5)^2} = 2.5 \leftarrow \text{smaller}$
 - Distance $[(5, 8), (6.5, 5.25)] = \sqrt{(5-6.5)^2 + (8-5.25)^2} = 3.1324$
 - Distance $[(5, 8), (1.5, 3.5)] = \sqrt{(5-1.5)^2 + (8-3.5)^2} = 5.7008$
- $\therefore A4 (5, 8)$ belongs to cluster (C1)

5] A5 (7, 5)

- Distance $[(7, 5), (3, 9.5)] = \sqrt{(7-3)^2 + (5-9.5)^2} = 6.020$
 - Distance $[(7, 5), (6.5, 5.25)] = \sqrt{(7-6.5)^2 + (5-5.25)^2} = 0.5590 \leftarrow \text{smaller}$
 - Distance $[(7, 5), (1.5, 3.5)] = \sqrt{(7-1.5)^2 + (5-3.5)^2} = 5.7008$
- $\therefore A5 (7, 5)$ belongs to cluster (C2)

6] A6 (6, 4)

- Distance $[(6, 4), (3, 9.5)] = \sqrt{(6-3)^2 + (4-9.5)^2} = 6.2649$
 - Distance $[(6, 4), (6.5, 5.25)] = \sqrt{(6-6.5)^2 + (4-5.25)^2} = 1.3462 \leftarrow \text{smaller}$
 - Distance $[(6, 4), (1.5, 3.5)] = \sqrt{(6-1.5)^2 - (4-3.5)^2} = 4.5276$
- $\therefore A6 (6, 4)$ belongs to cluster (C2)

7] A7 (1, 2)

- Distance $[(1, 2), (3, 9.5)] = \sqrt{(1-3)^2 + (2-9.5)^2} = 7.7620$
 - Distance $[(1, 2), (6.5, 5.25)] = \sqrt{(1-6.5)^2 + (2-5.25)^2} = 6.3884$
 - Distance $[(1, 2), (1.5, 3.5)] = \sqrt{(1-1.5)^2 + (2-3.5)^2} = 1.5811 \leftarrow \text{smaller}$
- $\therefore A7 (1, 2)$ belongs to cluster (C3)

8] A8 (4,9)

- Distance $[(4,9), (3,9.5)] = \sqrt{(4-3)^2 + (9-9.5)^2} = 1.1180 \leftarrow \text{smaller}$
 - Distance $[(4,9), (6.5, 5.25)] = \sqrt{(4-6.5)^2 + (9-5.25)^2} = 4.5069$,
 - Distance $[(4,9), (1.5, 3.5)] = \sqrt{(4-1.5)^2 + (9-3.5)^2} = 6.0415$.
- $\therefore A8 (4,9)$ belongs to cluster (C1).

9) After Iteration 3:

$$\text{Cluster C1} = [A1(2,10), A4(5,8), A8(4,9)].$$

$$\text{Cluster C2} = [A3(8,4), A5(7,5), A6(6,4)]$$

$$\text{Cluster C3} = [A2(2,5), A7(1,2)].$$

* Iteration 4:

$$C1 = \left[\left(\frac{2+5+4}{3} \right), \left(\frac{10+8+9}{3} \right) \right] = (3.6, 9).$$

$$C2 = \left[\left(\frac{8+7+6}{3} \right), \left(\frac{4+5+4}{3} \right) \right] = (7, 4.3)$$

$$C3 = \left[\left(\frac{2+1}{2} \right), \left(\frac{5+2}{2} \right) \right] = (1.5, 3.5).$$

1] A1 (2,10)

- Distance $[(2,10), (3,6.9)] = \sqrt{(2-3)^2 + (10-6.9)^2} = 1.8867 \leftarrow \text{smaller}$
 - ~~Distance $[(2,10), (7,4.3)] = \sqrt{(2-7)^2 + (10-4.3)^2} = 7.5822$.~~
 - Distance $[(2,10), (1.5, 3.5)] = \sqrt{(2-1.5)^2 + (10-3.5)^2} = 6.5192$
- $\therefore A1 (2,10)$ belongs to cluster (C1).

2] A2 (2,5)

- Distance $[(2,5), (3,6.9)] = \sqrt{(2-3)^2 + (5-6.9)^2} = 4.3081$
 - ~~Distance $[(2,5), (7,4.3)] = \sqrt{(2-7)^2 + (5-4.3)^2} = 5.0487$~~
 - Distance $[(2,5), (1.5, 3.5)] = \sqrt{(2-1.5)^2 + (5-3.5)^2} = 1.5811 \leftarrow \text{smaller}$
- $\therefore A2 (2,5)$ belongs to cluster (C3).

3] A3 (8, 4)

- Distance $[(8, 4), (3.6, 9)] = \sqrt{(8-3.6)^2 + (4-9)^2} = 6.6603$
 - Distance $[(8, 4), (7, 4.3)] = \sqrt{(8-7)^2 + (4-4.3)^2} = 1.0440$ < smaller
 - Distance $[(8, 4), (1.5, 3.5)] = \sqrt{(8-1.5)^2 + (4-3.5)^2} = 6.5192$
- $\therefore A3 (8, 4)$ belongs to cluster (C2).

4] A4 (5, 8)

- Distance $[(5, 8), (3.6, 9)] = \sqrt{(5-3.6)^2 + (8-9)^2} = 1.7204$ < smaller
 - Distance $[(5, 8), (7, 4.3)] = \sqrt{(5-7)^2 + (8-4.3)^2} = 4.2059$
 - Distance $[(5, 8), (1.5, 3.5)] = \sqrt{(5-1.5)^2 + (8-3.5)^2} = 5.7008$
- $\therefore A4 (5, 8)$ belongs to cluster (C1).

5] A5 (7, 5)

- Distance $[(7, 5), (3.6, 9)] = \sqrt{(7-3.6)^2 + (5-9)^2} = 5.2497$
 - Distance $[(7, 5), (7, 4.3)] = \sqrt{(7-7)^2 + (5-4.3)^2} = 0.7$ < smaller
 - Distance $[(7, 5), (1.5, 3.5)] = \sqrt{(7-1.5)^2 + (5-3.5)^2} = 5.7008$
- $\therefore A5 (7, 5)$ belongs to cluster (C2).

6] A6 (6, 4)

- Distance $[(6, 4), (3.6, 9)] = \sqrt{(6-3.6)^2 + (4-9)^2} = 5.5461$
 - Distance $[(6, 4), (7, 4.3)] = \sqrt{(6-7)^2 + (4-4.3)^2} = 1.0440$ < smaller
 - Distance $[(6, 4), (1.5, 3.5)] = \sqrt{(6-1.5)^2 + (4-3.5)^2} = 4.5276$
- $\therefore A6 (6, 4)$ belongs to cluster (C2).

7] A7 (1, 2)

- Distance $[(1, 2), (3.6, 9)] = \sqrt{(1-3.6)^2 + (2-9)^2} = 7.4672$
 - Distance $[(1, 2), (7, 4.3)] = \sqrt{(1-7)^2 + (2-4.3)^2} = 6.425$
 - Distance $[(1, 2), (1.5, 3.5)] = \sqrt{(1-1.5)^2 + (2-3.5)^2} = 1.5811$ < smaller
- $\therefore A7 (1, 2)$ belongs to cluster (C3).

8] A8 (4,9)

- Distance $[(4,9), (3.6, 9)] = \sqrt{(4-3.6)^2 + (9-9)^2} = 0.7 \leftarrow$ smaller
 - Distance $[(4,9), (7, 4.3)] = \sqrt{(4-7)^2 + (9-4.3)^2} = 5.575$.
 - Distance $[(4,9), (1.5, 3.5)] = \sqrt{(4-1.5)^2 + (9-3.5)^2} = 6.0418$.
- $\therefore A8 (4,9)$ belongs to cluster (C1).

• After Iteration 4:

$$\text{Cluster C1} = [A1(2,10), A4(5,8), A8(4,9)]$$

$$\text{Cluster C2} = [A3(8,4), A5(7,5), A6(6,4)]$$

$$\text{Cluster C3} = [A2(2,5), A7(1,2)]$$

* Therefore After Iteration 3 & 4 the cluster (3) remain unchanged *

* The final cluster :

$$\text{Cluster C1} = [A1(2,10), A4(5,8), A8(4,9)]$$

$$\text{Cluster C2} = [A3(8,4), A5(7,5), A6(6,4)]$$

$$\text{Cluster C3} = [A2(2,5), A7(1,2)]$$

