Skills_Project

December 2, 2022

1 Honours Differential Equations

1.1 Project Assignment

Due: Friday 2nd December 2022, noon

2 Pranith Praveen

3 s2097010

3.1 Question 1

[1]:
```python
#IMPORTS
import sympy as sym
import numpy as np
sym.init_printing()
from IPython.display import display_latex
import math
```

Please **clearly** indicate where you answer each sub question by using a markdown cell.

4 Part (a)

[2]:
```python
### Write your code here. You might want to add extra code cells if you prefer
x = sym.Function('x')
y = sym.Function('y')
t = sym.Symbol("t")
a = sym.Symbol("a")
b = sym.Symbol("b")
u = sym.Function('u')
v = sym.Function('v')

eq1 = sym.Eq(x(t).diff(t),y(t))
eq2 = sym.Eq(y(t).diff(t),-x(t)+a*(y(t)-(y(t))**3/3))
[eq1, eq2]
```

[2]:

$$\left[\frac{d}{dt}x(t) = y(t),\ \frac{d}{dt}y(t) = a\left(-\frac{y^3(t)}{3} + y(t)\right) - x(t)\right]$$

[3]:
```python
FG = sym.Matrix([eq1.rhs, eq2.rhs])
matJ = FG.jacobian([x(t), y(t)])
matJ
```

[3]:
$$\begin{bmatrix} 0 & 1 \\ -1 & a\left(1 - y^2(t)\right) \end{bmatrix}$$

[4]:
```python
lin_mat = matJ.subs({x(t):0, y(t):0})
lin_mat * sym.Matrix([u(t),v(t)])
```

[4]:
$$\begin{bmatrix} v(t) \\ av(t) - u(t) \end{bmatrix}$$

[5]:
```python
lin_mat.eigenvects()[0][2][0],lin_mat.eigenvects()[1][2][0] #eigenvectors
```

[5]:
$$\left(\begin{bmatrix} \frac{a}{2} + \frac{\sqrt{(a-2)(a+2)}}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} \frac{a}{2} - \frac{\sqrt{(a-2)(a+2)}}{2} \\ 1 \end{bmatrix} \right)$$

[6]:
```python
evals = list(lin_mat.eigenvals().keys())
evals #eigenvalues
```

[6]:
$$\left[\frac{a}{2} - \frac{\sqrt{(a-2)(a+2)}}{2}, \ \frac{a}{2} + \frac{\sqrt{(a-2)(a+2)}}{2} \right]$$

[7]:
```python
for i in range(-5,5):
    print("a = " + str(i))
    display_latex([evals[0].subs({a:i}) ,evals[1].subs({a:i}) ])
```

```
a = -5
```

$$\left[-\frac{5}{2} - \frac{\sqrt{21}}{2}, \ -\frac{5}{2} + \frac{\sqrt{21}}{2} \right]$$

```
a = -4
```

$$\left[-2 - \sqrt{3}, \ -2 + \sqrt{3} \right]$$

```
a = -3
```

$$\left[-\frac{3}{2} - \frac{\sqrt{5}}{2}, \ -\frac{3}{2} + \frac{\sqrt{5}}{2} \right]$$

```
a = -2
```

$$\left[-1, \ -1 \right]$$

```
a = -1
```

$$\left[-\frac{1}{2} - \frac{\sqrt{3}i}{2}, \ -\frac{1}{2} + \frac{\sqrt{3}i}{2} \right]$$

a = 0

$[-i,\ i]$

a = 1

$$\left[\frac{1}{2} - \frac{\sqrt{3}i}{2},\ \frac{1}{2} + \frac{\sqrt{3}i}{2} \right]$$

a = 2

$[1,\ 1]$

a = 3

$$\left[\frac{3}{2} - \frac{\sqrt{5}}{2},\ \frac{\sqrt{5}}{2} + \frac{3}{2} \right]$$

a = 4

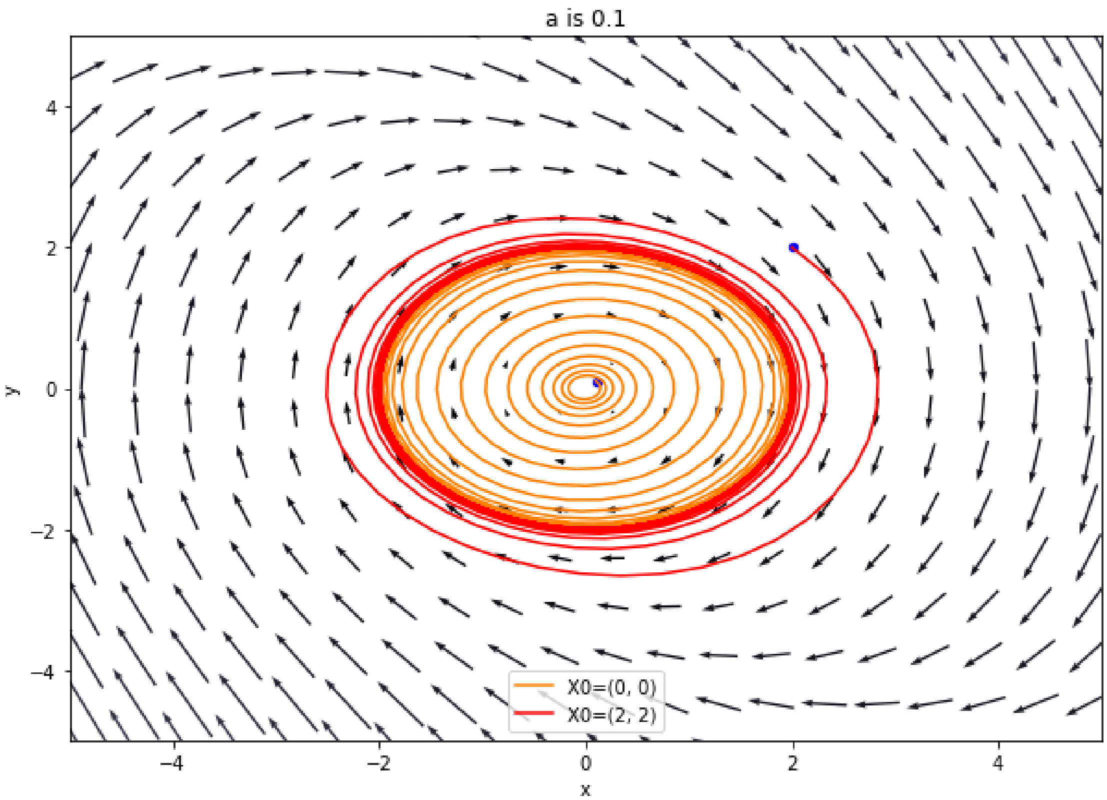
$\left[2 - \sqrt{3},\ \sqrt{3} + 2\right]$

From the above results , we can see that when a is lesser than -1 , it produces a pair of real negative eigen values (nodal sink) when it is 2 or greater , it produces positive real eigen values (source) , and when it is in [-1,2) , it produces imaginary eigen values and can produce either a spiral source or a spiral sink

5 Part (b)

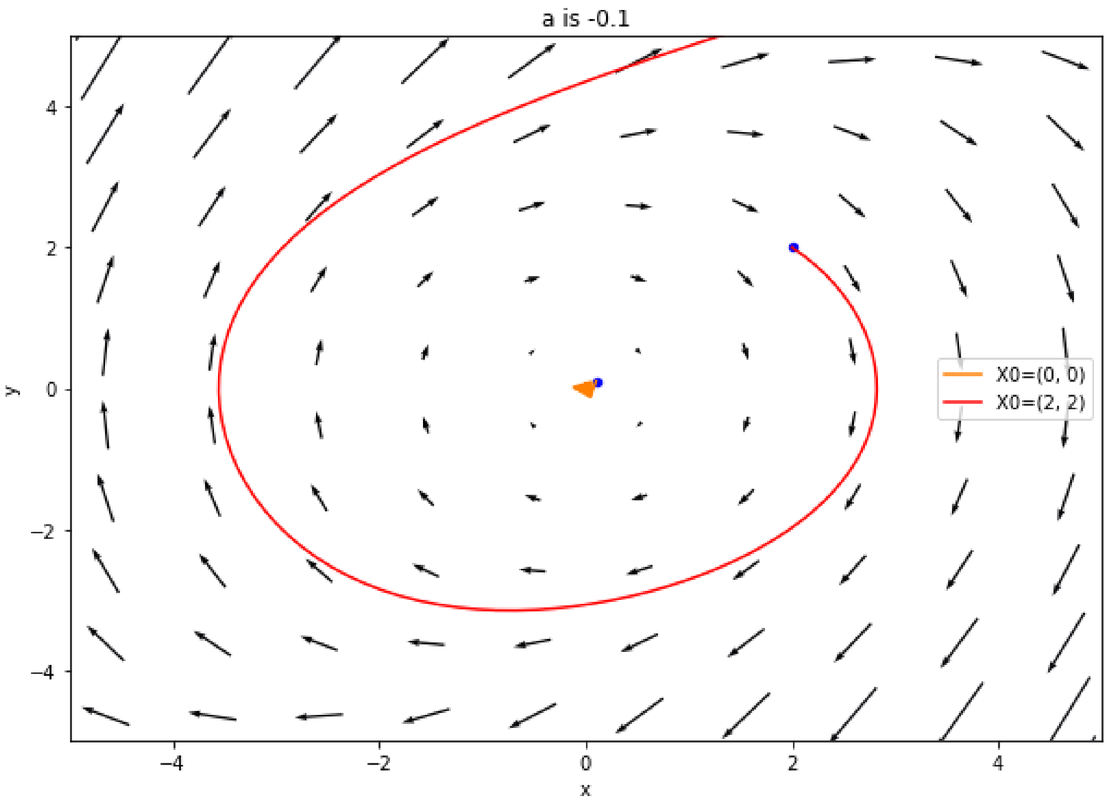
[8]:
```
import numpy as np
from matplotlib import pyplot as plt
from scipy.integrate import odeint
%matplotlib inline

# Define vector field
def vField(x,t):
    u = x[1]
    v = -x[0]+.1*(x[1]-((x[1]**3)/3))
    return [u,v]

# Plot vector field
X, Y = np.mgrid[-10:10:40j,-10:10:30j]
U, V = vField([X,Y],0)

# define colours for each vector based on their lengths
M = np.hypot(U, V)

fig, ax = plt.subplots(figsize=(10, 7))
ax.quiver(X, Y, U, V, M, scale=1/0.01, pivot = 'mid', cmap = plt.cm.bone)
```

```python
# Settings for trajectories
ics  = [[0.1,0.1],[2,2]]
durations = [[0,200],[0,150]]
vcolors = plt.cm.autumn_r(np.linspace(0.5, 1., len(ics)))  # colors for each␣
↪trajectory

# plot trajectories
for i, ic in enumerate(ics):
    t = np.linspace(durations[i][0], durations[i][1],1000)
    x = odeint(vField, ic, t)
    ax.plot(x[:,0], x[:,1], color=vcolors[i], label='X0=(%.f, %.f)' % (ic[0],␣
↪ic[1]) )

ic_x = [ic[0] for ic in ics]
ic_y = [ic[1] for ic in ics]
ax.scatter(ic_x, ic_y, color='blue', s=20)

plt.xlabel('x')
plt.ylabel('y')
plt.xlim(-5,5)
plt.ylim(-5,5)
plt.legend()
plt.title('a is 0.1')

plt.show()
```



[9]:
```
import numpy as np
from matplotlib import pyplot as plt
from scipy.integrate import odeint
%matplotlib inline

# Define vector field
def vField(x,t):
    u = x[1]
    v = -1*x[0]-0.1*(x[1]-(x[1]**3)/3)
    return [u,v]

# Plot vector field
X, Y = np.mgrid[-15:15:30j,-15:15:30j]
U, V = vField([X,Y],0)

# define colours for each vector based on their lengths
M = np.hypot(U, V)

fig, ax = plt.subplots(figsize=(10, 7))
ax.quiver(X, Y, U, V, M, scale=1/0.01, pivot = 'mid', cmap = plt.cm.bone)
```

```python
# Settings for trajectories
ics  = [[0.1,0.1],[2,2]]
durations = [[0,200],[0,6]]
vcolors = plt.cm.autumn_r(np.linspace(0.5, 1., len(ics)))  # colors for each trajectory

# plot trajectories
for i, ic in enumerate(ics):
    t = np.linspace(durations[i][0], durations[i][1],100)
    x = odeint(vField, ic, t)
    ax.plot(x[:,0], x[:,1], color=vcolors[i], label='X0=(%.f, %.f)' % (ic[0], ic[1]) )

ic_x = [ic[0] for ic in ics]
ic_y = [ic[1] for ic in ics]
ax.scatter(ic_x, ic_y, color='blue', s=20)

plt.xlabel('x')
plt.ylabel('y')
plt.xlim(-5,5)
plt.ylim(-5,5)
plt.legend()
plt.title('a is -0.1')

plt.show()
```



At a = 0.1 , it produces a spiral sink , and at a = -0.1 , it produces a spiral source , which is in accordance with the results from part (a)

5.1 Question 2

Please **clearly** indicate where you answer each sub question by using a markdown cell.

6 Part (a)

[10]:
```
## Write your code here. You might want to add extra code cells if you prefer
x = sym.Function('x')
y = sym.Function('y')
t = sym.Symbol("t")
a = sym.Symbol("a")
b = sym.Symbol("b")
u = sym.Function('u')
v = sym.Function('v')

eq1 = sym.Eq(x(t).diff(t),a - x(t) -b*x(t) +x(t)**2*y(t))
eq2 = sym.Eq(y(t).diff(t),b*x(t)-x(t)**2*y(t))
[eq1, eq2]
```

[10]: $\left[\frac{d}{dt}x(t) = a - bx(t) + x^2(t)y(t) - x(t),\ \frac{d}{dt}y(t) = bx(t) - x^2(t)y(t)\right]$

[11]:
```
FG = sym.Matrix([eq1.rhs, eq2.rhs])

display_latex(sym.solve(FG))
print("x(t) = a and y(t) = b/a")
```

$[\{a : x(t),\ b : x(t)y(t)\}]$

```
x(t) = a and y(t) = b/a
```

[12]:
```
FG = sym.Matrix([eq1.rhs, eq2.rhs])
matJ = FG.jacobian([x(t), y(t)])
display_latex(matJ)

print("subbing x(t) = a and y(t) = b/a")
lin_mat = matJ.subs({x(t):a, y(t):(b/a)})
display_latex(lin_mat)
```

$$\begin{bmatrix} -b + 2x(t)y(t) - 1 & x^2(t) \\ b - 2x(t)y(t) & -x^2(t) \end{bmatrix}$$

```
subbing x(t) = a and y(t) = b/a
```

$$\begin{bmatrix} b-1 & a^2 \\ -b & -a^2 \end{bmatrix}$$

[13]:
```
lin_mat.eigenvects()[0][2][0],lin_mat.eigenvects()[1][2][0] # Eigenvectors
```

[13]: $$\left(\begin{bmatrix} -\frac{a^2}{b} - \frac{-\frac{a^2}{2} + \frac{b}{2} - \frac{\sqrt{(a^2-2a-b+1)(a^2+2a-b+1)}}{2} - \frac{1}{2}}{b} \\ 1 \end{bmatrix},\ \begin{bmatrix} -\frac{a^2}{b} - \frac{-\frac{a^2}{2} + \frac{b}{2} + \frac{\sqrt{(a^2-2a-b+1)(a^2+2a-b+1)}}{2} - \frac{1}{2}}{b} \\ 1 \end{bmatrix} \right)$$

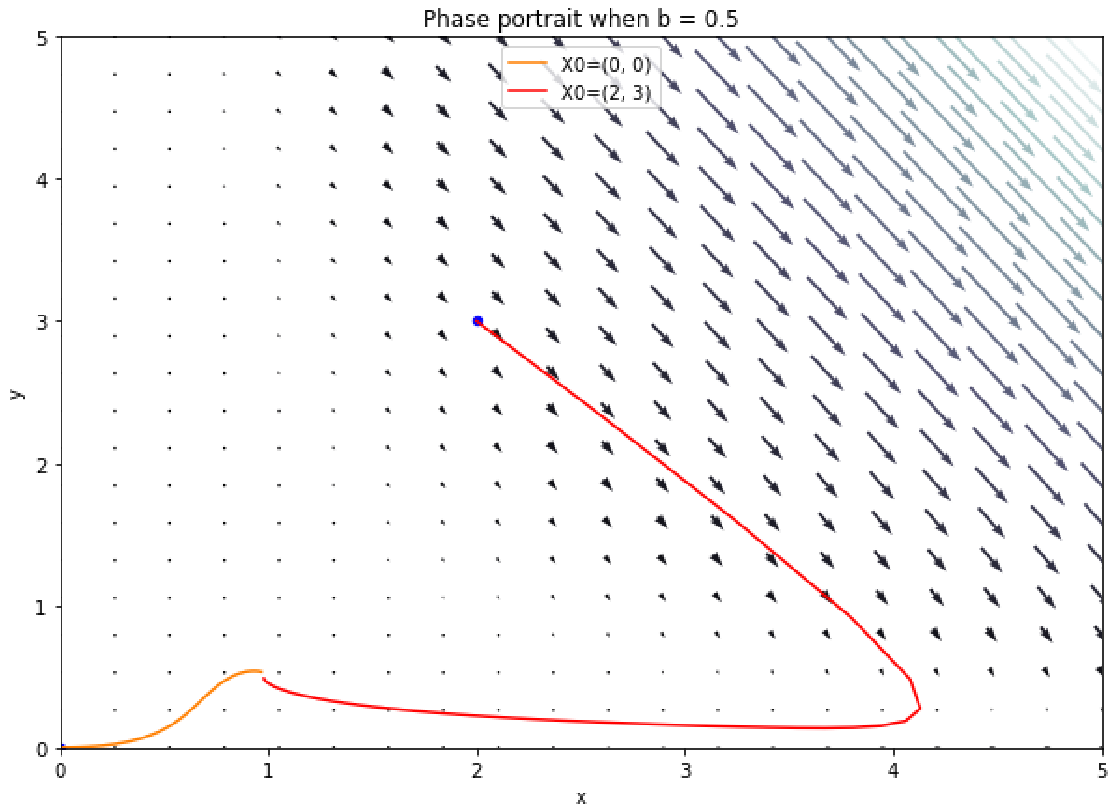
[14]:
```
evals = list(lin_mat.eigenvals().keys())# Eigenvals
evals
```

[14]: $$\left[-\frac{a^2}{2} + \frac{b}{2} - \frac{\sqrt{(a^2-2a-b+1)(a^2+2a-b+1)}}{2} - \frac{1}{2},\ -\frac{a^2}{2} + \frac{b}{2} + \frac{\sqrt{(a^2-2a-b+1)(a^2+2a-b+1)}}{2} - \frac{1}{2} \right]$$

7 Part (b)

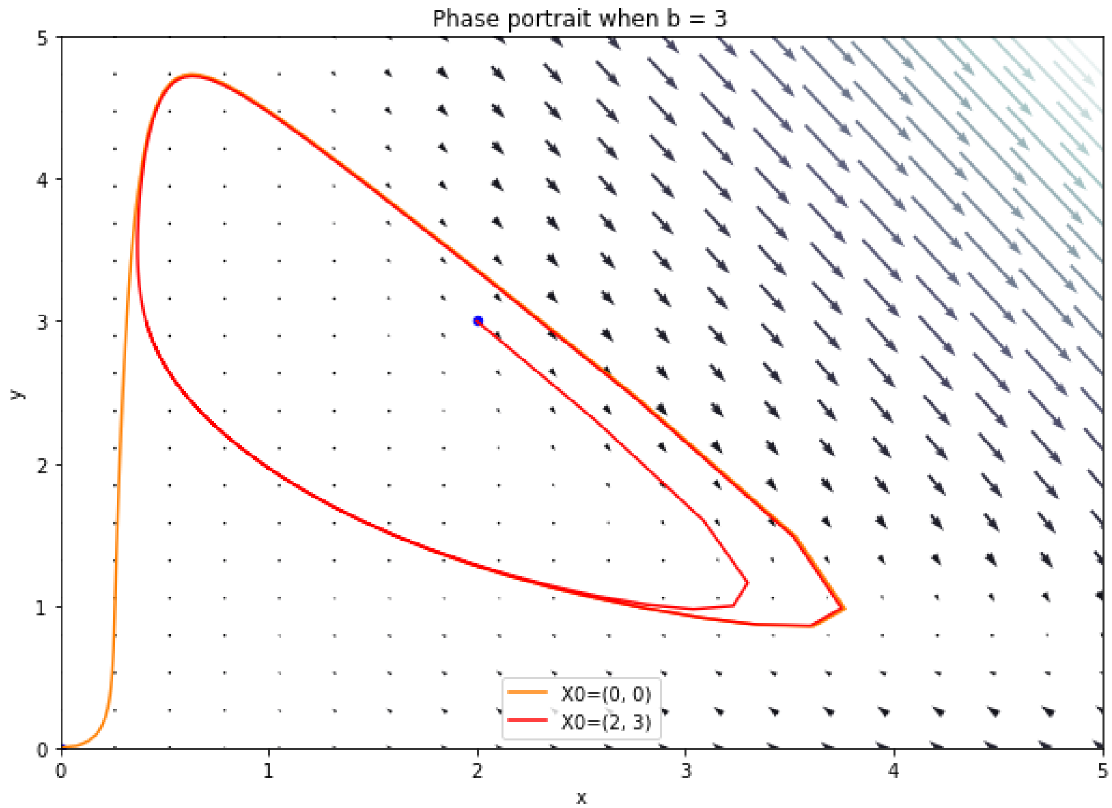
[15]:
```
import numpy as np
from matplotlib import pyplot as plt
from scipy.integrate import odeint
%matplotlib inline

# Define vector field
def vField(x,t):# b is 0.5
```

```
    u = 1 - x[0] - 0.5*x[0] + (x[0]**2)*x[1]
    v = 0.5*x[0] - x[0]**2*x[1]
    return [u,v]

# Plot vector field
X, Y = np.mgrid[0:5:20j,0:5:20j]
U, V = vField([X,Y],0)

# define colours for each vector based on their lengths
M = np.hypot(U, V)

fig, ax = plt.subplots(figsize=(10, 7))
ax.quiver(X, Y, U, V, M, scale=1/0.001, pivot = 'mid', cmap = plt.cm.bone)

# Settings for trajectories
ics  = [[0,0],[2,3]]
durations = [[0,5],[0,5]]
vcolors = plt.cm.autumn_r(np.linspace(0.5, 1., len(ics)))  # colors for each␣
↪trajectory

# plot trajectories
for i, ic in enumerate(ics):
    t = np.linspace(durations[i][0], durations[i][1],100)
    x = odeint(vField, ic, t)
    ax.plot(x[:,0], x[:,1], color=vcolors[i], label='X0=(%.f, %.f)' % (ic[0],␣
↪ic[1]) )

ic_x = [ic[0] for ic in ics]
ic_y = [ic[1] for ic in ics]
ax.scatter(ic_x, ic_y, color='blue', s=20)
# note: you can replace ic_x, ic_y with *list(zip(*ics)) but this is a bit␣
↪cryptic!

plt.xlabel('x')
plt.ylabel('y')
plt.xlim(0,5)
plt.ylim(0,5)
plt.legend()
plt.title('Phase portrait when b = 0.5')

plt.show()
```



[16]:
```
import numpy as np
from matplotlib import pyplot as plt
from scipy.integrate import odeint
%matplotlib inline

# Define vector field
def vField(x,t):#b is 3
    u = 1- x[0] - 3*x[0] + (x[0]**2)*x[1]
    v = 3*x[0] - x[0]**2*x[1]
    return [u,v]

# Plot vector field
X, Y = np.mgrid[0:5:20j,0:5:20j]
U, V = vField([X,Y],0)

# define colours for each vector based on their lengths
M = np.hypot(U, V)

fig, ax = plt.subplots(figsize=(10, 7))
ax.quiver(X, Y, U, V, M, scale=1/0.001, pivot = 'mid', cmap = plt.cm.bone)
```

```python
# Settings for trajectories
ics  = [[0,0],[2,3]]
durations = [[0,10],[0,10]]
vcolors = plt.cm.autumn_r(np.linspace(0.5, 1., len(ics)))  # colors for each␣
↪trajectory

# plot trajectories
for i, ic in enumerate(ics):
    t = np.linspace(durations[i][0], durations[i][1],100)
    x = odeint(vField, ic, t)
    ax.plot(x[:,0], x[:,1], color=vcolors[i], label='X0=(%.f, %.f)' % (ic[0],␣
↪ic[1]) )

ic_x = [ic[0] for ic in ics]
ic_y = [ic[1] for ic in ics]
ax.scatter(ic_x, ic_y, color='blue', s=20)
# note: you can replace ic_x, ic_y with *list(zip(*ics)) but this is a bit␣
↪cryptic!

plt.xlabel('x')
plt.ylabel('y')
plt.xlim(0,5)
plt.ylim(0,5)
plt.legend()
plt.title('Phase portrait when b = 3')

plt.show()
```

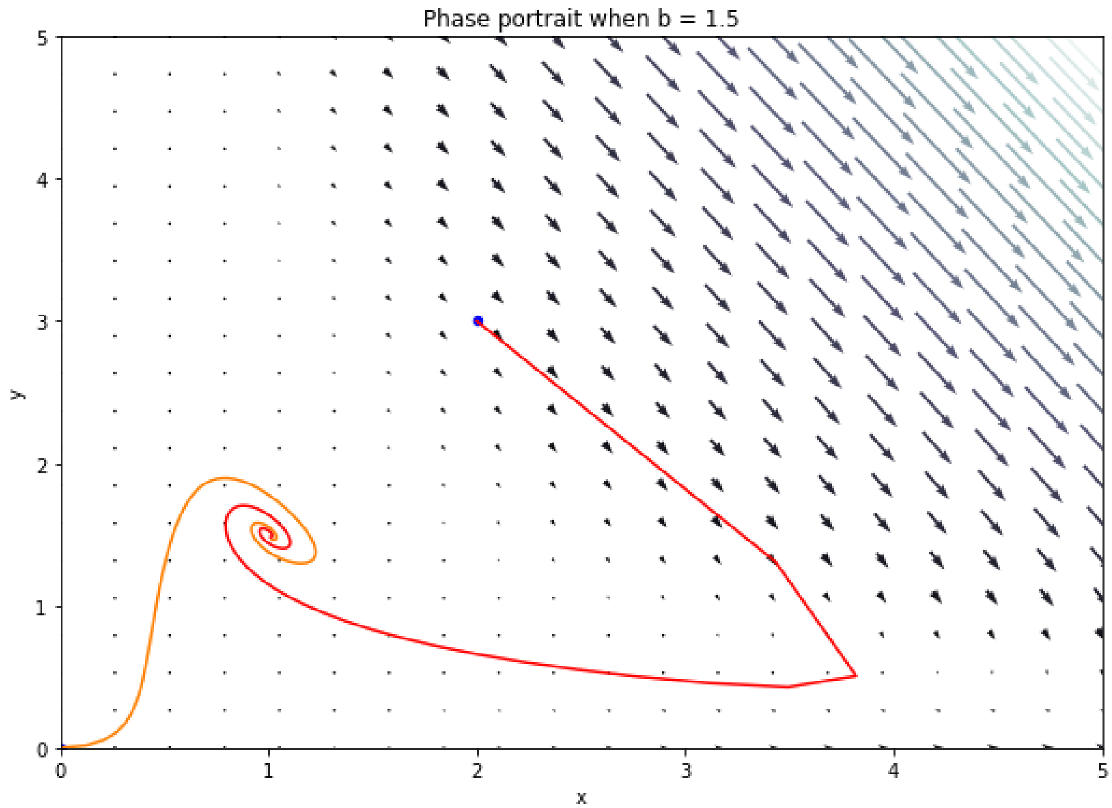


We get a spiral source for 3 and a spiral sink for 0.5

8 Part (c)

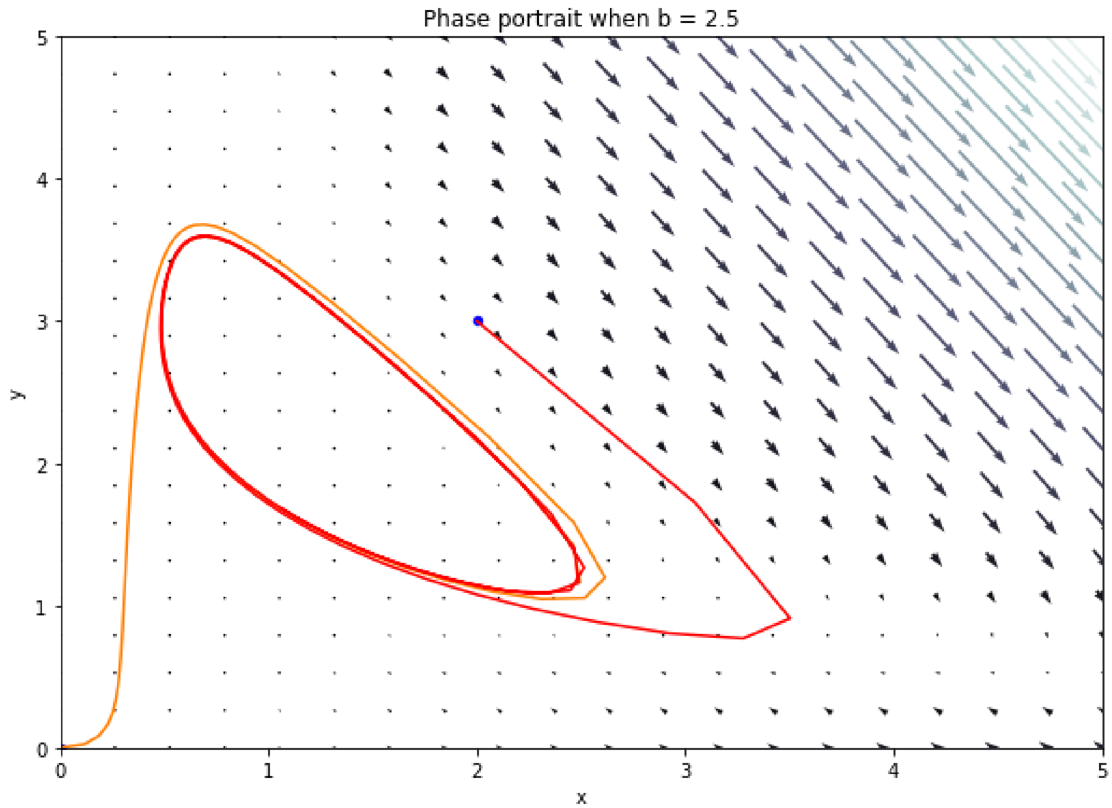
[17]:
```
import numpy as np
from matplotlib import pyplot as plt
from scipy.integrate import odeint
%matplotlib inline

# Define vector field
def vField(x,t): # b = 1.5
    u = 1 - x[0] - 1.5*x[0] + x[0]**2*x[1]
    v = 1.5*x[0] - x[0]**2*x[1]
    return [u,v]

# Plot vector field
X, Y = np.mgrid[0:5:20j,0:5:20j]
U, V = vField([X,Y],0)

# define colours for each vector based on their lengths
M = np.hypot(U, V)
```

```python
fig, ax = plt.subplots(figsize=(10, 7))
ax.quiver(X, Y, U, V, M, scale=1/0.001, pivot = 'mid', cmap = plt.cm.bone)

# Settings for trajectories
ics  = [[0,0],[2,3]]
durations = [[0,15],[0,15]]
vcolors = plt.cm.autumn_r(np.linspace(0.5, 1., len(ics)))  # colors for each␣
↪trajectory

# plot trajectories
for i, ic in enumerate(ics):
    t = np.linspace(durations[i][0], durations[i][1],100)
    x = odeint(vField, ic, t)
    ax.plot(x[:,0], x[:,1], color=vcolors[i], label='X0=(%.f, %.f)' % (ic[0],␣
↪ic[1]) )

ic_x = [ic[0] for ic in ics]
ic_y = [ic[1] for ic in ics]
ax.scatter(ic_x, ic_y, color='blue', s=20)
# note: you can replace ic_x, ic_y with *list(zip(*ics)) but this is a bit␣
↪cryptic!

plt.title('Phase portrait when b = 1.5')
plt.xlabel('x')
plt.ylabel('y')
plt.xlim(0,5)
plt.ylim(0,5)

plt.show()
```




```
[18]: import numpy as np
from matplotlib import pyplot as plt
from scipy.integrate import odeint
%matplotlib inline

# Define vector field
def vField(x,t): # b = 2.5
    u = 1 - x[0] - 2.5*x[0] + x[0]**2*x[1]
    v = 2.5*x[0] - x[0]**2*x[1]
    return [u,v]

# Plot vector field
X, Y = np.mgrid[0:5:20j,0:5:20j]
U, V = vField([X,Y],0)

# define colours for each vector based on their lengths
M = np.hypot(U, V)

fig, ax = plt.subplots(figsize=(10, 7))
ax.quiver(X, Y, U, V, M, scale=1/0.001, pivot = 'mid', cmap = plt.cm.bone)
```

```
# Settings for trajectories
ics  = [[0,0],[2,3]]
durations = [[0,15],[0,15]]
vcolors = plt.cm.autumn_r(np.linspace(0.5, 1., len(ics)))  # colors for each␣
 ↪trajectory

# plot trajectories
for i, ic in enumerate(ics):
    t = np.linspace(durations[i][0], durations[i][1],100)
    x = odeint(vField, ic, t)
    ax.plot(x[:,0], x[:,1], color=vcolors[i], label='X0=(%.f, %.f)' % (ic[0],␣
 ↪ic[1]) )

ic_x = [ic[0] for ic in ics]
ic_y = [ic[1] for ic in ics]
ax.scatter(ic_x, ic_y, color='blue', s=20)
# note: you can replace ic_x, ic_y with *list(zip(*ics)) but this is a bit␣
 ↪cryptic!

plt.title('Phase portrait when b = 2.5')
plt.xlabel('x')
plt.ylabel('y')
plt.xlim(0,5)
plt.ylim(0,5)

plt.show()
```



[19]:
```
evals[0].subs({a:1,b:2.5}) # when b is 2.5
```

[19]:

$0.25 - 0.968245836551854i$

[20]:
```
evals[0].subs({a:1,b:1.5}) # when b is 1.5
```

[20]:

$-0.25 - 0.968245836551854i$

[21]:
```
evals[0].subs({a:1,b:2})# when b is 2
```

[21]:

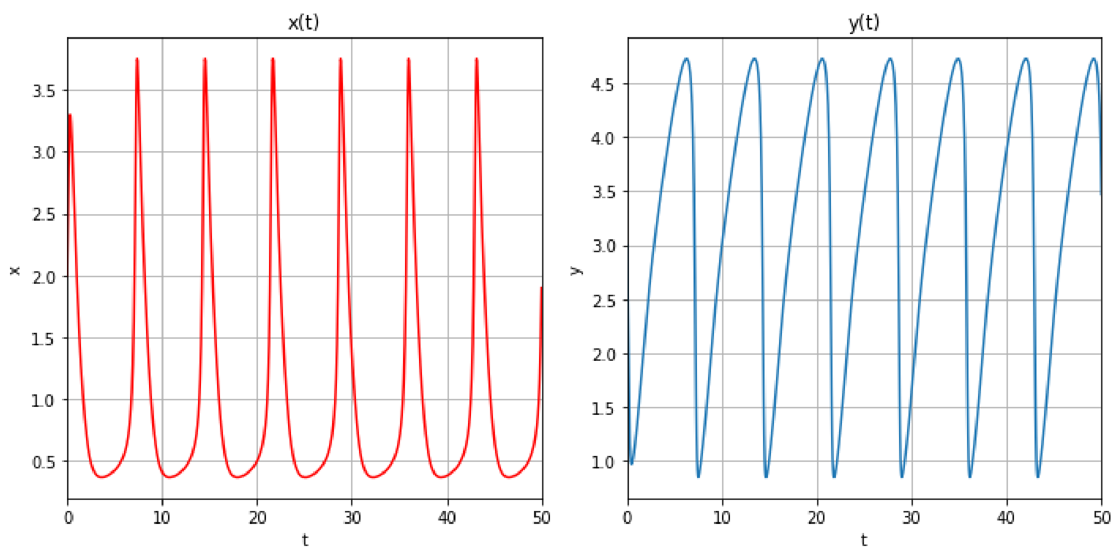
$-i$

From the above result we see that when b is 2 , we only get an imaginary part with a 0 real part and this is where the bifurcation takles place , and hence for values greater than 2 , a spiral source is formed and for values lesser than 2 a spiral sink is formed

9 Part (d)

[22]:
```
fig, axes = plt.subplots(1, 2, figsize=(10, 5))
def dX_dt(X, t):
    x1, x2 = X
    return [1- x1 - 3*x1 + x1**2*x2, 3*x1 - x1**2*x2]
```

```
X0 = [2, 3]
t = np.linspace(0, 50, 1000)
Xsol = odeint(dX_dt, X0, t)

axes[0].plot(t, Xsol[:, 0],'r')
axes[0].set_title("x(t)")
axes[0].set_xlabel("t")
axes[0].set_ylabel("x")
axes[0].set_xlim(0,50)
axes[0].grid(True)

axes[1].plot(t, Xsol[:, 1])
axes[1].set_title("y(t)")
axes[1].set_xlabel("t")
axes[1].set_ylabel("y")
axes[1].set_xlim(0,50)
axes[1].grid(True)

fig.tight_layout()

plt.show()
```



Write your written solution here. You may have to include extra markdown cells.

9.1 Question 3

Please **clearly** indicate where you answer each sub question by using a markdown cell.

10 Part (a)

[23]:
```
### Write your code here. You might want to add extra code cells if you prefer

def ModifiedEuler(vectorField,times,initialConditions):
    n = vectorField(initialConditions,times[0]).size
    x = np.zeros((times.size,n))
    x[0,:] = initialConditions
    for k, t in enumerate(times[:-1]):
        h = times[k+1]-t
        x[k+1,:] = x[k,:]+ h*vectorField(x[k,:]+0.5*h*vectorField(x[k,:],t),t +␣
 ↪0.5*h)
    return x
```

11 Part (b)

[24]:
```
def timesteps(start, stop, h):
    num_steps = math.ceil((stop - start)/h)
    return np.linspace(start, start+num_steps*h, num_steps+1)

def ModifiedEuler_step(vectorField, start, stop, h, ics):
    t = timesteps(start, stop, h)
    x = ModifiedEuler(vectorField, t, ics)
    return x, t

def eqn_dy_dt(y, t):
    return 5*t - 2*y**0.5

import pandas as pd
times = [0,0.1,0.2,0.3,0.4]
values = []
for t in times:
    eq_values, eq_times = ModifiedEuler_step(eqn_dy_dt,0,t,0.05,2)
    values.append(eq_values[-1][0])

pd.DataFrame(data = values, index = times, columns = ["Modified euler ,h = 0.
 ↪05"])
```

[24]:

	Modified euler ,h = 0.05
0.0	2.000000
0.1	1.751741
0.2	1.569657
0.3	1.449553
0.4	1.386962

12 Part (c)

[25]:
```
t = sym.symbols('t')
y = sym.Function('y')
eq1 = sym.Eq(y(t).diff(t), 5*t-2*(y(t)**0.5))
exact = sym.dsolve(eq1, y(t), ics={y(0):2},hint ='best')
exact
```

[25]:

$$y(t) = 2 - 2.82842712474619t + 3.5t^2 - 0.58925565098879t^3 - 0.208333333333333t^4 + 0.0220970869120796t^5 + O(t^6)$$

13 Part (d)

[26]:
```
def exact(t): # returning exact solution excluding the O(t^6)
      return 2 - 2.82842712474619*t + 3.5*t**2 - 0.58925565098879*t**3 - 0.
 ↪208333333333333*t**4 + 0.0220970869120796*t**5

def Euler(vectorField,times,initialConditions):
    n = vectorField(initialConditions,times[0]).size
    x = np.zeros((times.size,n))
    x[0,:] = initialConditions
    for k, t in enumerate(times[:-1]):
        x[k+1,:] = x[k,:]+(times[k+1]-t)*vectorField(x[k,:],t)
    return x

def Euler_step(vectorField, start, stop, h, ics):
    t = timesteps(start, stop, h)
    x = Euler(vectorField, t, ics)
    return x, t

tVals = [0, 0.1, 0.2, 0.3, 0.4, 0.5, 1.0]
me_vals = []
e_vals = []
exact_vals = []

for tval in tVals:
    me_values, eq_times = ModifiedEuler_step(eqn_dy_dt,0,tval,0.05,2)
    e_values, eq_times = Euler_step(eqn_dy_dt,0,tval,0.05,2)


    me_vals.append(me_values[-1][0])
    e_vals.append(e_values[-1][0])
    exact_vals.append(exact(tval))
```

[27]:
```python
import pandas as pd
data = {
    "Euler, h=0.05":e_vals,
    "Modified Euler, h=0.05":me_vals,
    "Exact":exact_vals
}
pd.DataFrame(data = data, index = tVals)
```

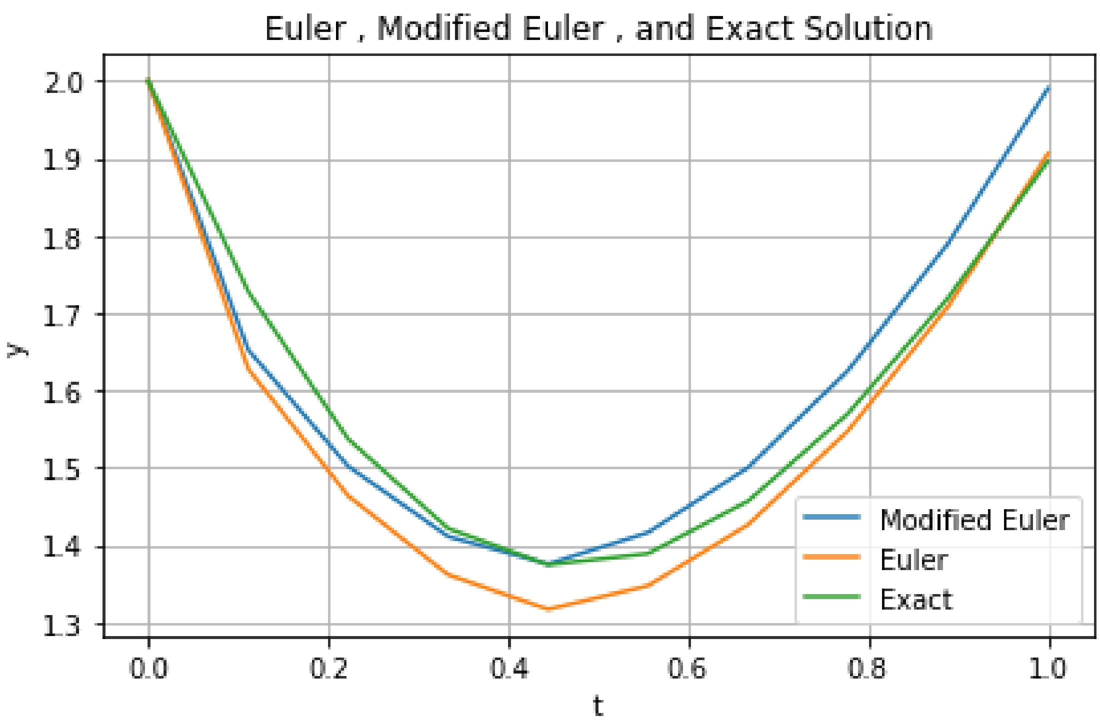
[27]:

	Euler, h=0.05	Modified Euler, h=0.05	Exact
0.0	2.000000	2.000000	2.000000
0.1	1.734749	1.751741	1.751547
0.2	1.537944	1.569657	1.569274
0.3	1.405438	1.449553	1.448928
0.4	1.332687	1.386962	1.385810
0.5	1.314973	1.377377	1.374799
1.0	1.906060	1.990196	1.896081

13.1 Question 4

Please **clearly** indicate where you answer each sub question by using a markdown cell.

[28]:
```python
tVals = list(np.linspace(0,1,10))
me_vals = []
e_vals = []
exact_vals = []

for tval in tVals:
    me_values, eq_times = ModifiedEuler_step(eqn_dy_dt,0,tval,0.05,2)
    e_values, eq_times = Euler_step(eqn_dy_dt,0,tval,0.05,2)


    me_vals.append(me_values[-1][0])
    e_vals.append(e_values[-1][0])
    exact_vals.append(exact(tval))

fig, axes = plt.subplots()

axes.plot(tVals, me_vals,label='Modified Euler',)
axes.plot(tVals, e_vals,label='Euler')
axes.plot(tVals, exact_vals,label='Exact')
axes.set(xlabel='t',ylabel = "y",
       title='Euler , Modified Euler , and Exact Solution')
axes.legend()
axes.grid(True)
fig.tight_layout()

plt.show()
```



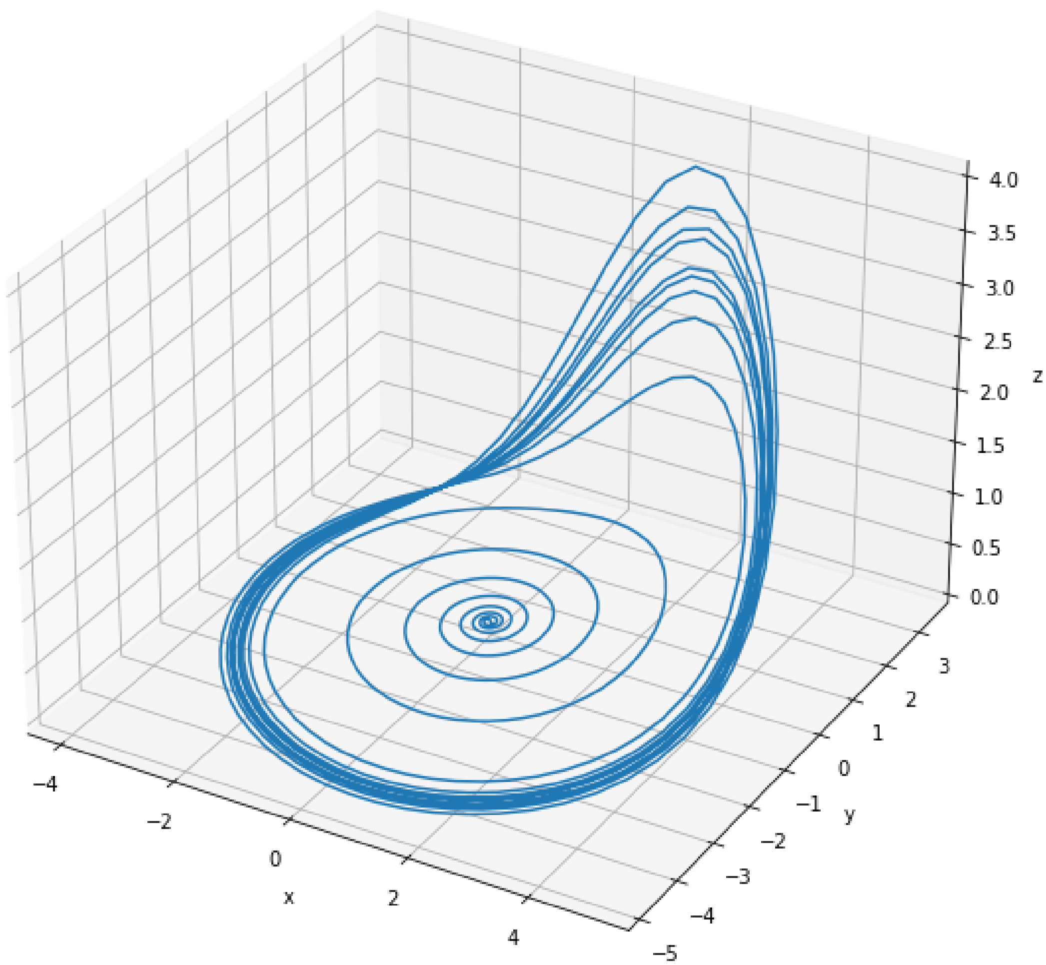
14 Part (a)

[29]:
```python
## Write your code here. You might want to add extra code cells if you prefer

def Rossler_dX_dt(X, t):
    x, y, z = X
    return [-y-z, x+(1/5)*y, (1/5) + (x-(5/2))*z]

X0 = [0, 0, 0]
t = np.linspace(0, 100, 1000)
Xsol = odeint(Rossler_dX_dt, X0, t)
```

Write your written solution here. You may have to include extra markdown cells.

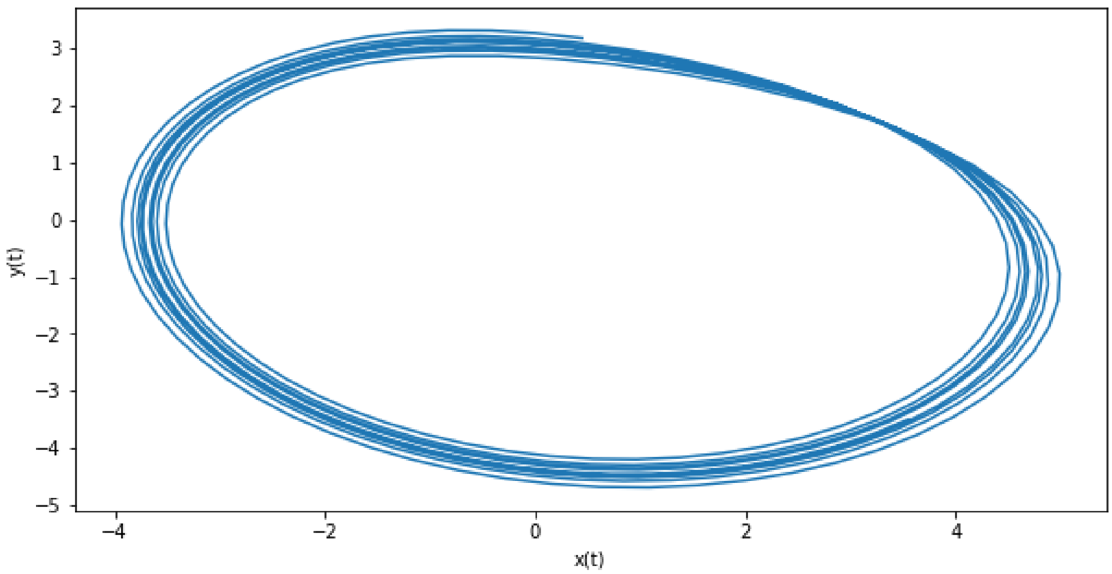
[30]:
```python
from mpl_toolkits.mplot3d import axes3d
import matplotlib.pyplot as plt
import numpy as np

'''
fig = plt.figure(figsize=(10, 10))
ax = fig.gca(projection='3d') #get matplotlib deprecation warning
'''
fig = plt.figure(figsize=(10, 10))
```

```
ax = fig.add_subplot(projection='3d')

plt.plot(Xsol[:, 0], Xsol[:, 1], Xsol[:, 2])

ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')

plt.show()
```



15 Part (b)

[31]:
```
fig = plt.figure(figsize=(10, 5))
ax = fig.add_subplot()

t = np.linspace(0, 100, 1000)
Xsol = odeint(Rossler_dX_dt, X0, t)
plt.plot(Xsol[:, 0][500:],Xsol[:, 1][500:])

ax.set_xlabel('x(t)')
ax.set_ylabel('y(t)')

plt.show()
```

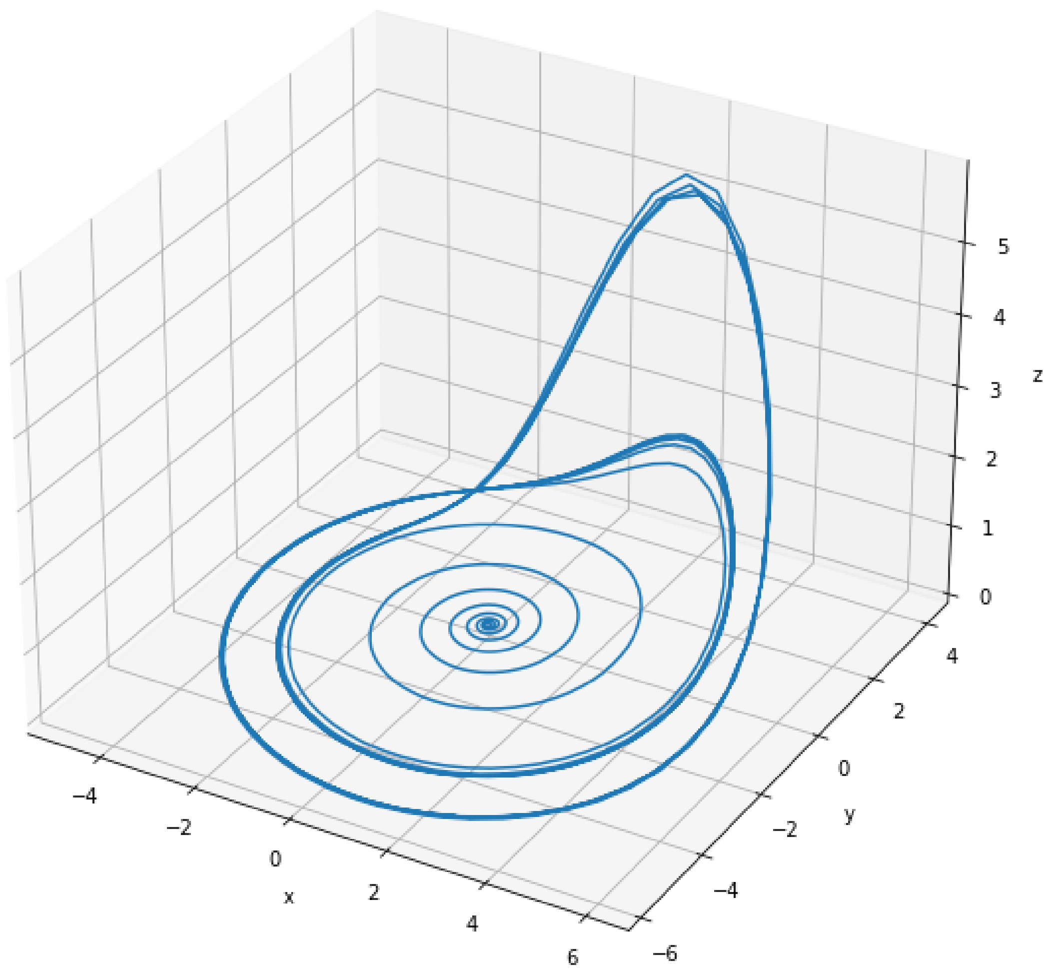


16 Part (c)

[32]:
```
def Rossler_dX_dt(X, t):# when 5/2 is 3
    x, y, z = X
    return [-y-z, x+(1/5)*y, (1/5) + (x-3)*z]

X0 = [0, 0, 0]
t = np.linspace(0, 100, 1000)
Xsol_3 = odeint(Rossler_dX_dt, X0, t)
```

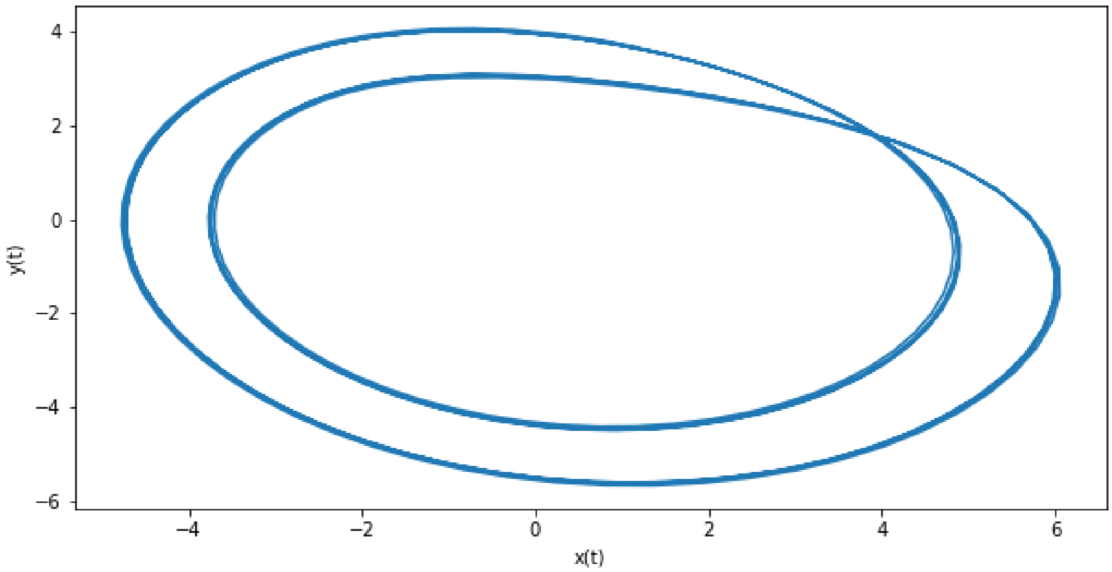
[33]:
```
fig = plt.figure(figsize=(10, 10))
ax = fig.add_subplot(projection='3d')
```

```
plt.plot(Xsol_3[:, 0], Xsol_3[:, 1], Xsol_3[:, 2])

ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')

plt.show()
```



[34]:
```
fig = plt.figure(figsize=(10, 5))
ax = fig.add_subplot()

t = np.linspace(0, 100, 1000)
Xsol = odeint(Rossler_dX_dt, X0, t)
```

```python
plt.plot(Xsol[:, 0][500:],(Xsol[:, 1][500:]))

ax.set_xlabel('x(t)')
ax.set_ylabel('y(t)')

plt.show()
```



[ ]:

```python

```