12/8/24

Band Theory.

3nm gold particle – semiconductor

band gap

metals – orbital overlapping is more – atomic distance (is less)
density more

Q) Why are we doping only semiconductors?

Phones – touchscreen – transparent conducting . . .
Indium tin oxide

mix < 2% → doping
mix > 2% → alloy

more than 2% → properties changed.

$Si \rightarrow 10^{23}$

intrinsic carrier conc. in Si ~ $\underline{10^{10}}$

if we do 10^5 doping, then carrier conc.

* intrinsic → no doping

* $10^{18} - 10^{21}$ → degenerate semiconductor

equal no. of holes & electrons → intrinsic

* if we do both p type & n type doping → compensation doping

metal ~ high carrier conc.

Doping bombardment of ions.

metal - semicond
- transistor

zener → heavily doped.

tunnelling → (i) barrier thin (< 5 nm)
(ii) momentum conserved
after junction/tunnel, unoccupied is there
(iii) finite barrier height

material	Resistivity	Conductivity ($\Omega^{-1} \cdot m^{-1}$)
silver	1.59×10^{-8}	6.29×10^{7}
⋮		
Glass	$10^{10} - 10^{14}$	$10^{-10} - 10^{-14}$

free e^- → not bound to any particular atom

metal ~ bond lengths small, inter-atom spacing is different

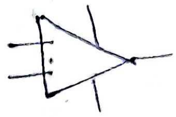
liquid metal - amorphous

solar spectrum peak ~ 1.5eV (1100 Å)
so band gap of less then 1.5 eV - material is suitable

	Band gap (eV)	Tech
Si	1.11	IC, Transistor,
Ge	0.67	Transistor, fiber optics solar cells
Ga As	1.43	Microwave IC, Solar cell
ZnO	3.37	LCD, Transistors, LEDs

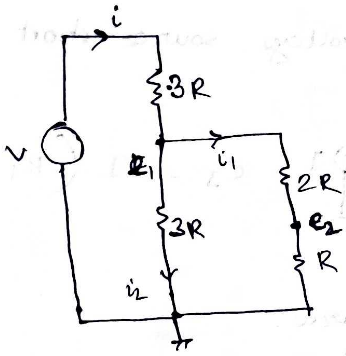
Transistor → ON state, OFF state
conc. of carriers - making it high or low

min 24 transistors



Photoelectric effect ~ very less efficiency

14/8/24

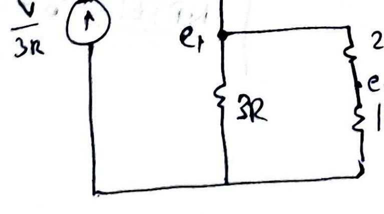


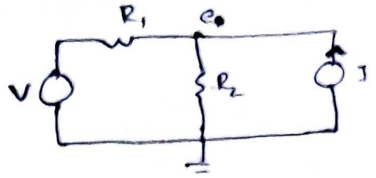
$i = i_1 + i_2$

$$\begin{array}{ccc} G & e & I \\ \begin{pmatrix} \frac{1}{3R}+\frac{1}{3R}+\frac{1}{2R} & -\frac{1}{2R} \\ -\frac{1}{2R} & \frac{1}{2R}+\frac{1}{R} \end{pmatrix} & \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = & \begin{pmatrix} V/3R \\ 0 \end{pmatrix} \end{array}$$

$$\frac{V-e_1}{3R} = \frac{e_1-e_2}{2R} + \frac{e_1}{3R}$$

[If no nonlinear element]





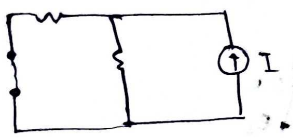
KCL

$$\frac{e-V}{R_1} + \frac{e}{R_2} - I = 0$$

$$e = \frac{R_1R_2}{R_1+R_2} I + \frac{R_2}{R_1+R_2} \cdot V$$

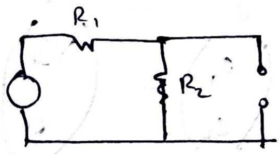
* Superposition ~~principle~~ principle

Part I : effect of current source :
by making voltage source short



$e_I = I \, (R_1 \parallel R_2)$

Part II effect of voltage source .
by making ~~voltage~~ current source ~~short~~ open



$$e_V = V \cdot \frac{R_2}{R_1+R_2}$$

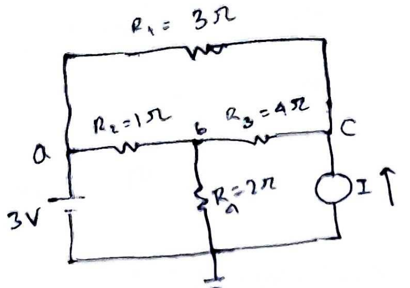
many current source , voltage source

take one current source → making all other current source short & all other voltage source open.

NPTEL Course note

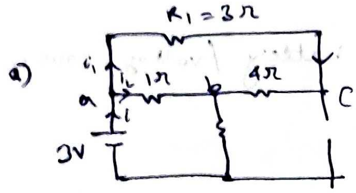
① Solve Newtons law with drag. using python

② 3x3 matrix diagonalisation



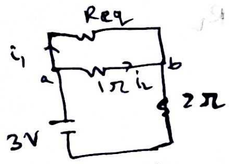
Find $I_{a \to b}$

a) I open b) V short



$$i = \frac{3 - V_b}{1} + \frac{3 - V_c}{3}$$

$R_{eq} = 7\Omega$



$$R' = \frac{7 \times 1}{7+1} = \frac{7}{8} + 2$$

$$\therefore i = \frac{V}{23/8} = \frac{3}{23/8} = 24/23 \text{ A}$$

$$I_{ab} = \frac{7}{8} \times \frac{24}{23} = \frac{R_2}{R_1 + R_2} \times \frac{24}{23}$$

$$I_{ab} = \frac{21}{23} \text{ A} = 0.91 \text{ A}$$

$V_{cg} \sim 2.61$ V

Viva resolution of measurement

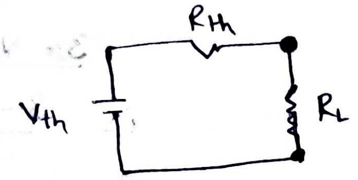
* presentation → make x axis (time scale) from zero.

* LC. & error.

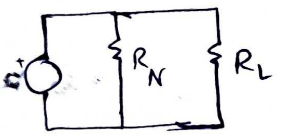
Thevenin Norton.

$\frac{V}{n}\left[\quad n \text{ networks} \quad \right] \frac{I}{n}.$

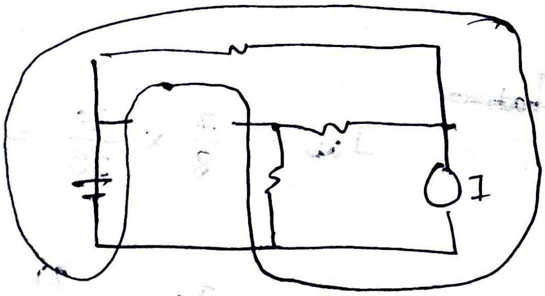
↳ replace networks with battery / voltage source



norton

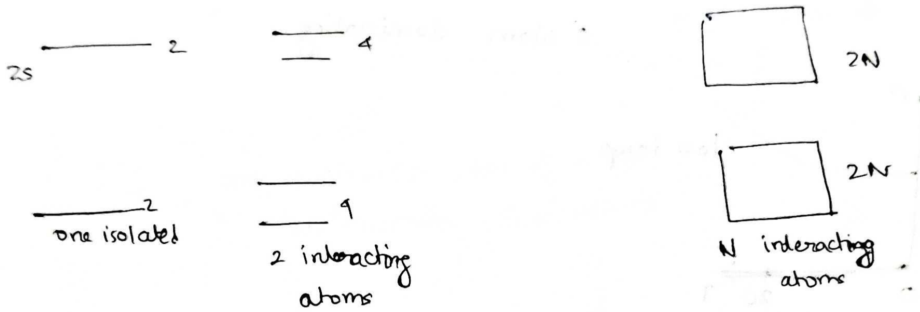


* Remove the load first



19/8/24

Band Theory



Faraday experiment : add more Gold atom - colour changes
electronic transition
variable band gap

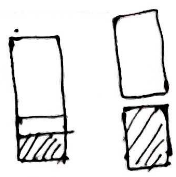
Band gap - consider last filled level.

valence band : band occupied by outermost electrons.

conduction band :

unoccupied states for conduction.

a) metal/conductor
b) Insulator
c) metal
d) semiconductor



(c) (d)

Conductor : VB partially filled
CB overlaps VB.

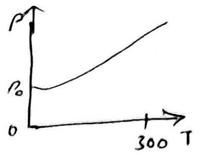
DOS → no. of states per unit energy per unit vol.

Resistivity

$e^- - e^-$ collisions

e^- - atoms.

defects, impurities



T↑ – KE↑

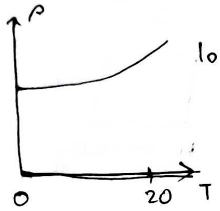
mean free path decreases

$\frac{1}{n\sigma}$

cross section of atom is more

Prob of collision is less for $e^- - e^-$ compared to e^- - atom

e^- atom dominating

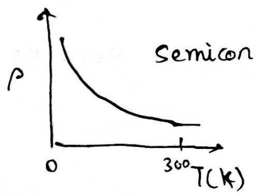


low temp

T↓ KE.↓ (low)

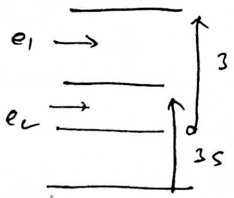
collisions ↓

— almost flat — depends on defects, impurities



semiconductor

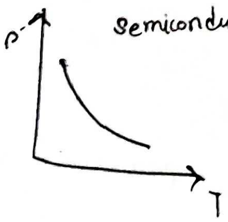
ρ_0 → how much min- mean free path
→ defects



3p e^- move more freely

effective mass less

→ depends on electronic structure



semiconductor

carrier conc. ↓ with T↓

e^--hole pair – exciton binding energy

intrinsic carrier conc.

depends on band gap

Ge, band gap is less.

carriers ↓ conductivity ↓

FD.
↳ prob. of occupancy
↓
carrier conc. → DOS × (FD stat)

T↓ Prob. ↓

2D only 2 direction, lot of atoms
other direction, few atoms.

H.W DOS for 1D, 2D, 3D,

DOS

DOS = no. of states per unit energy per unit vol.

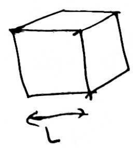
N atoms → N orbitals — 2N states

unit $J^{-1} m^{-3}$ or $eV^{-1} cm^{-3}$

Phase transition from bulk to nanomaterial

Expression for g(E)
scanning tunneling microscope

Assumption uniform potential V = 0,



$$E = \frac{h^2}{8mL^2}(n_x^2 + n_y^2 + n_z^2)$$

$$n^2 = n_x^2 + n_y^2 + n_z^2$$

$$n_x, n_y, n_z > 0$$

no. of states

$$S = (\text{vol. of sphere}) \cdot \frac{1}{8}$$

$$= \frac{4}{3}\pi n^3 \cdot \frac{1}{8} = \frac{1}{6}\pi n^3$$

$$n = \left(\frac{8EmL^2}{h^2}\right)^{1/2}$$

spin degen → $S(x) = \frac{\pi}{3} n^3$

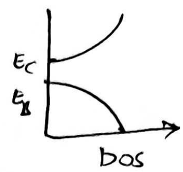
$$S(E) = \frac{\pi}{3}\left(\frac{8mEL^2}{h^2}\right)^{3/2}$$

$\text{Vol.} = L^3$

$$S_v(E) = \frac{\pi}{3} \cdot \frac{(8mE)^{3/2}}{h^3}$$

$$g(E) = \frac{dS_v(E)}{dE}$$

$$\boxed{g(E) = 8\pi\sqrt{2}\left(\frac{m}{h^2}\right)^{3/2}\sqrt{E}}$$ 3D solid



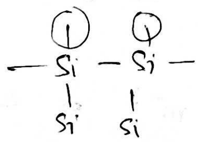
$$g(E) = \frac{\pi(8m)^{3/2}}{3\,h^3} \cdot \frac{d}{dE} \cdot \left(E^{3/2}\right)$$

BZ & Band structure for 1D & 2D.

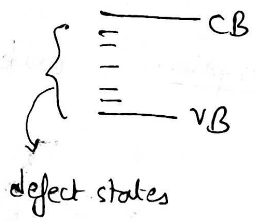
<u>Semiconductor</u>

Amorphous
no long range order
↓
(amorphous silica glass)

Crystalline
(order in 3D direction)



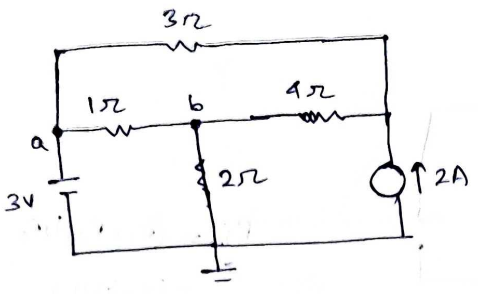
Amorphous - more dangling bonds



- Tune the carriers in semiconductor using doping

defects → carrier conc.

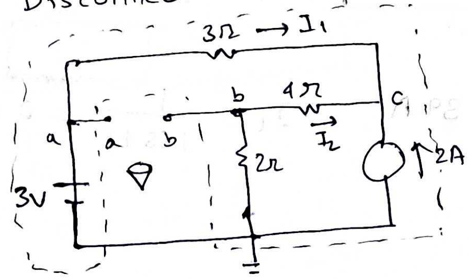
21/8/24.



Principle of Thevenin - Norton

Using

Step 1. Disconnect R_L

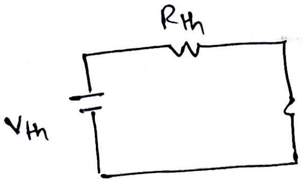


Step 2 At node C., $2 + I_1 + I_2 = 0$

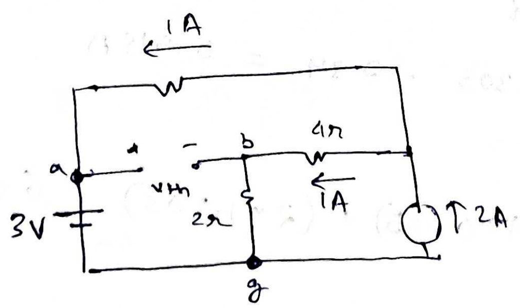
$$2 + \frac{3 - V_c}{3} + \frac{0 - V_c}{6} \Rightarrow V_c = 6V$$

$I_1 = 1A$ from 'c' to 'a'

$I_2 = 1A$ from c to b to g

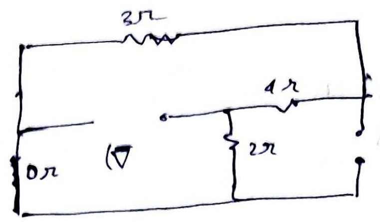


Step 3



$V_{th} = 3 - (2 \times 1) = 1V$

Step 4   Replace all sources by internal resistance



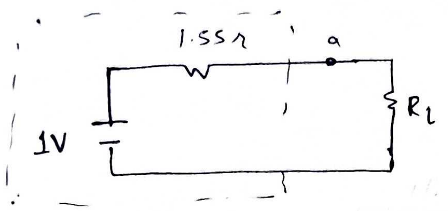
$R_{Th} = (R_1 + R_3) \| R_4$

$= (3+4) \| 2$

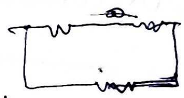
$= \frac{14}{9}$

$= 1.55$

Step 5

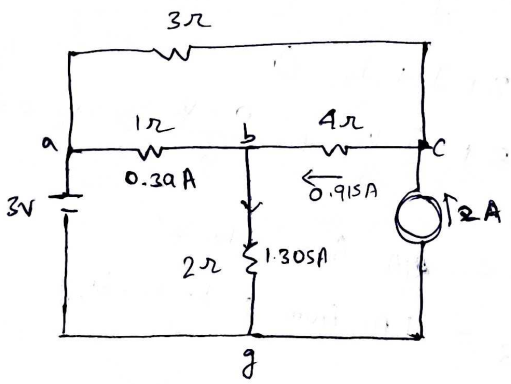


$I_L = 0.39\text{ A}$



$I_L = \frac{1}{1.55+1} = 0.39\text{ A}$

Step 6



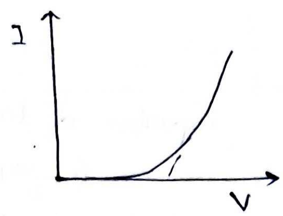
$V_{bg} = 3 - 1 \times 0.39 = 2.61\text{ V}$

$I_{bg} = \frac{2.61 - 0}{2} = 1.305\text{ A}$

$I_{cb} = 1.305 - 0.39 = 0.915\text{ A}$

$V_{cg} = (4 \times 0.915) + (2 \times 1.305) = 6.27\text{V}$

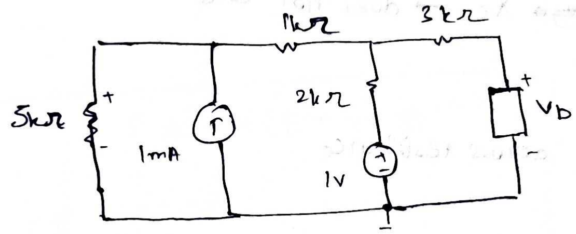
Diode resistance.



non linear resistor

Tried to plot

x axis ln I

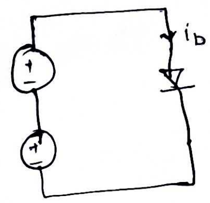


$i_D =$

MOSFET

kV^2

AC part amplified



Diode current.

$$i_D = I_s\left(e^{(0.7V + 0.001\,V\sin(\omega t))/V_{Th}} - 1\right).$$

$$i_D = I_s\left(e^{(V_D + \Delta V_D)/V_{Th}} - 1\right)$$

Aggarwal.
Pg 212-220.

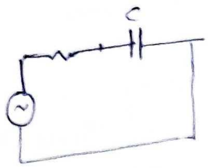
DC, high freq. AC amp.

Eg. 4.20

max power transfer

Filter circuits

R, C



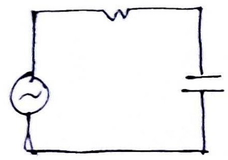
capacitor dc impedance
AC open
divider circuit

$X_C \; \frac{1}{2\pi f C}$

$f \downarrow \rightarrow$ ~~low~~ high X_C → does not conduct

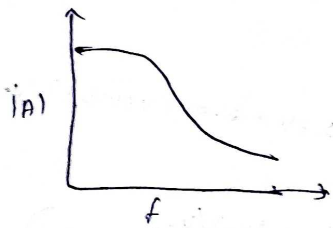
~~low pass~~

measure across resistance

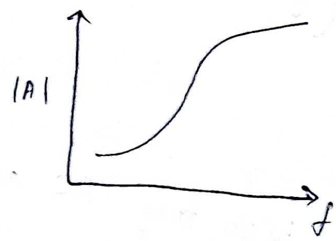


band pass.

low pass



high pass

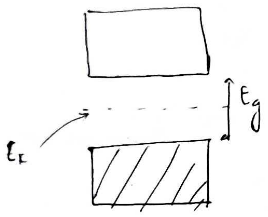


Low pass of high pass filter in ~~parallel~~

Intrinsic semiconductor

Fermi energy - energy reqd to remove an e^- from surface of metal to vacuum level
(sphere of influenc)

E_F
↳ defn change for semiconductor
2 type of carriers.



Intrinsic
E_F — nearly at centre. (there is offset

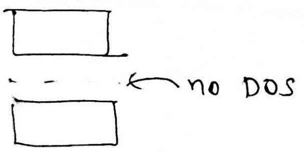
m^* is more for holes.

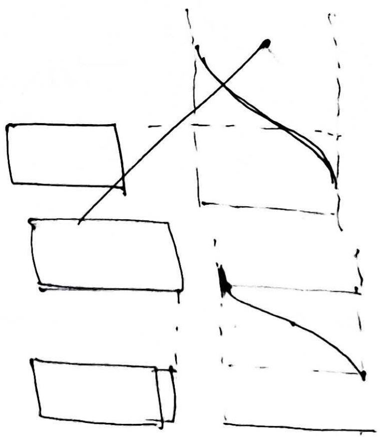
no. of holes = no. of e^-

law of mass action $n \times p = n_i^2$
at given temp.

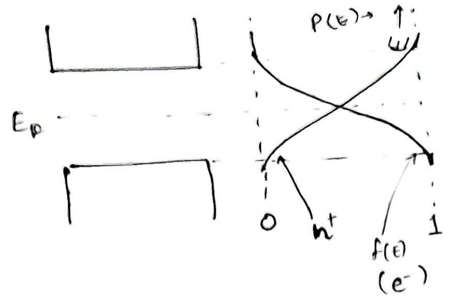
* n type $\xrightarrow{to}$ Intrinsic. (by changing temp)

more e^-, n type E_F, closer to CB → effective position, COM
p type E_F closer to VB





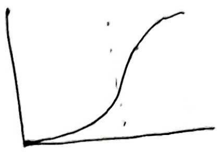
Semiconductor
↳ FD
independent of DOS.



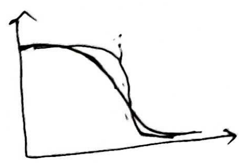
$e^- \rightarrow f(E)$

$h^+ \rightarrow 1 - f(E)$

$P(E) = 1 \longrightarrow$ filled



recombination ⇝ e^--hole gives $\underline{\gamma}$ photon



generation & recombination takes place multiple times

$$\mu_e = \frac{e\tau_e}{m_e^*}$$

scattering time ↑

$$\mu_h = \frac{e\tau_h}{m_h^*}$$

μ_e is less.

$$\sigma = ne\mu_e + pe\mu_h$$

At temp T

$$\frac{3}{2}kT = \frac{1}{2}m_e v_{th}^2$$

$\tau_e \sim 2 \times 10^{-13}\ s$

$$v \sim 1.16 \times 10^3\ m/s.$$

$v \times \tau_e \rightarrow$ distance $\underline{\sim 23\,nm}$ (without scattering)

Scattering ↓ length ⟵ ~~Temp~~ Temp ↓

⟵ $\underline{\text{dopants, conc.}}$

⟵ defects

$\underline{\text{Carrier conc.}}$

no. of carriers per unit vol.

$$n = \int_{band} g(E)\, f(E)\, dE$$

$\chi \rightarrow$ E.A

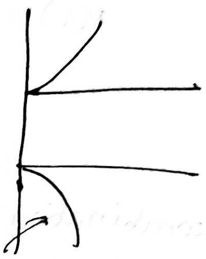
$$n = \int_{E_c}^{E_c+\chi} g(E)\, f(E)\, dE$$

In principle $\to \int_0^\infty g(E)\cdot f(E)\, dE$

$$g_{CB} = \frac{8\pi\sqrt{2}}{h^3}\cdot(m^*_e)^{3/2}(E - E_C)^{1/2}$$

$$n = N_C\, e^{-\frac{E_C - E_F}{kT}}$$

$$N_C \sim 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$



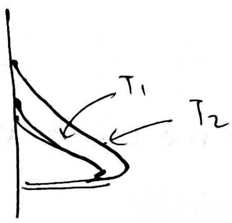
$N_C \to$ effective DOS at CB edge.

$$g = (\ \ )\,(E_V - E)^{1/2}$$

$$\boxed{n_i = \sqrt{N_C N_C}\; e^{-E_g/2kT}}$$

Si $\quad T\uparrow$ more scattering $\to \sigma \downarrow$

$$\begin{cases} m_e^* = 0.7\, m_e \\ m_h^* \sim 1.4\, m_e \end{cases}$$



$T_2 > T_1$

$T_2 \to$ less sharp

$\underline{n = p = n_i} \to$ get E_{Fi}

28/8/24

Electronics

Y Liao Microwave devices

lossless ~~Lossy~~ dielectric $\sigma = 0$

lossy media $\rightarrow \sigma \neq 0$

$\hookrightarrow \nabla^2 E = j\omega\mu(\sigma + j\omega\epsilon)E$

3 ~~med~~ lyres

find E_x, H_y

$\gamma = \sqrt{\cdot}$

Good conductor

$\sigma >> \omega\epsilon$

conductivity

δ skin depth.

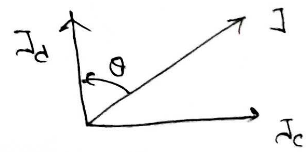
$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

propagation constant $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

Gold anticorrosive, silver best conductor, copper can rust

skin depth for poor conductor $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

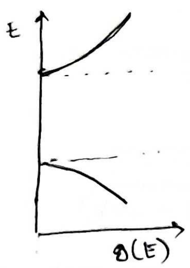
loss tangent $\rightarrow \frac{\sigma}{\omega\epsilon}$



phase shift of capacitive & resistive part

	Seawater	Dry sand	~~Conductor~~ Concrete
σ	4	2×10^{-4}	2×10^{-5}
ϵ_r	20	4	3

2/9/24



intrinsic $n = p = n_i$

$$n_i = \sqrt{N_c N_v}\, \exp\left(-\frac{E_g}{2kT}\right)$$

E_g dependent on T

→ $e^{-(\,)}$ depends on T and dominates

T ↑ vibrational energy ↑ oscillates more

band gap ↓

Ge – highest carrier conc. – low bandgap

GaAs – lower carrier conc. – higher bandgap

intrinsic → Si – 10^{10} room temp.

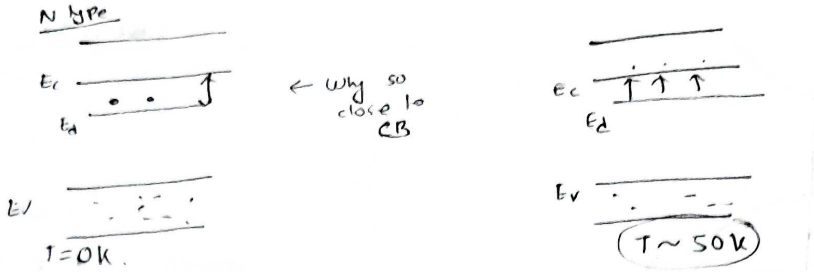
we need 10^{15} → so from 300 K → 600 K

III col element B, Al, Ga → ptype	V col. elements P, As, Sb.

Q) when doped, does it become extrinsic immediately

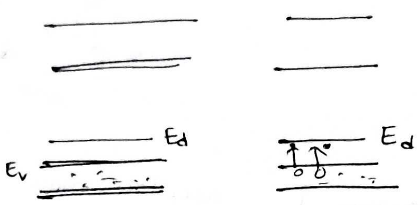
No. Not immediately at 0 K or 20 K.

N type



need to give sufficient energy to make extra electron available.

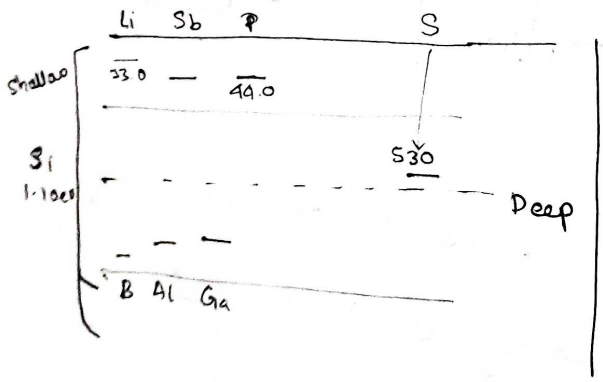
P type



Band → huge DOS

Q) Why we are not choosing group 2 (like Mg, Ca) or group VI

Ans: It forms defect levels.



Li, Sb, P
energy required is less.

consider shallow - focus on lowest possible energy
avoid deep level defect.

Ionisation energy for extra electron from P.

P^+ with extra e^- approximate as hydrogen with one e^-

N type.

$$E_b = \frac{-m_e^* e^4}{8 \varepsilon_0^2 \varepsilon_r^2 h^2}$$

B atom sites every 10^6 Si atoms

10^{23} Si

$\underline{\underline{10^6}}$

$\therefore$ dopant $\rightarrow \frac{10^{23}}{10^6} \rightarrow \underline{\underline{10^{17}}}$

$N_D \rightarrow$ Density of donor atoms used for doping

$np = n_i^2$ intrinsic

<u>N type material</u>

$n \sim N_D$

$N_D >>> n_i$ $\quad p \sim \frac{n_i^2}{N_D}$

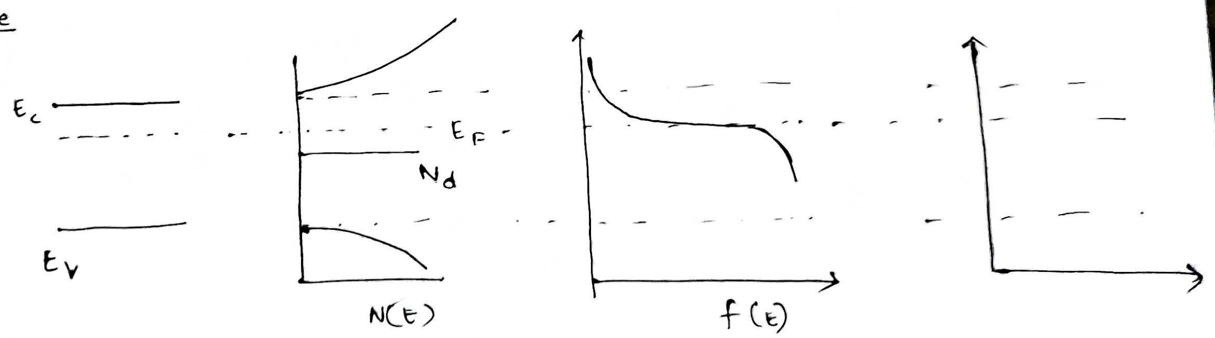
<u>P type material</u>

$p \sim N_A$

$n \sim \frac{n_i^2}{N_A}$

$N_A >>> n_i$

<u>N type</u>



Assignment

Determine E_{Fn} & E_{Fi} for n type & p type semiconductors wrt intrinsic semiconductor, Fermi level position

$$n = N_c \exp\left(-\frac{(E_c - E_{Fn})}{kT}\right)$$

$$n_i = N_c \exp\left[-\frac{(E_c - E_{Fi})}{kT}\right]$$

T func.

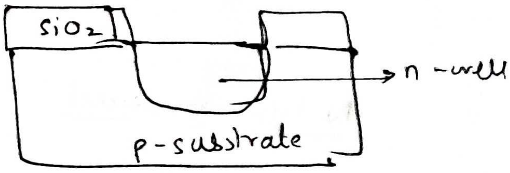
Impurity doping

- Diffusion
- ion implantation.

Hall effect

Compensation doping

p type - chargeless



n-well $\begin{cases} Si \sim 10^{15} \\ p\ type \sim 10^{17} \\ n\ type \sim 10^{19}\ /cm^3 \end{cases}$

Seawater	Dry sand	Concrete
high conductivity dissolved salts High dielectric strength varies with salinity and temp. polar water molecules and ions	lacks free ions & moisture low conductivity Low dielectric constant absence of water non polar nature of sand	~~can~~ vary with moisture content & presence of conductive additives ~~diele~~ ε_r higher than that of dry sand.

need to know dielectric const - developed with growth of radar

measurement - using coaxial transmission line, using reflection technique.

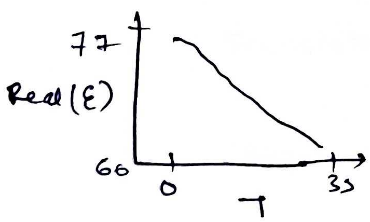
reflection, transmission, waveguide transmission

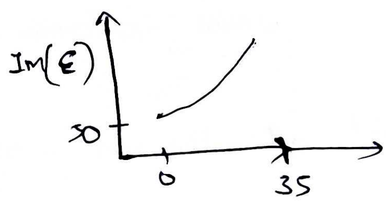
~~measured~~ seawater b/w 9.3 & 48.2 GHz [1974]

resonant cavity - permittivity from shift in reso freq. & change in Q factor

(seawater introduced into cavity small)

$$\varepsilon' - 1 = 2C\left(\frac{\Delta f}{f}\right) \qquad \varepsilon'' = C\Delta\left(\frac{1}{Q}\right)$$





$\varepsilon = K \cdot \varepsilon_0$

identical, dimensionless

$K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$

- Ionic Content
- Moisture content
- material density (with polar molecules)

7/9/24

H.W. Plot σ_f (eq 2-6-1)

Thin film - surface resistance

$$R_s = \frac{1}{t\sigma_f} \ \Omega/\square$$

$\square \rightarrow$ per sq. of length

few atomic layer

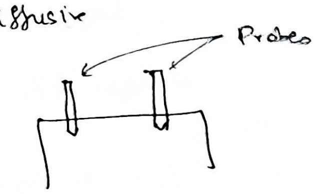
$t << p$

ballistic transport

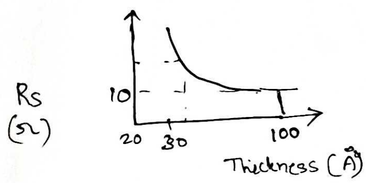
not diffusive

→ Attenuation

$= 40 - 20\log(R_s)$ dB



- R_s vs thickness for gold film on substrate.



Mesoscopic Quantum Transport

Supriyo Dutta.

Eg. 2-6-5, 2-6-6.

calculations.

Ch-2.

Assignment Problems 2-11 to 2-19

8(2-18) → take some MHz f.

Transistor - dependent source

superposition - take out the 'independent source'

9/9/24

compensation doping
→ p-n junction made

intrinsic – thermally generated carriers

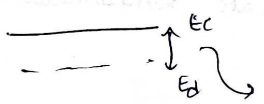
doping conc. > intrinsic carrier conc.

$n_i \gg n$
(intrinsic – dominating)

$n_i \sim 10^{10}$

doping $> 10^{10}$

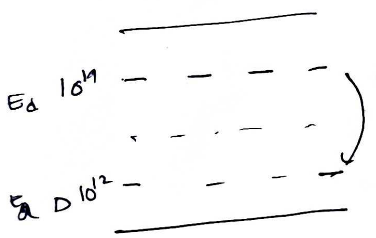
$\left.\begin{matrix} N_d \\ N_a \end{matrix}\right\}$ conc. of dopants



$T > T_0 \qquad E_c - E_d \simeq kT_0$

Then e^- go from E_d to E_c

Temp reqd.



trapped ⟶ only from donors

charge neutrality

$n_0 + N_a^- = p_0 + \underline{N_d^+}$

Nd → how much doped

$N_d^+ = (N_d - n_d)$
↓
unionised.

Q) $N_D = 6 \times 10^{16}$

$N_A = 3 \times 10^{16}$ compensated Doped.

what is type of semicond.? ~~Type of~~

find out carrier conc.

Ans: $N_D > N_A$ ⇒ n type.

$$n_0 = \left(\frac{N_D - N_A}{2}\right) + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

density of Si $\sim 10^{23}$

$n_i \simeq 10^{10}$

$n \simeq 3\times10^{16}$ $\quad n_i \sim 10^{10}$

$$p = \frac{10^{20}}{3\times10^{16}} = \frac{1}{3}\times10^{4} = 3.3\times10^{3}$$

Degenerate.

E_F inside CB for n type.

→ happens at typical dopant conc. of 10^{19} & $10^{20}\ cm^{-3}$ then degenerate semiconductor.

every extrinsic ⟶ intrinsic.

① by doping

or

② increase temp., intrinsic carrier ↑

n type $\sigma = ne\mu_e$

p type $\sigma = pe\mu_h$

Scattering time $\tau = \dfrac{1}{S\, v_{th} N_s}$ $\quad v_{th}$ - mean speed of e^- (thermal vel.)

N_s - no. of scatterers (or scattering events) per unit vol.

$$\mu_e = \frac{e\tau_e}{m_e^*}$$

$$\mu_h = \frac{e\tau_h}{m_h^*}$$

$\times 10^8$

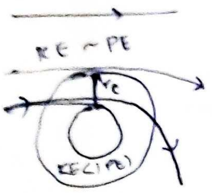
Extrinsic

$$\frac{3}{2}kT = \frac{e^2}{4\pi\epsilon_0\epsilon_r r_c} \qquad r_c \propto T^{-1}$$

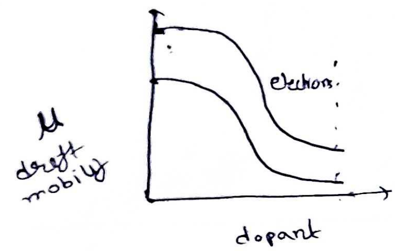
$$S = \pi r_c^2 \propto T^{-2}$$

scattering time due to impurities

$$\tau = \frac{1}{S v_{Th} N_s} \qquad \tau_I = \frac{1}{(\pi r_c^2) \cdot v_{th} N_I} \propto \frac{1}{T^{-2} T^{1/2}} \propto T^{3/2}$$



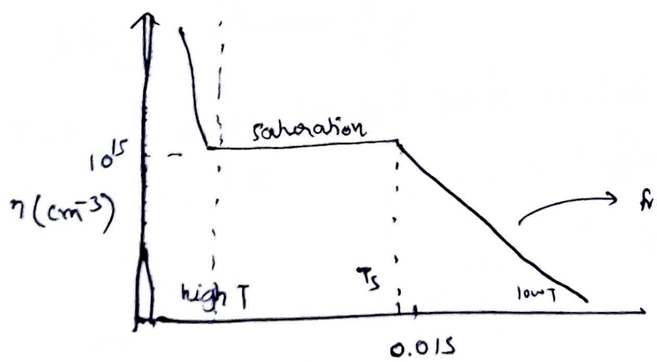
e^- mobility & hole mobility is due to movement of e^-



compromise mobility by doping

dope - charge carrier conc. - modulate - we can have on & off
0 & 1

so we used doped in devices

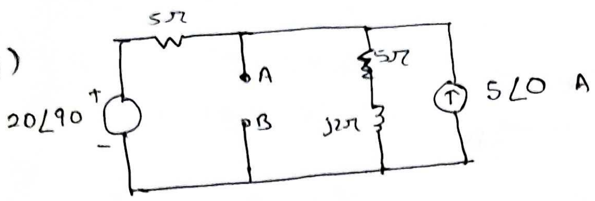


from T_s to high T, extrinsic carrier do not increase
intrinsic increases
from 10^9 to 10^{15}

from low T (0 to T_s), extrinsic carrier ↑
90% ionised.

Superposition - Thevenin - Norton

Lec 4
(Tille academy)

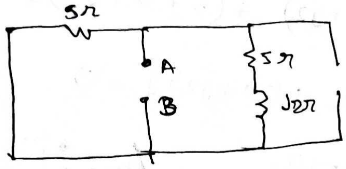


R_{th} or Z_{th}

V_{th}

Then Thevenin circuit

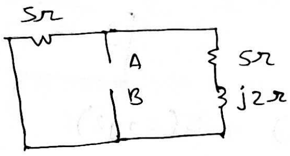
Z_{th}



→ current source ← open

short circuit voltage source

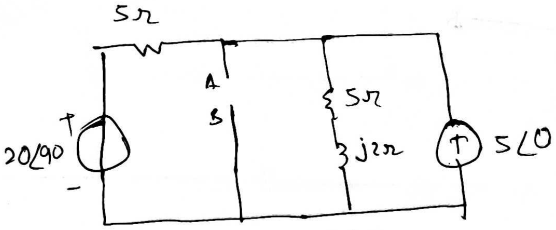
↓



$$Z_{th} = \frac{5 \times (5+j2)}{5+(5+j2)} = \frac{25+10j}{10+j2} = \frac{\sqrt{725}\angle 21.8}{\sqrt{109}\angle 11.3}$$

$$= 2.64\angle 10.5$$

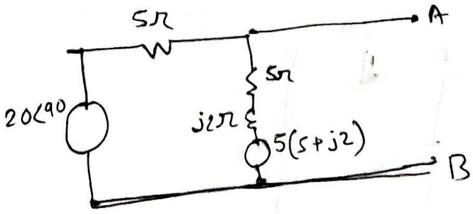
V_{th}

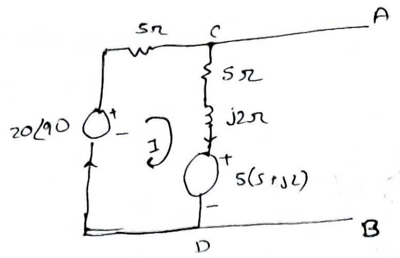


current source → voltage source

V_s = (current source) × (Z parallel to I source)

$$V_s = (5\angle 0)\,(5+j2) = 5\,(5+j2)\,V$$





$V_{CD} = V_{AB}$

Apply KVL, I enters −ve terminal for 20∠90
so voltage source −ve

$$-20\angle 90 + 5(5+j2) + (5+5+j2)I = 0$$

$$\Rightarrow (25 + 10j - 20j) + (10+j2)I = 0$$

$$\Rightarrow I = \frac{-25-10j}{10+j2} = \frac{\sqrt{725}\angle}{\sqrt{104}\angle}$$

625
100

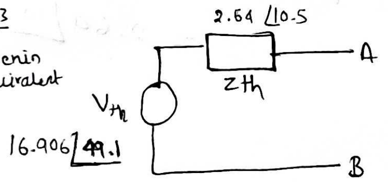
$$\boxed{I = 2.64\angle 146.8}$$

$$V_{th} = I(5+j2) + 5(5+j2)$$

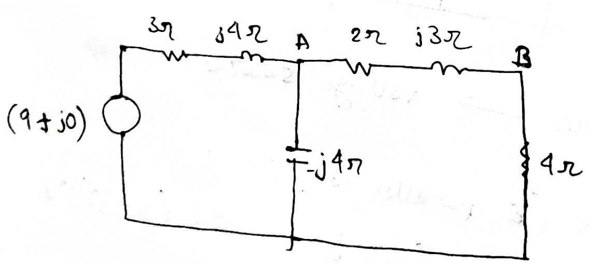
$$= 16.906\angle 49.1 \text{ V}$$

Step 3

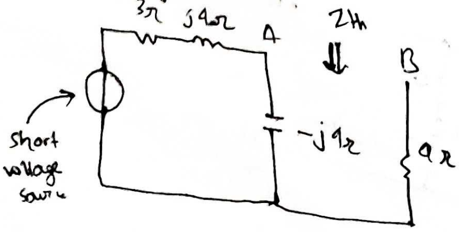
Thevenin equivalent



Lec 5



Calculation of Z_{th}



$$Z_{th} = \left[(3+j4) \parallel (-j4)\right] + 4$$

$$= \frac{(3+j4).(-j4)}{(3+j4) - j4} + 4$$

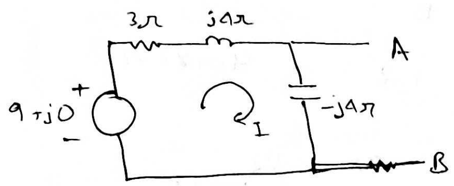
$$Z_{th} = \frac{16-12j}{3} + 4$$

$$Z_{th} = 9.33 - 4j = \underline{10.15 \angle -23.19\,\Omega}$$

Step 2

V_{th} ~~AB~~ across open terminal AB

6.25
1.00

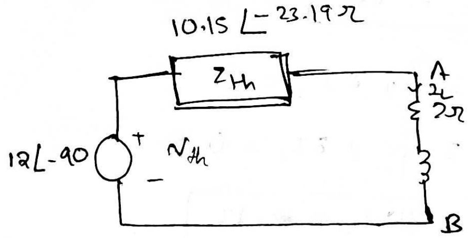


$$-(9+j0) + (3+j4+j4)\,I = 0$$

$$\Rightarrow 3I = 9 \qquad \boxed{I = 3}$$

$$V_{th} = I.(-j4\Omega) = 12 \angle -90 \text{ V} \quad \text{or} \quad -12j \text{ V}$$

Step 3



$$I_L = \frac{V_{th}}{Z_{th} + (2+j3)} = \frac{12\angle -90}{10.15\angle -23.19 + 2+j3}$$

$$I = 1.054 \angle \quad \text{A}$$

$$\frac{12\angle -90}{11.32 + -3.996j + 3j}$$

$$= \frac{12\angle -90}{11.32 - 0.996j}$$

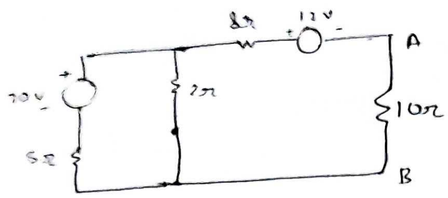
$$= \frac{12\angle -90}{11.36\angle\theta}$$

$$I = 1.05\angle\theta_2$$

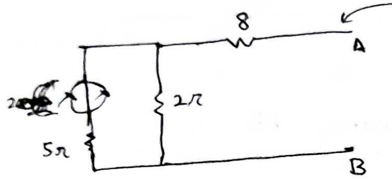
Norton Theorem

Z_{th}, I_N

1)



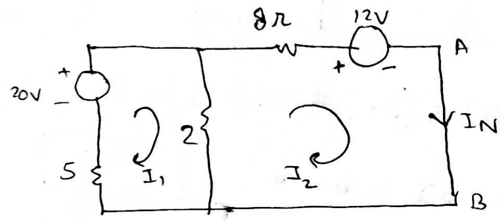
short circuit   V source



step 1 $R_{th} = \frac{5\times 2}{7} + 8 = 9.428\,\Omega$

Step 2

calculate I_N



loop ① $-20 + 5I_1 + 2(I_1 - I_2) = 0$

$$\boxed{7I_1 - 2I_2 = 20}$$

loop ②

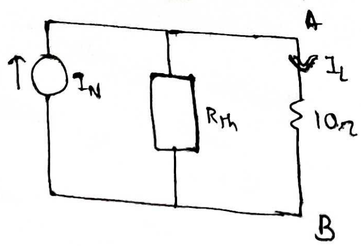
$12 + 2(I_2 - I_1) + 8I_2 = 0$

$12 + 2I_2 - 2I_1 + 8I_2 = 0$

$$\boxed{-2I_1 + 10I_2 = -12}$$

$I_2 = -\frac{2}{3}\,A$

Norton Equivalent



$I_L = -0.323A$