

CSX Corporation

- Based in Jacksonville, Florida USA.
- Industry: Railroads
- Market Cap.: 61.273 Billion

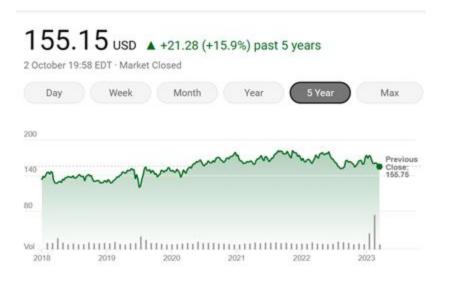


Johnson & Johnson

- Based in New Brunswick, New Jersey USA.
- Industry: Pharmaceutical and Medical technology.
- Market Cap.: 373.603 Billion







Johnson & Johnson

ANNUALIZED DAILY RETURNS

6.75%

VARIANCE OF ANNUALIZED DAILY RETURN

0.04261

STANDARD DEVIATION

0.206

(√Variance)



CSX Corporation

ANNUALIZED DAILY RETURNS

5.47%

VARIANCE OF ANNUALIZED DAILY RETURN

0.0939

STANDARD DEVIATION

0.301

(√Variance)



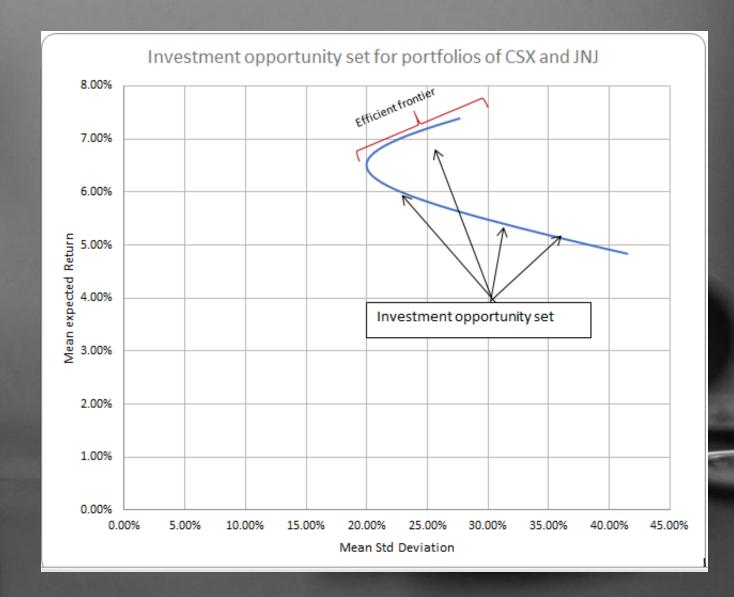
Q3. Investment Opportunity Set and Efficient Frontier for CSX and J&J

COVARIANCE BETWEEN JNJ AND CSX OVER 5 YEARS

0.0281

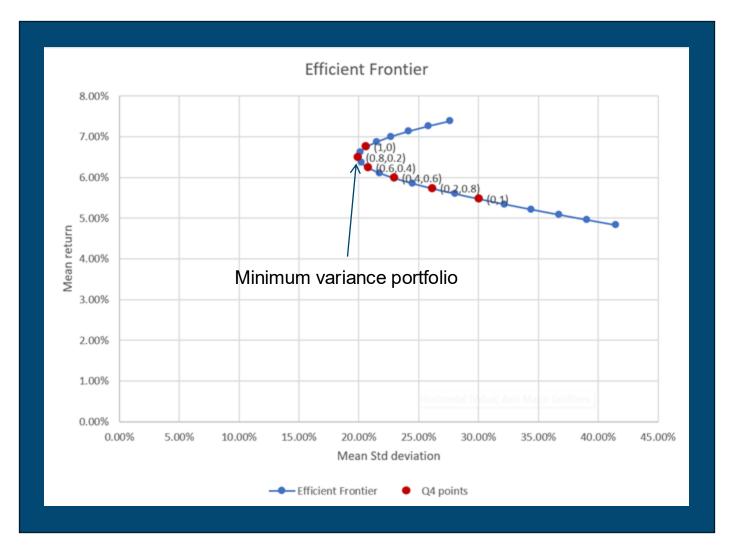
CORRELATION COEFFICIENT

0.4523



Q4: Portfolio Weights and Portfolio Statistics

	Portfolio weights		Portfolio Statistics	
	JNJ	CSX	Return	St Dev
-	1_	0	_6.75%_	_20.64% .
	0.8	0.2	6.50%	19.97%
	0.6	0.4	6.24%	20.80%
	0.4	0.6	5.99%	22.98%
	0.2	0.8	5.73%	26.18%
	0	1	5.47%	30.06%



Q5. Minimum Variance Portfolio



- The minimum variance portfolio aims to minimize risk and maximize return for a given combination of assets.
- For a 2-asset universe, minimum variance portfolio is obtained by minimizing the portfolio variance equation

we set
$$\frac{d\sigma_p}{dw_1} = 0$$
 and get:

$$W_{1} = \frac{\sigma_{2}^{2} - \sigma_{12}}{\sigma_{1}^{2} + \sigma_{2}^{2} - 2\sigma_{12}}; W_{2} = 1 - W_{1}$$



Minimum Standard deviation 19.96%

Interpretation: The higher proportion of JNJ is expected since the stock provides a greater average rate of return for lower volatility compared to CSX.

Q6: Tangency portfolio

$$Sharpe\ Ratio = rac{R_p - R_f}{\sigma_p}$$

where:

 $R_p = \text{return of portfolio}$

 $R_f = \text{risk-free rate}$

 $\sigma_p = \text{standard deviation of the portfolio's excess return}$

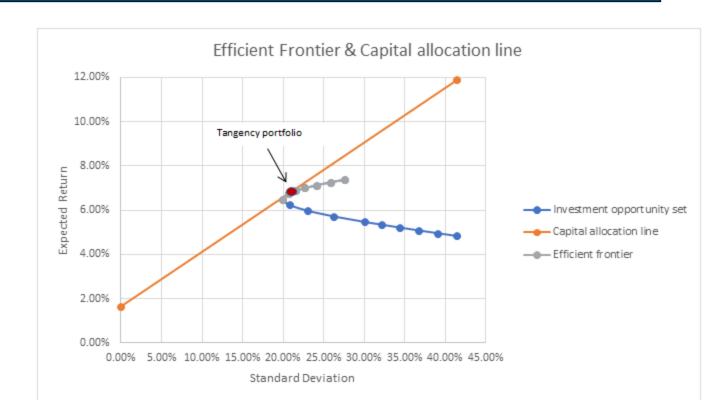
We maximize Sharpe ratio to obtain the optimal portfolio

Using

$$R_p=W_AR_A+W_BR_B$$
, $\sigma_p^2=W_A^2\sigma_A^2+W_B^2\sigma_B^2+2W_AW_B\sigma_A\sigma_B$, and $W_B=1-W_A$

•
$$W_A = \frac{(R_A - R_f)\sigma_B^2 - (R_B - R_f)\sigma_{AB}}{(R_A - R_f)\sigma_B^2 + (R_B - R_f)\sigma_A^2 - (R_A - R_f + R_B - R_f)\sigma_{AB}};$$

• $W_{INI} = 0.947; W_{CSX} = 0.053$



Q7: Utility function maximization

- Utility: $U = r k \times \sigma^2$
- For 3 asset (2 risky; 1 risk-free), we use:
- $r_c = w_p r_p + w_f r_f$
- Since std dev of risk-free asset is 0: $\sigma_c^2 = w_p^2 \sigma_p^2$,
- Maximizing U, we get:
- $U(w_p) = (w_p r_p + w_f r_f) k(w_p^2)$
- Setting $\frac{dU}{dw_n} = 0$:

$$w_p = \frac{r_p - r_f}{2k\sigma_p^2}$$

• $\frac{d^2U}{dw_p^2} = -2k\sigma_p^2 < 0$, which confirms that this value of w_p maximizes the Utility.

Setting k =2 (moderate (but positive) risk-aversion), we obtain:

$$w_p = 0.305; w_f = 0.694$$

- $w_{INI} = (0.947 * w_p) = 0.289;$
- $w_{CSX} = (0.053 * w_p) = 0.016;$
- $w_f = 0.694$

As k increases, the weight of the risk-free asset increases in the portfolio

Q8: Beta calculation using OLS

We chose S&P 500 as the market index for our calculations

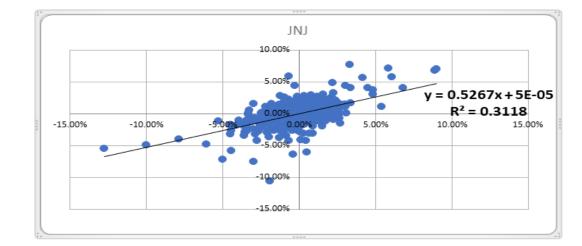
Mean annualized market log return
$$(R_M) = 8.91\%$$

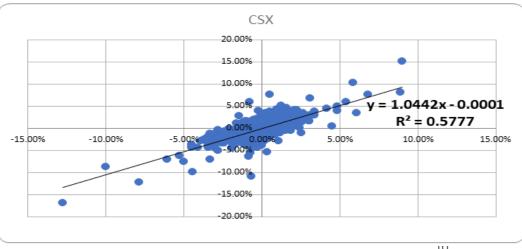
Market variance = 0.0479
Market standard deviation = 0.2188

- We chose S&P 500 as the market index for our calculations.
- Beta is a measure of the systemic risk (volatility) of the stock as compared to the volatility of the market as a whole.

$$Beta(\beta_a) = \frac{Covariance(R_a, R_m)}{Variance(R_m)}$$

- We approximate the Beta for CSX and JNJ below by fitting an OLS regression model between each stocks excess return $(R_{stock} - R_f)$ vs the excess market return $(R_M - R_f)$
- $\beta_{INI} = 0.5267; \ \beta_{CSX} = 1.0442$
- The R^2 values indicate the amount of variance in the stock's return that is explained by the market movement





Used together, β and R^2 give a detailed picture of the expected performance.

Q9:Return prediction and Jensen's alpha

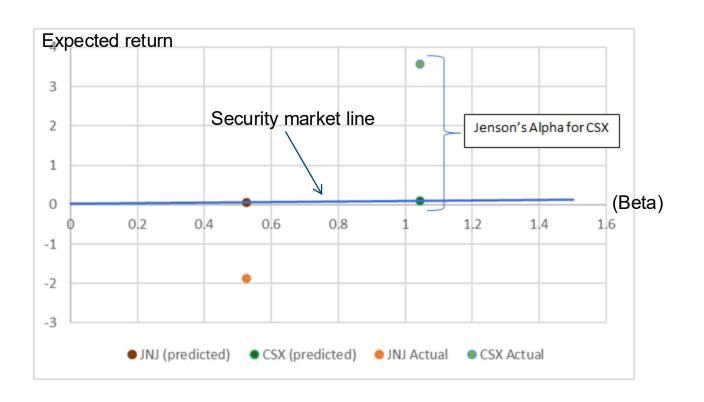
$$Alpha = R_i^{actual} - R_i^{expected as per CAPM}$$

•
$$R_{INI}^{expected} = 1.64 + 0.5267(8.91 - 1.64) = 5.47\%$$

•
$$R_{JNJ}^{expected} = 1.64 + 0.5267(8.91 - 1.64) = 5.47\%$$

• $R_{CSX}^{expected} = 1.64 + 1.0442(8.91 - 1.64) = 9.23\%$

- Actual annualized log return on Sep 1:
- $R_{JNJ}^{actual} = ln \left(\frac{160.48}{161.68} \right) * 252 = -187.73\%$
- $R_{CSX}^{actual} = lnln\left(\frac{30.63}{30.2}\right) * 252 = 356.28\%$
- $alpha_{JNJ} = R_{JNJ}^{actual} R_{INJ}^{expected} = (-187.73\%) -$ 5.47% = -193.20%
- $alpha_{CSX} = R_{CSX}^{actual} R_{CSX}^{expected} = 356.28\% -$ 9.23% = 347.04%



As the actual returns are significantly different in magnitude due to annualization factor as compared to the expected values for both JNJ and CSX.

