

## Solution to Assignment:1

### Answer to Q3:

To find the maximum bandwidth of a path between two switching centers  $a$  and  $b$  in a telephone network graph, we can use a modified version of Dijkstra's algorithm.

Here's the algorithm:

1. Initialize a priority queue  $q$  and a distance array  $d$  to store the maximum bandwidth values for each vertex.
2. Set all distances in  $d$  to negative infinity, except for  $a$ , which is set to positive infinity.
3. Enqueue  $a$  into  $q$  with a priority of positive infinity.
4. While  $q$  is not empty, do the following:
  - Dequeue a vertex  $v$  from  $q$ .
  - For each neighbour  $n$  of  $v$ , do the following:
    - Let  $bw$  be the bandwidth of the edge between  $v$  and  $n$ .
    - Let  $max$  be the maximum bandwidth value between  $v$  and  $n$ .
    - If  $max$  is greater than the maximum bandwidth value recorded for  $n$ , update  $n$ 's maximum bandwidth value in  $d$  and enqueue  $n$  into  $q$  with a priority of  $max$ .
5. Return the maximum bandwidth value recorded for  $b$  in  $d$ .

### Answer to Q4:

The Floyd-Warshall algorithm can be used. Here is an algorithm for calculating the transitive reduction:

1. Initialize a matrix  $R$  of size  $n \times n$ , where  $n$  is the number of vertices in the graph. Set all elements of  $R$  to 0 initially.
2. For each edge  $(u, v)$  in the graph, set  $R[u][v]$  to 1.
3. For each vertex  $k$  from 1 to  $n$ , do the following:
  - a. For each pair of vertices  $u$  and  $v$ , if  $u$  is not equal to  $k$  and  $v$  is not equal to  $k$ , check if  $R[u][k]$  and  $R[k][v]$  are both 1.
  - b. If both  $R[u][k]$  and  $R[k][v]$  are 1, then there is a path from  $u$  to  $v$  through  $k$ . In this case, set  $R[u][v]$  to 0.
4. Return the resulting matrix  $R$  as the transitive reduction of the graph.

The time complexity of this algorithm is  $O(n^3)$ , where  $n$  is the number of vertices in the graph. The algorithm iterates through all possible vertex pairs and checks if there is a transitive edge through each intermediate vertex. The Floyd-Warshall algorithm efficiently computes the transitive closure of a graph, which can be used to derive the transitive reduction.

