ESO 208: Computational Methods in Engineering

Programming Assignment->1

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Section : J8

Question 1:

(i). Bisection method:-

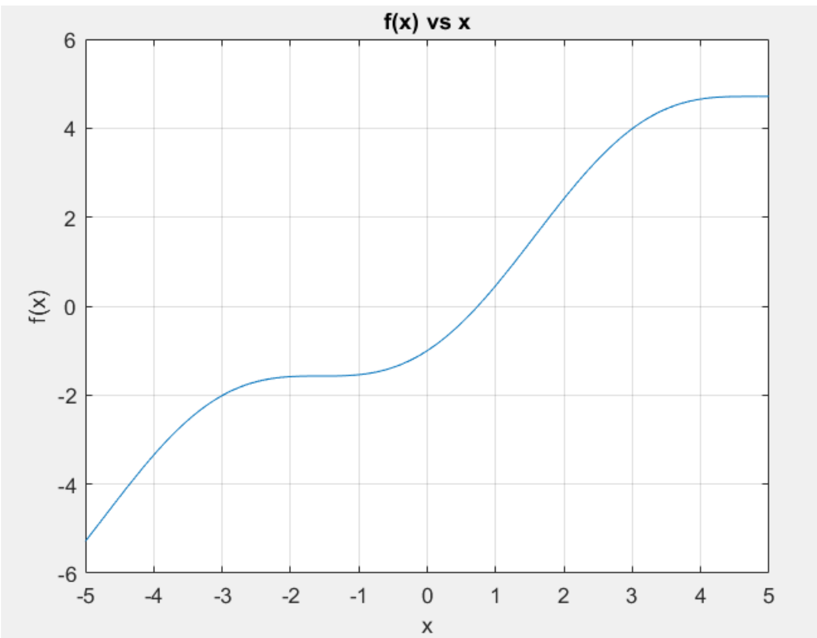
Test case 1 :

f(x) = x - cos(x)
Initial bracket = (0,1)
Maximum iteration = 50
Maximum error% = 0.01%

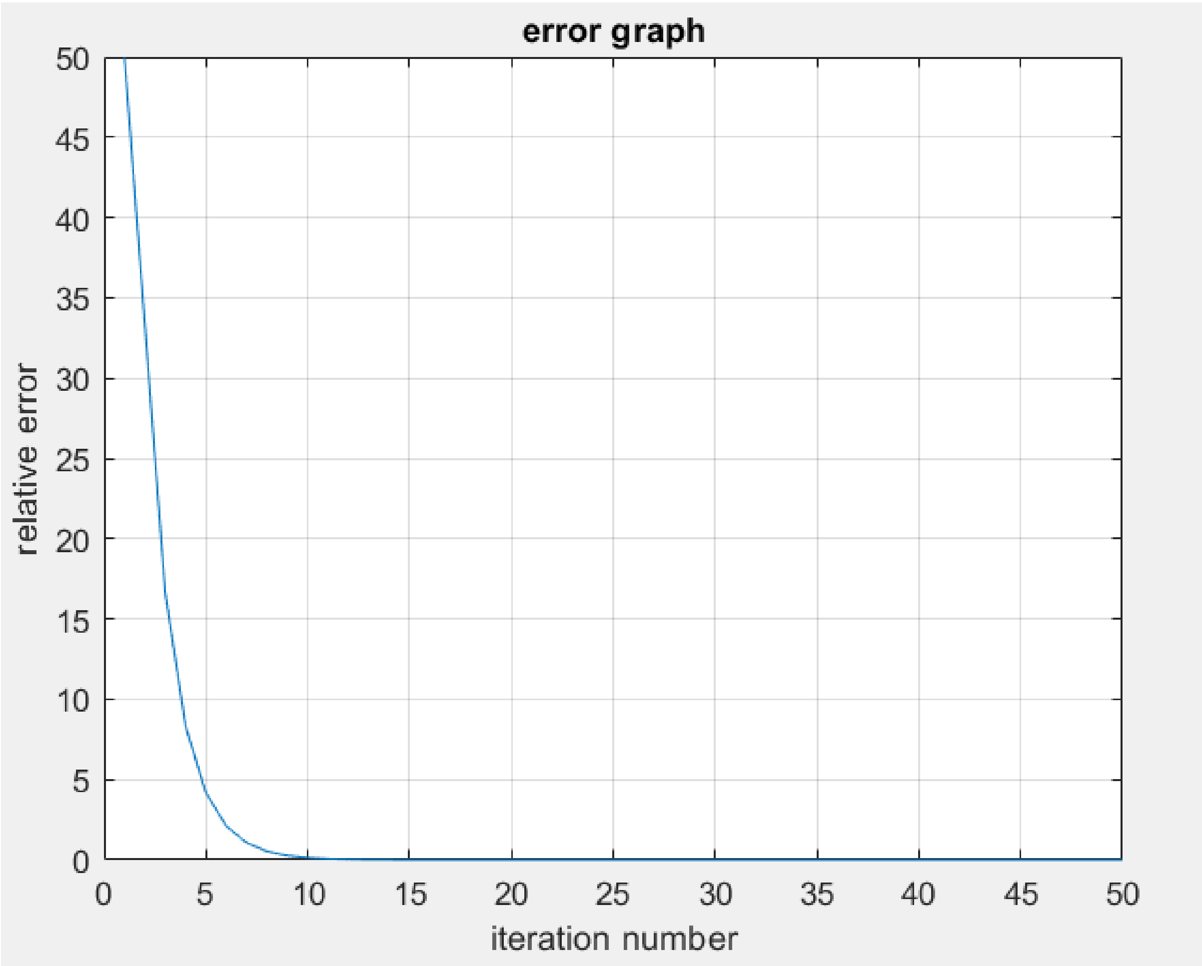
Result:-

The root of the function = 0.739075

F(x) vs x graph :-



Relative approximate error vs number of iteration graph:-



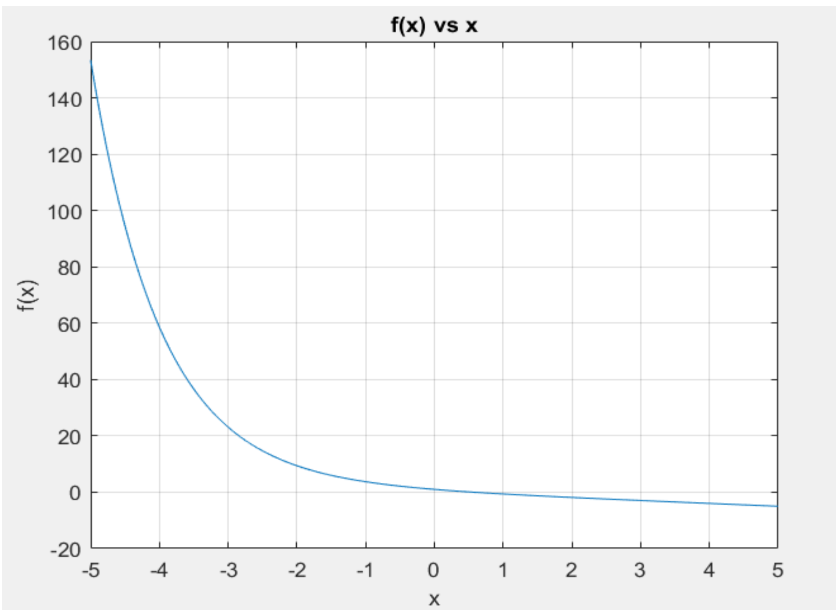
Test case 2:-

f(x) = exp(-x) - x
Initial bracket = (0,1)
Maximum iteration = 50
Maximum error% = 0.01%

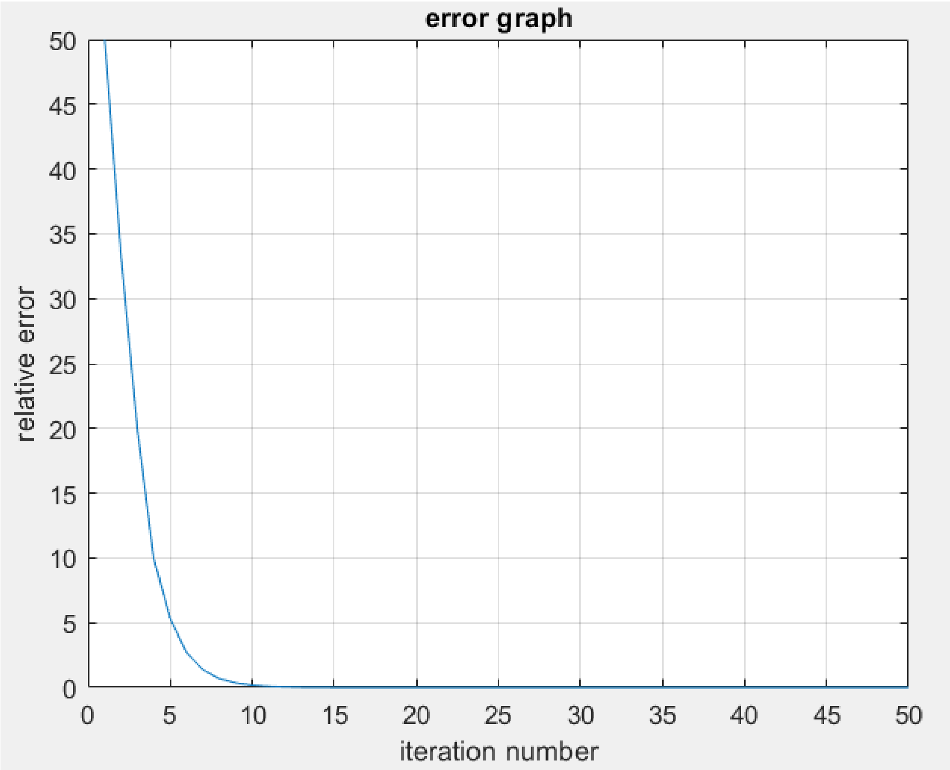
Result:-

The root is 0.567169

f(x) vs x graph:-



Relative approximate error vs number of iteration graph:-



Comment on convergence and stability:-

Convergence:- It has a linear rate of convergence, we can predict the number of iterations required.

$$\frac{\left|e^{(i+1)}\right|}{\left|e^{(i)}\right|} = \frac{1}{2}$$

This implies linear rate convergence and having an error constant 0.5

Stability:- This method can be a bit slow sometimes, but this method is very stable. Whatever the initial guess, it always converges to a solution.

(ii). **False position method:-**

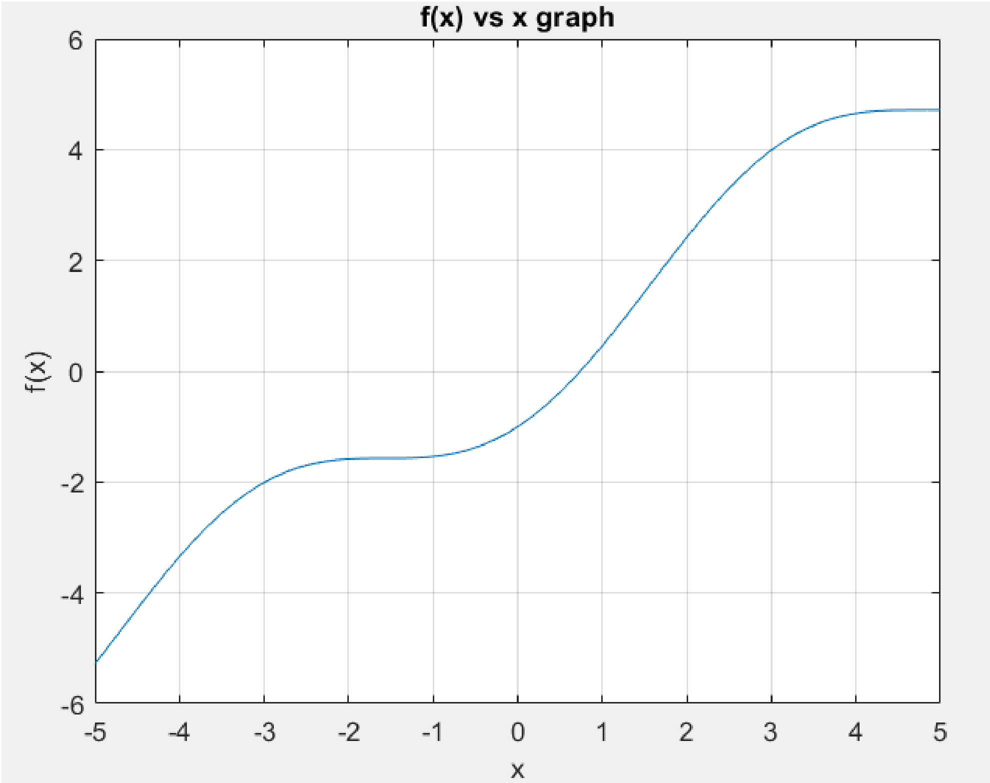
Test case 1:-

f(x) = x - cos(x)
Initial bracket = (0,1)
Maximum iteration = 50
Maximum error% = 0.01%

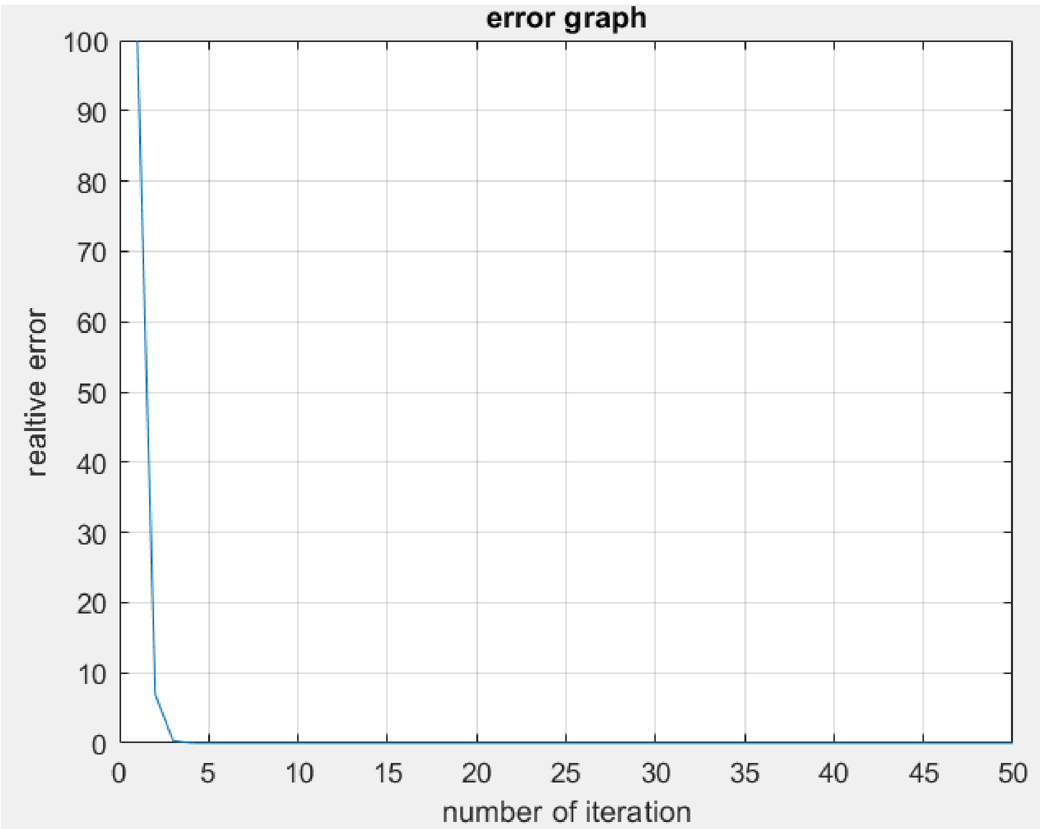
Result:-

The root is 0.739085

F(x) vs x graph:-



Relative approximate error vs number of iteration graph:-



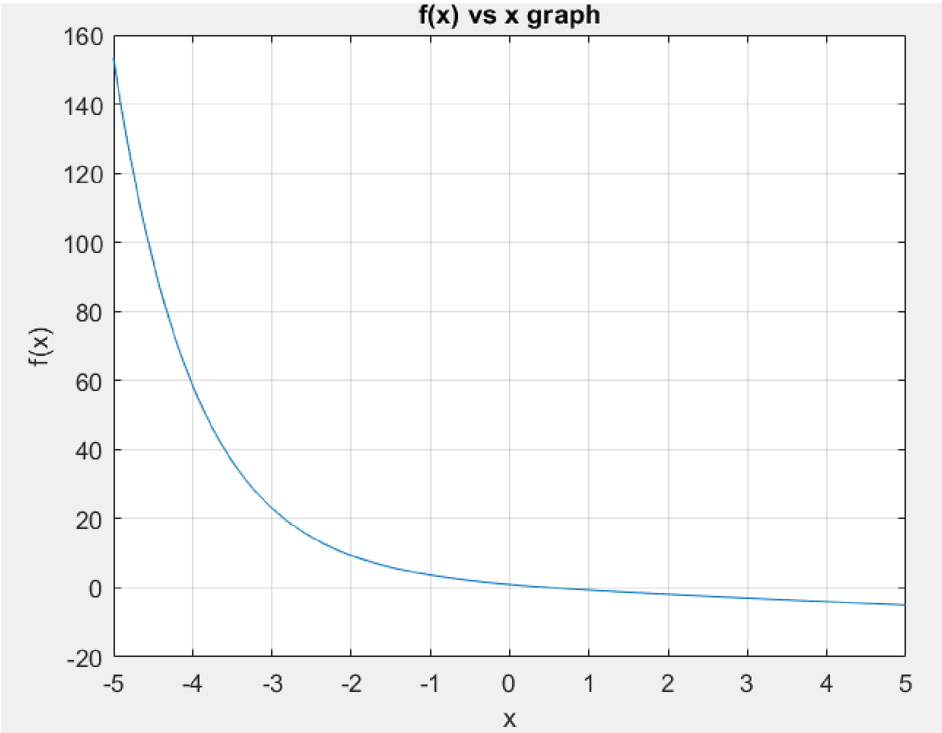
Test case 2:

f(x) = exp(-x) - x
Initial bracket = (0,1)
Maximum iteration = 50
Maximum error% = 0.01%

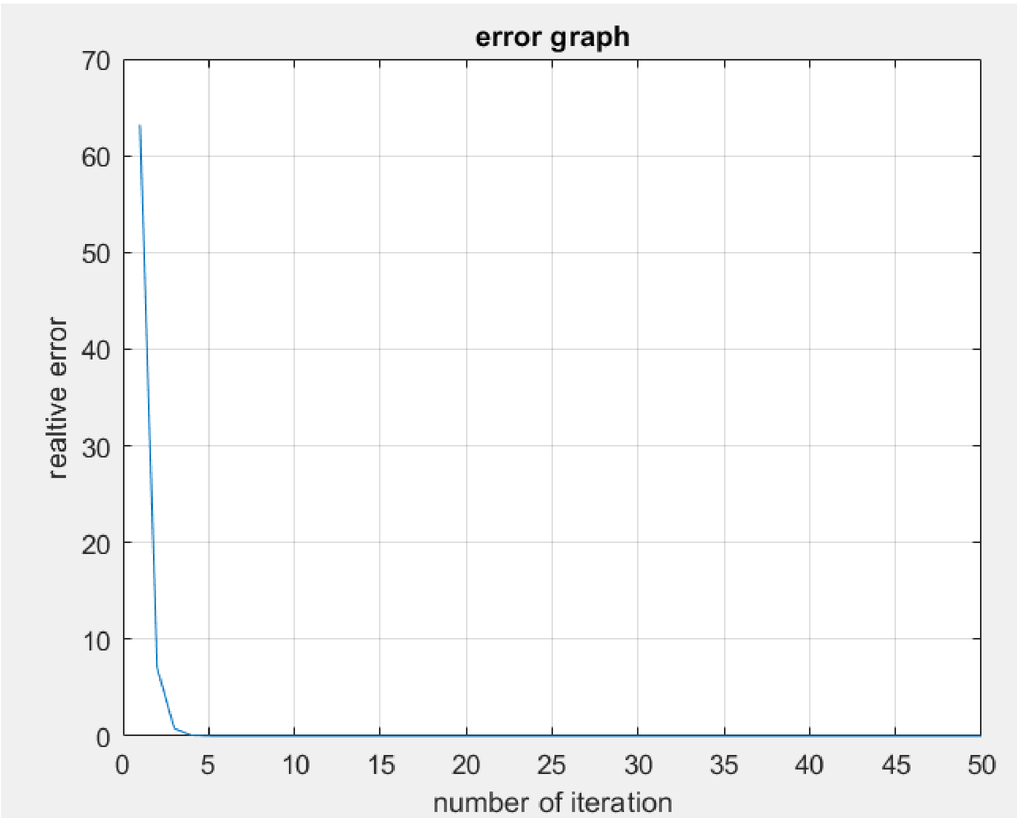
Result:-

The root is 0.567150

f(x) vs x graph:-



Relative approximate error vs number of iteration graph:-



Comment on convergence and stability:-

Convergence:- It has approximately 1.618. But for some problems it takes Less time than bisection. This implies that for some problems, the convergence rate is faster than the bisection method.

Stability:- False position method is a very stable method, it always converges to a solution. It has an error less than the bisection method. (error = 8.997e-4)

(iii). Fixed point method:-

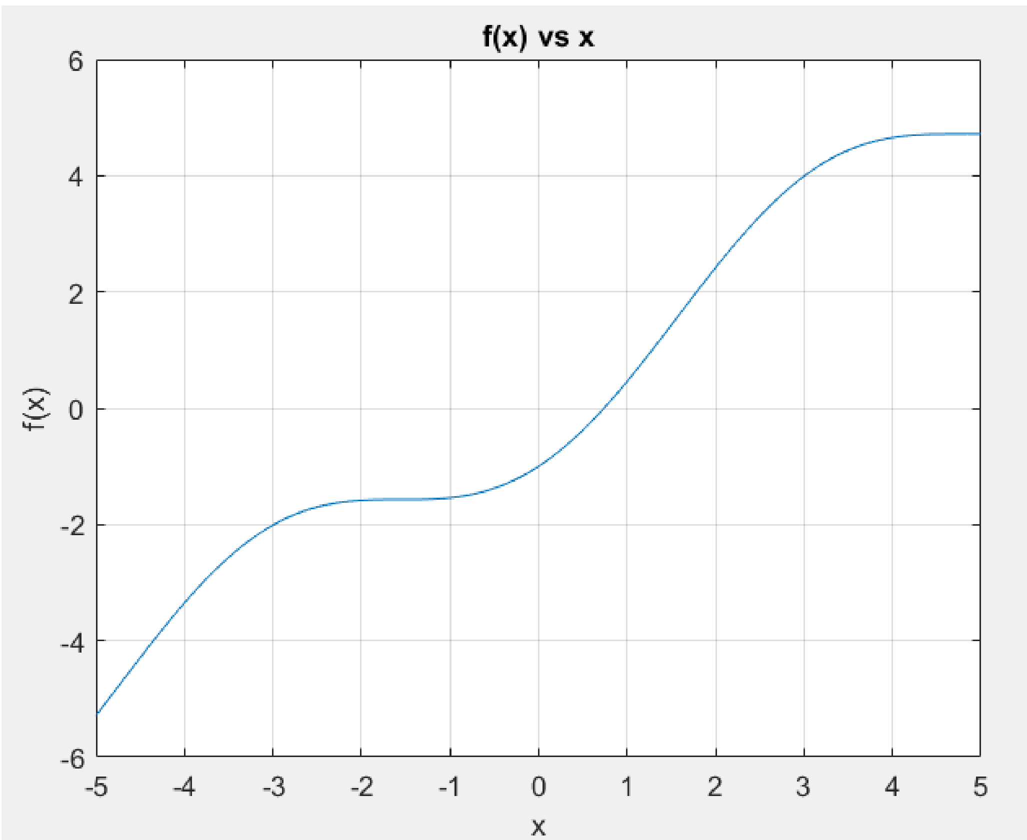
Test case 1:-

f(x) = x - cos(x)
Initial guess = 0
Maximum iteration = 50
Maximum error% = 0.01%
phi(x) = cos(x)

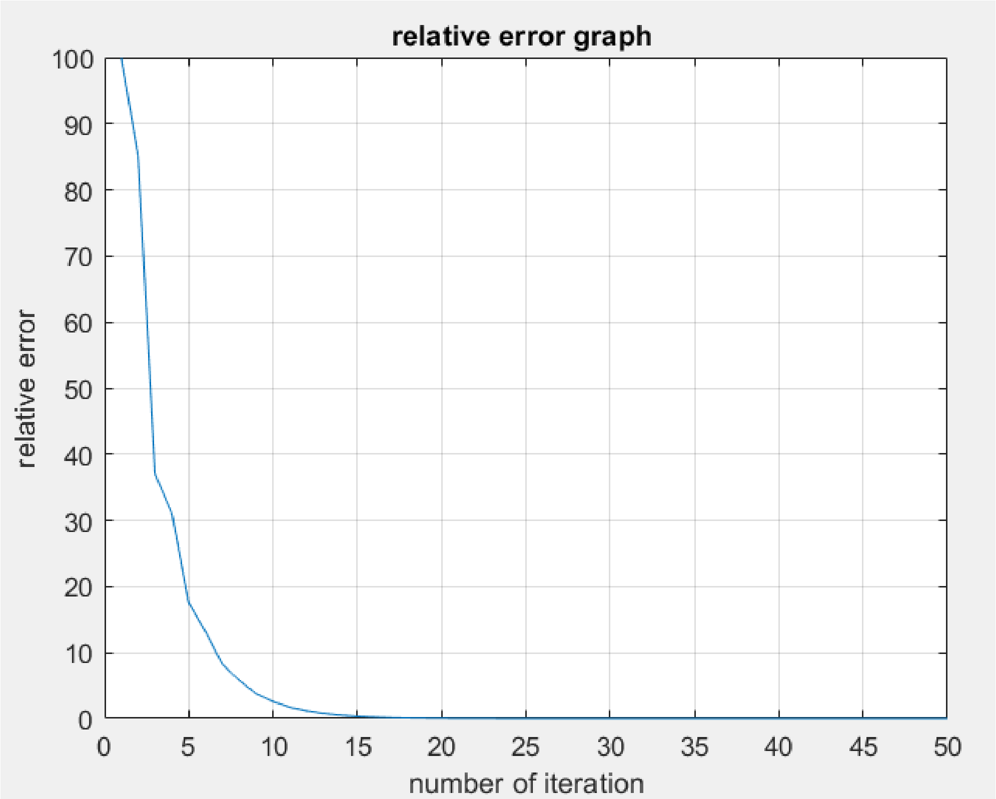
Result:-

The root is 0.739106

F(x) vs x graph:-



Relative approximate error vs number of iteration graph:-



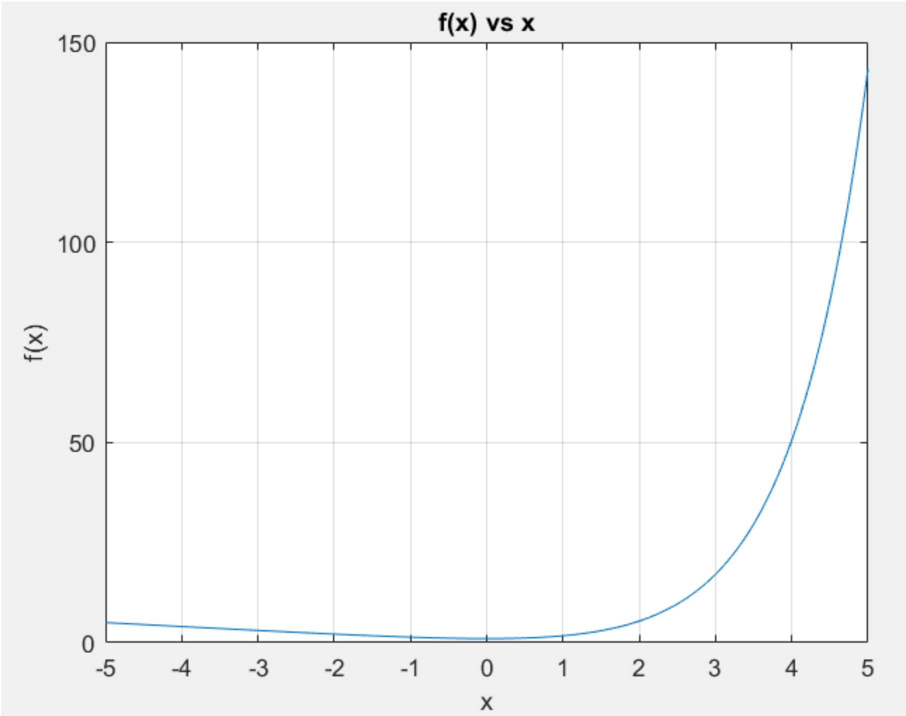
Test case 2:

f(x) = exp(-x) - x
Initial guess = 0
Maximum iteration = 50
Maximum error% = 0.01%
Phi (x) = exp(-x)

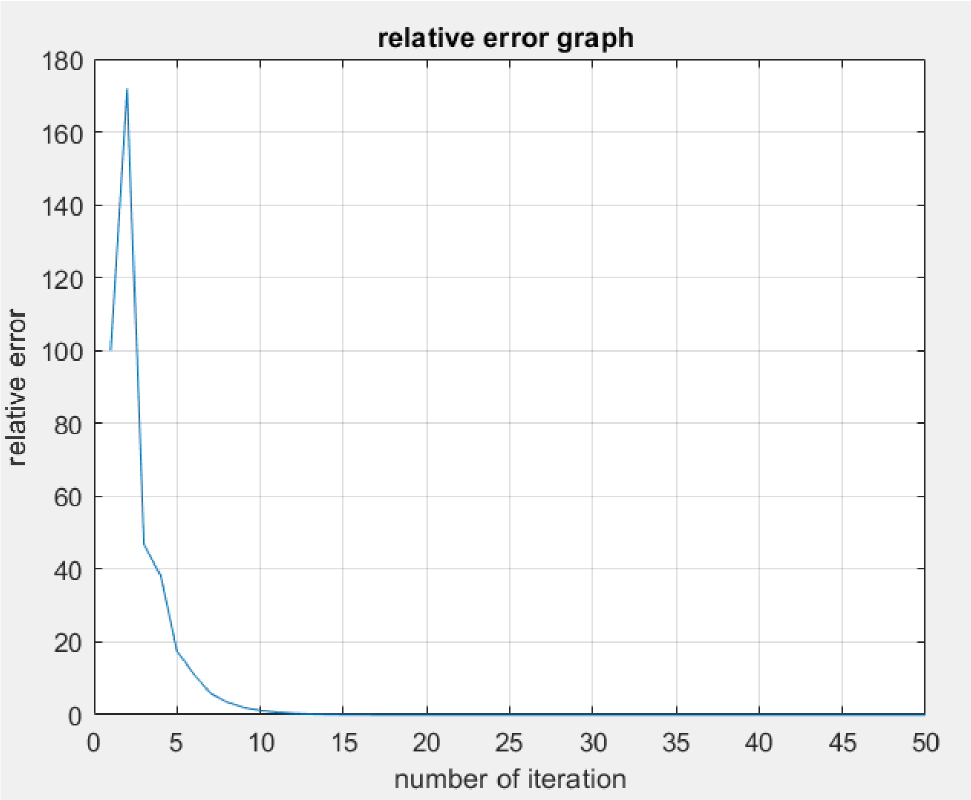
Result:-

The root is 0.567157

f(x) vs x graph:-



Relative approximate error vs number of iteration graph:-



Comment on convergence and stability:-

Convergence:- It has a linear rate of convergence. But it does not guarantee that it will converge to a solution. When $|g'(x)| < 1$, then the method definitely converges to a solution.

Stability:-

This method is not much stable as it guarantees convergence only when $|g'(x)| < 1$. In other cases, it does not guarantee, but it may or may not converge.

(iv). Newton Raphson:-

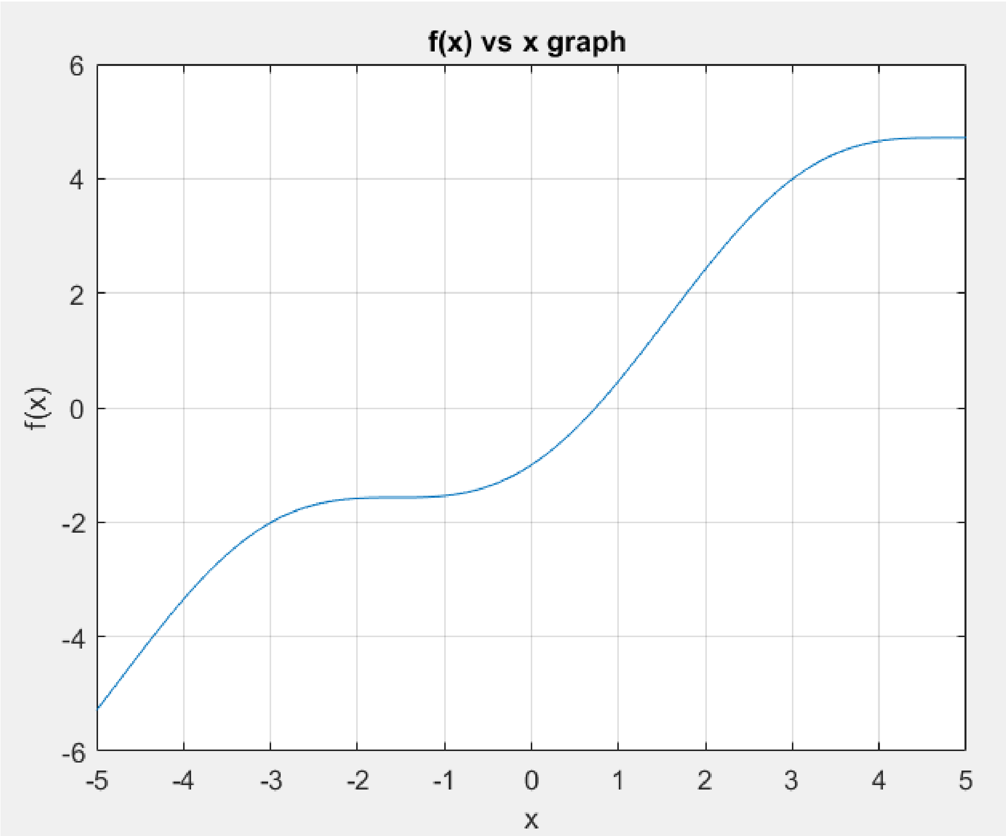
Test case 1:-

$f(x) = x - \cos(x)$
Initial guess = 0
Maximum iteration = 50
Maximum error% = 0.01%
$f'(x) = 1 + \sin(x)$

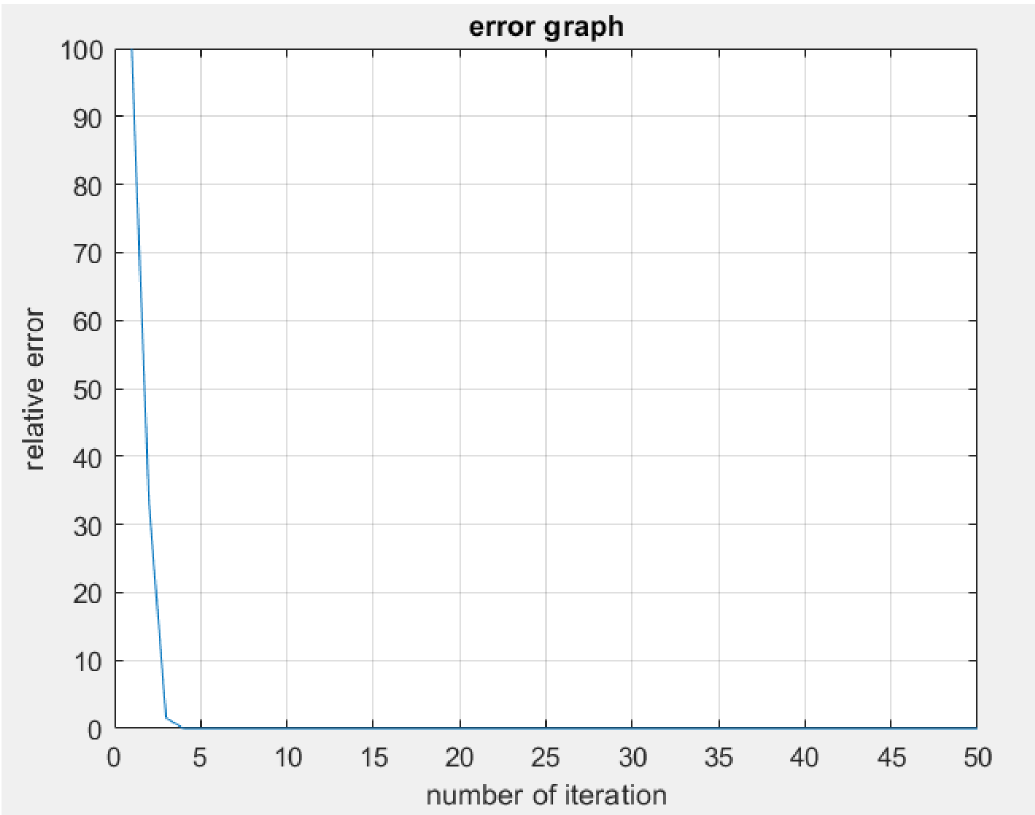
Result:-

The root is 0.739085

F(x) vs x graph:-



Relative approximate error vs number of iteration graph:-



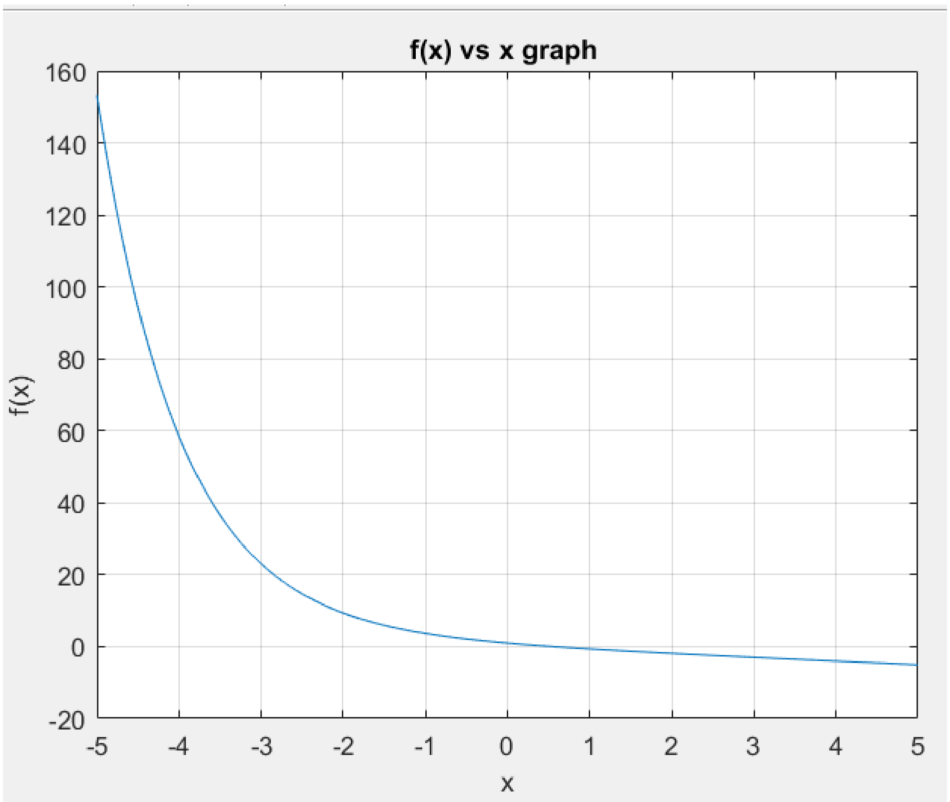
Test case 2:-

f(x) = exp(-x) - x
Initial guess = 0
Maximum iteration = 50
Maximum error% = 0.01%
f'(x) = -exp(-x) - 1

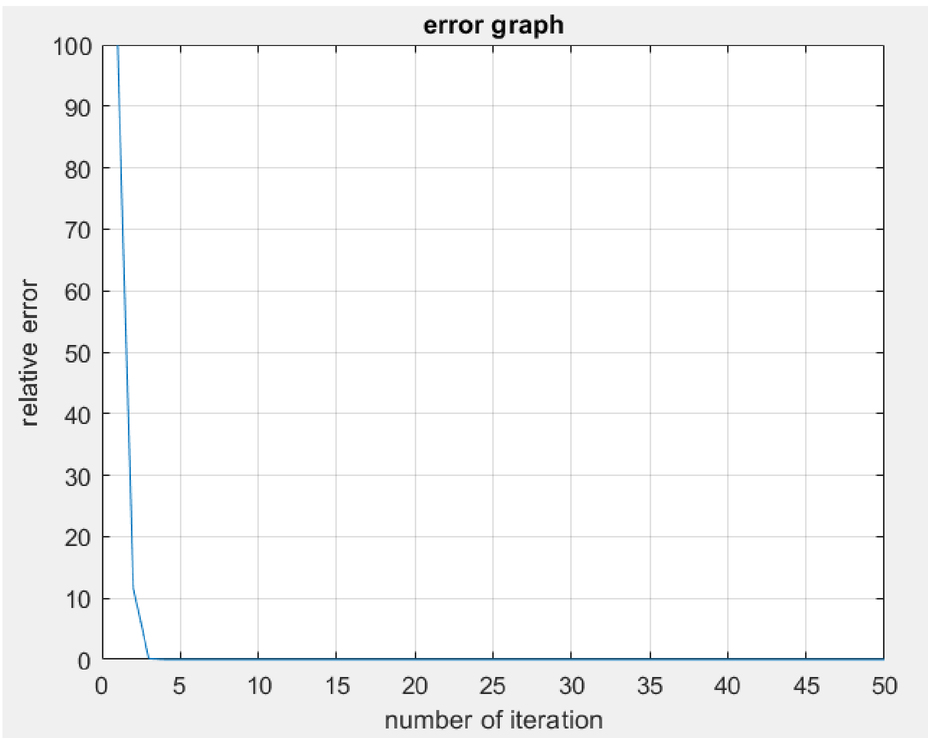
Result:-

The root is 0.567143

F(x) vs x graph:-



Relative approximate error vs number of iteration graph:-



Comment on convergence and stability:-

Convergence:- It has a quadratic rate of convergence, It is one of the fastest method.

Stability:- This method converges to a solution in a very less number of iterations. This method converges to a solution if the first differential of f(x) is not zero. This means newton raphson is unstable in some cases.

(v) Secant method:-

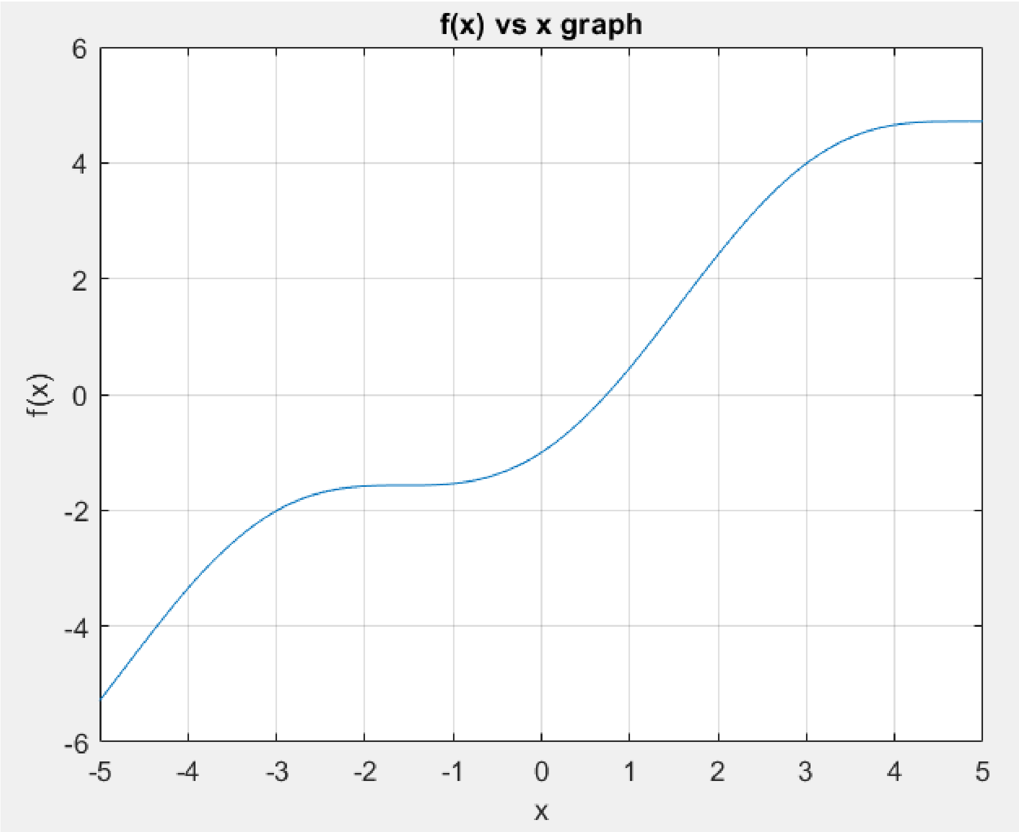
Test case 1:-

f(x) = x - cos(x)
Initial bracket = (0,1)
Maximum iteration = 50
Maximum error% = 0.01%

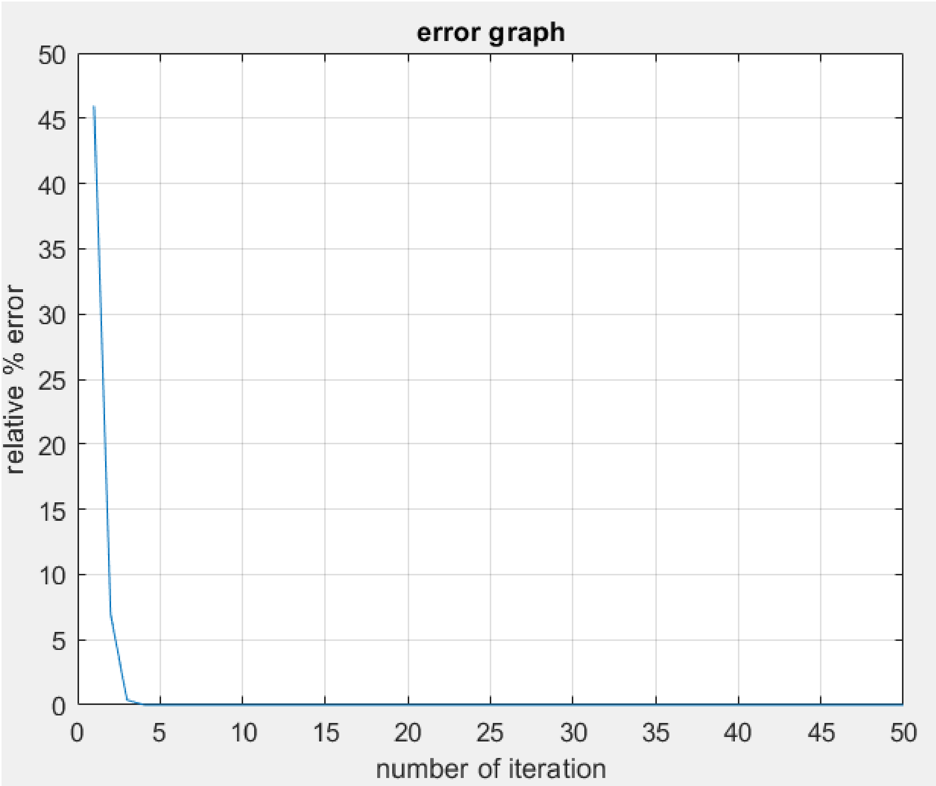
Result:-

The root is 0.739085

F(x) vs x graph:-



Relative approximate error vs number of iteration graph:-



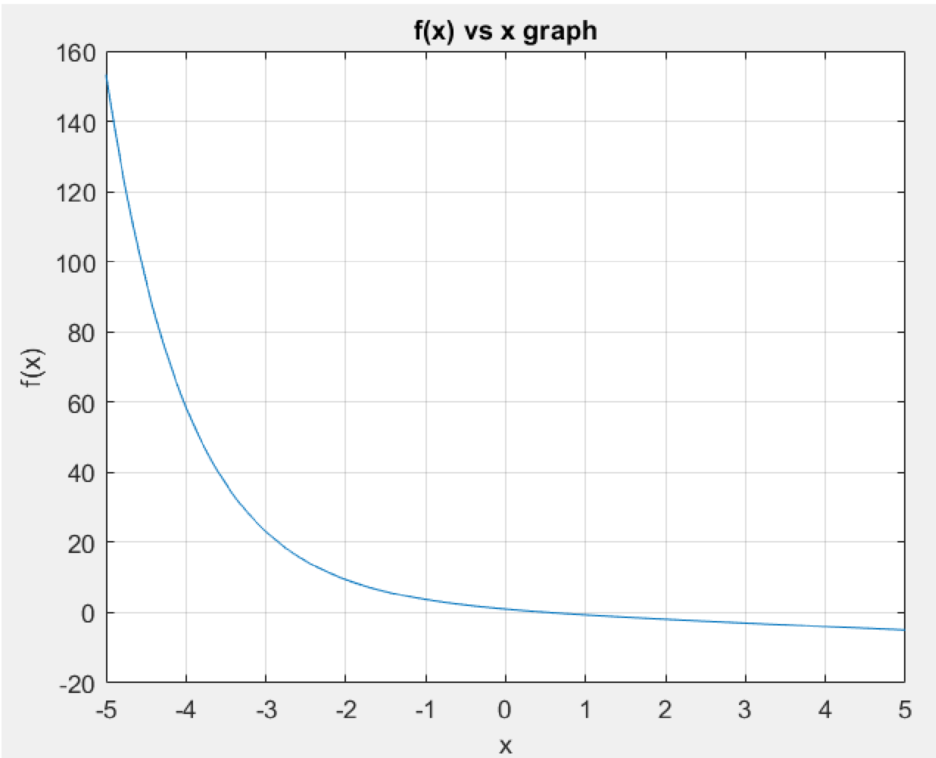
Test case 2:-

f(x) = exp(-x) - x
Initial bracket = (0,1)
Maximum iteration = 50
Maximum error% = 0.01%

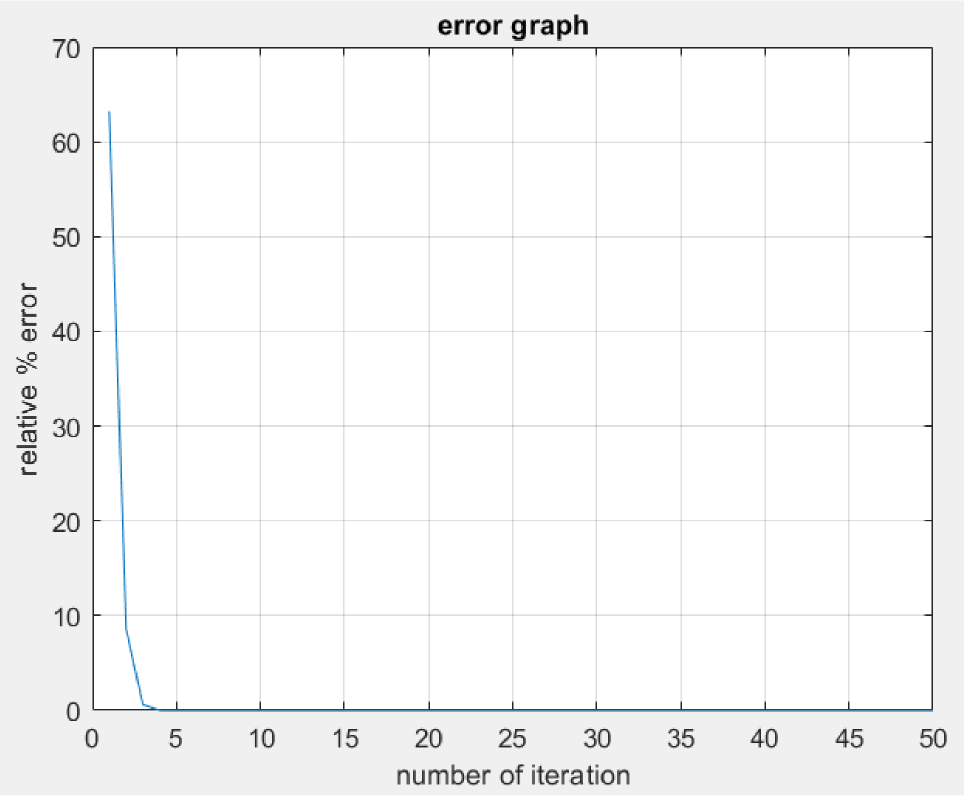
Result:-

The root is 0.567143

F(x) vs x graph:-



Relative approximate error vs number of iteration graph:-



Comment on convergence and stability:-

Convergence:- It has a rate of convergence between 1 and 2 . (approx 1.62). This method is less accurate than newton raphson.

Stability:- In this method, The root may not remain bracketed and there is always a possibility of instability, so the method can fall. The secant method works well when we choose the initial guess near the root.

Question 2:-

(i) Muller method:-

Test case 1:-

$f(x) = x^4 - 7.4x^3 + 20.44x^2 - 24.184x + 9.6448 = 0$

Initial guesses = (-1,0,1)

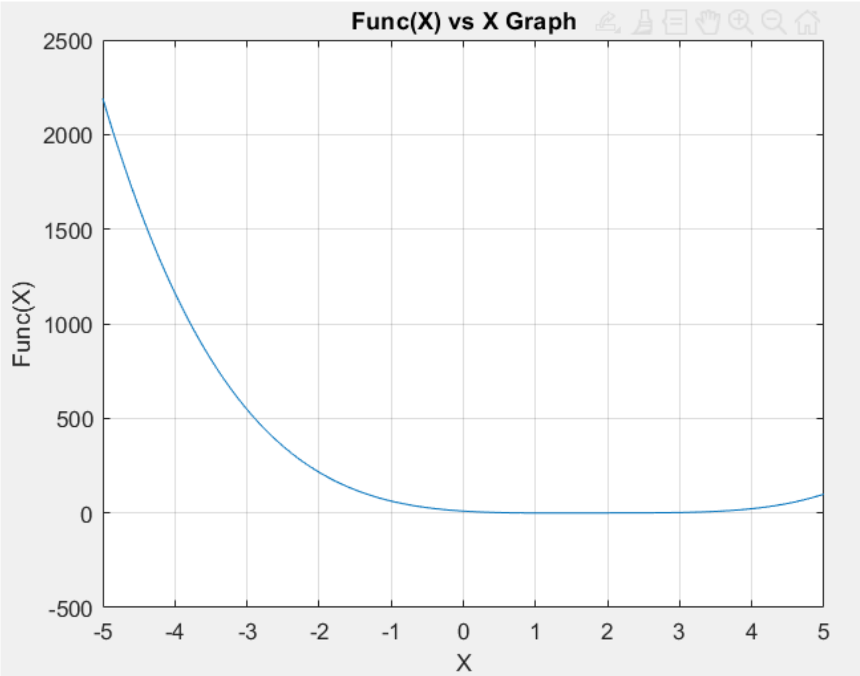
Maximum iterations = 50

Maximum relative approximate error = 0.01%

Result:-

The root is 0.800019.

f(x) vs x graph:-



Test case 2:-

$f(x) = x^4 - 7.4x^3 + 20.44x^2 - 24.184x + 9.6448 = 0$

Initial guesses = (0,1,2)

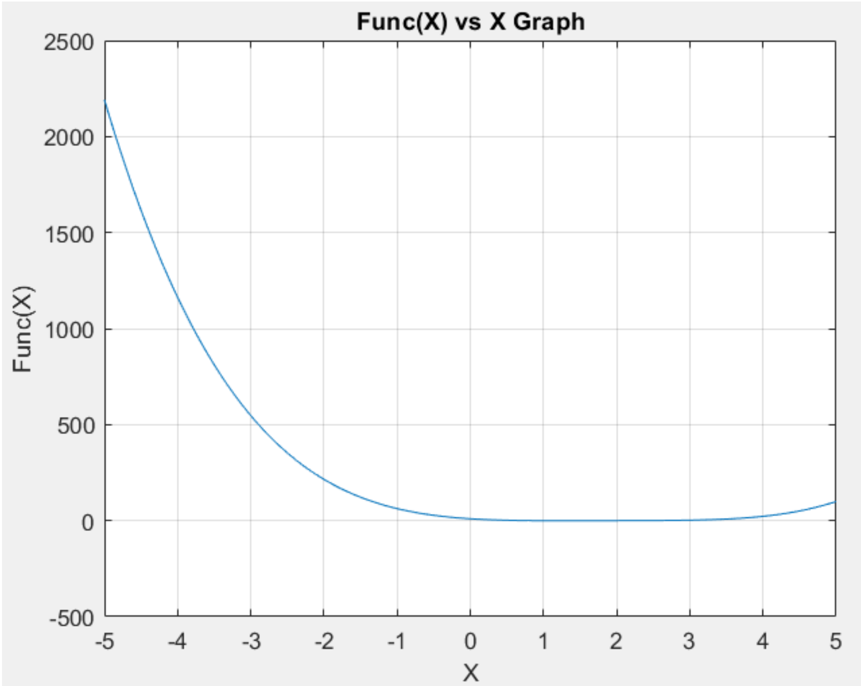
Maximum iterations = 50

Maximum relative approximate error = 0.01%

Result:-

The root is 2.199999

f(x) vs x graph:-



Comment on convergence and stability:-

Convergence:- It has a rate of convergence 1.82, It is faster than secant but Slower than newton raphson.

Stability:- It may or may not converge to a solution and therefore unstable.

(ii). Bairstow method:-

Test case 1:-

$f(x) = x^4 - 7.4x^3 + 20.44x^2 - 24.184x + 9.6448 = 0$

Start with $\alpha_0 = -5$, $\alpha_1 = 4$

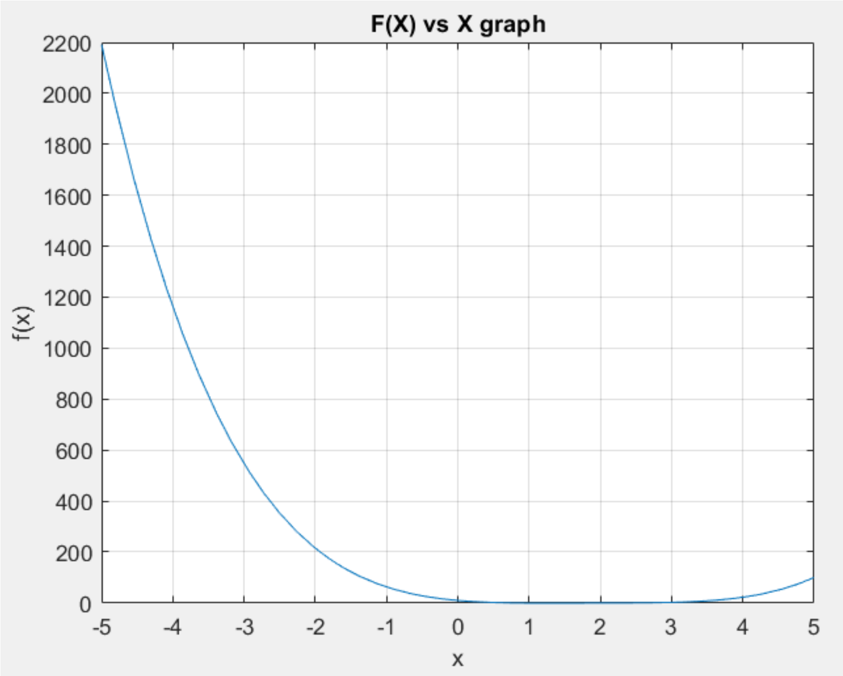
Maximum iterations = 50

Maximum relative approximate error = 0.01%

Result:-

The roots are (2.200000 + 1.280000i) , (2.200000 - 1.280000i) , 0.800061 , 2.199918

f(x) vs x graph:-



Test case 2:-

$f(x) = x^4 - 7.4x^3 + 20.44x^2 - 24.184x + 9.6448 = 0$

Start with $\alpha_0 = -2, \alpha_1 = 2$

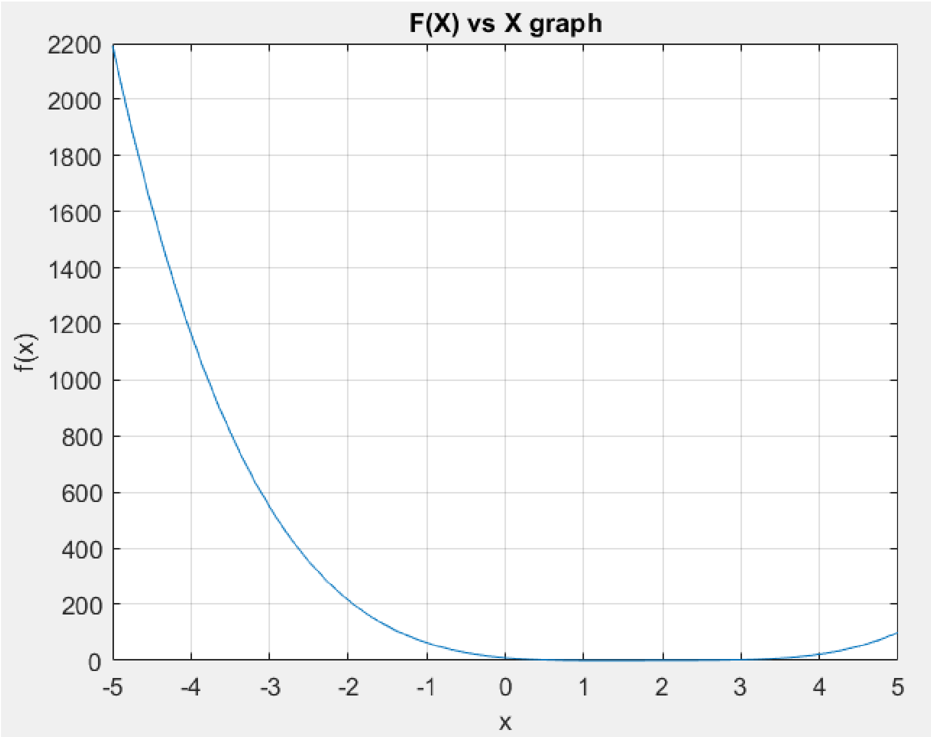
Maximum iterations = 50

Maximum relative approximate error = 0.01%

Result:-

The roots are 2.200000 , 0.800000 , (2.200000 + 0.800000i) , (2.200000 - 0.800000i)

f(x) vs x graph:-



Comment on convergence and stability:-

Convergence:- It gets the local quadratic convergence of newton's method. So it has a quadratic rate of convergence.

Stability:- Generally this method is stable, but it is unstable for odd degree polynomials and has only one real root.