Assignment 3

Tuesday, 6 December 2022 3:04 AM

Submitted by → Sarthak Chauhan
Enrollment no. → MTE4220003

Questions solved in this Assignment.

① 1 Example of Either SVD or PCA
② " " " K-means
③ " " " Naive Bayes

Q1) SVD Example

$$A = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$$

$$AA' = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix} \times \begin{bmatrix} 4 & 3 \\ 0 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 16+0 & 12+0 \\ 12+0 & 9+25 \end{bmatrix}$$

$$\begin{bmatrix} 16 & 12 \\ 12 & 34 \end{bmatrix}$$

→ finding Eigen vector for AA'

$$|AA' - \lambda I| = 0$$

$$\begin{vmatrix} 16-\lambda & 12 \\ 12 & 34-\lambda \end{vmatrix} = 0$$

$$(544 - 50\lambda + \lambda^2) - 144 = 0$$

$$\lambda^2 - 50\lambda + 400 = 0$$

$$(\lambda - 10)(\lambda - 40) = 0$$

$$\therefore \lambda = 10, 40$$

• Eigen Vector for $\lambda = 40$

$$A.A' - \lambda I = \begin{bmatrix} 16 & 12 \\ 12 & 34 \end{bmatrix} - 40\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -24 & 12 \\ 12 & -6 \end{bmatrix}$$

now reducing the matrix.

$R_1 \leftarrow R_1 \div -24$

$R_2 \leftarrow R_2 - 12 \times R_1$

$$= \begin{bmatrix} 1 & -.5 \\ 0 & 0 \end{bmatrix}$$

System associated with Eigen value $\lambda = 40$

$$(A.A' - 40I)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -.5 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 - \frac{1}{2}x_2 = 0$$

$$x_1 = \frac{1}{2}x_2$$

Eigen Vector

$$v = \begin{bmatrix} .5x_2 \\ x_2 \end{bmatrix} \quad \text{If } x_2 = 1$$

then $v_1 = \begin{bmatrix} .5 \\ 1 \end{bmatrix}$

• Eigen Vector for $\lambda = 10$

$$|A.A' - \lambda I| = \begin{bmatrix} 16 & 12 \\ 12 & 32 \end{bmatrix} - 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 12 & 24 \end{bmatrix}$$

now reducing the matrix.

Interchange $R_1 \leftrightarrow R_2$

$R_1 / 12 \rightarrow R_1$
$R_2 - 6 \times R_1 \rightarrow R_2$

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$(A.A' - 10I)\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 + 2x_2 = 0$$

$$x_1 = -2x_2$$

$$v = \begin{bmatrix} -2x_2 \\ x_2 \end{bmatrix}$$

let $x_2 = 1$

$$v_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

• for v_1, $L = \sqrt{.5^2 + 1^2} = 1.11$

normalising $u_1 = \left[\frac{0.5}{1.11}, \frac{1}{1.11}\right]$

$$= (.4472, .8944)$$

• for v_2, $L = \sqrt{(-2)^2 + 1^2} = 2.23$

normalising $u_2 = \left(\frac{-2}{2.23}, \frac{1}{2.23}\right)$

$$= (-0.8944, .4472)$$

Solution

$$\Sigma \begin{bmatrix} \sqrt{40} & 0 \\ 0 & \sqrt{10} \end{bmatrix} = \begin{bmatrix} 6.32 & 0 \\ 0 & 3.16 \end{bmatrix}$$

$$V = [v_1, v_2] = \begin{bmatrix} -0.707 & 0.707 \\ 0.707 & 0.707 \end{bmatrix}$$

U is found using formula $u_i = \frac{1}{\sigma_i} A.v_i$

$$\therefore U = \begin{bmatrix} -.447 & .894 \\ -.894 & -.447 \end{bmatrix}$$

• K-means Clustering

Marks of students in T1

$\{4, 7, 8, 6, 14, 15, 10, 9\}$

Marks of student in T2

$\{18, 11, 5, 2, 7, 9, 12, 15\}$

K = 2   2 cluster centers randomly

$C_1 = (8,5)$, $C_2 = (15,9)$

$ED_1 = \sqrt{(4-8)^2 + (18-5)^2} = 13.60$ C1

$\sqrt{(4-15)^2 + (18-9)^2} = 14.21$

$ED_2 = \sqrt{(7-8)^2 + (18-9)^2} = 6.08$ C1

$\sqrt{(7-15)^2 + (11-9)^2} = 8.24$

$ED_3 = 0$

$ED_4 = \sqrt{(6-8)^2 + (2-5)^2} = 3.60$ C1

$\sqrt{(6-15)^2 + (2-9)^2} = 11.40$

$ED_5 = \sqrt{(14-8)^2 + (7-5)} = 6.32$ C2

$\sqrt{(14-15)^2 + (7-9)^2} = 2.23$

$ED_6 = 0$

$ED_7 = \sqrt{(10-8)^2 + (12-5)^2} = 7.28$ C1

$\sqrt{(10-15)^2 + (12-5)^2} = 8.60$

$ED_8 = \sqrt{(9-8)^2 + (15-5)^2} = 10.04$ C2

$\sqrt{(9-15)^2 + (15-9)^2} = 8.48$

$$\begin{bmatrix} C_1 = (4,18), (7,11), (6,2), (10,12) \\ C_2 = (14,7), (9,15) \end{bmatrix}$$ Answer

Naive Bayes Classification

	Purple	white	Support	total
Brinjal	100	350	50	500
Coconut	250	350	400	1000
others	50	100	200	350
total	400	800	650	1850

Item = { Purple, white, support } = x

$$P(A/B) = \frac{P(B/A).P(A)}{P(B)}$$

Brinjal $P(x \mid \text{Brinjal}) = 0.2 \times 0.7 \times 0.1 = 0.014$

$$P(\text{Purple} \mid \text{Brinjal}) = \frac{\frac{100}{400} \cdot \frac{400}{1850}}{\frac{500}{1850}} = 0.2$$

$$P(\text{white} \mid \text{Brinjal}) = \frac{\frac{350}{800} \cdot \frac{800}{1850}}{500/1850} = 0.7$$

$$P(\text{Support} \mid \text{Brinjal}) = \frac{\frac{50}{650} \cdot \frac{650}{1850}}{\frac{500}{1850}} = 0.1$$

$$P(\text{Purple} \mid \text{coconut}) = \frac{\frac{250}{400} \cdot \frac{400}{1850}}{\frac{1000}{1850}} = 0.25$$

$$P(\text{white} \mid \text{coconut}) = \frac{\frac{350}{800} \cdot \frac{800}{1850}}{\frac{1000}{1850}} = 0.35$$

$$P(\text{Support} \mid \text{coconut}) = \frac{\frac{400}{650} \cdot \frac{650}{1850}}{\frac{1000}{1850}} = 0.4$$

$$P(\text{Purple} \mid \text{others}) = \frac{\frac{50}{400} \cdot \frac{400}{1850}}{350/1850} = 0.14$$

$$P(\text{white} \mid \text{others}) = \frac{\frac{100}{800} \cdot \frac{800}{1850}}{350/1850} = 0.28$$

$$P(\text{Support} \mid \text{others}) = \frac{\frac{200}{650} \cdot \frac{650}{1850}}{350/1850} = 0.57$$

Max Probability = 0.03 which is of coconut